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MIXING IN STARS

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ABSTRACT

Three cases for mixing not present in standard stellar models are presented: Light element depletion in low mass main-sequence stars, deep mixing in massive stars, and deep mixing in low mass giants. The review begins with the mixing indicators and the predictions of standard models. The observational evidence for anomalous mixing is then presented, followed by the physics of mixing outside the standard model. The status of theoretical models that include extra mixing is then examined.

1. INTRODUCTION

Stellar evolution is driven by changes in chemical composition caused by nuclear reactions. The extent to which the interiors of stars are mixed strongly influences their evolution and such properties as age of clusters, chemical evolution, and the relationship between the initial and surface abundances of stars. In standard stellar models, convection is the sole mixing agent, and these models make strong predictions about the surface abundances of stars as a function of mass, composition, and age.

To first order, the agreement between observation and standard stellar models is encouraging. For some time, however, there have been indications that the standard stellar model is incomplete. The long-standing problem of explaining the surface lithium abundances in low mass stars is one good example, and the evidence for C $\rightarrow$ N processing in the envelopes of red giants is another. In both cases, the data appear to require not just more mixing than expected in standard models, but mixing in mass ranges and phases of evolution where it is not expected. In the massive star regime, there is evidence for extensive mixing that could dramatically influence the evolution of at least the most rapidly rotating stars. Processes not included in standard models, such as mixing

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driven by rotation, are required to understand the observed depletion pattern. By extension, some important properties inferred from studies of stars could potentially be affected, including globular cluster ages, the inferred primordial abundances of light elements, and chemical evolution. The purpose of this review is twofold: first, to assess the observational evidence for mixing not present in standard stellar models and, second, to examine the current state of the theoretical explanations for the data and the potential importance of effects not included in standard stellar models.

2. DIAGNOSTICS OF MIXING

The surface abundances of stars provide indicators of mixing to a wide range of temperatures. Figure 1 shows the normalized abundances of some of the

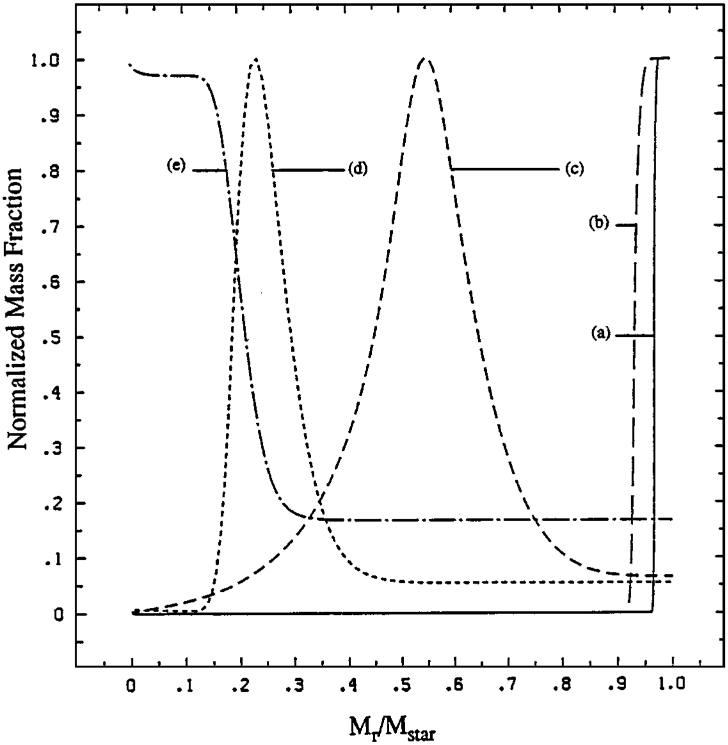

Figure 1 Normalized abundances of ^{7}Li (a), Be (b), ^{3}He (c), 13C (d), and 14N (e) as a function of mass fraction within the Sun (from Pinsonneault 1988).

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species of interest in the Sun. Some species, such as ^{6}Li and ^{7}Li, are destroyed at relatively low temperatures; others, like the CNO elements, are transformed at much higher temperatures.

2.1 *Outer Layers: Li, Be, B*

LITHIUM-7 ^{7}Li is produced in the Big Bang, and the primordial ^{7}Li abundance is of considerable interest for testing Big Bang nucleosynthesis (BBN) and $\Omega_{\rm baryon}$ (Boesgaard & Steigman 1985, Alcock et al 1987, Malaney & Fowler 1988, Audouze & Silk 1989, Krauss & Romanelli 1990, Mathews et al 1990, Reeves et al 1990, Walker et al 1991, Wilson & Matteucci 1992, Copi et al 1995, Dearborn et al 1996). It is fragile in stellar interiors, surviving only to $\sim 2.5 \times 10^6$ K at typical envelope main-sequence (MS) densities. For stellar structure purposes, the extent of lithium depletion is the quantity of theoretical interest; measuring the amount of lithium depletion requires knowledge of both the initial and current abundance.

