QED Radiation in Vincia

Rob Verheyen

With Peter Skands & Ronald Kleiss

Work in progress

IMAPP

Nikhef

Radboud University

Introduction

Vincia is a parton shower plugin for Pythia based on antenna factorization

Currently Vincia only does QCD radiation

We want to include QED radiation too

Giele, Kosower, Skands:1102.2126

Gehrmann, Ritzmann, Skands:1108.6172

Current approaches to photon radiation

DGLAP

- Resums collinear photon logarithms
- Interleaving with QCD shower

YFS

- Resums soft photon logarithms
- Collinear logarithms can be included, but not resummed
- Afterburner to add soft photons

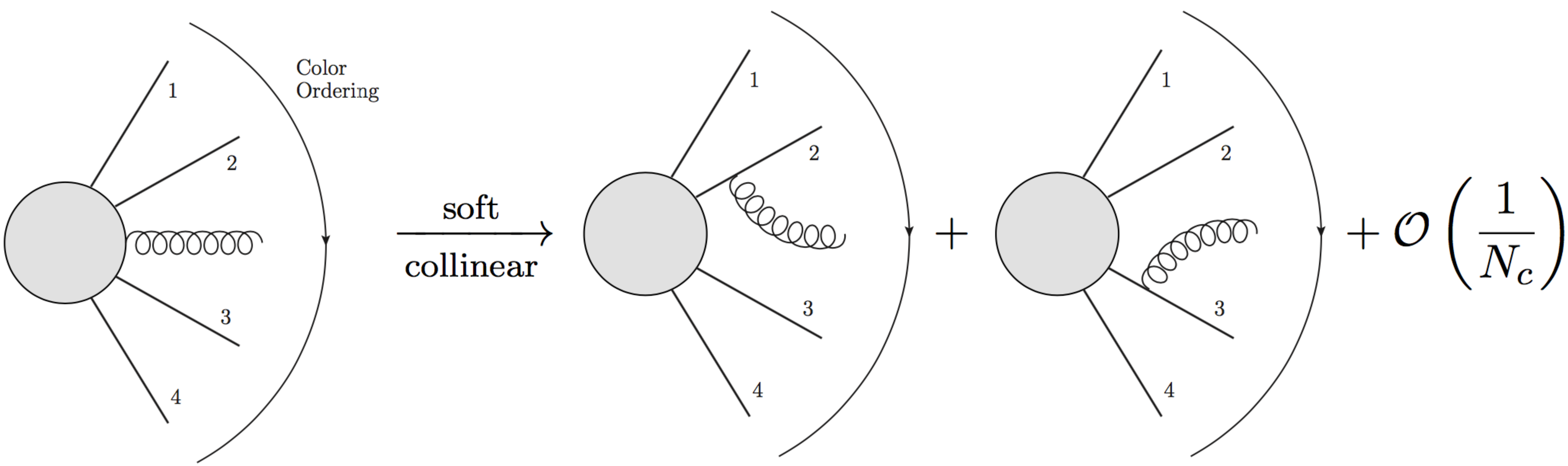
I'll discuss the three algorithms for photon emission we're implementing

Leading Color Gluon Emission

Factorization

$$|M(..,p_a,k,..)|^2 \xrightarrow{p_a \| k} g^2 C \frac{P(z)}{p_a \cdot k} |M(..,p_a+k,..)|^2$$

$$|M(..,p_a,k,p_b,..)|^2 \xrightarrow{k \to 0} g^2 C \left[\frac{2 p_a \cdot p_b}{(p_a \cdot k)(k \cdot p_b)} - \frac{m_a^2}{(p_a \cdot k)^2} - \frac{m_b^2}{(p_b \cdot k)^2} \right] |M(..,p_a,p_b,..)|^2$$



Leading Color Gluon Emission

Factorization

$$|M(..,p_a,k,p_b,..)|^2 \approx g^2 C\, a_e^{QCD}(p_a,k,p_b)|M(..,p'_a,p'_b,..)|^2$$

