Starvation Freedom in Multi-hop Network Topologies

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Summary

Congestion control often uses game theory to model senders competing for bandwidth in a network.

In this context, prior work was able to bound conditions for bandwidth starvation when senders optimize different utility functions in a network with a single link.

This project expands that result to networks with multiple links. In order to describe the utility function in these networks, we model delay both deterministically and with M/M/1 queues.

The Congestion Control Game

Senders compete for bandwith on links with given capacities.

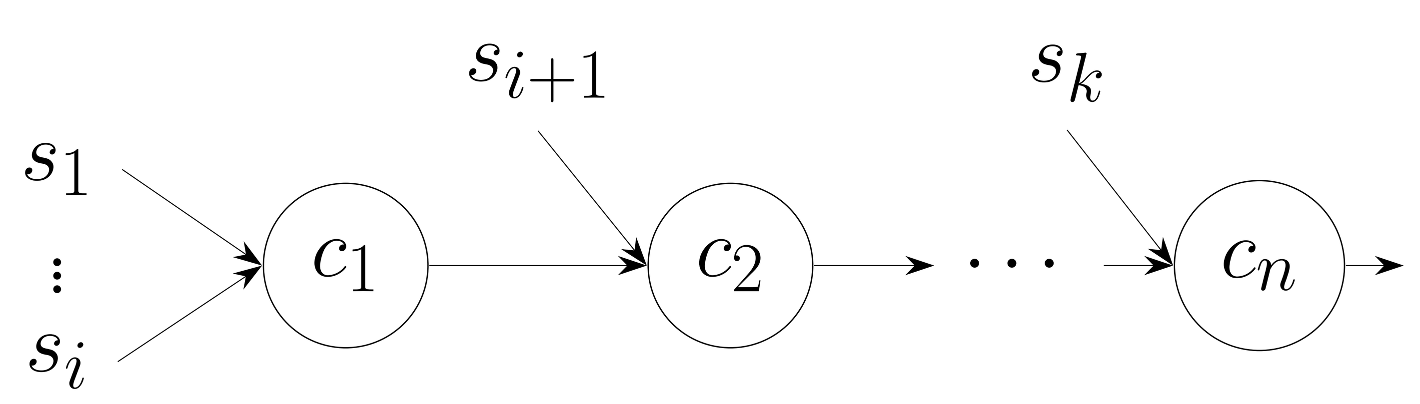


Figure 1. Multi-hop Network with Multiple Senders

The Utility Function

Sender i optimizes u_i as a function of its sending rate s_i and the aggregate load Q

$$u_i(s_i, Q) = T(s_i, Q) - \alpha_i s_i D(s_i, Q)$$

Each sender is rewarded for throughput, determined by T, and penalized for delay, given by D, which is scaled by a delay sensitivity parameter α.

Goal

To model delay in multi-hop networks in two ways.

