SE499 Project

User advisory control for improving sustainability in mobile energy-harvesting network

Student name: Yuzhou Chen

Student ID: 20423002

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Report Summary

During the earthquake period, users are guided to certain fixed locations where they must stay until the period is over. Despite the impact of the earthquake, users would still like to have their mobile devices connected to the Internet as long as possible. However, networks in earthquake area are constrained by factors such as the location of base stations and the limitation of energy.

The output power of available base stations is limited. Base stations cannot work on their full strength; they need to go to sleep mode and recharge after consuming all energy. The power consumption of each base station depends on the farthest user it covers and the allowed BER (bit error rate) tolerance. The higher the output power of a station is, the faster the station consumes its energy and therefore, the ratio of recharging time to operating time becomes larger.

Ultimately, the problem is to make sure that on average, users have the maximum connection time given the limited energy of the base stations.

This project investigates the problem with sufficient depth and constructs a rigorous mathematical model that fits the situation. It also applies various Calculus techniques to find the optimal locations where users are guided to and the most efficient coverage area of each base station during the earthquake period such that the average connection time is maximized as much as possible.

1. Introduction

Natural disasters such as earthquakes cause vast amount of damage to networks in the area. Countries such as Japan face the threat of earthquake very often and even if earthquake resistance is implemented in buildings, a large earthquake can still be catastrophic. Energy sources such as power plants can easily be damaged or become unfunctional during the earthquake period. As a result, base stations for mobile networks must operate in energy harvesting mode during earthquake period.

Figure 1.1 shows a typical energy-harvesting access network architecture, where base stations harvest energy from external energy sources such as TV towers and portable power transmitters, and provides network connection to the users.

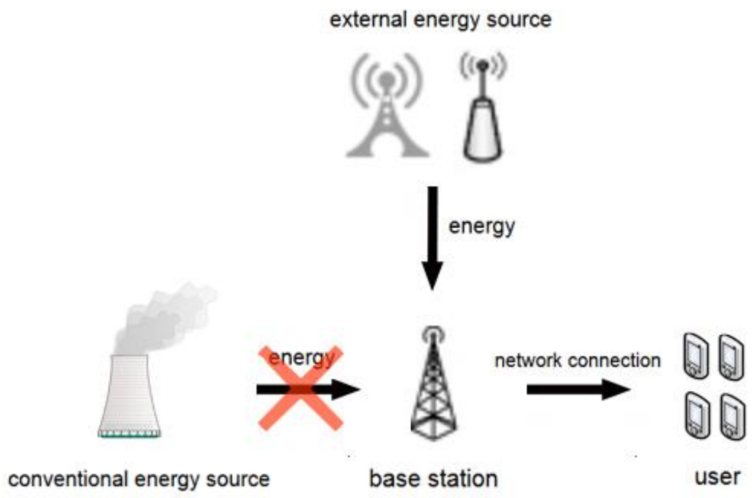


Figure 1.1 Energy-harvesting access network during earthquake

Due to low harvested energy and high consumption, base stations cannot provide stable and long-term connection. It is important that they operate in an efficient way to improve the experience of the users, who are guided to certain fixed locations during the earthquake period.

This paper analyzes the problem in depth, constructs a mathematical model of the above, and proposes a method to find the best locations to guide the users to and the corresponding operation mode for base stations to optimize user connection time. Section 2 to 4 introduce the settings of the problem; section 5 models the problem and provides an example; finally section 6 and 7 propose ways to solve the problem

2. Background Information

2.1 Output Power of Base Station

The output power of a base station follows the log-distance path loss model, which states the total path loss (the reduction of power during the transmission) is

$$PL = PL_0 + \omega log_{10} \frac{d}{d_0}$$

where PL is the total path loss, PL_0 is the path loss measured at distance d_0, d is the actual distance of the transmission. For $d_0 = 1$ meter, the typical measured path loss for a base station is around 57db [1]. ω is the path loss exponent which is dependent on the medium of transmission; the value is 2 for free space.

In our model we use a reference distance $d_0 = 1$ meter. Since the output power of the base station is equal to the sum of power received by the user and the path loss, we have the following relationship:

$$C_{tx} = C_{rx} + PL_0 + \omega log_{10} d$$

where C_{tx} is the output power of the base station and C_{rx} is the received power of the user, both in dbm.

For our purpose, we are interested in the output power in watts instead of dbm so we need to perform some conversion:

$$P_{tx} = \frac{10^{\frac{C_{tx}}{10}}}{1000} = \frac{10^{\frac{C_{rx}}{10}}}{1000} \cdot 10^{\frac{PL_0}{10}} \cdot d^{\frac{\omega}{10}} = P_{rx} \cdot 10^{\frac{PL_0}{10}} \cdot d^{\frac{\omega}{10}}$$

where P_{tx} is the output power of the base station and P_{rx} is the received power of the user, both in watts.

2.2 User Receiver Model

As shown in the previous section, the output power of the base station is a function of the power received by the user. The more power the user needs, the more power the base station has to consume. The power needed by the user depends on the signal to noise ratio (SNR) per bit $\frac{E_b}{N_0}$ and the system noise.

$$C_{rx} = \frac{E_b}{N_0} + N_{sys}$$

Since the signal to noise ratio $\frac{E_b}{N_0}$ is also related to the bit error rate (BER) in an AWGN channel as shown in the equation

$$BER = \frac{1}{2} erfc(\sqrt{\frac{E_b}{N_0}})$$

we can inverse the equation to obtain the SNR as a function of BER

$$\frac{E_b}{N_0} = \mathrm{erfc}^{-1}(2BER)^2$$

and therefore obtain the received power by user as a function of BER

$$C_{rx} = \frac{E_b}{N_0} + N_{sys} = \mathrm{erfc}^{-1}(2BER)^2 + N_{sys}$$

again, the unit of C_{rx} is dbm so we need to convert it to P_{rx} in watts

$$P_{rx} = \frac{10^{\frac{C_{rx}}{10}}}{1000} = \frac{10^{\frac{\mathrm{erfc}^{-1}(2BER)^2 + N_{sys}}{10}}}{1000}$$

3. Energy Harvesting Mode

3.1 Architecture

Now that both the base station output model and the user receiver model is explained, this section will explain the energy-harvesting operation. Due to the damage caused by the earthquake, base stations do not have sufficient energy supply and therefore operates in energy-harvesting mode by relying on external energy sources.

There are many types of energy harvesting such as Radio Frequency (RF) based, solar panels and large wind turbines. Due to the damage caused by the earthquakes, we suppose RF based energy harvesting is being deployed since dedicated RF energy sources can be small and mobile [2] while other types mentioned above are not easy to recover from earthquake damage quickly.

In a typical RF based energy harvesting network, the energy sources will continue supplying fixed power and network nodes will harvest energy from the sources to support their operations. There are two types of RF sources: dedicated RF sources and ambient RF sources. Dedicated RF sources include devices such as Powercaster Transmitters; they usually operate with small transmission power but are very mobile and can be deployed to meet specific demand. Ambient RF sources include TV towers and radio stations; they are not intended for energy transfer but can still provide free and stable power over time.

The typical harvesting rate is 189 microwatts from an RF transmitter source 5 meters away and 60 microwatts from a TV tower source 4 kilometers away [2]. In our model, we suppose we can deploy ten dedicated RF transmitter sources within 5 meters range of each base station and there is one or two TV tower or radio station within four kilometer range. This provides us with a total of $1.95mw$ to $2.01mw$ of harvested power.

On the other hand, the power consumption for the base station can be large in operating mode. Although power is being continuously harvested even during the operating mode, the base station needs to at minimum supply power for necessary components in order to operate properly. A typical base station consists of a Remote Radio Unit (RRU) component and a Base Band Unit (BBU) component [3][4].