In metal-poor stars, the discovery of measurable ^{7}Li (Spite & Spite 1982) at a level 10 times lower than the solar meteoritic value of 3.31 ± 0.04 (Anders & Grevesse 1989) (on the logarithmic scale where $H = 12$) raised the exciting possibility of a direct measurement of the primordial ^{7}Li abundance. It remains controversial whether the observed abundances in metal-poor stars reflects their initial abundance or if they have been nearly uniformly depleted from a higher initial value. Little depletion is expected from standard stellar models (Deliyannis et al 1990), whereas models that include extra mixing can predict depletion factors as large as a factor of 10 (Pinsonneault et al 1992). The uncertainty in the initial abundance of halo stars makes it difficult to distinguish unambiguously between different stellar structure models. Disk stars provide a much cleaner test of theoretical models because the lithium depletion pattern can be mapped out as a function of age.

Abundances in the interstellar medium (ISM) are consistent with the solar meteoritic value (Lemoine et al 1993). Although higher abundances were initially reported in T Tauri stars, non–local thermodynamic equilibrium (non–LTE) effects and complications due to the presence of accretion disks appear to be responsible for anomalously high abundances (Magazzu et al 1992). The open cluster system also shows evidence for a nearly uniform initial abundance, on the order of 3.2–3.3 (Soderblom 1995).

LITHIUM-6, BERYLLIUM, AND BORON ^{6}Li, Be, and B are produced primarily through cosmic-ray nucleosynthesis, with minimal primordial abundances expected from standard BBN (Reeves et al 1970, Vangioni-Flam et al 1990, Walker et al 1993), although a higher initial abundance may be possible for inhomogeneous BBN (Kajino & Boyd 1990). They differ dramatically in their sensitivity to destruction in stellar interiors: ^{6}Li is destroyed at even lower temperatures than ^{7}Li, whereas Be and B survive to much higher temperatures ($\sim$3.5 and 5 million K respectively). ^{6}Li may have been detected in some hot halo stars, and if it is present, it can be used to place constraints on the degree of mixing in these stars (Section 4.1, Halo Stars).

There has been a fair amount of work on Be and B in metal-poor stars, with the goal of testing chemical evolution models (Duncan et al 1992, Ryan et al 1992, Boesgaard & King 1993, Thorburn & Hobbs 1996). The spectral features used to measure beryllium and boron abundances are in ultraviolet spectral regions that are difficult to model, and much of the recent observational work has focused on identifying the spectral features in the vicinity of the beryllium and boron features.

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2.2 *Deep Mixing: ^{3}He, CNO, and ^{4}He*

HELIUM-3 AND HELIUM-4 ^{3}He and ^{4}He are important for BBN but difficult to measure directly in stars. The observational situation for measurements of ^{3}He in HII regions has been reviewed by Wilson & Rood (1994); see also Balser et al (1994). ^{4}He measurements are possible in hot stars.

CNO The solar surface CNO isotope ratios and the relative surface C, N, and O abundances differ dramatically from the nuclear equilibrium values. ISM measurements vary somewhat (e.g. Wilson & Mateucci 1992, Wilson & Rood 1994) but are also uniformly different from nuclear equilibrium. The mixing of CNO-processed material into the surface convection zone can therefore be measured by examining these ratios. C$\rightarrow$N processing is completed at lower temperatures than O$\rightarrow$N processing, but observational evidence exists for changes in the surface oxygen abundances of stars (Kraft 1994). CNO burning tends to occur at temperatures characteristic of the nuclear burning core rather than of the envelope. Observational evidence for surface CNO anomalies is seen in giants and massive stars; this indicates deep mixing.

3. PROPERTIES OF STANDARD MODELS

3.1 *Pre–Main-Sequence Light Element Depletion*

In a pioneering work, Iben (1965) explored the evolution of pre–main-sequence (pre-MS) stars. Low mass pre-MS stars are cool and luminous and have deep surface convection zones. They evolve to the MS over a Kelvin-Helmholtz time scale. During the course of the pre-MS, their surface convection zone can be hot and dense enough at the base to burn light elements such as Li and Be, even in stars that on the MS have relatively thin surface convection zones (Bodenheimer 1965, 1966). See Strom (1994) for a review of pre-MS

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lithium burning in standard models. More recent studies of pre-MS stars have emphasized the role of deuterium burning in the creation of a stellar birthline (Stahler 1988, 1994), and intermediate mass stars may emerge on the birthline close to the MS (Palla & Stahler 1993). This does not qualitatively change the light element depletion pattern because pre-MS light element depletion is only significant for low mass stars (below $\sim$1.2 $M_\odot$ at solar composition).

