Modeling Delay in Multi-hop Networks

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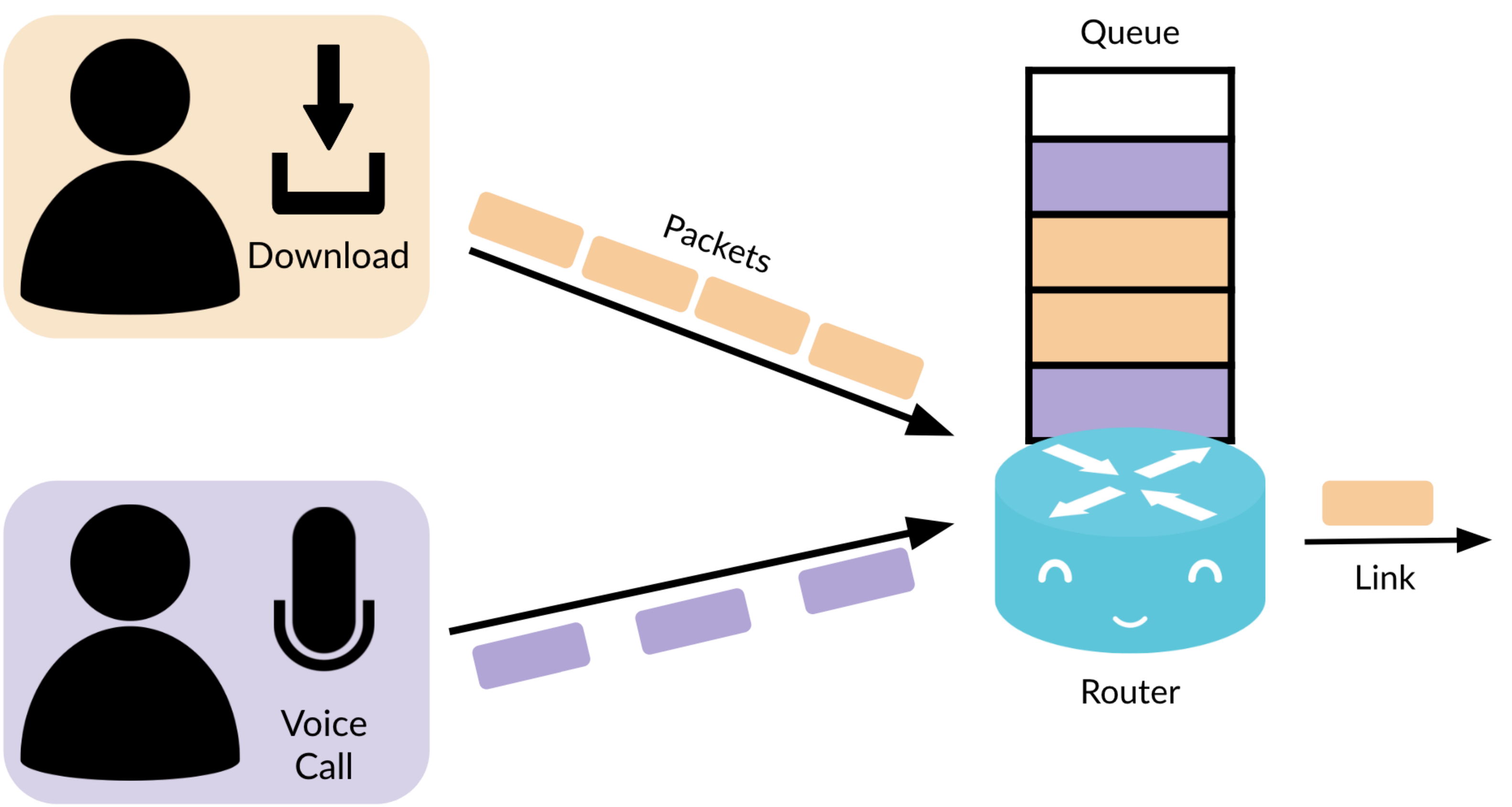
Motivation

When people send data through a network, they often share bandwidth and experience *delay* as queues build on routers.



How fast should they send? Too fast and there's delay. Too slow and throughput suffers.

Prior work models this to find optimal sending rates using game theory [2].