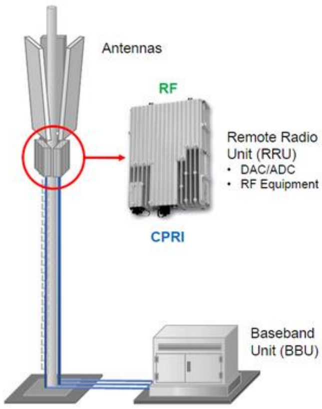


Figure 3.1 A typical base station architecture [4]

The RRU component is responsible for the power output amplifying process which depends on the power received by the user. The BBU is responsible for the physical layer so that the baseband processor can properly communicate with the antenna. According to earlier research, the baseband unit consumes $5mw$ power for a wireless base station [5].

The power consumption is larger than the harvested energy, so the base station has to go to sleep mode [6] once all the energies are consumed. During sleep mode, it does not need to consume energy for baseband amplifier and therefore can slowly but safely recharge its energy. As a consequence, the base station operates periodically between operating mode and sleep mode.

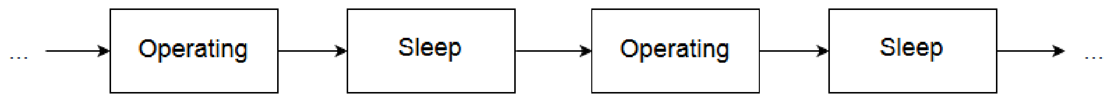


Figure 3.2 The Energy Hartesting Mode

3.2 Mathmatical Model

In our model, we define C to be the rate of energy harvesting and P_b to be the rate of power consumption for the BBU component. The total power consumption during the sleep mode is 0 and therefore the net power is simply

$$Net\ Power(Sleep\ mode) = C$$

However, in operating mode we have power consumption for both RRU and BBU so the net power is

$$Net\ Power(Operating\ mode) = C - P_{tx} - P_b$$

4. Definitions

4.1 Motivation

As shown in the earlier sections, the equations are getting complicated and a large amount of variables are involved in the calculations.

In this section we define several concepts to simplify later calculations.

4.2 Standard Power Units

We first define the concept of standard power units S_{rx} and S_{tx}, which are necessary to define later concepts. We use the P_{rx} when BER is set to 10^{-9} as a standard receiver power unit.

$$Standard\ Power\ Unit\ (S_{rx}) = \frac{10^{\frac{\mathrm{erfc}^{-1}(2\times10^{-9})^2+N_{sys}}{10}}}{1000} = 0.0629 \cdot 10^{\frac{N_{sys}}{10}}$$

Also, we use the P_{tx} when $d = 1$ and the receiver power is at S_{rx} as a standard base station output power unit.

$$Standard\ Power\ Unit\ (S_{tx}) = S_{rx} \cdot 10^{\frac{PL_0}{10}} \cdot 1^{\frac{\omega}{10}} = 0.0629 \cdot 10^{\frac{PL_0 + N_{sys}}{10}}$$

4.3 Bit Error Coefficient

For any other BER, the bit error coefficient would be the ratio of the P_{rx} of that BER to S_{rx}. We denote the coefficient to be u in this report.

$$u = \frac{P_{rx}}{S_{rx}} = \frac{\frac{10^{\frac{\mathrm{erfc}^{-1}(2BER)^2+N_{sys}}{10}}}{1000}}{0.0629 \cdot 10^{\frac{N_{sys}}{10}}} = \frac{10^{\frac{\mathrm{erfc}^{-1}(2BER)^2}{10}}}{62.9}$$

Note that N_{sys} is cancelled out in the calculation and the bit error coefficient u depends only on the BER given.

Using the bit error coefficient definition, we can represent the received power by the user using the simple equation.

$$P_{rx} = uS_{rx}$$

The output power can therefore be represented in a simple way as well as shown in the following calculation.

$$P_{tx} = P_{rx} \cdot 10^{\frac{L_0}{10}} \cdot d^{\frac{\omega}{10}} = uS_{rx} \cdot 10^{\frac{L_0}{10}} \cdot d^{\frac{\omega}{10}} = u\left(S_{rx} \cdot 10^{\frac{L_0}{10}} \cdot 1^{\frac{\omega}{10}}\right) \cdot d^{\frac{\omega}{10}} = u \cdot d^{\frac{\omega}{10}} \cdot S_{tx}$$

4.4 Power consumption and charge ratio

We also define the baseband consumption ratio B to be the ratio of base band consumption to the harvested power.

$$B = \frac{P_b}{C}$$

We also define the transmission consumption ratio T to be the ratio of a standard output power unit of the base station to the harvested power.

$$T = \frac{S_{tx}}{C}$$

The ratio of the actual output to harvested power is therefore

$$\frac{P_{tx}}{C} = \frac{ud^{\frac{\omega}{10}}S_{tx}}{C} = ud^{\frac{\omega}{10}}T$$

Using these definitions, we can rewrite the operating mode output power as a multiple of C

$$Net\ Power(Operating\ mode) = C - P_{tx} - P_b = (1 - ud^{\frac{\omega}{10}}T - B)C$$

As mentioned in earlier sections, the harvested power is much less than the BBU power consumption during the earthquake period, so B is generally greater than one and the net power during operation mode is generally negative.

4.5 Active Time ratio

The active time ratio (ATR) is defined as the ratio of time spent in operating mode to time spent in a complete operating-sleep period. If we let the total energy harvested (and used) during the the whole period be Ω then the length of time in operating mode is

$$\left|\frac{\Omega}{Net\ Power(operating\ mode)}\right| = \left|\frac{\Omega}{(1 - ud^{\frac{\omega}{10}}T - B)C}\right| = \frac{\Omega}{(B + ud^{\frac{\omega}{10}}T - 1)C}$$

Note that the absolute value is evaluated to negative because we can safely assume $B > 1$ in the disaster area. The length of time in sleep mode is

$$\left|\frac{\Omega}{Net\ Power(sleep\ mode)}\right| = \left|\frac{\Omega}{C}\right| = \frac{\Omega}{C}$$

Hence the ATR, by definition is

$$\frac{\frac{\Omega}{(B + ud^{\frac{\omega}{10}}T - 1)C}}{\frac{\Omega}{(B + ud^{\frac{\omega}{10}}T - 1)C} + \frac{\Omega}{C}} = \frac{1}{B + ud^{\frac{\omega}{10}}T}$$

4.6 Table of Variables and Constants

In the next section we will show an example of calculation to find the best location in a two base station case. Table 4.1 and 4.2 show a list of numbers we are dealing with so far and the value we choose for the calculations in later sections.

Symbol	Meaning	Typical value	Value to be used in our calculations
PL_0	Path loss at the reference distance	$57db$	$57db$
ω	Path loss exponent	1 to 4	2
N_{sys}	System noise	$-100dbm$ to $-150dbm$	$-130dbm$
C	Harvested power	$1.95mw$ to $2.01mw$	$2mw$
P_b	BBU consumption	$5mw$	$5mw$
S_{rx}	Receiver standard power	Calculated from N_{sys}	$6.29 \times 10^{-15}w$
S_{tx}	BS output standard power	Calculated from S_{rx} and PL_0	$3.15 \times 10^{-9}w$
B	BBU consumption ratio	Calculated from P_b and C	2.5
T	Transmission consumption ratio	Calculated from S_{tx} and C	1.58×10^{-6}

Table 4.1 List of constants

Symbol	Meaning	Typical values	Value to be used in our calculations
BER	Bit error rate	1×10^{-5} to 1×10^{-9}	1×10^{-8}
u	Bit error coefficient	Calculated from BER	0.6
d	Distance to the user	Various	Various

Table 4.2 List of variables

5. The Problem

5.1 Input and Output

Our objective is to determine the best location(s) to place the users so that the ATR is maximized. The inputs are the constants and variables listed in section 4.6, along with the locations of all the base stations. The output is the location(s) where the users should be guided to during the disaster period.