Recent calculations of pre-MS lithium depletion for standard stellar models have been presented by Proffitt & Michaud (1989), Pinsonneault et al (1990), D'Antona & Mazzitelli (1993), and Swenson et al (1994). The light element depletion pattern is a sensitive function of mass and composition, but there are some broad patterns that have been found by all investigators. A sample set of lithium depletion isochrones is shown in Figure 2*a* at 100 Myr for a range in [Fe/H] and in Figure 2*b* for different ages at two values of [Fe/H], 0.0 and +0.15 (relevant for the Hyades cluster). The major features of the theoretical models are as follows:

- The degree of light element depletion increases with decreased mass. Lower mass stars retain deep convective envelopes for a longer time and contract towards the MS more slowly. For the lowest mass stars, this trend reverses (Nelson et al 1993); the survival of lithium in old objects has been used both as a test for brown dwarfs (Rebolo et al 1996) and as an age indicator for open clusters (Basri et al 1996).

- Pre-MS lithium depletion decreases with lower metal abundance at a given mass. Even though the relevant masses for the onset of significant light element depletion are significantly lower for halo metal abundances, the drop in lithium abundance occurs at a comparable MS effective temperature (see Deliyannis et al 1990 for a detailed discussion of lithium depletion in low metal abundance standard models). As can be seen from Figure 2, even for near-solar metal abundance there is a strong sensitivity of pre-MS lithium depletion to metal abundance, a point emphasized by Swenson et al (1994) (see also Chaboyer et al 1995b).

- The magnitude of pre-MS depletion is a sensitive function of the input physics; this can be seen by comparing lithium depletion results from different investigators and by parameter variation studies (Swenson et al 1994).

- Both pre-MS and MS beryllium depletion are confined to the lowest mass stars (below 0.5 $M_\odot$ at solar composition).

We can therefore anticipate that there will be a mass- and composition-dependent pre-MS depletion of lithium apparent in open clusters; beryllium is not expected to be depleted except in the lowest mass stars.

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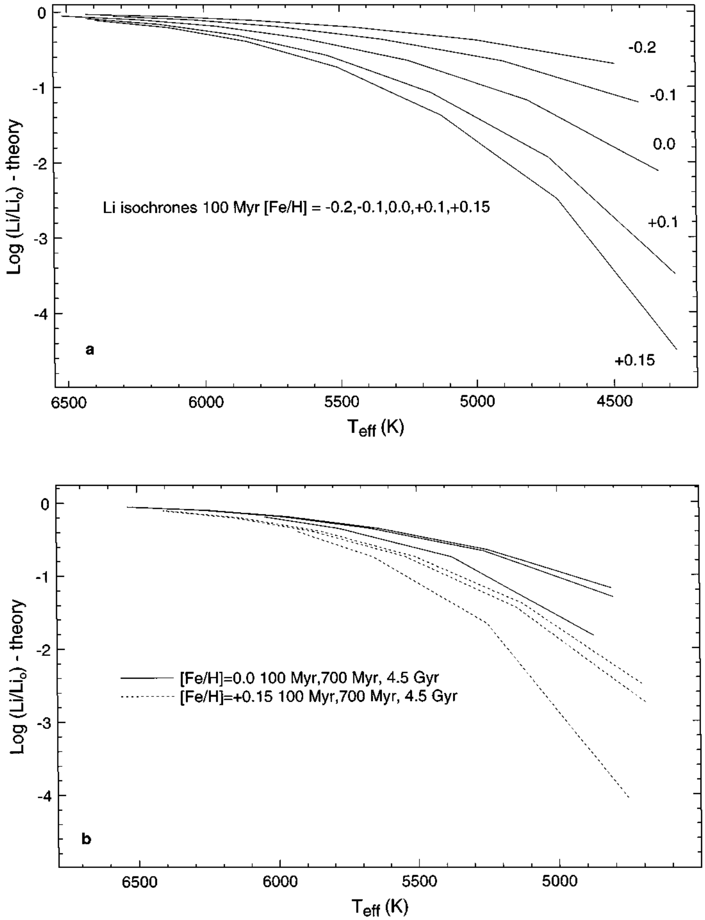

Figure 2 Pre-MS (*top*) and MS (*bottom*) ^{7}Li depletion isochrones from standard models. Theoretical lithium depletion factors as a function of T_{eff} for [Fe/H] $= -0.2, -0.1, 0, +0.1$, and $+0.15$ at an age of 100 Myr are illustrated in the top panel. Lithium depletion for [Fe/H] $= 0$ (*solid lines*) and [Fe/H] $= +0.15$ (*dashed lines*) are shown for ages of 100 Myr, 700 Myr, and 4.5 Gyr in the *bottom panel*.

