COMP8620: MC-AIXI-CTW
Group 3

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Abstract

We outline an implementation of Veness et al.'s Monte Carlo AIXI approximation[**?**] (MC-AIXI-CTW), and report our simulation results on a number of toy domains.

Contents

1 Introduction

Recall that the AIXI agent is defined by its actions, which for each cycle k are given by

$$a_k^{\text{AIXI}} = \arg\max_{a_k} \sum_{o_k r_k} \cdots \max_{a_m} \sum_{o_m r_m} [r_k + \cdots + r_m] \, \xi \left(o_1 r_1 \ldots o_m r_m | a_1 \ldots a_m \right),$$

where the o_n and r_n are the observation and reward provided by the environment at cycle n, and ξ is a Bayesian mixture model for the environment.

Following Veness et al., we approximate a_k^{AIXI} using Monte Carlo tree search (upper confidence bound) to approximate the expectimax, and we compute a mixture over variable-order Markov models using the context-tree weighting algorithm.

We present a lightweight C++ implementation of MC-AIXI-CTW, along with implementations of a number of simple games: Pacman, Tic-Tac-Toe, Biased Rock-Paper-Scissor, Extended-Tiger, and Cheesemaze.

2 MC-AIXI-CTW Implementation

In this section we outline the implementation of the agent, and its interface with its environments.

2.1 Agent-environment interaction loop

For each agent-environment interaction cycle, we run the following for each experiment (see Algorithm **??**):

Algorithm 1 Main loop.

```
1:  while cycle < max_cycles do
2:      while environment is not finished do
3:          generate (o, r) from environment
4:          update agent model with (o, r)
5:          if explore then
6:              a ← randomAction()
7:          else
8:              a ← MCTS()                          ▷ Monte Carlo Tree Search
9:          end if
10:         perform action a
11:         update agent history with a
12:         cycle++
13:     end while
14:     reset environment
15: end while
```

The $MCTS$ algorithm follows as in Algorithms 1-4 in Veness et al., and is found in `search.cpp`. Model updates are handled in methods in `agent.cpp`, which interfaces with the context tree defined in `predict.cpp`.

2.2 Monte Carlo Tree Search (MCTS) Algorithm

2.2.1 High level description

Since the environment is only partially observable, we have no explicit notion of state; instead, we only have a history of actions and percepts $h = (a_1 o_1 r_1 \ldots a_n o_n r_n)$. For the purposes of choosing the optimal action, we treat each possible (hypothetical) history as a node in the search tree, with the root being the tip of the current (realised) history.

The search tree is comprised of alternate layers of decision nodes and chance nodes with the root being a decision node. The maximum branching possible from decision nodes is the number of available actions in the given environment while

the maximum branching possible from chance nodes is equal to the number of possible observations times the number of possible rewards. We do however restrict branching in general to 100 to avoid memory issues. This number is configurable in `search.cpp`.

Each node is implicitly labelled by its history as chance nodes record the hypothetical action taken while decision nodes record the hypothetical observation and reward from the environment. The expected value of each node in the search tree is equal to the expected total (non-discounted) reward that would be accumulated from that node, where the expectation is under the agent's current policy and the agent's current model for the behavior of the environment.

Thus, for each node we keep a value estimate $\hat{V}$, and a count of the number of times T that node has been visited in search. This is used to determine how we explore the search space using the UCB algorithm, which, for each decision node picks (assuming $T(ha) > 0$ for all actions)

$$a_{\text{UCB}} = \arg\max_{a \in \mathcal{A}} \left\{ \frac{1}{m(\beta - \alpha)} \hat{V}(ha) + C\sqrt{\frac{\log T(h)}{T(ha)}} \right\},$$

where $\mathcal{A}$ is the set of all permissible actions, m is the search horizon, $\beta - \alpha$ is the difference between the minimal and maximal instantaneous reward, and C is a parameter controlling the propensity of the agent to explore less frequently-seen histories.

Note that the expectimax is a stochastic, partially observable game between the agent and its environment. In the following, call nodes corresponding to agent moves 'decision nodes', and nodes corresponding to Nature's moves 'chance nodes'.