5.2 An example：Two base station case

In order to illustrate the problem, this section shows an example where there are two base stations in the area. We also introduce several types of solutions including the sharing mode, and analyze these solutions through comparisons. As shown in the table in section 4.6, we use a path loss exponent value of 2, a BBU consumption ratio of 2.5, a transmission consumption ratio of 1.58×10^{-6} and a bit error coefficient of 0.6 in this example.

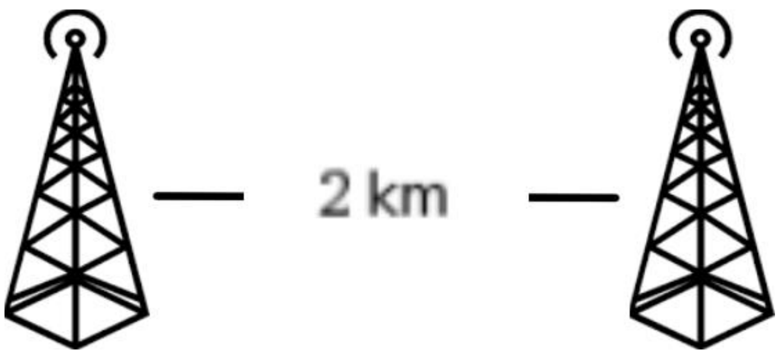


Figure 5.1 Problem scenario for two base stations

Figure 5.1 shows the scenario for our problem. There are two base stations located in the area, 2 kilometers apart from each other. The problem is to find the optimal location to place the users so that the active time ratio is maximized.

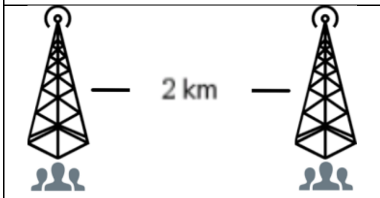
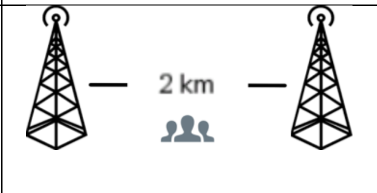
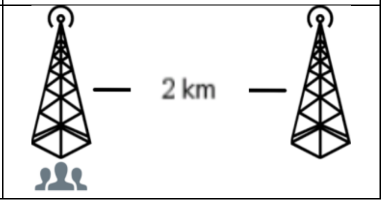
Naïve solution	Symmetric sharing	Asymmetric sharing
		

Table 5.1 Three possible solutions

Table 5.1 shows three possible solutions to the problem. In the naïve solution, users are guided near each base station and each base station operates on its own without interfering each other. In this case, we take the distance of users to each base station $d = 0$. Note that we cannot actually achieve $d = 0$ in reality but a small distance such as $d = 1$ will have only ignorable effect in our calculation so we use $d = 0$ to approximate any small distance near the base station for the sake of simplicity of calculation. The ATR of each location is therefore calculated by

$$ATR = \frac{1}{B + ud^{\frac{\omega}{10}}T} = \frac{1}{2.5 + 0.6 \cdot 0^2 \cdot (1.58 \times 10^{-6})} = 0.4$$

Since each location has the same ATR as shown above, the average ATR is

$\frac{0.4+0.4}{2} = 0.4$ as well. This means each user must wait for 60 minutes to have 40 minutes of internet connection.

5.3 Sharing Mode

We can certainly do better than that. A more advanced solution employs the sharing mode. In this solution, users are all guided to the midpoint between the two base stations (i.e. 1 kilometer away from each). During the sharing mode, base stations operate alternatingly such that each base station's operating mode covers the other one's sleeping mode as much as possible. Figure 5.2 shows a graphical representation of such sharing.

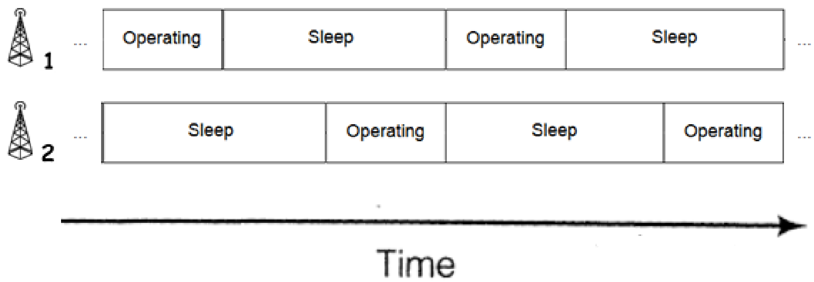


Figure 5.2 Sharing mode

At a first glimpse, it might seems like this solution works twice as efficient as the naïve solution. However, this is not true. Since the ATR of each base station depends on the distance squared in the denominator, when the distance gets large, the sharing mode solution can become worse than the naïve solution. The ATR for this solution is calculated as follows.

$$ATR = \frac{2}{B+ud^{\frac{\omega}{10}}T} = \frac{2}{2.5+0.6\cdot 1000^2\cdot(1.58\times 10^{-6})} = 0.580$$

note that the numerator is 2 instead of 1 because of the sharing in Figure 4.2. As shown in the calculation, the ATR of this solution is indeed better than the naïve solution despite the distance being a thousand time larger. However, as discussed earlier, this result is not guaranteed to be true; for example, if the base stations are four kilometers away from each other, then the result would become 0.31 which is less than the naïve solution value of 0.4.

5.4 Asymmetric sharing

It is not hard to find out that there exists a solution that is guaranteed to be better than the naïve solution if we think a little bit. In section 5.3 we introduced symmetric sharing but in reality the sharing mode can be asymmetric as well. If we guide all the user to a location near one base station, and let the two base stations operate in sharing mode, then the ATR can be calculated as

$$ATR = \frac{1}{B + ud_1^{\frac{\omega}{10}}T} + \frac{1}{B + ud_2^{\frac{\omega}{10}}T}$$

$$= \frac{1}{2.5 + 0.6 \cdot 0^2 \cdot (1.58 \times 10^{-6})} + \frac{1}{2.5 + 0.6 \cdot 2000^2 \cdot (1.58 \times 10^{-6})} = 0.559$$

If we look at the equation, we can realize that the ATR is simply the

ATR of naïve solution plus an extra term. This shows, mathematically, the asymmetric sharing solution is guaranteed to be better than the naïve solution.

The ATR value is slightly worse than the symmetric sharing solution as shown in the calculation. However, if the distance between base stations becomes longer, this solution becomes the best of the three solution. Table 5.2 shows a comparison of the three solutions over various distance.

Distance between base stations	ATR of solution		
	Naïve solution	Symmetric sharing	Asymmetric sharing
$500m$	0.4	0.781	0.765
$1km$	0.4	0.731	0.690
$2km$	0.4	0.580	0.558
$3km$	0.4	0.432	0.491
$4km$	0.4	0.318	0.457
$5km$	0.4	0.237	0.438

Table 5.2 Comparison of solutions

5.5 Asymmetric synchronization

One problem faced by the asymmetric sharing solution is that the period of the closer base station has a longer operating-sleep period than the farther base station. If we employ the same type of energy harvesting scheme as shown before, the sharing would become like in Figure 5.3

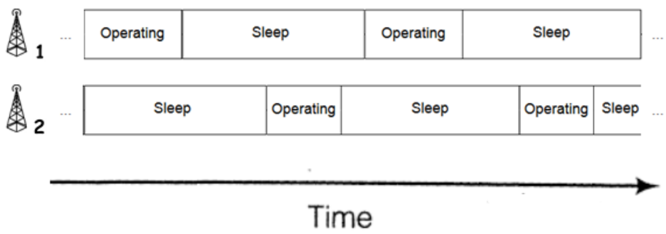


Figure 5.3 Asymmetric sharing

If this is the case, then at some point the operating mode part of the two base stations will overlap with each other and therefore the theoretical ATR value calculated in Table 5.2 is not in fact achievable.