$2 \to 3$ branching

Ordering scale

$$t = p_\perp^2 = 4\,\frac{p_a{\cdot}k\, p_b{\cdot}k}{m^2}$$

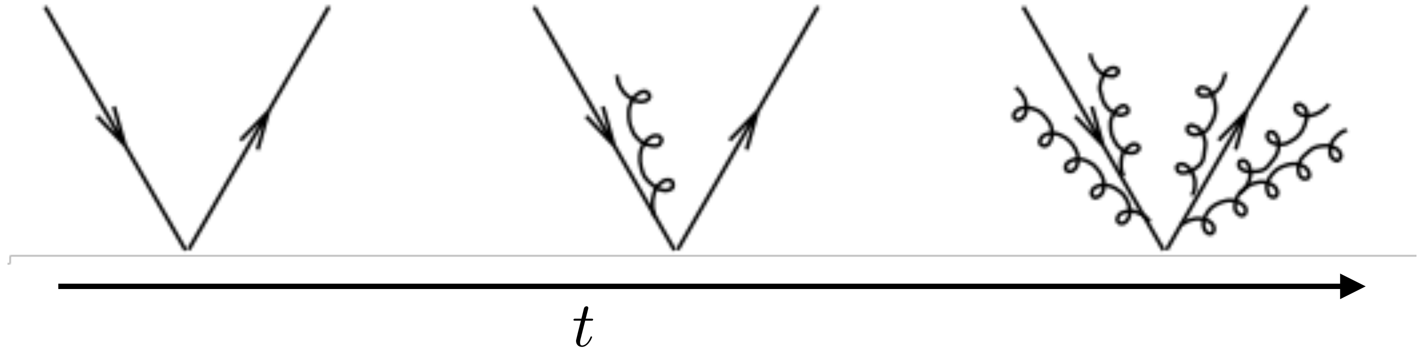


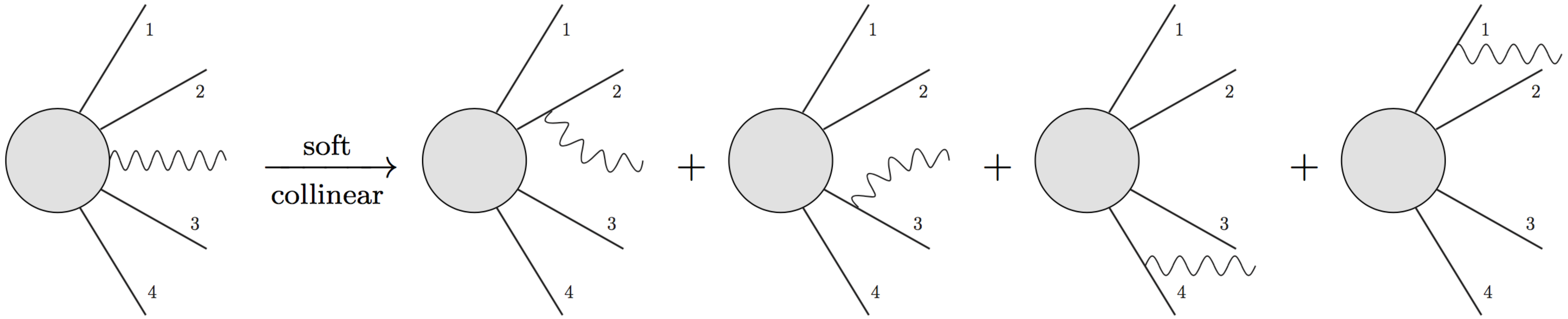
Illustration: G. Salam

Photon Emission

Factorization

$$|M(..,p_a,k,..)|^2 \xrightarrow{p_a \| k} e^2 Q_a^2 \frac{P(z)}{p_a \cdot k} |M(..,p_a+k,..)|^2$$

$$|M(\{p\},k)|^2 \xrightarrow{k\to 0} -e^2 \sum_{[a,b]} Q_a Q_b \left[\frac{2p_a \cdot p_b}{(p_a \cdot k)(k \cdot p_b)} - \frac{m_a^2}{(p_a \cdot k)^2} - \frac{m_b^2}{(p_b \cdot k)^2} \right] |M(\{p\})|^2$$



Photon Emission

Factorization

$$\boxed{|M(\{p\},k)|^2 \approx e^2 a_e^{QED}(\{p\},k)\,|M(\{p'\})|^2}$$

$$a_e^{QED}(\{p\},k) = -\sum_{[a,b]} Q_a Q_b\, a_e^{QCD}(p_a,k,p_b)$$

$n \to n+1$ branching

Ordering scale

$$t = p_\perp^2 = 4\,\frac{p_a \cdot k\, p_b \cdot k}{m^2}$$

Photon emissions are a multi-scale problem

Goal: recast this $n \to n+1$ branching into a (set of) $2 \to 3$ branchings

Option 1:
Pairing

Incoherent Pairing

Pythia-like approach: Include only one antenna function for every fermion

$$a_e^{QED}(\{p\},k) = Q_{f_1^+} Q_{f_1^-} a_e^{QCD}(p_{f_1^+}, k, p_{f_1^-}) + Q_{f_2^+} Q_{f_2^-} a_e^{QCD}(p_{f_2^+}, k, p_{f_2^-}) + \dots$$