1. Reasoning deterministically for a single sender
2. Using Poisson processes for arbitrary senders

Deterministic Delay with One Sender across Multiple Links

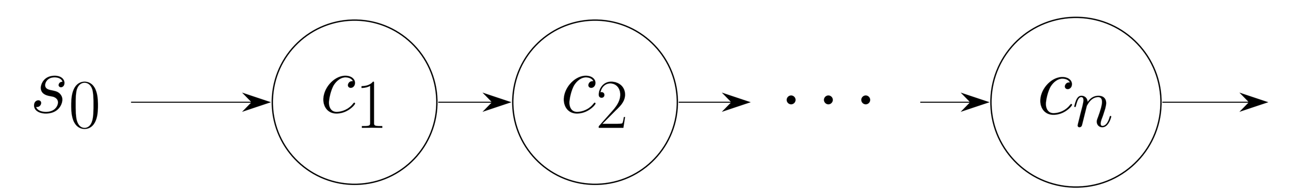


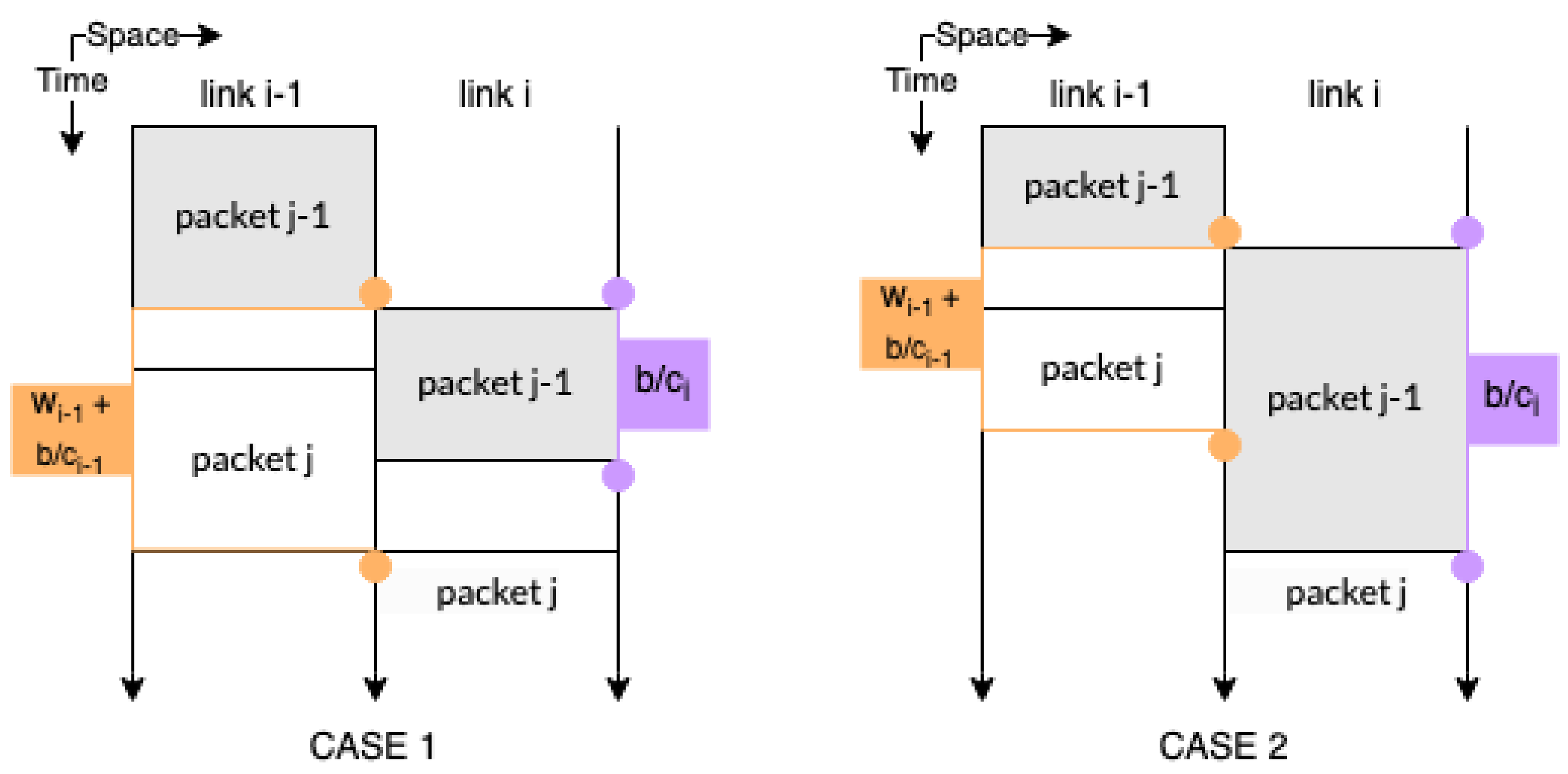
Figure 2. One sender into an arbitrarily long sequence of links.

p	# of packets	$c_{\min}$	smallest of all link capacities
b	# of bits per packet	w_i	idle time per packet on link i
c_i	capacity of link i	s_0	initial pace set by the sender

$$\text{end-to-end time for } p \text{ packets} = \frac{pb}{c_{\min}} + \sum_{i \neq \min}^{n} \frac{b}{c_i}$$

First term: Time per packet on the slowest link
Second term: Time it takes the first packet to traverse all links

Proof Sketch



Between two arbitrary links, we observe how packet spacing changes.

- **Case 1:** $\frac{b}{c_{i-1}} + w_{i-1} \geq \frac{b}{c_i} \rightarrow$ Packet spacing does not change.
- **Case 2:** $\frac{b}{c_{i-1}} + w_{i-1} < \frac{b}{c_i} \rightarrow$ Packet spacing increases to $\frac{b}{c_i}$.

The slowest link determines overall packet spacing.

Deterministic Drawback

When multiple senders enter the network, it is hard to reason deterministically about the interleaving of packets from different senders. Instead we use Poisson processes.

Delay using Poisson Processes across Multiple Links

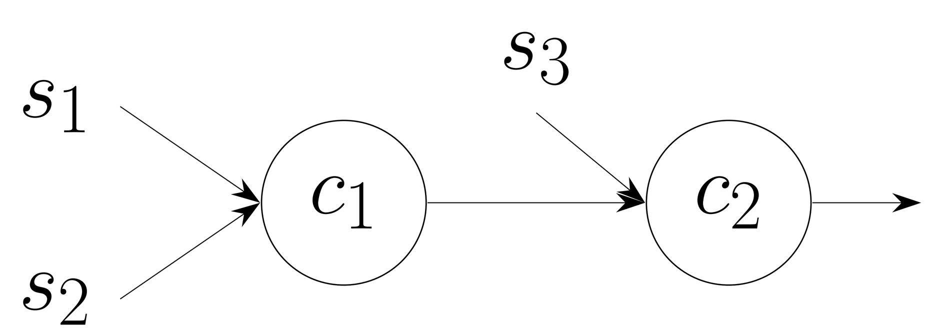


Figure 3. A simple network with multiple senders.

Poisson Process

Sending rates s_i and service rates c_i are random and memoryless, as are departures from any link[CITE Mor].

Merge two Poisson processes by adding their sending rates.

Average Delay

T_{s_i} is the average time a packet from s_i spends in the Network[CITE Mor]

$$\mathbb{E}[T_{s_1}] = \mathbb{E}[T_{s_2}] = \frac{1}{c_1 - (s_1 + s_2)} + \frac{1}{c_2 - (s_1 + s_2 + s_3)}$$

$$\mathbb{E}[T_{s_3}] = \frac{1}{c_2 - (s_1 + s_2 + s_3)}$$

Considerations

- Is a Poisson model realistic?
- Can we make the delay aggregative with an upper bound?
- How can we circumvent the stability requirement for the Poisson model?

Moving Forward

After calculating delay, we need to make it aggregative and revisit premises involving concavity in order to apply prior results which bound starvation.

References