Fortunately, there is a solution to this problem. We can just order the closer base station to decrease its maximum total harvested energy Ω. Once the Ω is decreased, since the length of time of operating mode and sleep mode is $\frac{\Omega}{(B+ud^{\frac{\omega}{10}}T-1)C}$ and $\frac{\Omega}{C}$, both mode will have their length of time reduced without changing the ATR. That is, we can "shrink" the operating-sleep period of the closer base station without affecting its ATR. Figure 5.3 shows the synchronized version of Figure 5.3, where the period of the first base station is shrinked.

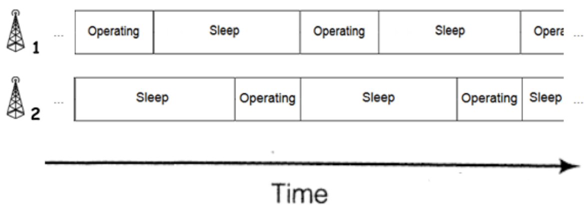


Figure 5.3 Asymmetric sharing synchronized

6. Mathematical Formalization

6.1 The general case

In the example in section 5, we discussed a scenario that consists of two base stations. In this section, we work with the general case where there can be an arbitrary number of base stations at arbitrary locations. We shall let the number of base stations be n and the i_{th} base station be denoted by BS_i for later discussions in this section. Also we define the coverage radius of BS_i, c_i, to be the distance such that users at this distance from BS_i can receive signal from BS_i with a bit error coefficient of exactly u. In other words, c_i is the distance such that any location within the circle centered at the location of BS_i with radius c_i can receive signal from BS_i with an acceptable BER. Figure 6.1 shows a possible solution in an $n = 3$ scenario.

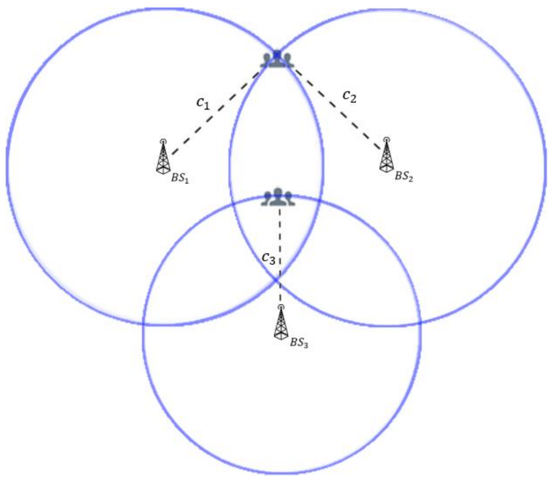


Figure 6.1 Coverage radius

The users at the middle location are in the coverage radius of all three base stations while those at the top location are in the coverage radius of BS_1 and BS_2. We say a location is covered by a base station if the location is in the coverage radius of that base station.

In section 5 we showed, mathematically, that in the $n = 2$ case, the asymmetric sharing solution is guaranteed to be better than the naïve solution no matter how the base stations are arranged. This gives us the intuition of the following theorem.

Theorem 1

There exists no multiple-location solution that is better than all single-location solutions.

Proof of Theorem 1

Suppose the opposite, that is, there exists a solution that the users are guided to at least two different locations that are better than all single-location solution. Let the number of locations be m $(m \geq 2)$, and let the ATR at the i_{th} location be a_i for $i = 1, 2, 3, \ldots, m$ respectively. The average ATR of this solution is therefore

$$\frac{1}{m}\sum_{i=1}^{m} a_i$$

Now we choose the location with the largest ATR. Suppose it is at the k_{th} location where the ATR is a_k. Then we have $a_k \geq a_i$ for all $i = 1, 2, 3, \ldots, m$. Also, consider the set of base stations that covers the k_{th} location $S = \{i \in [n] \mid BS_i \text{ covers } k_{th} \text{ location}\}$, then we have

$$a_k = \sum_{i \in S} \frac{1}{B + u{c_i}^2 T}$$

Now consider a new solution where all users are guided to the k_{th} location and all the base stations operate in sharing mode at the k_{th} location, with coverage radius exactly the distance to the k_{th} location. Since there is only one user location (k_{th}), the average ATR is simply

$$\sum_{i \in [n]} \frac{1}{B + u{d_i}^2 T}$$

where d_i is the distance from BS_i to the k_{th} location, which is also the coverage radius of the new solution. Now since for $i \in S$, BS_i covers k_{th} location in the original solution, we have $c_i \geq d_i$. Therefore, we have

$$\sum_{i \in [n]} \frac{1}{B + u{d_i}^2 T} \geq \sum_{i \in S} \frac{1}{B + u{d_i}^2 T} \geq \sum_{i \in S} \frac{1}{B + u{c_i}^2 T} = a_k = \frac{1}{m}\sum_{i=1}^{m} a_k \geq \frac{1}{m}\sum_{i=1}^{m} a_i$$

Note the first inequality comes from the fact that S is a subset of $[n]$. This is exactly the intuition that comes from the two base station example discussed in section 5. The inequality shows that the new one-position solution we found has an ATR at least as good as the original solution, which contradicts with the hypothesis that the original solution is better than all single-location solutions.

By Theorem 1, we can conclude that multiple-location solutions such as the one in Figure 6.1 cannot be the best solution (however, there is possibility that the best single-location solution shares the same ATR as a multiple-location solution). Although the proof of theorem 1 is not a constructive proof which gives us the best solution, we can convince ourselves that if we find the best single-location solution, it is guaranteed to be the overall best solution as well.

Corollary of Theorem 1

An optimal single-location solution is also a global optimal solution.

6.2 Formalization of problem

Now that we only need to find the optimal single-location solution, we can formalize our problem based on the assumption that there is only one location for the users. We use the Cartesian coordinate system to formalize the problem. In Cartesian coordinate system, we know that

$$d_i{}^2 = (x - x_i)^2 + (y - y_i)^2$$

by Pythagorean Theorem, where d_i is the distance between the users and BS_i as discussed before, (x, y) is the location of the users and (x_i, y_i) is the location of BS_i.

Using this result, we can formalize the problem mathematically as the following optimization problem:

Problem

Given B, T, u and x_i, y_i where $i = 1, 2, 3, \dots, n$. Find x, y that maximize the objective function $f(x, y)$ where

$$f(x, y) = \sum_{i=1}^{n} \frac{1}{B + uT((x - x_i)^2 + (y - y_i)^2)}$$

Under this formalization, we can show that we are guaranteed that we can find the optimal single-location solution.

Theorem 2

$f(x, y)$ is continuous, differentiable everywhere and has a global maximum.

Proof of Theorem 2

(1) $f(x, y)$ is continuous and differentiable on R^2 because sum of continuous and differentiable functions is continuous and differentiable. For each part of $f(x, y)$,

$$f_i(x, y) = \frac{1}{B + uT((x - x_i)^2 + (y - y_i)^2)}$$

can be easily shown to be continuous on R^2 using the fact that $\frac{1}{1+x^2+y^2}$ is continuous and differentiable. By simple change of variable. i.e, let $x' = \sqrt{\frac{B}{uT}}(x + x_i)$ and $y' = \sqrt{\frac{B}{uT}}(y + y_i)$ we can convert $f_i(x, y)$ to constant multiple of $\frac{1}{1+x^2+y^2}$ which shows $f_i(x, y)$ is continuous and differentiable.