2.2.2 Class structure

To represent our search nodes, we define a base `SearchNode` class, from which ChanceNode and DecisionNode inherit. ChanceNode and DecisionNode each have a `sample` method defined on them; each of these methods is mutually recursive. For a given node n, we keep its children in a dictionary keyed on the action or (observation,reward) used to generate each child.

2.2.3 Other Considerations

In addition to the psuedocode presented in Veness et al., we implemented a solution to allow us to decide on a per cycle basis whether or not to build a search tree from scratch. In addition, we constructed routines which given an action taken by the agent and an observation/reward received from the environment, would prune the search tree accordingly.

The pruning of the search tree removes all subtrees begining with the chance nodes directly below the root which correspond to the actions not taken by the agent and as such are impossible future paths. Additionally, all subtrees begining with descision nodes below the chance node (corresponding to the action taken) that do not match the observation/reward received from the environment are pruned. As cycles progress throughout an experiment, this represents a significant reduction in memory consumption.

Our final solution however did not implement pruning as our experimental results showed unusual behaviour whereby the agent would appear to get stuck in certain game states.

2.2.4 Efficiency/performance

Between calls to `search`, we retain much of the search tree, pruning those nodes that are now inaccessible from the realised (a, o, r) tuple that happened during the cycle. This allows us to avoid re-generating similar search trees from similar positions.

2.3 Context Tree Weighting (CTW)

2.3.1 High level description of algorithm

2.3.2 Class structure

2.3.3 Code snippets

2.3.4 Efficiency/performance

3 Environments

To test the effectiveness of the agent we developed 5 environment simulations which range in complexity from the relatively simple cheese maze to the much more complicated partially observable pacman game. The details of these environments with respect to their behaviour and implementation is discussed below. For technical convenience, our agent only handles non-negative rewards; for this reason, our environments are all implemented so as to have their minimum reward set to zero. In this way, our implementations differ from the environment specifications. For the purpose of plotting and analysis of the results, and for optimality calculations, we transform all rewards back to their original specified ranges, so as to facilitate comparison with Veness's benchmark results. Note that this additive reward transformation has no effect on the agent's behavior.

In all of the environments, the agent is given a static interface of possible actions it can perform. At any given game state, some (indeed many) of these actions may be illegal; in all games, illegal moves are punished with significant penalties.

3.1 Cheese Maze

Cheese Maze is an episodic, deterministic and partially observable game in which the agent is a mouse trying to find a piece of cheese in a two dimensional maze. The objective is to find the cheese in the fewest amount of moves, while avoiding running into walls. The episode ends when the agent finds the cheese. The actions available to the agent are:

Action	Code
Move Up	0
Move Right	1
Move Left	2
Move Down	3

The transformed rewards are given by

Action effect	Reward
Game start	0
Agent bumps into wall	0
Agent moves into free cell	9
Agent finds cheese	20

The configuration of the maze and initial states of the agent and the cheese are read from the configuration file, and are identical to those specified in Figure 5 in Veness et al. The maze is represented as the list of nodes as they would be visited by the depth first search algorithm. Mouse position and cheese position are simply the order of the node.
Each node of the maze is a structure which stores the percept corresponding to that node, along with an array of pointers to its neighbours. In case there is a wall on a particular side of the node then that pointer will be `NULL`.

The optimal strategy in each episode of Cheese Maze is to perform the sequence of actions(`game start` $\rightarrow$ `right` $\rightarrow$ `down` $\rightarrow$ `down`). This yields a total reward of 8 utils in 4 cycles, giving an upper bound for the average reward per cycle of 2. Average reward per cycle is the main metric with which we evaluate the agent's performance in this and all other environments.

3.2 Extended Tiger

Extended Tiger is an episodic stochastic game. This environment simulates two doors, behind one of which is a tiger, and the other a pot of gold. At the start of each episode the tiger is placed behind one of the doors with probability 0.5. The agent begins sitting down and it can perform the following actions:

Action	Code
Stand	0
Listen	1
Open left door	2
Open right door	3

When the agent performs the `listen` action, the environment provides an observation that correctly identifies the location of the tiger with probability p (parametrised in the configuration file). Otherwise, its observation is 0:

Observation	Code
Agent does not perform listen action	0
Tiger behind left door	1
Tiger behind right door	2

The agent is penalised for performing illegal moves, or for opening the door with the tiger. The episode ends when one of the doors is opened. The transformed rewards the agent receives in this environment are as follows:

State	Action	Reward
sitting	stand	99
sitting	open door	90
sitting	listen	99
standing	stand	90
standing	open door with tiger	0
standing	open door with gold	30
standing	listen	90

Clearly the optimal strategy in Extended Tiger will be the sequence (`game start` → `listen n times` → `stand` → `open door`). To play optimally given a fixed false observation probability of $q = 1 - p$, it is sufficient to maximise our expected reward with respect to n using this sequence of moves. Hence the expected average reward under the optimal policy can be approximated by

$$\mathbb{E}[r_{\text{avg}}] = \frac{30(1-q)^n - 100(1-(1-q)^n) - n - 1}{n+3},$$

where the denominator is equal to the number of actions taken, and the numerator approximates the expected reward for opening the door signalled by the majority of observations. In case of ties, the agent would need to listen more to collect more data, but we omit this case for simplicity's sake, since we are interested in a loose upper bound with which to benchmark our performance. With a false observation probability of $q = 0.15$, the optimal number of listen actions n is 2.

3.3 Tic-Tac-Toe

Tic-Tac-Toe is a fully observable adversarial game. In this environment, the agent plays numerous games of Tic-Tac-Toe against an opponent who plays randomly. The agent's moves are coded as 0-8 referring to the different positions of the board:

0	1	2
3	4	5
6	7	8

The agent's observations completely describe the current state of the board using 2 bits for each cell of the board:

00	Cell empty
01	Agent's cell
02	Opponent's cell

The agent's (additively transformed) rewards are as follows:

Status	Reward
Illegal move	0
Game is a draw	4
Game won by agent	5
Agent lost	1

In our setup, the agent has the first move. Under the optimal policy, and against a random opponent, the agent should win within 3 or 4 moves with high probability. Assuming no illegal moves are taken, the optimal strategy yields an average reward per cycle (loosely) upper bounded by $\frac{2}{3}$, since a win corresponds to a reward of 2. Taking into account draws and longer games, we expect the true tighest upper bound to be somewhere in the range $[0.5, \frac{2}{3}]$.

3.4 Biased Rock-Paper-Scissors

This is an adversarial, non-episodic game. The agent plays against an opponent who plays randomly, except when they win a round, in which case they will play the same move in the following round with probability 1. We encode the actions of the agent and the environment as:

Rock	0
Paper	1
Scissors	2

The agent receives a reward of 1 for a win, 0 for a draw and -1 for a loss.

In terms of an optimal agent strategy, an upper bound on reward per cycle for Biased Rock-Paper-Scissors is derived as follows. Let X_t be a random variable representing the outcome of cycle t. Under an optimal strategy, the agent will win after the environment won in cycle $t-1$ as the environments action in cycle t is predictable. Otherwise, the agent should play randomly. Now, the probability of the agent's winning at time t is given by

$$\begin{aligned} P(X_t = \text{win}) &= \sum_{X_{t-1}} P(X_t = \text{win}, X_{t-1}) \\ &= \sum_{X_{t-1}} P(X_t = \text{win} | X_{t-1}) P(X_{t-1}), \end{aligned}$$

which corresponds to a 1^{st} order Markov chain with the following right stochastic transition matrix

$$\mathbf{P} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \tag{1}$$

Therefore, we want a steady state distribution for $\mathbf{P}$, i.e. a solution to $\boldsymbol{\pi} = \mathbf{P}\boldsymbol{\pi}$. This is given by the eigenvector $\boldsymbol{\pi}^*$. The average reward at time t is then

$$\begin{aligned} \mathbb{E}[r_{\text{avg}_t}] &= 1 \cdot P(X_t = \text{win}) + 0 \cdot P(X_t = \text{draw}) - 1 \cdot P(X_t = \text{lose}) \\ &= \boldsymbol{\pi}^*_{\text{win}} - \boldsymbol{\pi}^*_{\text{lose}} \\ &= 0.25. \end{aligned}$$

3.5 Partially Observable PacMan

In contrast to standard PacMan, our implementation is partially observable, which makes it more difficult to play optimally, but more tractable computationally. The agent does not know the structure of the maze but only receives a 4 bit wall configuration of the node it is currently in. The agent also receives a 4 bit observation of the presence of any ghosts in its direct line of sight. The location of food pellets can change every episode as every free cell has a 50% chance of having food in it. The agent receives a 3 bit observation indicating the presence of food within a Manhattan distance of 2, 3 or 4. It also receives a 4 bit string indicating food in its line of sight similar to the ghosts. The agent also knows whether it is under the effect of the power pill.