Competition between independent radiators

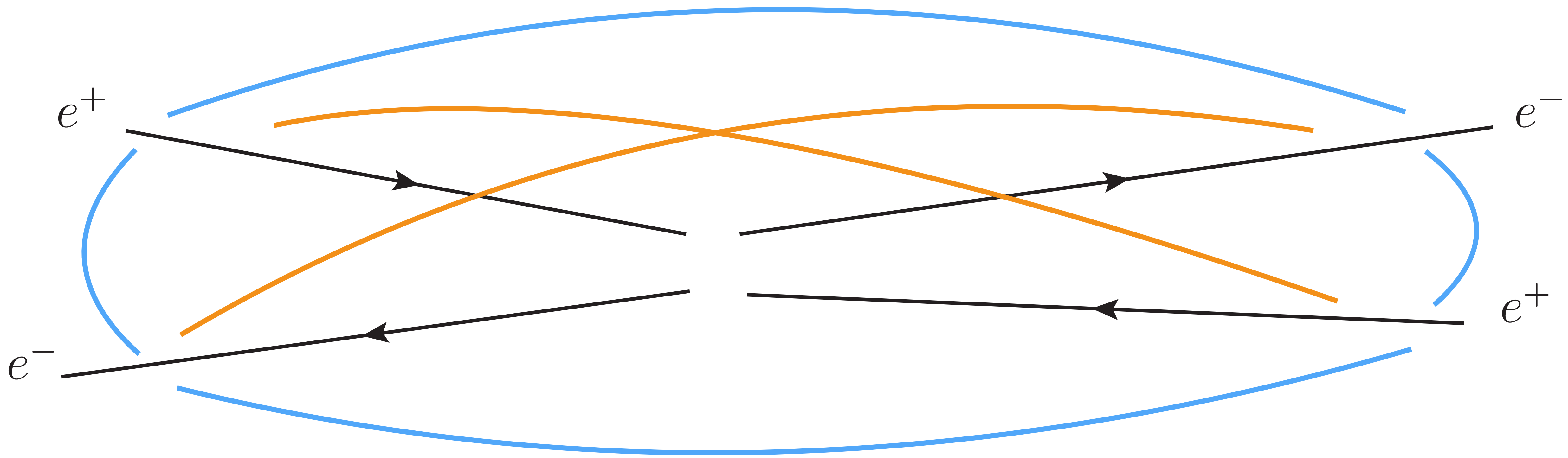
- Correct collinear behaviour
- Only includes some eikonal factors

Pair up the fermions to minimise $m^2_{f_1^+ f_1^-} + m^2_{f_2^+ f_2^-} + \dots$

Incoherent Pairing

Photon radiation should decrease as the angle between opposite charges decreases

Emission scales are kinematically restricted by the antenna mass



Pair up the fermions to minimise $m^2_{f_1^+ f_1^-} + m^2_{f_2^+ f_2^-} + \ldots$

Brute force $\mathcal{O}(n!)$ complexity…

The Hungarian Algorithm

Turns out this is a well-known problem from graph theory!

$$
\begin{array}{cc}
 & \begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix} \\
\begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix} & \begin{bmatrix} m^2_{f_1^+ f_1^-} & m^2_{f_1^+ f_2^-} & m^2_{f_1^+ f_3^-} \\ m^2_{f_2^+ f_1^-} & m^2_{f_2^+ f_2^-} & m^2_{f_2^+ f_3^-} \\ m^2_{f_3^+ f_1^-} & m^2_{f_3^+ f_2^-} & m^2_{f_3^+ f_3^-} \end{bmatrix}
\end{array}
$$

The Hungarian Algorithm

Let's look at an example to see how it works

$$
\begin{array}{c c}
 & \begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix} \\
\begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix} & \begin{bmatrix} 35 & 50 & 30 \\ 5 & 15 & 10 \\ 35 & 50 & 20 \end{bmatrix}
\end{array}
$$

The Hungarian Algorithm

Step 1: Subtract the lowest row element from all rows

$$
\begin{array}{c c c}
 & \begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix} & \\
\begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix} & \begin{bmatrix} 5 & 20 & 0 \\ 0 & 10 & 5 \\ 15 & 30 & 0 \end{bmatrix} & \begin{matrix} \color{red}{-30} \\ \color{red}{-5} \\ \color{red}{-20} \end{matrix}
\end{array}
$$

The Hungarian Algorithm

Step 2: Subtract the lowest column element from all rows

$$\begin{array}{cc} & \begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix} \\ \begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix} & \begin{bmatrix} 5 & 10 & 0 \\ 0 & 0 & 5 \\ 15 & 20 & 0 \end{bmatrix} \\ & \quad\;\; {\color{red}-10} \end{array}$$