(2) $f(x, y)$ has a global maximum because $0 < f_i(x, y) < \frac{1}{B}$ for all i and hence $0 < f(x, y) < \frac{n}{B}$. Also $\lim_{(x,y)\to(\pm\infty,\pm\infty)} f(x, y) = 0$. Hence $f(x, y)$ has a global maximum.

Corollary of Theorem 2

One of the critical point of $f(x, y)$ is the global maximum. (Obviously if a global maximum exists and is not a non-differentiable point then it has to be a critical point)

6.3 Partial Derivative method

Now we can use the partial derivative method to find the optimal solution of the problem. To find the maximum of $f(x, y)$, we can simply let $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$ and solve for x and y. This will give one of the following three possibilities: a maximum, a minimum or a saddle point. By corollary to theorem 2, we know that one of the solutions to $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$ must be the global maximum. Consequently, the solution that provides the maximum value of $f(x, y)$ is the global maximum.

6.4 Two base station case revisited

This section shows how to find the optimal solution for the example in section 5 using the partial derivative method. Recall that we have two base stations 2 kilometers apart and the values we use are shown in section 4.6. We first choose a suitable coordinate system where BS_1 is located at $(0, 0)$ and BS_2 is located at $(2000, 0)$. Note that in most cases where there are three or more base stations, we have both x and y being non-zero values and that is the reason we choose to use the Cartesian coordinate system (it is hard to directly deal with formulas involving d_i). However since calculations will get messy in such cases we show the simple case where $y = 0$ in this example.

After assigning Cartesian coordinates for each base stations, we write down the objective function $f(x, y)$

$$f(x,y) = \frac{1}{2.5 + 0.6 \cdot (1.58 \times 10^{-6}) \cdot (x^2 + y^2)} + \frac{1}{2.5 + 0.6 \cdot (1.58 \times 10^{-6}) \cdot ((x - 2000)^2 + y^2)}$$

$$f(x,y) = \frac{1}{2.5 + 9.48 \times 10^{-7} \cdot (x^2 + y^2)} + \frac{1}{2.5 + 9.48 \times 10^{-7} \cdot ((x - 2000)^2 + y^2)}$$

After that, we calculate the partial derivatives

$$\frac{\partial f}{\partial x} = \frac{-1.9 \times 10^{-6} x}{(2.5 + 9.48 \times 10^{-7} \cdot (x^2 + y^2))^2} + \frac{-1.9 \times 10^{-6} (x - 2000)}{(2.5 + 9.48 \times 10^{-7} \cdot ((x - 2000)^2 + y^2))^2}$$

$$\frac{\partial f}{\partial y} = \frac{-1.9 \times 10^{-6} y}{(2.5 + 9.48 \times 10^{-7} \cdot (x^2 + y^2))^2} + \frac{-1.9 \times 10^{-6} y}{(2.5 + 9.48 \times 10^{-7} \cdot ((x - 2000)^2 + y^2))^2}$$

Letting both of them equal zero, we can obtain the following three real solutions using Newton's Method [8] with two variables. (Source code in C is attached in Appendix)

$$(x, y) = (579.14, 0) \text{ or } (1000, 0) \text{ or } (1420.86, 0)$$

Substitute in these values back into $f(x, y)$ and we obtain that $f(579.14, 0) = f(1420.86, 0) = 0.581$, $f(1000, 0) = 0.580$ so the two points $(579.14, 0)$ or $(1420.86, 0)$ are the optimal solutions. In this case, it does not matter which point we choose, and we can also guide some users to one of the locations and the remaining to the other (remember that Theorem 1 does not rule out the possibility of a multi-location solution that shares the same average ATR as the optimal single-location solution).

One thing to notice in this example is that the optimal solution only differs from the symmetric sharing solution, i.e. the $(1000, 0)$ solution, by 0.001. However this is not always the case, if the distance between the stations is 3000 kilometers instead of 2000 kilometers, then the optimal solution is $f(179.04, 0) = f(2820.96, 0) = 0.495$ while the symmetric sharing solution is $f(1500, 0) = 0.431$ which is significantly less than the optimal solution. When the distance is small (such as 200 meters), the optimal solution can actually coincide with the symmetric sharing one. However, no matter what the distance is, the existence of at least one real solution is guaranteed by the corollary of Theorem 2.

7. Constrained Version

7.1 The constraint

In reality, base stations have a limit on the output power P_{tx}. Due to this limit, the maximum possible coverage radius of a base station is also limited. That is, we cannot let a base station covers users that are too far away from it. Typically, a base station has a limit of 5 to 35 kilometers in reality [7. p183]. In our model, we denote this limit D.

7.2 Theorem 1 Revised

With the maximum coverage distance constraint introduced, it might seem less intuitive that an optimal one-position solution is also a global optimal solution since one-position solutions are more likely to violate the constraint, but we can still prove Theorem 1 to be true which implies its corollary even with the constraint. The following is a revised proof for theorem 1, taking into consideration the constraint. Revised parts are underlined.

Proof of Theorem 1 Revised

… (The beginning of the proof is the same as the original proof)

Now consider a new solution where all users are guided to the k_{th} location. <u>For each $i = 1, 2, 3, \ldots, \mathrm{n}$, let BS_i operate in sharing mode at the k_{th} location with coverage radius exactly the distance to the k_{th} location if that distance is less than or equal to D and let it stop working if that distance is longer than D. We let S' be the set of base station that the distance is within the limit, $S' = \{i \in [n] \mid d_i \leq D\}$.</u> Since there is only one user location (k_{th}), the average ATR is simply

$$\underline{\sum_{i \in S'} \frac{1}{B + u{d_i}^2 T}}$$

where d_i is the distance from BS_i to the k_{th} location, which is also the coverage radius of the new solution. Now since for $i \in S$, BS_i covers k_{th} location in the original solution, we have $c_i \geq d_i$. Therefore, we have

$$\sum_{i\in \boldsymbol{S}'}\frac{1}{B+u{d_i}^2T} \geq \sum_{i\in S}\frac{1}{B+u{d_i}^2T} \geq \sum_{i\in S}\frac{1}{B+u{c_i}^2T} = a_k = \frac{1}{m}\sum_{i=1}^{m} a_k \geq \frac{1}{m}\sum_{i=1}^{m} a_i$$

The first inequality comes from the fact that S is a subset of S'. It is not hard to prove this. Suppose $i \in S$, then BS_i covers k_{th} location in the original solution, this means the distance between BS_i and k_{th} location is less than or equal to D in the original solution. Since both solution share the same arrangement of base stations and the k_{th} location we are dealing with is the same location for both solution, hence the distance between BS_i and k_{th} location is the same for both solutions as well. Therefore $d_i \leq D$. Therefore $i \in S'$. We have shown $i \in S \Rightarrow i \in S'$ which is the definition of $S \subseteq S'$.

…(Later parts are the same as original proof)

Now that we have shown the optimal single-location solution is still a global solution under the distance constraint, our goal is still to find a single pair of $(x,\ y)$ to guide the users to.