Action	Reward
Agent runs into a wall	-10
Agent caught by a ghost	-50
Agent moves into empty cell	-1
Agent eats a food pellet	10
Agent collects all the food	100

To obtain non-negative rewards for our agent, we transform this environment by adding 60 to all reward percepts. This is because the agent can potentially be simultaneously caught by a ghost and run into a wall in the same cycle, resulting in the penalties being added.

The optimal strategy for PacMan is unknown, and so an upper bound on agent performance is unknown. Given that this is the most complex and demanding of our five environments, we can assume that without long training times and finely tuned agent learning parameters, learning in this game will be very difficult.

4 Experiments

4.1 Experimental Setup

Having implemented the MC-AIXI-CTW agent and five game environments, we proceed to run simulations. Our primary objective here is to test the agent against each of the environments, with the agent having no prior knowledge. During each experiment, we run the agent-environment loop for a large number of cycles (typically in the range of $[5 \times 10^3, 3 \times 10^4]$), while maintaining the state of the agent's context tree model betwene cycles and episodes.

For each experiment, the agent's actions are controlled according to a training-evaluation schedule. During the training phase, the agent chooses a random action with probability ϵ, and performs the normal Monte Carlo expectimax search otherwise. Over the course of the experiment, the exploration rate decays exponentially as $\epsilon_t = \epsilon_0 \gamma^t$. During evaluation cycles, the agent plans using the Monte Carlo tree search, and does no explicit exploration. We evaluate the agent's performance by plotting its average reward per cycle against *evaluation* cycles. Typically, we alternate between 1000 cycles of training, followed by 200 cycles of evaluation, and repeat until the total allocated number of cycles is exhausted.

In all of the following plots, we report the *running* average reward per cycle $\bar{R}$, and the *rolling* average reward per cycle $\tilde{R}$, which are defined as

$$\bar{R}_t = \frac{1}{t} \sum_{i=1}^{t} R_i$$

$$\bar{R}_t^w = \frac{1}{w} \sum_{i=t-w}^{t} R_i.$$

4.2 Experiment Summary

Here we summarise the parameters used in our experiments.

	Bits[1]	CT-depth	Cycles	Timeout[2]	Horizon	Exp. Rate	Exp.-Decay	UCB-weight[3]
Cheesemaze	11	96	10	0.5	8	0.999	0.99977	1.4
	11	96	10	1.6	6	0.999	0.99977	1.4
	11	144	2.5	3	8	0.999	0.99908	1.4
	11	48	10	0.1	4	0.999	0.99977	1.4
	11	96	10	0.5	8	0.999	0.99977	1.4
Biased Rock-Paper-Scissors	6	32	10	2.5	4	0.999	0.9998	1.4
	6	48	5	5	8	0.999	0.9995	1.4
	6	96	4.5	4	4	0.999	0.9995	1.4
	6	96	10	0.5	4	0.999	0.999	1.4
Extended Tiger	13	96	16	2.5	6	0.999	0.9999	1.4
	13	96	2.5	12	4	0.999	0.99908	1.4
	13	96	25	0.8	4	0.999	0.999908	1.4
	13	52	5	8	4	0.999	0.99954	1.4
Tic-Tac-Toe	25	96	4.5	4	9	0.9999	0.99949	1.4
	25	192	5	8	9	0.9999	0.99954	1.4
	25	256	20	8	9	0.9999	0.99988	1.4
	25	512	25	3	9	0.9999	0.999908	1.4
	25	512	30	2	9	0.9999	0.99992	1.4
Pacman	26	256	10	4	8	0.99	0.999	1.5
	26	512	20	2	4	0.9999	0.99988	1.4
	26	320	10	4	8	0.99	0.999	1.4
	26	256	10	1	6	0.9999	0.99977	1.4

Table 1: MC-AIXI-CTW Experiments

4.3 Cheesemaze

Here we present the environmental setup and learning rate of our best results training AIXI on the Cheesemaze environment.

Interpreting the results, we conclude that ...