Problem

Prior work studies multiple senders competing on *one link* [2]. **In real networks, senders share multiple links. How can we extend this model to multiple senders across multiple links?**

Goals

1. Model delay for multiple senders
2. Model delay across multiple links
3. Model in a way that is simple to integrate with game theory

Approach

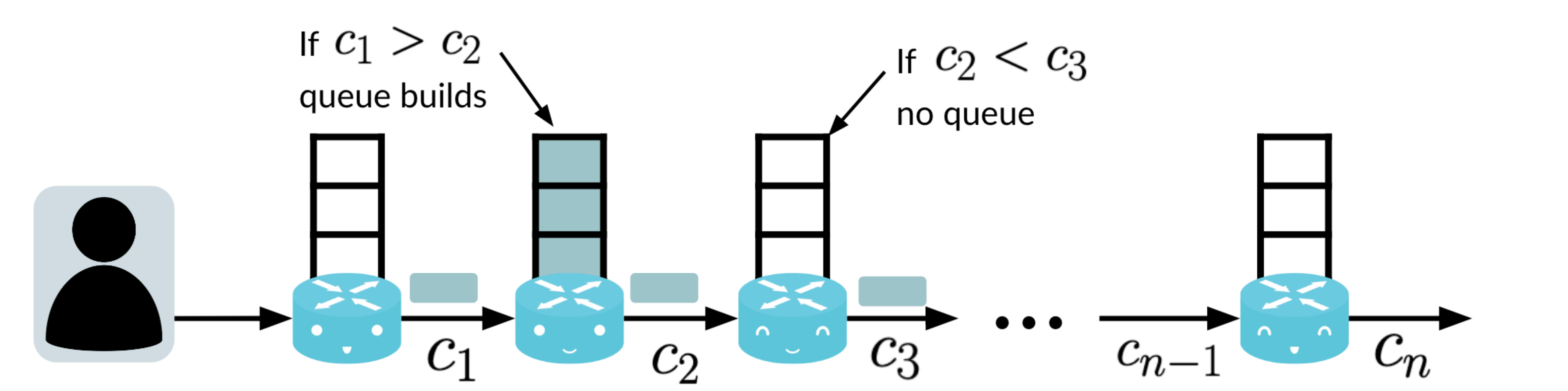
First, analyze **one sender across multiple hops** deterministically.

Next, analyze **multiple senders across multiple hops** with queueing theory.

Delay with One Sender and Multiple Hops

Setup

Links have capacities, c_i, that determine its sending speed.



Delay is caused by time on **queues**, **transmission** time, and **propagation** time (speed of light)

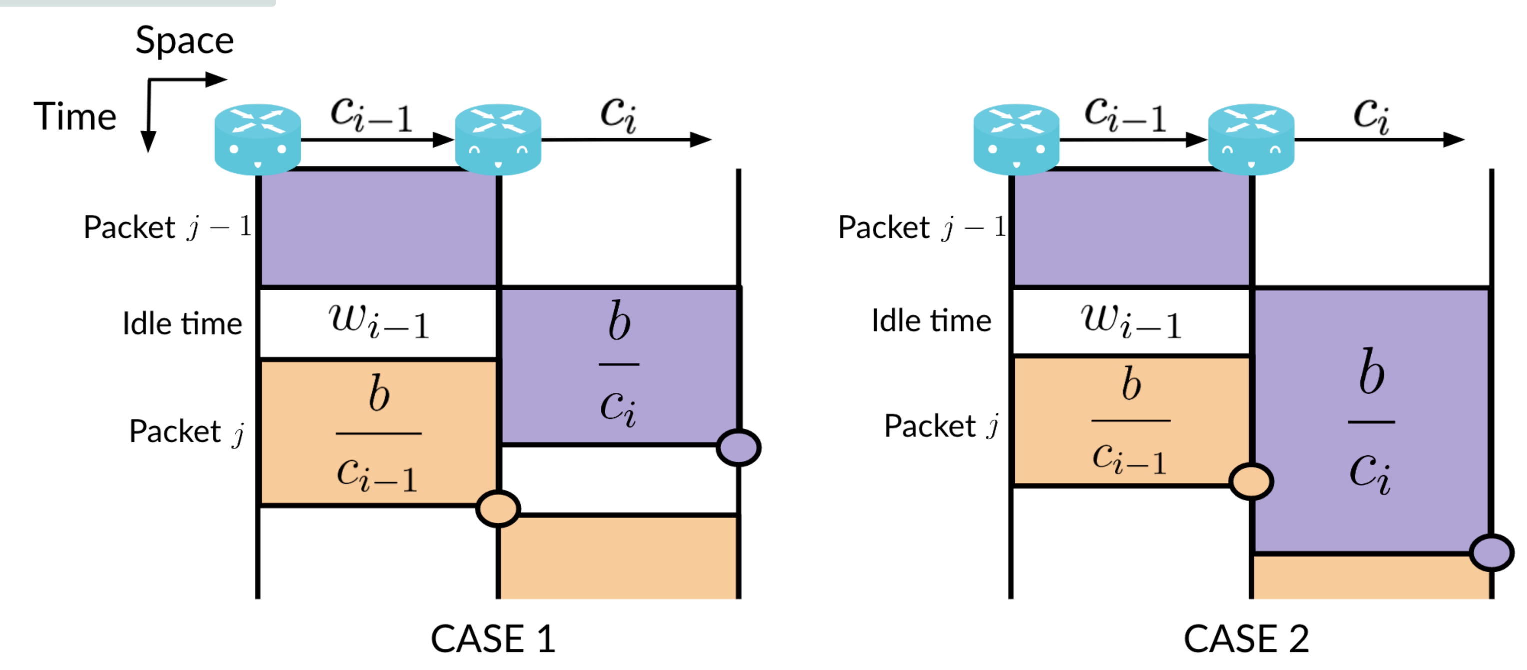
How long does it take p packets of b bits each to traverse the network? It's not obvious how long packets spend on queues.