7.3 Formalization with constraint

The problem is that, with the constraint introduced, there can be an arbitrary number of base station that is unused in the optimal solution. To formalize this fact, we introduce a vector g to indicate whether a base station is used in the solution or not, i.e, $g_i = 1$ means BS_i is used and $g_i = 0$ means BS_i is unused. Our problem now becomes the following constrained optimization problem,

Problem

Given B, T, u and x_i, y_i where $i = 1, 2, 3, \dots, n$. Find x, y and $g_1, g_2, g_3, \dots, g_n$ that maximize the objective function $f(x, y)$ where

$$f(x,y) = \sum_{i=1}^{n} \frac{g_i}{B + uT((x - x_i)^2 + (y - y_i)^2)}$$

Subject to the constraints

$$(x - x_i)^2 + (y - y_i)^2 \le D^2 \; for \; i = 1, 2, \dots, n$$

$$g_i \in \{0, 1\} \; for \; i = 1, 2, \dots, n$$

7.4 The Lagrange Multiplier Method

We first consider a simpler version of the problem: if the vector g is given, is it possible to find the maximal solution? To be short, the answer is yes. Essentially, we want to find a point that is inside a bounded area which maximize our objective function. This can be solved perfectly using the method of Lagrange Multiplier. The Lagrange Multiplier method essentially treats points on the given boundary as variable and calculates the critical points of the objective function with respect to that variable. Before introducing the algorithm to compute the optimal solution, we first take a look at the following theorem.

Theorem 3

Suppose there exists a common area among all circles (i.e. an area that satisfy all constraints), then an optimal solution inside the area exists and must be one of the following three:

(1) A critical point of $f(x, y)$

(2) A point found by Lagrange Multiplier method on one of the circle specified by the constraint, i.e. $(x - x_i)^2 + (y - y_i)^2 = D^2$

(3) An intersection point of two circles

Proof of Theorem 3

First, an optimal solution must exist since it is a bounded area and is differentiable everywhere except the boundary (by theorem 2).

(1) If the optimal location is not on the boundary of a circle, then it must

be strictly inside the constrained area, which implies it is a critical point.

(2) If the optimal solution is on the boundary of a circle but not an intersection point of two circles, then it is on the boundary of only one circle and is therefore a local maximum with respect to the circle. This means the Lagrange Multiplier method can find it.

(3) The only option left is an intersection of two circles.

Now that we have narrow down the optimal solution to the three possibilities stated in theorem 3, we can employ the following algorithm to calculate the optimal value.

Algorithm – FindOptimalPoint

Input : g, Output : Optimal location or constraint cannot be satisfied

(1) Find all critical points of $f(x, y)$

(2) Find all Lagrange Multiplier solutions with respect to each circle specified by the constraint

(3) Find all pairwise intersections of the circles

(4) Loop through all the points calculated in steps (1) to (3) and record

the point that has the maximum $f(x, y)$ value and that is inside all circles.

(5) If no such point is found then it means an area that satisfies all the constraints does not exist.

The time complexity of FindOptimalPoint is $O(n^3)$ as there are at most $n(n-1) + O(n)$ points to check and each check loops through n circles to check whether it is contained in all circles.

7.5 Example

This section illustrates how one can use the algorithm in section 7.4 method to find a solution to a three base station example.

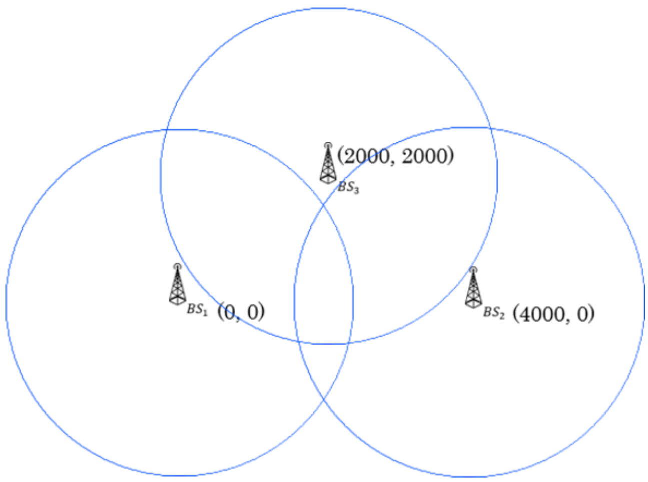


Figure 7.2 Example setting (blue circles denote the constraints)

Figure 7.2 shows the scenario we are concerned about. Given $g =$

$(1,1,1)$ and $D = 2500$, we want to find the optimal location to place users subject to the constraints $(x - x_i)^2 + (y - y_i)^2 \leq D^2\ for\ i = 1, 2,3$.

We first perform the similar Partial Derivative method as shown in section 6.4 to find the two critical points (Source code is attached in Appendix): $(259.42, 163.24)$ and $(3740.58, 163.24)$ .It is easy to see that neither of the point is in the constrained area.

Then we perform the Lagrange Multiplier method to find the Lagrangian critical points for each circle. First we write down the partial derivatives for $f(x, y)$.

$$\frac{\partial f}{\partial x} = \frac{-1.9 \times 10^{-6} x}{(2.5 + 9.48 \times 10^{-7} \cdot (x^2 + y^2))^2} + \frac{-1.9 \times 10^{-6}(x - 4000)}{(2.5 + 9.48 \times 10^{-7} \cdot ((x - 4000)^2 + y^2))^2} + \frac{-1.9 \times 10^{-6}(x - 2000)}{(2.5 + 9.48 \times 10^{-7} \cdot ((x - 2000)^2 + (y - 2000)^2))^2}$$

$$\frac{\partial f}{\partial y} = \frac{-1.9 \times 10^{-6} y}{(2.5 + 9.48 \times 10^{-7} \cdot (x^2 + y^2))^2} + \frac{-1.9 \times 10^{-6} y}{(2.5 + 9.48 \times 10^{-7} \cdot ((x - 4000)^2 + y^2))^2} + \frac{-1.9 \times 10^{-6}(y - 2000)}{(2.5 + 9.48 \times 10^{-7} \cdot ((x - 2000)^2 + (y - 2000)^2))^2}$$

For the first circle, the boundary function is

$$h(x, y) = x^2 + y^2$$

Therefore,

$$\frac{\partial h}{\partial x} = 2x$$

$$\frac{\partial h}{\partial y} = 2y$$

Now applying the Lagrange Multiplier λ, along with our constraint function, we obtain the following three equations:

$$\frac{-1.9\times10^{-6}x}{(2.5+9.48\times10^{-7}\cdot(x^2+y^2))^2}+\frac{-1.9\times10^{-6}(x-4000)}{(2.5+9.48\times10^{-7}\cdot((x-4000)^2+y^2))^2}+\frac{-1.9\times10^{-6}(x-2000)}{(2.5+9.48\times10^{-7}\cdot((x-2000)^2+(y-2000)^2))^2}=2\lambda x$$

$$\frac{-1.9\times10^{-6}y}{(2.5+9.48\times10^{-7}\cdot(x^2+y^2))^2}+\frac{-1.9\times10^{-6}y}{(2.5+9.48\times10^{-7}\cdot((x-4000)^2+y^2))^2}+\frac{-1.9\times10^{-6}(y-2000)}{(2.5+9.48\times10^{-7}\cdot((x-2000)^2+(y-2000)^2))^2}=2\lambda y$$

$$x^2+y^2=2500^2$$

Note that if we divide the first equation by the second equation we can obtain

$$\frac{\frac{-1.9\times10^{-6}x}{(2.5+9.48\times10^{-7}\cdot(x^2+y^2))^2}+\frac{-1.9\times10^{-6}(x-4000)}{(2.5+9.48\times10^{-7}\cdot((x-4000)^2+y^2))^2}+\frac{-1.9\times10^{-6}(x-2000)}{(2.5+9.48\times10^{-7}\cdot((x-2000)^2+(y-2000)^2))^2}}{\frac{-1.9\times10^{-6}y}{(2.5+9.48\times10^{-7}\cdot(x^2+y^2))^2}+\frac{-1.9\times10^{-6}y}{(2.5+9.48\times10^{-7}\cdot((x-4000)^2+y^2))^2}+\frac{-1.9\times10^{-6}(y-2000)}{(2.5+9.48\times10^{-7}\cdot((x-2000)^2+(y-2000)^2))^2}}=\frac{x}{y}$$

$$x^2+y^2=2500^2$$

Now we have two equations and two unknowns, we can use the two-variable Newton's Method to find the value of x and y (The code is attached in Appendix). The x and y we found is $(-2236.8,-1116.6)$. We perform the similar operations for the other two circles as well and found $(6236.8,-1116.8)$ $(2000,\ 4500)$ and $(2000,-500)$. After checking with the three circles, only the last point is inside the constrained area. In this case,

$f(x, y) = 0.425$.