Radboud University

The Hungarian Algorithm

Step 3: Find the minimal line covering

$$
\begin{array}{cc}
 & \begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix} \\
\begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix} & \begin{bmatrix} 5 & 10 & 0 \\ 0 & 0 & 5 \\ 15 & 20 & 0 \end{bmatrix}
\end{array}
$$

If the line covering is maximal (n=3), pairing with cost 0 can be found

The Hungarian Algorithm

Step 4: Find the lowest uncovered element

$$
\begin{array}{cc}
 & \begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix} \\
\begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix} & \begin{bmatrix} 5 & 10 & 0 \\ 0 & 0 & 5 \\ 15 & 20 & 0 \end{bmatrix}
\end{array}
$$

The Hungarian Algorithm

Step 4: Subtract that number from all uncovered element
Add it to all doubly covered elements

$$\begin{array}{ccc} & -5 & \begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix} & \\ \begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix} & & \begin{bmatrix} 0 & 5 & 0 \\ 0 & 0 & 10 \\ 10 & 15 & 0 \end{bmatrix} & \begin{array}{c} \\ +5 \\ \\ \end{array} \end{array}$$

And go back to step 3

The Hungarian Algorithm

$$
\begin{array}{cc}
 & \begin{bmatrix} f_1^- & f_2^- & f_3^- \end{bmatrix} \\
\begin{bmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \end{bmatrix} & \begin{bmatrix} 0 & 5 & 0 \\ 0 & 0 & 10 \\ 10 & 15 & 0 \end{bmatrix}
\end{array}
$$

Now we are able to find an optimal pairing!

$\mathcal{O}(n^3)$ complexity, so not computationally prohibitive

Option 2:
Coherent

Coherent Emission

$2 \to 3$ branching

Separate phase space into *sectors*

$$|M(\{p\},k)|^2 \approx \sum_{[i,j]} \left(-\sum_{[a,b]} Q_a Q_b a_e^{QCD}(p_a,k,p_b) \right) \theta(p_{\perp\, ij}^2) |M(..,p_i',p_j',..)|^2$$

1 if $p_{\perp\, ij}^2$ is the smallest
0 otherwise

Equivalent to ordering in

$$t = \min\left(p_{\perp\, ij}^2\right) = \min\left(4\frac{p_i\cdot k\, p_j\cdot k}{m^2}\right)$$