3.2 *Main Sequence Changes in Surface Abundance*

MAIN SEQUENCE: LIGHT ELEMENT SURVIVAL On the MS, the depth of the surface convection zone, and therefore the temperature at the base of the surface convection zone, is a strong function of mass (see Kippenhahn & Weigart 1990 for a discussion of the basis of this phenomenon in stellar structure). As a result, only the lowest mass stars (below 0.9 $M_\odot$ at solar composition) are expected

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to have surface convection zones deep and dense enough to burn significant amounts of lithium on the MS, as can be seen in Figure 2*b*. The top line corresponds to a typical age (100 Myr) for young systems; the second line (at 700 Myr) indicates the expected depletion pattern at intermediate ages where there is extensive open-cluster lithium data (Hyades, Praesepe, Ursa Major, Coma). The bottom line indicates the extent of depletion at 4.5 Gyr, which is comparable in age to the Sun and the open cluster M 67.

Little lithium depeletion is predicted for solar-mass models and above on the MS. The location of the base of the solar surface convection zone inferred from helioseismology is $0.713 \pm 0.003\, R_\odot$ (Christensen-Dalsgaard et al 1991), not hot enough to burn Li on the MS. This confirms the absence of MS lithium depletion in solar composition stars comparable to, and by extension more massive than, the Sun (Christensen-Dalsgaard et al 1992). Because of the steep dependence of the temperature at the base of the convection zone on mass, this result is insensitive to the assumed input physics. This is in direct contradiction to the observational data for open cluster stars and the Sun (Section 4.1); the interesting situation for halo stars is discussed in Section 4.1 (Halo Stars).

MASSIVE STARS There is an extensive literature on the evolution of massive stars; see Maeder & Conti (1994) for a review. Mass loss dominates stellar evolution on the upper MS, which produces objects in late phases of evolution with numerous and well-documented surface abundance anomalies (e.g. Wolf-Rayet stars). However, MS models, even with mass loss, are not expected to show surface processing of elements for $M < 50\, M_\odot$ (Maeder & Conti 1994). Evidence for mixing in massive MS stars not predicted by standard models is presented in Section 4.2.

3.3 *Post-Main Sequence: First Dredge-Up*

Post-Main Sequence (post-MS) stars develop deep surface convection zones as they travel from the MS to the first ascent giant branch (Iben 1967, 1974, Sweigart & Gross 1978, Kippenhahn & Weigart 1990). Once on the giant branch, the surface convection zone of low mass stars reaches a maximum depth and then gradually retreats in mass as material from the envelope falls onto the hydrogen-burning shell, is converted into helium, and is deposited in a degenerate core. A region with ~0.3 solar masses is not incorporated into the surface convection zone at solar composition, almost independent of the total stellar mass. In standard models, there will therefore be a first dredge-up between the MS and the giant branch where the surface abundances change to reflect the average abundances of the entire MS star outside the nuclear burning core. A second dredge-up can occur after helium core burning on the asymptotic giant branch (AGB) (Iben & Renzini 1983, Lattanzio et al 1991).

Recent calculations of the surface abundances predicted from the first dredge-up in standard models can be found in Dearborn (1992), Schaller et al (1992), Bressan et al (1993), El Eid (1994), and Wasserburg et al (1995). The mass and composition dependence of the first dredge-up constrains deep internal mixing on the MS, which could significantly alter the internal composition profile of species such as ^{13}C without altering their surface abundances (Section 4.3).

An important prediction of standard models is that changes in abundance on the upper giant branch are not expected for basic stellar structure reasons. The clear presence of such changes in low mass giants is an indication for deep mixing of their surface layers and is the third major piece of evidence for mixing outside the framework of the standard stellar model (Section 4.3). Oxygen isotope anomalies may also be an indicator of slow mixing in AGB stars.

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3.4 *Mixing in Standard and Nonstandard Models*

In standard stellar models, convection is the sole mixing mechanism, and the time scale for convection is far less than the nuclear burning time scale. The strong temperature dependence of nuclear burning therefore implies that essentially complete burning of fragile species will precede significant processing of species that burn at higher temperatures. For example, complete destruction of ^{6}Li will occur before significant ^{7}Li destruction, which in turn would be completely destroyed before significant Be destruction. Similar considerations apply to species that burn at higher temperatures, such as B, ^{3}He, and the CNO species. In the Sun, the surface Li^{7} is depleted by a factor of $\sim$100, and Be is depleted by a factor of $\sim$2 with respect to meteorites. Such a depletion pattern is not consistent with short time scale mixing to an arbitrary depth.