Lastly we calculate the intersection of the three circles. The calculations are trivial and will not be included. As a result, the Intersections are $(2000, 1500)$, $(2000, -1500)$, $(-457.8, 2457.7)$, $(2457.7, -457.8)$, $(1542.3, -457.8)$, $(4457.7, 2457.7)$. The three points $(2000, 1500)$, $(2457.7, -457.8)$ and $(1542.3, -457.8)$ are in the constrained area and the respective $f(x, y)$ for each of them are 0.602, 0.439 and 0.439.

From the four values 0.425, 0.602, 0.439 and 0.439 we take the maximum which is 0.602, and therefore the optimal solution is $(2000, 1500)$.

7.6 The decision problem

The solution we found above is only the solution for $g = (1, 1, 1)$. Unfortunately, the best solution for $g = \{1, 1, 1\}$ case is not necessarily the global optimal solution. As a simple example, consider the case where the constraint circles do not share a common area; in this case $g = \{1, 1, 1\}$ does not even have a solution.

The naïve method to find the optimal solution is very simple: go through all possible assignments of g, find the optimal solution for each assignment using the algorithm illustrated in section 7.4 and 7.5, and take the best solution out of them. However, since there are 2^n possible assignments the computation times grow exponentially. Even with 20 base stations, the number of iterations grow above a million.

This problem is very similar to the Binary Integer Programming problem, which in general is NP-Hard. With the objective function and the constraints both being non-linear, and an additional layer of variables, the problem is likely to be even harder than a normal BIP problem. Nevertheless, we can still try to find, a good solution even if not the best. The good news is that the nature of the constraint gives us a good starting point.

7.7. Approximation Methods

We shall notice that, a large D with a dense set of base stations usually means the optimal solution with distance constraint is not much different from the optimal solution without the constraint, while a small D usually means there are a lot of disjoint circles which we can take advantage of. In this section, we will discuss two techniques that suit each case respectfully.

Firstly, for large D with dense base stations case, we can utilize the local search algorithm to give us a very good solution, if not the best.

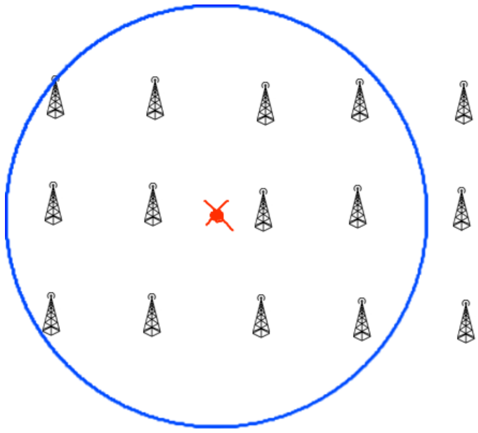


Figure 7.3 large D with dense base stations

We define $G = \{V, E\}$ to be the search graph where each node represents a state, or an assignment of g, $V = \{(g_1, g_2, \ldots, g_n) \mid g_i \in \{0, 1\}\}$ and an edge exists between two nodes v_1, v_2 if v_1 and v_2 differs by exactly one entry. Since we want to maximize the objective function $f(x, y)$, we let the cost function (to be minimized) at node v be $-f_{max}(v)$ where $f_{max}(v)$ is the value of $f(x, y)$ if $FindOptimalPoint(v)$ returns a valid point and -9999 otherwise. To choose our initial state, we perform the following procedure.

Algorithm - ComputeInitialState

(1) Treat the problem as if there is no constraint, use the partial derivative method to find the global maximum at location l.

(2) For $i = 1, 2, \ldots, \text{n}$, let $g_i = 1$ if the distance between BS_i and l is

less than D and $g_i = 0$ otherwise.

(3) The resulting assignment g is our initial state / starting node.

Like the example shown in Figure 7.3, in a large D with dense base stations case, the initial assignment calculated above (the red dot) should be very close to the global optimal solution and it should take relatively small number of iterations to reach the optimal solution or at least a very good solution.

Next, we look at the case where D is small. This means there will be a lot of disjoint circles, like shown in Figure 7.4.

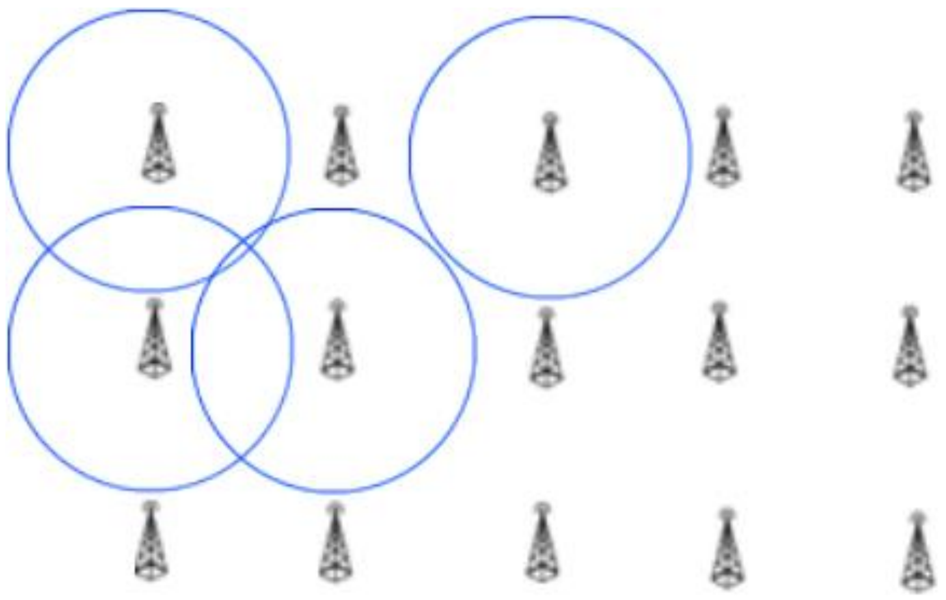


Figure 7.4 Small D with lots of disjoint circles

First, let us take a look at the following lemma.

Lemma

If the constrained areas (i.e, the circles) of two base stations BS_i and BS_j are disjoint, then for any assignment of g where g_i and g_j are both 1, $FindOptimalPoint$ will return no solution.

Proof

Since g_i and g_j are both 1, $FindOptimalPoint$ will check whether any found critical points or intersections to be in both circles for BS_i and BS_j, which cannot be possible since the circles do not intersect.

Using the result of this Lemma, we can employ the following algorithm to find the global maximal solution with a relatively good time complexity.

Algorithm – FindGlobalMaximumPoint

Input：$\boldsymbol{g_{in}}$ an assignment of $\boldsymbol{g}$

$\boldsymbol{d_{in}}$ a flag array for each entry of $\boldsymbol{g}$ meaning "determined"

Output：optimal point or nothing

(1) Base case: if there exists no i where $g_{in} = 1$ and d_{in}= false, then return $FindOptimalPoint(g_{in})$.

