Starvation Freedom in Multi-hop Network Topologies

Rachel Wilson [1] Pratiksha Thaker [1] Justine Sherry [1]

[1]Carnegie Mellon University

Summary

Congestion control often uses game theory to model senders competing for bandwidth in a network.

In this context, prior work was able to bound conditions for bandwidth starvation when senders optimize different utility functions in a network with a single link.

This project expands that result to networks with multiple links. In order to describe the utility function in these networks, we model delay both deterministically and with M/M/1 queues.

The Congestion Control Game

Senders compete for bandwith on links with given capacities, choosing a rate each round.

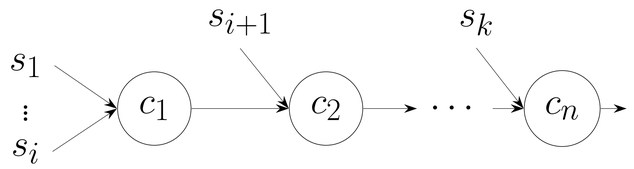


Figure 1. Multi-hop Network with Multiple Senders

The Utility Function

Each sender is rewarded for throughput, determined by T, and penalized for delay, given by D, which is scaled by a delay sensitivity parameter α.

$$u_i(s_i, Q) = T(s_i, Q) - \alpha_i s_i D(s_i, Q)$$

Sender i optimizes u_i as a function of its sending rate s_i and the aggregate *load* Q.

Goal

How do we model delay in networks with multiple links?

1. Deterministically
2. Using Poisson processes

Deterministic Delay across Multiple Links

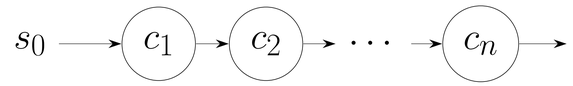


Figure 2. One sender into an arbitrarily long sequence of links.

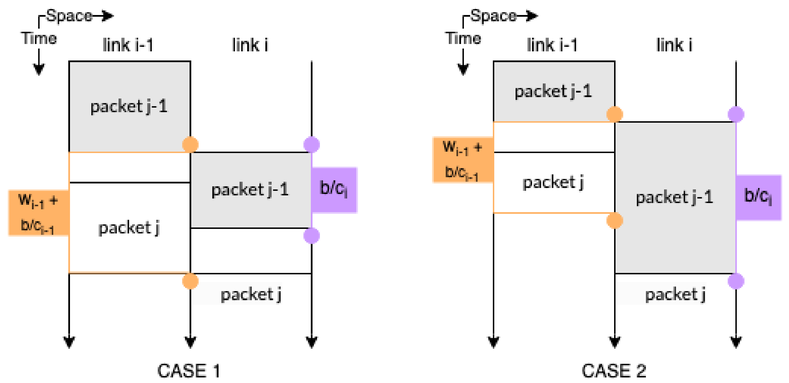
p	# of packets
b	# of bits per packet
c_i	capacity of link i

$c_{\min}$	smallest of all link capacities
w_i	idle time per packet on link i
s_0	initial pace set by the sender

$$\text{end-to-end time for } p \text{ packets} = \frac{pb}{c_{\min}} + \sum_{i \neq \min}^{n} \frac{b}{c_i}$$

First term: Time per packet on the slowest link
Second term: Time it takes the first packet to traverse all links

Proof Sketch



Between two arbitrary links, we observe how packet spacing changes.

- **Case 1:** $\frac{b}{c_{i-1}} + w_{i-1} \geq \frac{b}{c_i} \rightarrow$ Packet spacing does not change.
- **Case 2:** $\frac{b}{c_{i-1}} + w_{i-1} < \frac{b}{c_i} \rightarrow$ Packet spacing increases to $\frac{b}{c_i}$.

The slowest link determines overall packet spacing.

Concern

When multiple senders enter the network, it is hard to reason about the interleaving of packets deterministically. Instead we use Poisson processes.

Delay using Poisson Process across Multiple Links

DIAGRAM

cool easy math

Considerations

Is this realistic???

Moving Forward

Using this to actually bound starvation

References