In general, models with mixing over a time scale comparable to or greater than the nuclear burning time scale show a very different pattern. In the presence of slow mixing, a species such as lithium will be destroyed before it ever reaches very high temperatures, even if the mixing persists to depths sufficient to destroy more resilient species such as beryllium. As a result, the degree of lithium or beryllium depletion will depend, respectively, on the distance Li-poor or Be-poor material must travel to reach the surface and the time scale for mixing across this region. Simultaneous mixing of lithium and beryllium is therefore expected in solar models with mild envelope mixing on general grounds (e.g. Pinsonneault et al 1989).

The assumption of instantaneous mixing to an arbitrary depth, which is sometimes made when investigating the effects of mixing in stars (Vandenberg & Smith 1988, Sweigart 1997), is therefore severely limited in its physical application when applied to species that burn at different temperatures. Such models can indicate how deep mixing may be (the stated purpose of Vandenberg & Smith 1988, for example), but there is no a priori reason to expect the detailed properties to agree with full calculations. Even models that permit both the time scale and depth to vary (Wasserburg et al 1995) do not necessarily reflect the underlying physical picture; the time scale for rotational mixing, for example, depends strongly on the distance from the surface convection zone and the internal stellar rotation.

4. THE OBSERVATIONAL PATTERN

4.1 *Mixing on the Main Sequence: Low Mass Stars*

The challenge of explaining the light element abundances of low mass stars is an old one; Greenstein & Richardson (1951) first proposed extra mixing as the solution of the low solar photospheric lithium abundances relative to that on the Earth and in meteorites. A good review of early work on lithium can be found in Zappala (1972). The lithium problem in low mass stars was already recognized in much of its present form by the time of the Zappala article: The lithium abundances in the young Pleiades were higher than the Hyades, which in turn were higher than the Sun; this contradicted the expectations from standard models.

The advent of modern charge coupled device (CCD) detectors led to a large increase in both the accuracy and quantity of the observational data. There is an extensive literature on lithium observations in low mass stars (see the series of papers in *Memorie della Societa Astronomica Italiana*, 1995, Vol. 66). Observational reviews include Hobbs & Pilachowski (1988) for an overview of the open cluster data at the time; Strom (1994) for a review of pre-MS and young cluster lithium abundances; Balachandran (1994, 1995) for reviews of lithium on the MS in open clusters, with a special emphasis on the F star “lithium dip” in the latter; and Pallavicini (1994) for lithium in post-MS stars. Carlsson et al (1994) also summarizes the observational lithium literature. The most extensive data set for metal-poor stars is that of Thorburn (1994); see Molaro et al (1995) and Ryan et al (1996) for recent discussions of the observational situation in halo stars.

Beryllium and boron data are becoming available for some stars as well. Because these species burn at different depths than lithium, they provide powerful additional constraints on the allowed classes of theoretical models (Pinsonneault et al 1989, Boesgaard & King 1993, Deliyannis 1995, Primas 1996).

In this review, I concentrate on low mass MS stars in open clusters, which provide the best laboratory for testing the reliability of the light element depletion pattern inferred from stellar models. The observed depletion pattern is a complex function of mass, composition, and age; in addition, there is strong observational evidence for variations in abundance among cluster stars of the same mass, composition, and age. Clusters provide the cleanest means of quantifying these dependencies. In particular, there are numerous possible solutions

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for the abundances of any one cluster or the Sun, but the requirement that the overall pattern seen in low mass stars be reproduced is a far more challenging theoretical task. An approximately constant initial lithium abundance is also appropriate for the open cluster system.

By contrast, the initial abundance of metal-poor stars is the quantity of interest, with considerable cosmological impact. The validity of different models for relating the initial and current abundances in metal-poor stars must therefore be established in the open cluster system, where such models can be distinguished, rather than relying on subtle distinctions between different classes of models in their predictions for the halo stars themselves. Note that chemical evolution considerations (Steigman 1996), as well as observations of the survival of the isotope Li^6 in some metal-poor stars (Smith et al 1993, Hobbs & Thorburn 1994), can place interesting additional constraints on any possible depletion in metal-poor stars relative to their initial value.

The overall properties of the open cluster system can be summarized as follows. The mean trend in young open clusters is consistent with that expected from pre-MS depletion in standard models. There is a large and unexpected dispersion in lithium abundance for young cool stars, possibly correlated with rotation. For older clusters, there is anomalous MS depletion for late F, G, and K stars. The rate of excess depletion decreases with increased age in these stars. A range in lithium abundance at fixed T_{eff} is seen in all of the older open clusters. This implies that mass, composition, and age do not uniquely determine the surface lithium abundance of a star. The majority of the halo star data shows relatively little scatter.