But there's a problem…

Sudakov Veto Algorithm

Want to sample from

$$f(t)\exp\left(-\int_t^u d\tau f(\tau)\right)$$

Find $g(t) \geq f(t)$

Set $u = t_{\text{start}}$

Sample t from $g(t)\exp\left(-\int_t^u d\tau g(\tau)\right)$

Accept with probability $\frac{f(t)}{g(t)}$

Set $u = t$

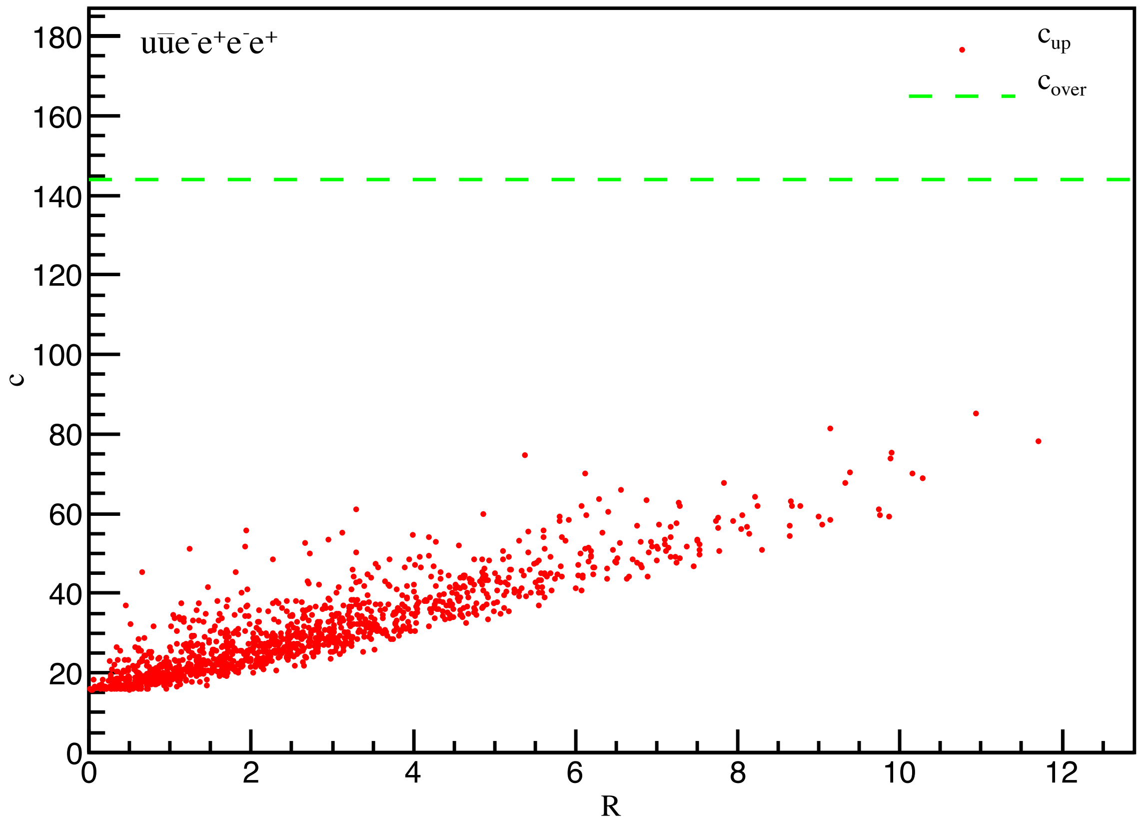
Done $\longrightarrow t$

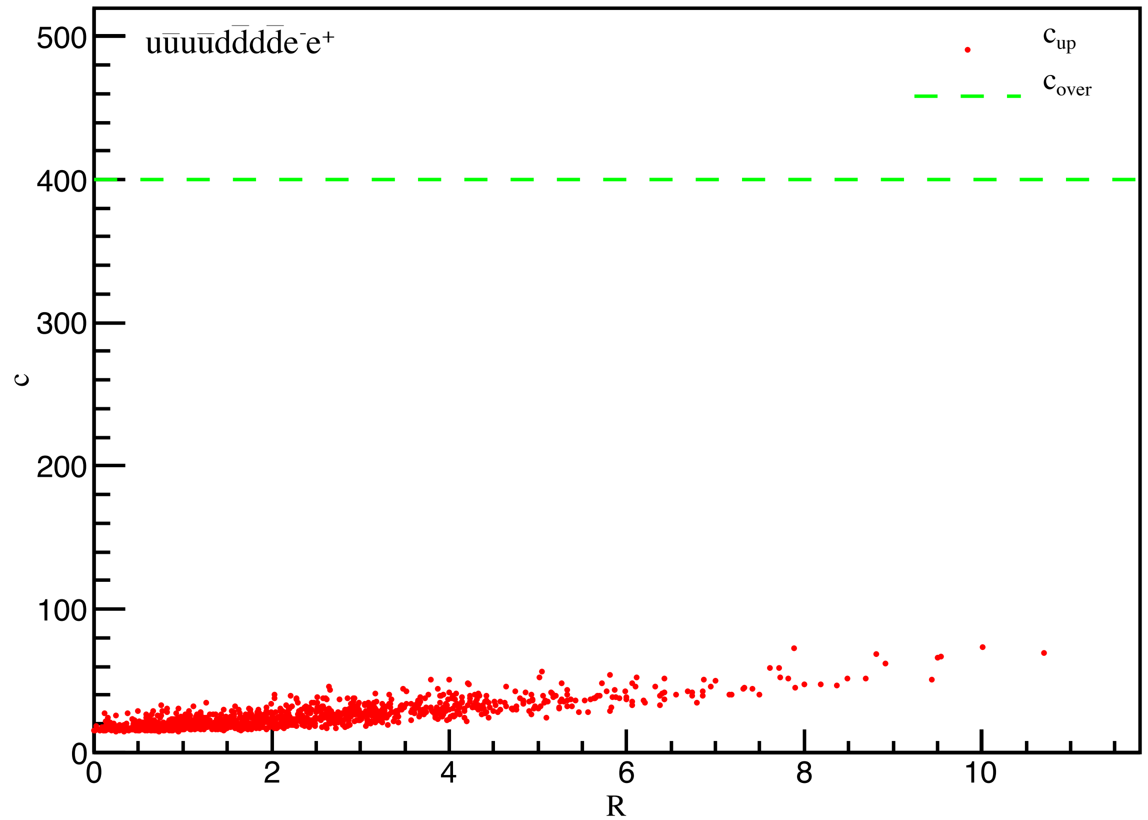
Coherent Emission

We need an overestimate for the branching kernel

$$a_e^{QED} = -\sum_{[a,b]} Q_a Q_b \, a_e^{QCD}(p_a, k, p_b)$$

It's possible to find one, but…





The algorithm is slow!