Result for One Sender

Suprisingly, the order of links does not matter. **End-to-end latency only depends on the slowest link with capacity $c_{\min}$.**

$$\text{Total time} = \frac{(p-1)b}{c_{\min}} + \sum_i \frac{b}{c_i}$$

Proof Sketch



Between two links, who finishes first? Does time spent per packet change?

Case 1: $w_{i-1} + \frac{b}{c_{i-1}} \geq \frac{b}{c_i} \longrightarrow$ No change in time between packets

Case 2: $w_{i-1} + \frac{b}{c_{i-1}} < \frac{b}{c_i} \longrightarrow$ Time increases to the rate of the slower link

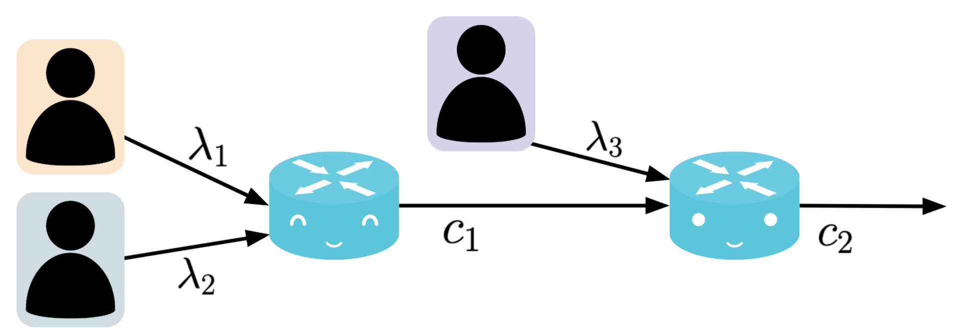
Across all links, the time per packet is stretched to the slowest rate.

Delay with Multiple Senders and Multiple Hops

Challenge

When multiple senders share bandwidth, it's hard to reason deterministically about the interleaving of their packets. How do we solve this?

Poisson Processes



Poisson processes model arrival rates λ and link capacities c. If stable ($\lambda < c$), they have nice properties [1]:

- If the input is Poisson(λ), so is the output.
- Poisson(λ_1) + Poisson(λ_2) = Poisson($\lambda_1 + \lambda_2$)

Result for Multiple Senders

When stable, $\mathbb{E}[T_{\lambda_i}]$ is the average time a packet from λ_i spends in the network [1].

$$\mathbb{E}[T_{\lambda_1}] = \mathbb{E}[T_{\lambda_2}] = \frac{1}{c_1 - (\lambda_1 + \lambda_2)} + \frac{1}{c_2 - (\lambda_1 + \lambda_2 + \lambda_3)}$$

$$\mathbb{E}[T_{\lambda_3}] = \frac{1}{c_2 - (\lambda_1 + \lambda_2 + \lambda_3)}$$

Conclusion and Next Steps

We can now easily model delay for multiple senders over multiple hops. It remains difficult to integrate this into a game theoretic framework.

Towards integration with game theory, we must assess the stability assumption and bound the delay so it does not depend on subsets of senders' rates. This will ultimately provide minimum throughput guarantees for senders.

References

[1] Mor Harchol-Balter. *Performance modeling and design of computer systems: queueing theory in action*. Cambridge University Press, 2013.

[2] Pratiksha Thaker, Matei Zaharia, and Tatsunori Hashimoto. Don't hate the player, hate the game: Safety and utility in multi-agent congestion control. In *Proceedings of the Twentieth ACM Workshop on Hot Topics in Networks*, pages 140–146, 2021.