A population of highly Be-depleted stars exists; most of these stars lie in the mid-F star region. The Sun and α Cen A and B also appear to be depleted with respect to the solar meteoritic value. ^{6}Li detections have been claimed in some metal-poor stars. I quantify the observational picture in systems of progressively greater age below.

YOUNG CLUSTERS Young clusters provide a powerful test of pre-MS lithium depletion from standard models. The observational data also show some interesting phenomena not expected from standard models. Lithium data has been obtained for the young (30-Myr) cluster IC 2391 (Stauffer et al 1989) and the 50-Myr cluster α Per (Balachandran et al 1988, 1996, Zapatero et al 1996). The best-studied young cluster by far is the 70- to 110-Myr Pleiades. An extensive set of observations was performed by Soderblom et al (1993b) (this paper contains an extensive discussion of the history of the study of lithium in the cluster). Further observations were obtained by Jones et al (1996b). Russell (1996) studied the question of the dispersion in abundance for cool Pleiades stars (see also Stuik et al 1996).

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The lithium abundances in the Pleiades and α Per are compared with standard models in the top two panels of Figure 3. The dashed line represents the effects of even a small amount of overshoot (0.05 pressure scale heights), which can have a dramatic effect on pre-MS depletion (e.g. Swenson et al 1994). The most important features are as follows:

- The mean depletion trend in both clusters is in good agreement with that predicted by standard models (Strom 1994, Jones et al 1996b). The revised α Per abundances (Balachandran et al 1996) are much more similar to the Pleiades than the pattern inferred from earlier work (Balachandran et al 1988). The consistency between the data and models strongly constrains increased pre-MS depletion as a solution for the abundance patterns seen in older systems (see for instance Pinsonneault et al 1990, Michaud & Charbonneau 1991, Chaboyer et al 1995b).
- A large dispersion is seen in lithium abundances for stars cooler than 5300 K. Most of the overabundant stars are rapid rotators, but not all, and most are chromospherically active (Soderblom et al 1993b). This phenomenon has proven challenging to explain theoretically (Section 6.1). Soderblom et al (1993b) examined a variety of means of explaining this range through observational error and concluded that the observed spread in abundance was real. This conclusion was recently challenged by Russell (1996), who examined the abundances for a subset of the Soderblom et al (1993b) sample derived from a weaker spectral feature (at 6104 Å) rather than the typical 6708-Å resonance line (see also Stuik et al 1996). It appears unlikely that the large observed range can be entirely removed, but the magnitude of the effect is controversial.
- A small depression exists in the lithium abundance pattern for mid-F stars. Although the effect appears small in comparison with the steep mass dependence for cool stars, it is highly statistically significant (as can be seen by noting the small abundance errors for these stars in Figure 3*a*). This is the earliest detection of the dramatic F star lithium dip first discovered by Boesgaard & Tripicco (1986).

INTERMEDIATE-AGE CLUSTERS There are five systems with ages ranging from 200–700 Myr with large lithium samples: M34 (200 Myr), Ursa Major (300 Myr), Coma (500 Myr), the Hyades, and Praesepe (both 500–700 Myr).

The Hyades has been the focus of much theoretical and observational work, owing to both its closeness and the wealth of other data available for the system (see Thorburn et al 1993 for a summary of earlier work). Thorburn et al (1993) performed a large survey in the Hyades designed to carefully quantify the

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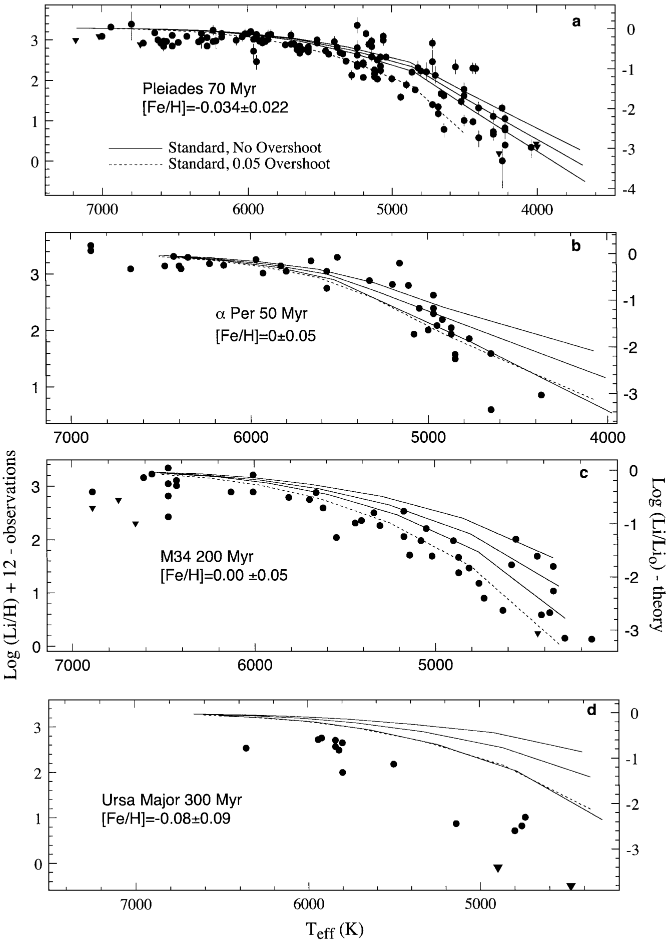