Bits	CT-depth	Cycles	Timeout	Horizon	Exp. Rate	Exp.-Decay	UCB-weight
11	96	10	0.5	8	0.999	0.999769818	1.4

Table 2: Environment Setup

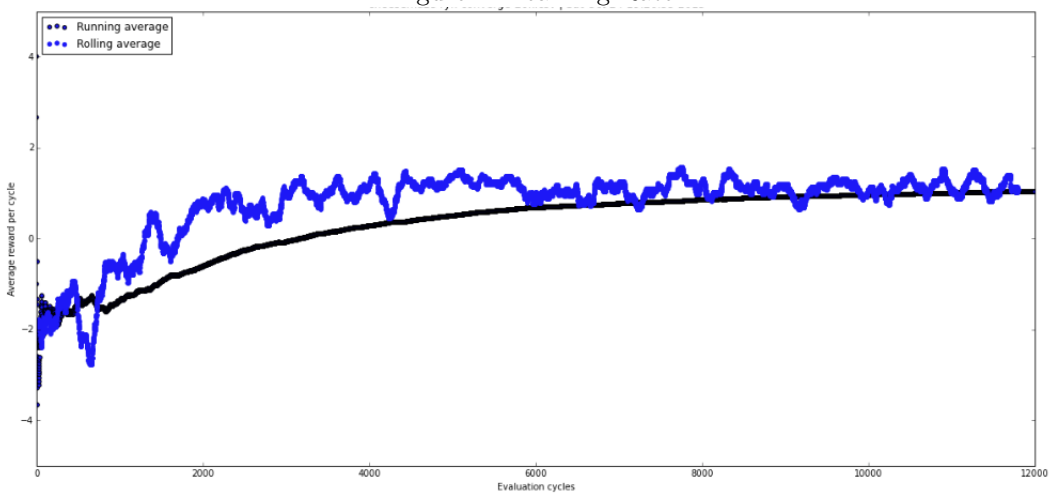
4.4 Extended Tiger

Here we present the environmental setup and learning rate of our best results training AIXI on the Extended Tiger environment.

Interpreting the results, we conclude that ...

[1] total bit length of an `ora` cycle
[2] timeout value for each select action
[3] UCB exploration bias parameter

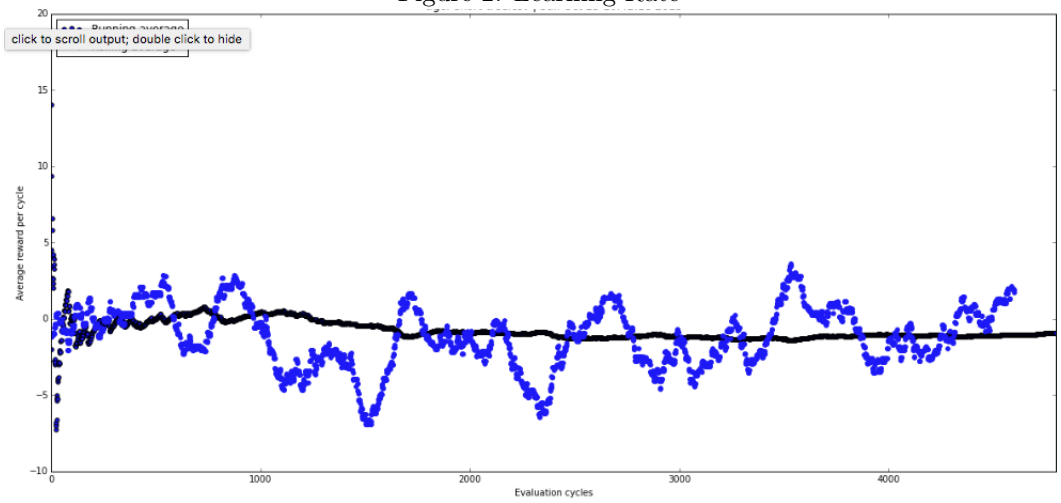
Figure 1: Learning Rate



Bits	CT-depth	Cycles	Timeout	Horizon	Exp. Rate	Exp.-Decay	UCB-weight
11	96	10	0.5	8	0.999	0.999769818	1.4