Option 3:
Coherent Weighted

Sudakov Veto Algorithm

Want to sample from

$$f(t)\exp\left(-\int_t^u d\tau\, f(\tau)\right)$$

Find $g(t) \ngeq f(t)$

Set $u = t_{\text{start}}$

Sample t from $g(t)\exp\left(-\int_t^u d\tau\, g(\tau)\right)$

Accept with probability $p(t)$

Apply weight

$$\frac{1}{p(t)}\frac{f(t)}{g(t)}$$

Done $\longrightarrow t$

Set $u = t$

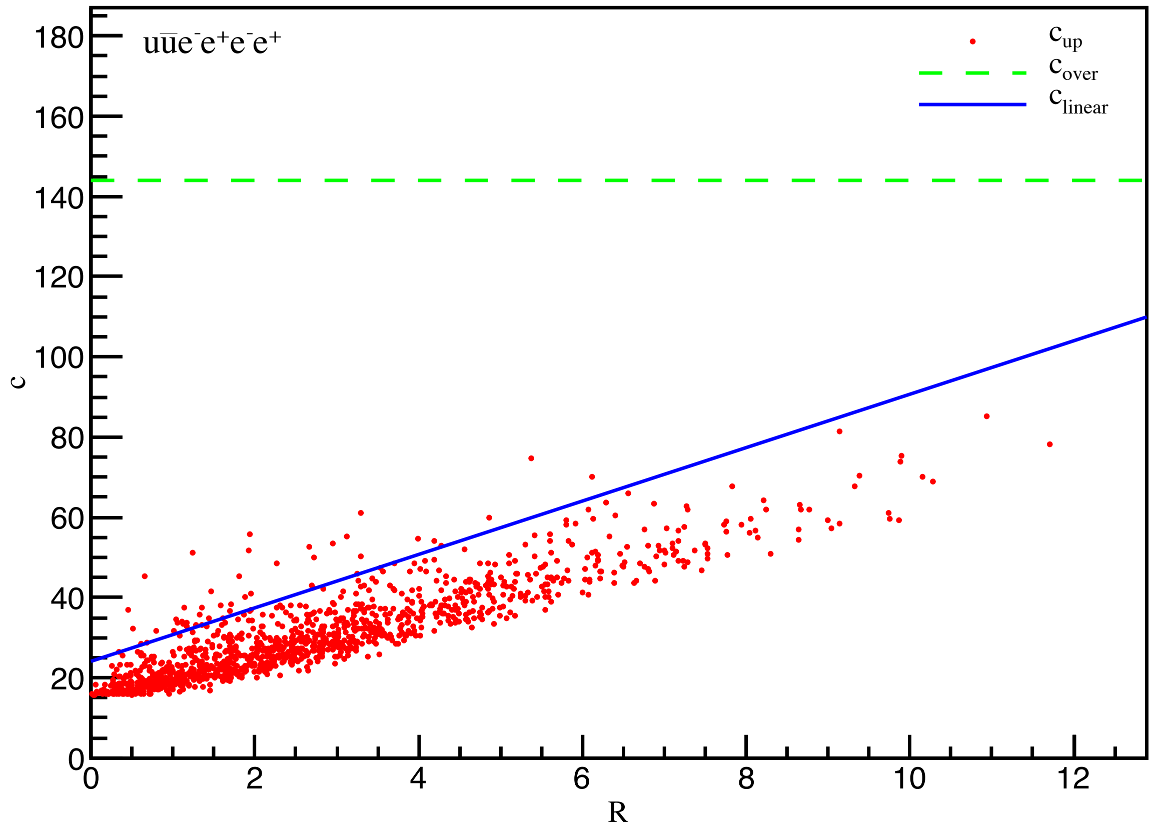
Apply weight