Figure 3 Standard models compared with the Pleiades (*top panel*, data from Soderblom et al 1993b); α Per (*second panel from top*, data from Balachandran et al 1996); M34 (*third panel from top*, data from Jones et al 1996a,b), and Ursa Major (*bottom panel*, data from Soderblom et al 1993a). The *central solid line* indicates standard model depletion for the mean cluster [Fe/H] and the *solid lines above and below* represent 1-σ errors in the [Fe/H] from Boesgaard & Friel (1990). The *dashed line* represents standard models with the addition of 0.05 pressure scale heights of overshoot.

dispersion in lithium abundance. Soderblom et al (1995) looked at lithium in K stars in the cluster, and Barrado y Navascues & Stauffer (1996) examined Hyades binary abundances.

Lithium in the younger (300 Myr) Ursa Major system was studied by Boesgaard et al (1988). Later work by Soderblom et al (1993c) found that most of the F stars in the Boesgaard et al sample were nonmembers, but it extended the sample down to much lower T_{eff}. Praesepe was examined by Boesgaard & Budge (1988) and Soderblom et al (1993a) (see also Balachandran 1995). Recent observations in M34 by Jones et al (1996a) provide valuable new insights into the process of lithium depletion in low mass stars. At 180–250 Myr, this cluster is young enough to retain a range of rotation rates at a given mass but old enough to have experienced MS depletion.

The data for M34 and Ursa Major are compared with standard models in the bottom two panels of Figure 3; Coma, the Hyades, and Praesepe are compared with standard models in the top two panels of Figure 4. Important features include the following:

- A dramatic lithium dip is seen in the mid-F stars. A hint of this phenomenon can be seen in the early photographic work of Wallerstein et al (1965). Jones et al (1996a) point out that this dip is already deep by the age of M34. It extends more than two orders of magnitude down in the Hyades, and is seen in all of the intermediate age systems that have data in the 6400–6800 K region. The existence of such a feature is an unambiguous contradiction of standard stellar models, as the convection zones in this mass range are far too thin to burn lithium.

- The mean abundance trend is significantly below that predicted by standard models in all of the systems, by an amount that increases with increased age. This is, in fact, the same conclusion reached by Zappala (1972) with more limited data. Even the models with overshoot sufficient to fit the lower bound of the Pleiades distribution fall above the mean cluster trends in Figures 3 and 4. Increasing depletion for clusters in this age range would result in inconsistencies for young clusters while not removing the observed discrepancy in the mean trend for older systems.

- A dispersion in abundance exists at a fixed T_{eff} for stars near 6000 K, and anomalous abundances are found for at least some cooler stars, for Coma, Praesepe, the Hyades, and Ursa Major. M34 shows an intriguing pattern: a larger lithium dispersion for hotter stars than the Pleiades and a smaller dispersion for cooler stars than the Pleiades.

The claim is sometimes made in the literature that clusters such as the Hyades show no dispersion (for example, Swenson et al 1994, Russell 1996). Thorburn