Table 3: Environment Setup

Figure 2: Learning Rate



4.5 TicTacToe

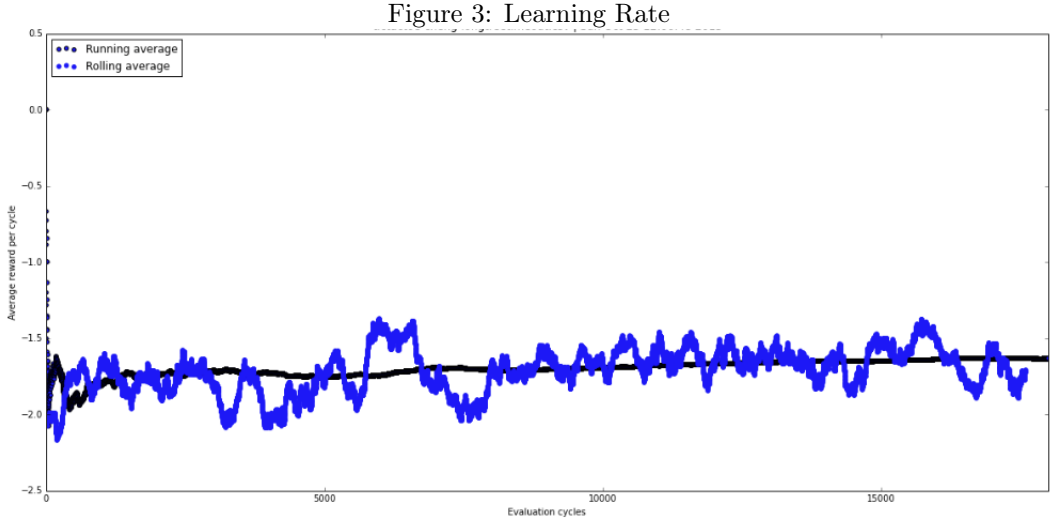
Here we present the environmental setup and learning rate of our best results training AIXI on the Tic-Tac-Toe environment.

Interpreting the results, we conclude that ...

Bits	CT-depth	Cycles	Timeout	Horizon	Exp. Rate	Exp.-Decay	UCB-weight
11	96	10	0.5	8	0.999	0.999769818	1.4

Table 4: Environment Setup

Figure 3: Learning Rate



4.6 Biased Rock-Paper-Scissors

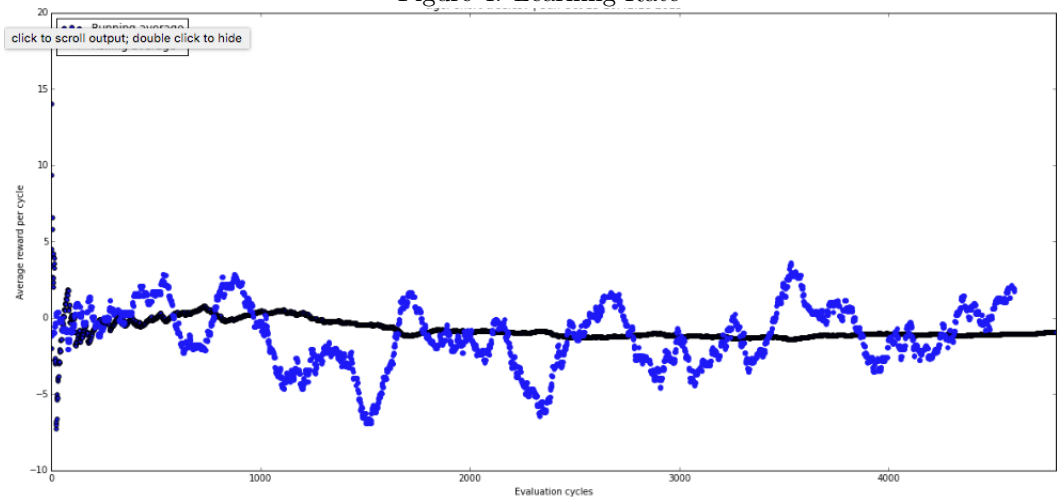
Here we present the environmental setup and learning rate of our best results training AIXI on the Biased Rock-Paper-Scissors environment.

Interpreting the results, we conclude that ...