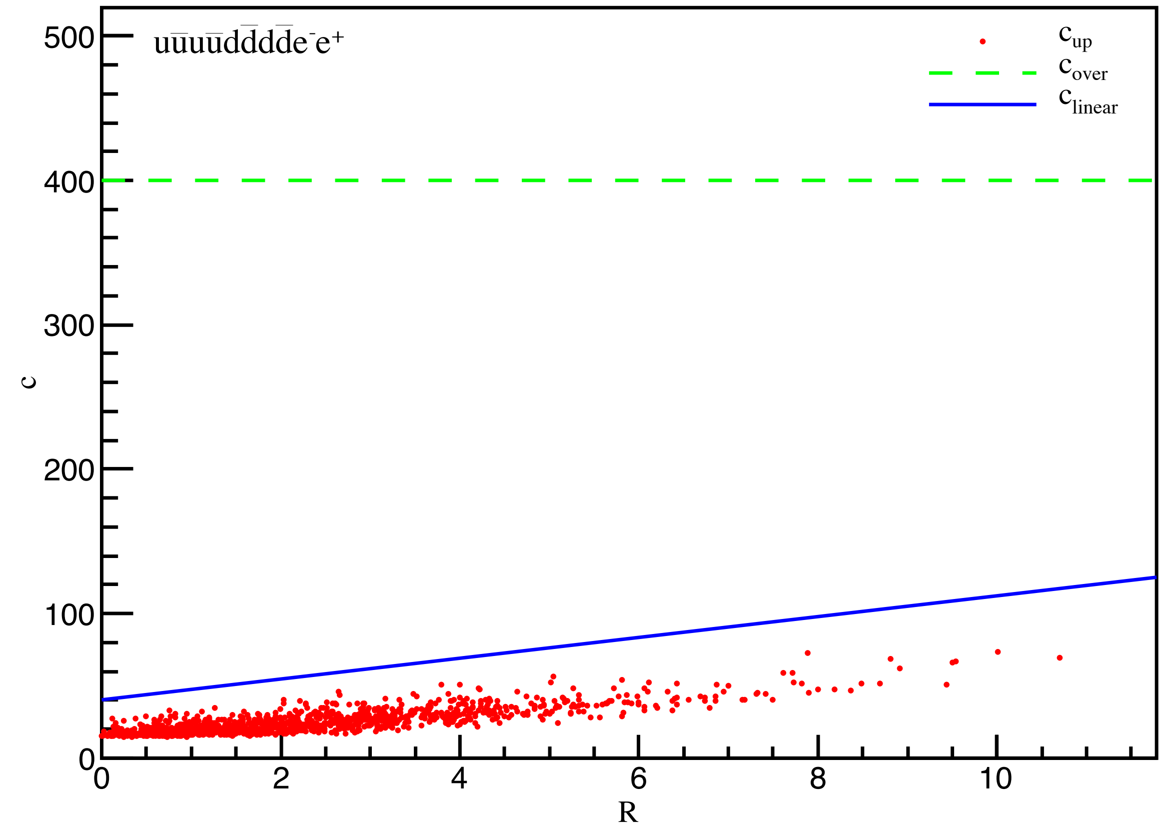
$$\frac{1}{g(t)}\frac{g(t)-f(t)}{1-p(t)}$$

Coherent Weighted Emission

Use an event-based incomplete overestimate

$$p(t) = \tanh\left(\frac{f(t)}{g(t)}\right)$$





$$R = -\sum_{[a,b]} Q_a Q_b \left(1 - \cos(\theta_{ab})\right)$$

Much faster, but events are weighted

Summary

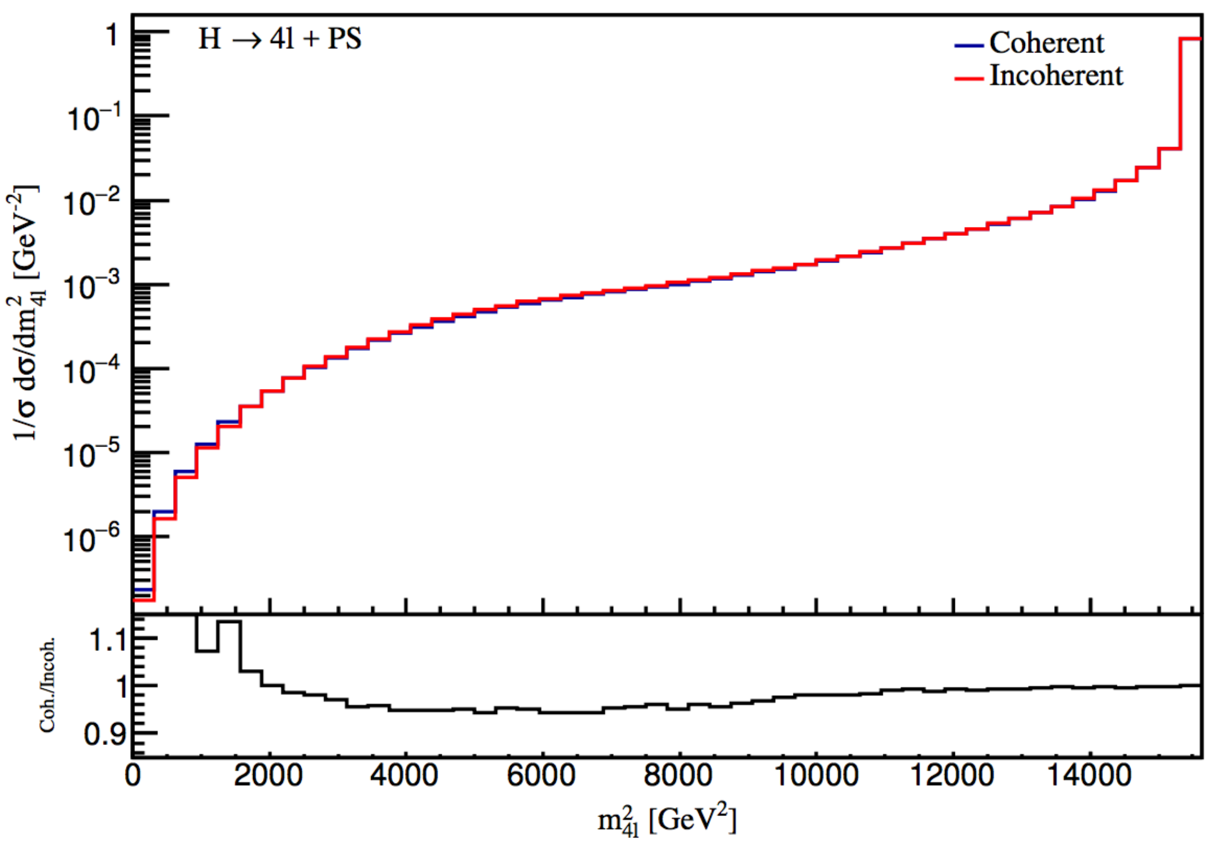
We're implementing three ways of doing photon emissions