(2) Choose a base station BS_k such that $g_{in_k} = 1$ and g_{in_k} is not

marked "determined" and let K be the set of base stations whose constraint circle intersects with BS_k's circle. Let d_{rec} be a copy of d_{in}.

(3) If BS_k shares constraint area with all other base stations, apply $FindOptimalPoint(g_{in})$, let the result be r_1, i.e, (x_1, y_1) or nothing. Also mark d_{rec_k} = true.

(4) Let g_K be the assignment of g such that $g_{K_i} = 1$ if and only if $g_{in_i} = 1$ and $(i = k$ or $i \in K)$. Recursively compute

$FindGlobalMaximumPoint\ (g_K,\ d_{rec})$ and let the result be r_2.

(5) Let g_H be the assignment of g such that $g_{H_k} = 0$ and $g_{H_i} = g_{in_i}$ for $i \neq k$. Recursively compute

$FindGlobalMaximumPoint\ (g_H,\ d_{rec})$ and let the result be r_3.

(6) Return the best result from r_1, r_2, r_3. (Take the result that maximize f)

At first glance, the algorithm might not be easy to understand. The idea is that for any base station, if we choose to include it, then we can exclude all base stations that do not share a common constraint area with it.

Otherwise we exclude the base station. Figure 7.5 shows a graphical representation of the algorithm. As shown in the recursive tree, we always all the algorithm with an initial input of all ones, i.e. $g = (1,1,1,\ldots,1)$, which means all base stations are initially included. During each iteration, if we cannot find any base station that does not share common area with another base station, then we call $FindOptimalPoint$ to find the actual optimal solution.

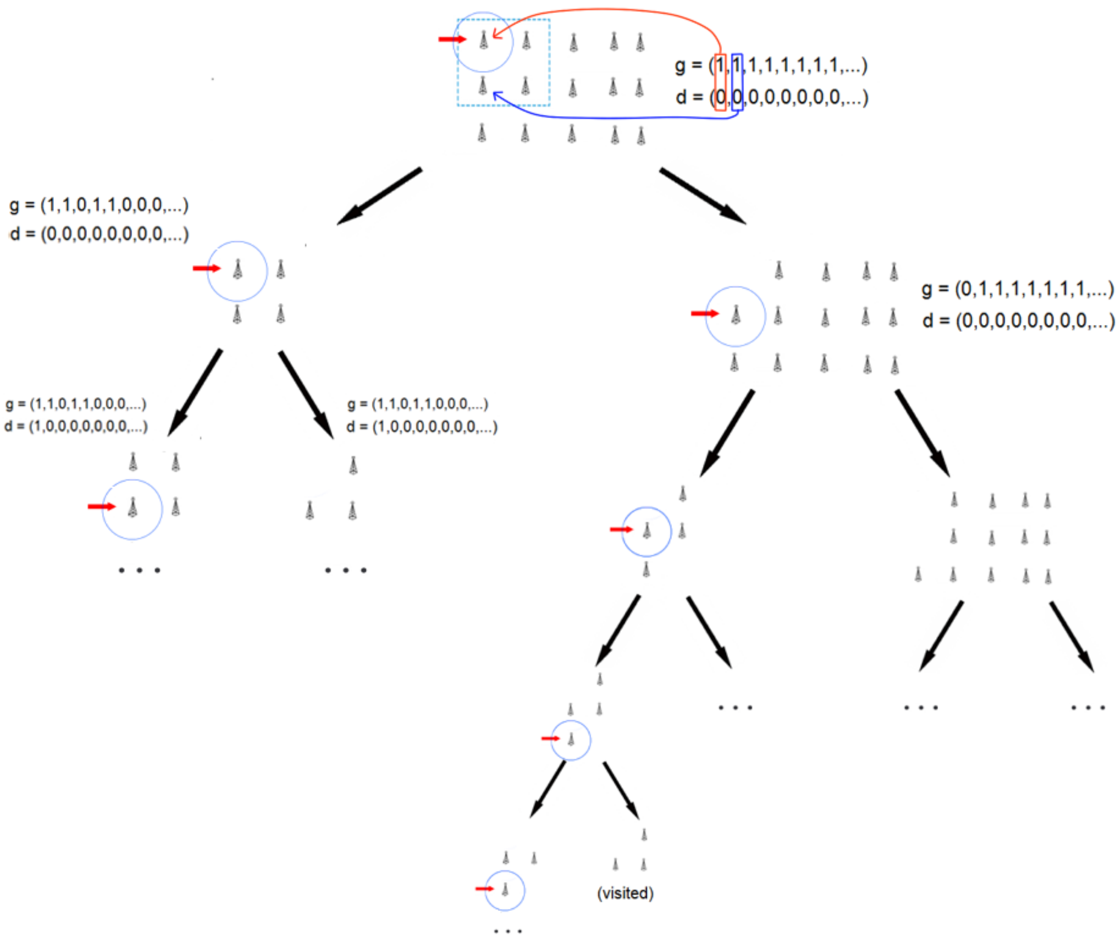


Figure 7.5 Recursion Tree of $FindGlobalMaximumPoint$

For example, in the first iteration, we take the top left base station, and break in to two cases: (left) we include it or (right) we do not include it. If we include it, then we only check base stations that share a common constraint area with it (the blue box); otherwise we check all other base stations. Now if we go down another level from the left side, this time since all other base stations' constraint circles intersect with the top left one's circle, we can safely mark the top left as "determined" and will never choose it again in this subtree. In each iteration, we can only choose a base station where the g entry is 1 and d entry is 0; if such base station does not exist, then we return the best location for the current g assignment.

If each base station shares common constraint area with at most k other base stations, then the left child of the recursive tree is $O(2^k)$ and hence the running time of the algorithm is

$$T(n) = O(n) + O(2^k) + T(n-1) \Rightarrow T(n) \in O(n^2 + n \cdot 2^k)$$

Note the $O(n)$ term accounts for the process to check whether each base station shares common constraint area with the chosen base station or not, as well as searching for a non-determined and still included base station. This polynomial time complexity is much better than the $O(2^n)$ complexity of the naïve method. It is also better than the brute force method that checks every g assignment with at most k ones, which has time complexity $O(n^{k+1})$ provided $k+1 << \frac{n}{2}$.

8. Conclusion

In order to provide network connection during the earthquake period, base stations need to operate in energy harvesting mode. Despite the lack of energy caused by the damage of the earthquake, the aim to maximize user connection time has never changed.

The first half of this paper analyzes various variables and details of signal transmission and energy harvesting and establishes a well-defined mathematical model of the problem that fits the situation. The second half of this paper focus on solving the problem in a rigorous manner using various Calculus techniques. It also discusses the constrained case which is important to model the problem accurately despite its hardness to solve. Although a perfect solution has not been found for such case, two input-dependent methods are proposed to solve the problem in different scenarios.

9. Recommendations

Readers should have a clear understanding of how an energy harvesting network operates, what the problem is and knows how to solve the unconstrained version step by step after reading this report.

Future studies include investigating energy-harvesting methods other than the RF based model, developing better algorithms for the constrained case, formally proving its NP-Hardness and possibly include other constraints such as movement of users.

Appendix

Source code to calculate the solutions in section 6.3 and section 7.5 can be found at the following GitHub page.

https://github.com/rc41392/TwoVariableNewtonMethod/blob/master/findroot.c

Usage: findroot x0 y0, where x0 and y0 are the starting point for the Newton's Method. Data such as coordinate and function definitions are written in the code and needs to be manually changed. Lagrange Multiplier functions are also defined but should never be changed as it calculates via rules of differentiation using the original functions.

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