Bits	CT-depth	Cycles	Timeout	Horizon	Exp. Rate	Exp.-Decay	UCB-weight
11	96	10	0.5	8	0.999	0.999769818	1.4

Table 5: Environment Setup

Figure 4: Learning Rate



4.7 Pacman

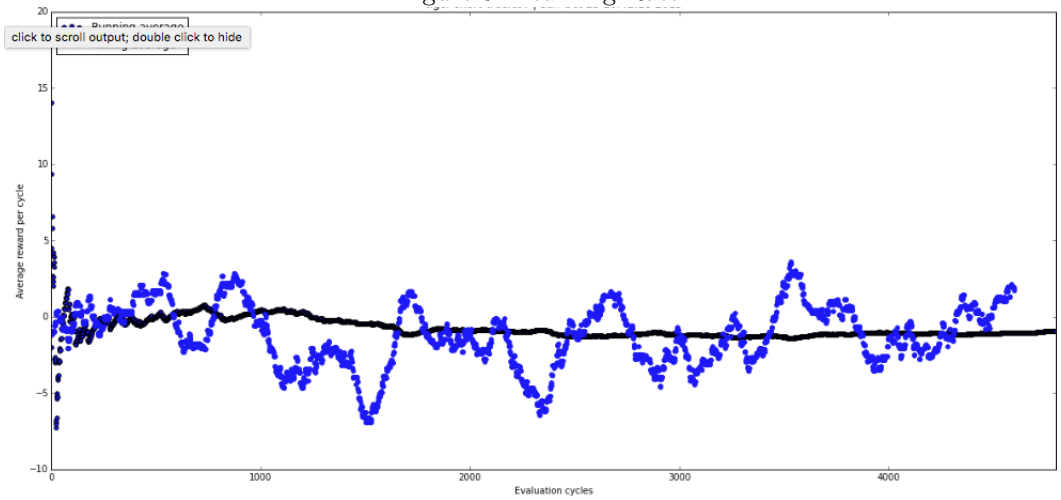
Here we present the environmental setup and learning rate of our best results training AIXI on the Pacman environment.

Interpreting the results, we conclude that ...

Bits	CT-depth	Cycles	Timeout	Horizon	Exp. Rate	Exp.-Decay	UCB-weight
11	96	10	0.5	8	0.999	0.999769818	1.4

Table 6: Environment Setup

Figure 5: Learning Rate



4.8 Cross Domain Simulation Results

- ◇ Cheesemaze and Biased Rock-Paper-Scissors
- ◇ Cross domain simulation on more difficult environments...