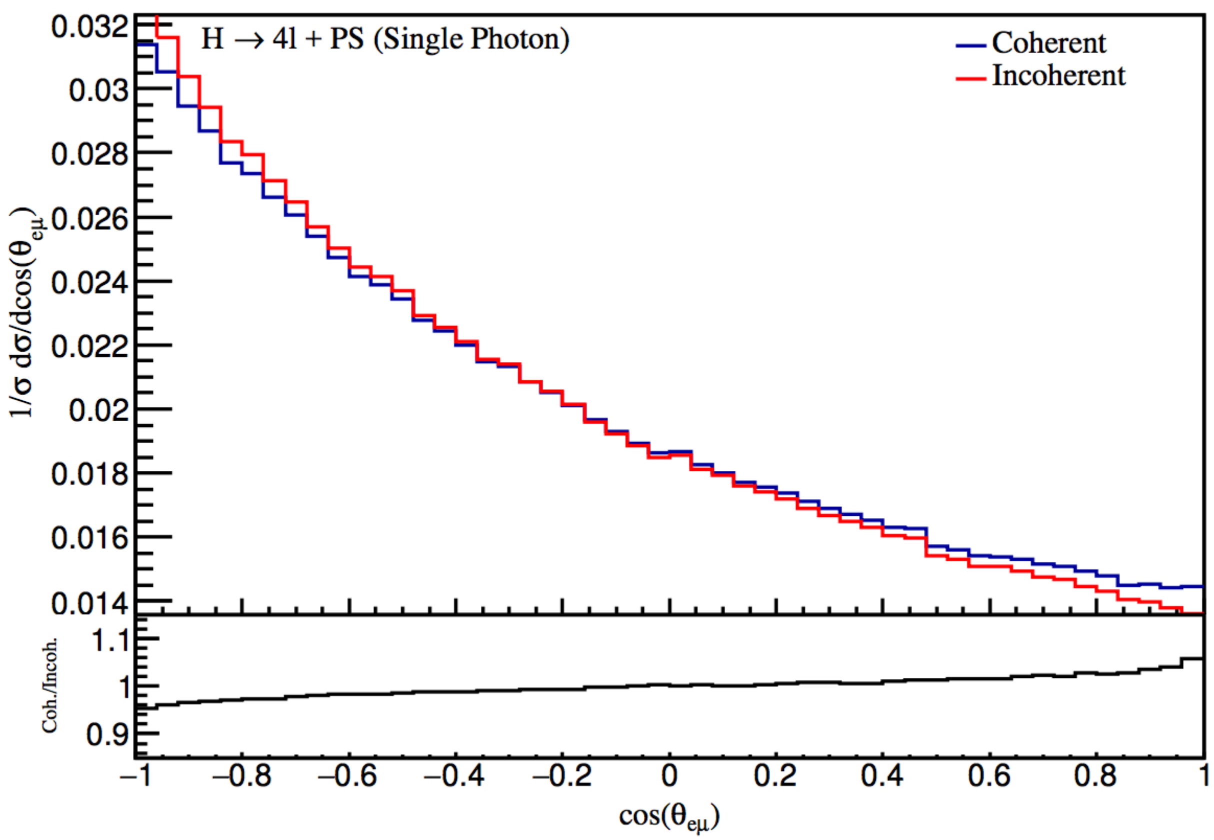
1. Incoherent Pairing
   - Fast
   - Not coherent, but has most important eikonals
2. Coherent Unweighted
   - Slow
   - Fully coherent
3. Coherent Weighted
   - Fast
   - Weighted events

Extra Slides

Radboud University

Comparison - Coherence





Comparison - DGLAP equation