Figure 6: Learning Rate

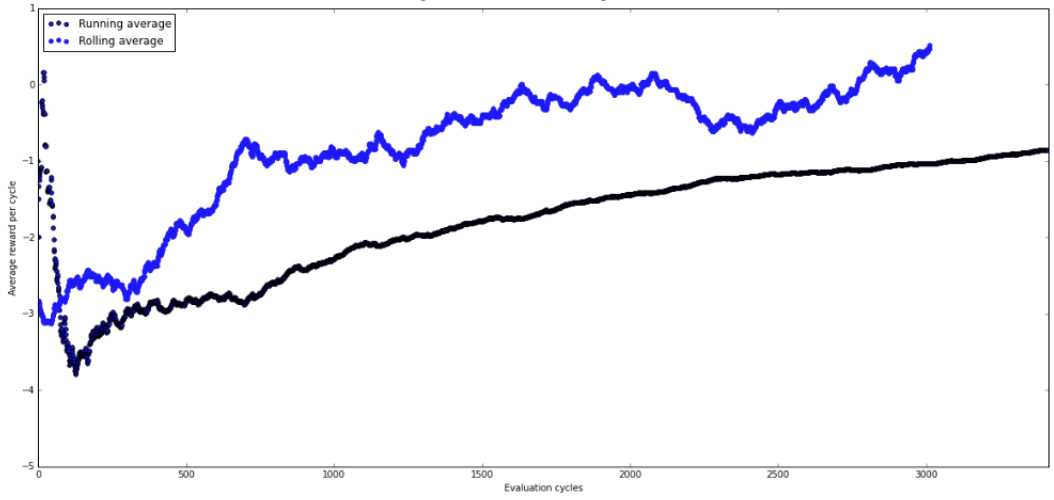


Figure 7: Learning Rate

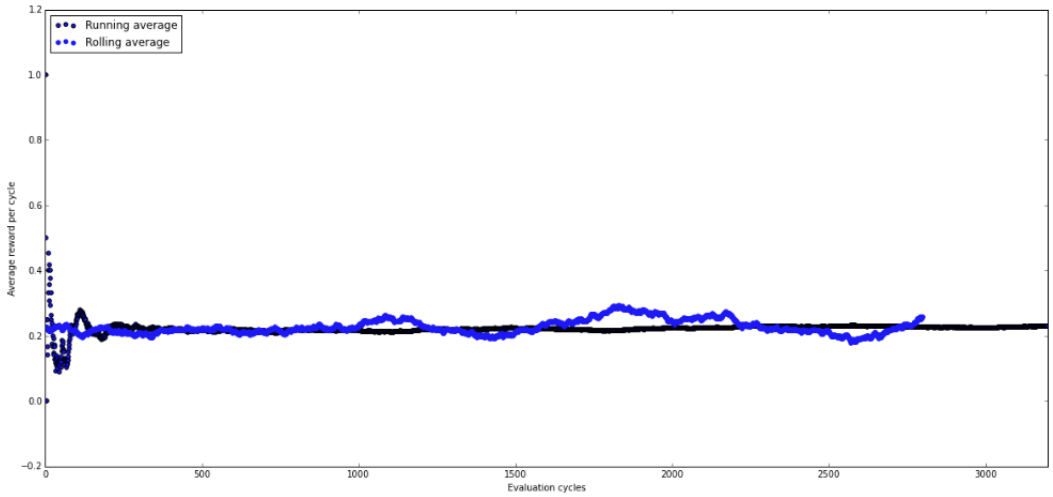
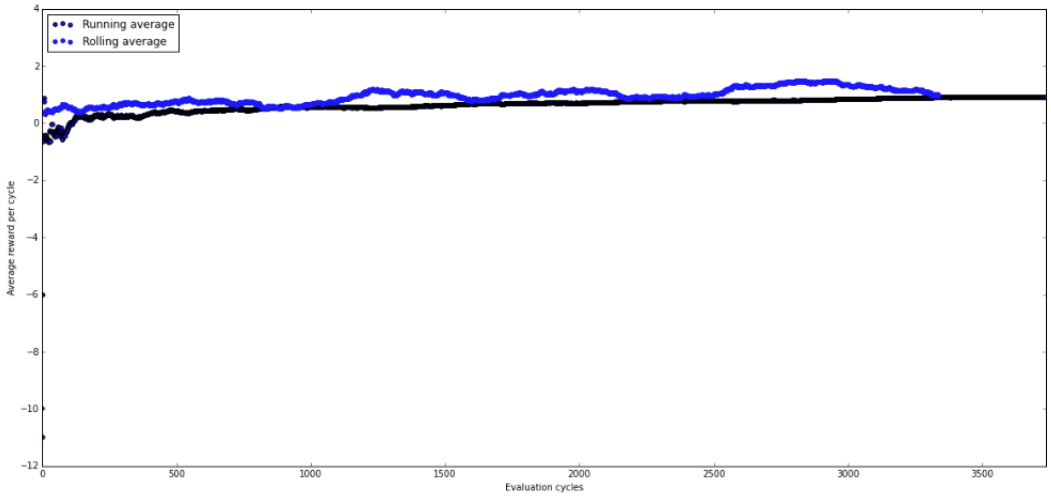


Figure 8: Learning Rate



5 Conclusion

Appendix

User Manual

To build from source on linux, run `g++ -std=c++0x *.cpp -o aixi` **TODO check**
To run, invoke ./`aixi` `$ENVNAME.conf`.

A Files

The report archive should contain the following:

`MC-AIXI-CTW-Grp3.zip`

```
\report
    report.pdf // this report
    report.tex
    cheesemaze_best.png // results plots
    tiger_best.png
    rockpaper_best.png
    tictactoe_best.png
    pacman_best.png
\src
    main.hpp
    main.cpp
    environment.hpp
    environment.cpp
    agent.hpp
    agent.cpp
    search.hpp
    search.cpp
    predict.hpp
    predict.cpp
    util.hpp
    util.cpp
    README.md
    cheesemaze.conf // environment configuration files
    rockpaper.conf
    tictactoe.conf
    coinflip.conf
    tiger.conf
```

B Plots