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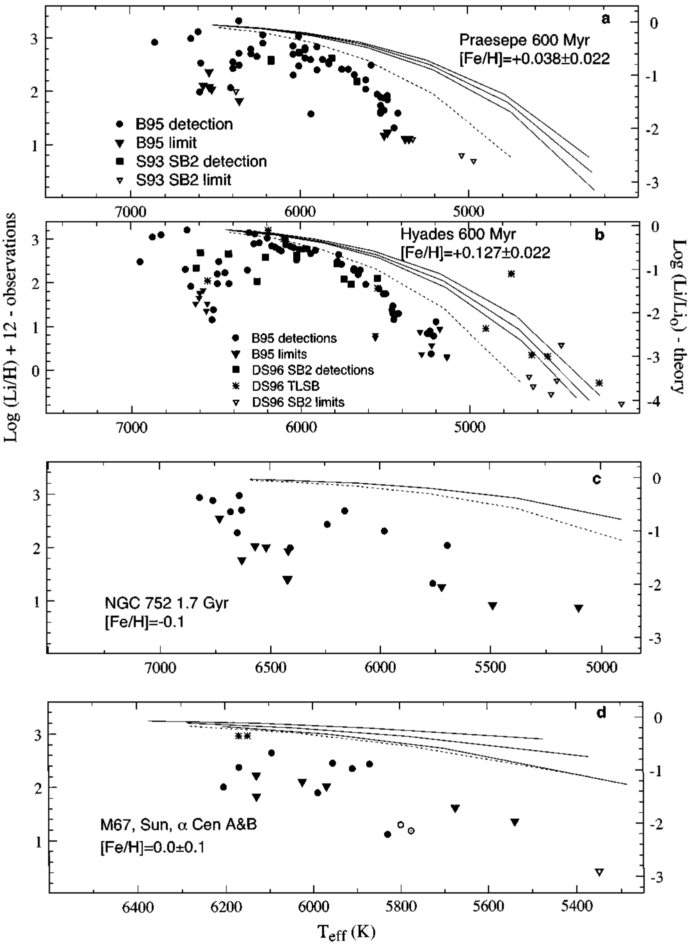

Figure 4 Standard models compared with Praesepe (*top panel*, data from Balachandran 1995), the Hyades (*second panel from top*, data from Balachandran 1995, Barrado y Navascues & Stauffer 1996), and NGC 752 (*third panel from top*, data from Balachandran 1995). In the *bottom panel*, data for M67 (Balchandran 1995, Deliyannis et al 1994) are the *solid points*, with tidally locked binaries in M67 represented by *asterisks*. Lithium abundances for the Sun (Anders & Grevesse 1989) and α Cen A and B (Soderblom & Dravins 1984) are also shown; the *solar symbol* represents the Sun, α Cen A is the *open circle*, and α Cen B is an upper limit (*open triangle*). Additional cluster [Fe/H] data taken from Friel & Boesgaard (1992). Tidally locked binaries in the Hyades are represented by *asterisks*.

et al (1993) showed that the range in abundance for late F stars was greatly in excess of their errors, with *average* deviations from the mean trend at the 5-σ level. A similar conclusion can be drawn from other clusters in this age range. In cooler stars, the strong dependence of lithium on mass makes detecting a modest dispersion difficult. Even so, there are clear examples of G and K stars that are normal in every other way except lithium (e.g. vB 9 in the Hyades) and of stars that differ in their lithium content but otherwise have similar spectra. It is true, and important, that there is a mean trend obeyed by most stars, i.e. that the range in depletion for most objects is less than the average depletion at a given T_{eff}.

- The cool star region where a large dispersion is seen in young systems is detected only in the younger M34 and Ursa Major systems. The Ursa Major sample shows scatter but with a small observational sample (Soderblom et al 1993c). The relative pattern in M34, namely cool faster spinners with higher abundances, is similar to that of the younger clusters, but the difference is less marked (Jones et al 1996a). Soderblom et al (1995) found that the mixture of detections and upper limits seen for the cool Hyades stars by Thorburn et al (1993) were all only upper limits when seen at higher resolution, and they concluded that there was no dispersion for cool Hyades stars. The absence of detections does imply a lack of distinctly underdepleted stars but does not permit a measurement of the intrinsic dispersion in their sample.

- Tidally locked binaries have higher abundances than single stars of the same T_{eff} (Soderblom et al 1990, Thorburn et al 1993, Barrado y Navascues & Stauffer 1996 for the Hyades). The abundances of tidally locked binaries are normal in the Pleiades (Soderblom et al 1993b), so differing pre-MS depletion is not the cause of this phenomenon. The different behavior of tidally locked systems implies that the rotational history of stars influences their surface lithium abundances (Deliyannis et al 1990, Zahn 1994, Ryan & Deliyannis 1995). Significantly, the tidally locked binaries in older systems have abundances consistent with those predicted by standard models, as can be seen in Figure 5*c*. Barrado y Navascues & Stauffer (1996) reanalyzed the binary data in the Hyades and obtained some new measurements as well; they found that all of the cool short period binaries had abundances well above the mean trend from detached binaries and single stars. Two tidally locked binaries in Praesepe, KW 181 (Soderblom et al 1993a) and KW 367 (King & Hiltgen 1996), appear to have normal abundances. In both cases, however, the authors note that there is independent evidence that the systems may not be tidally synchronized, which is required in theoretical models for lithium underdepletion.

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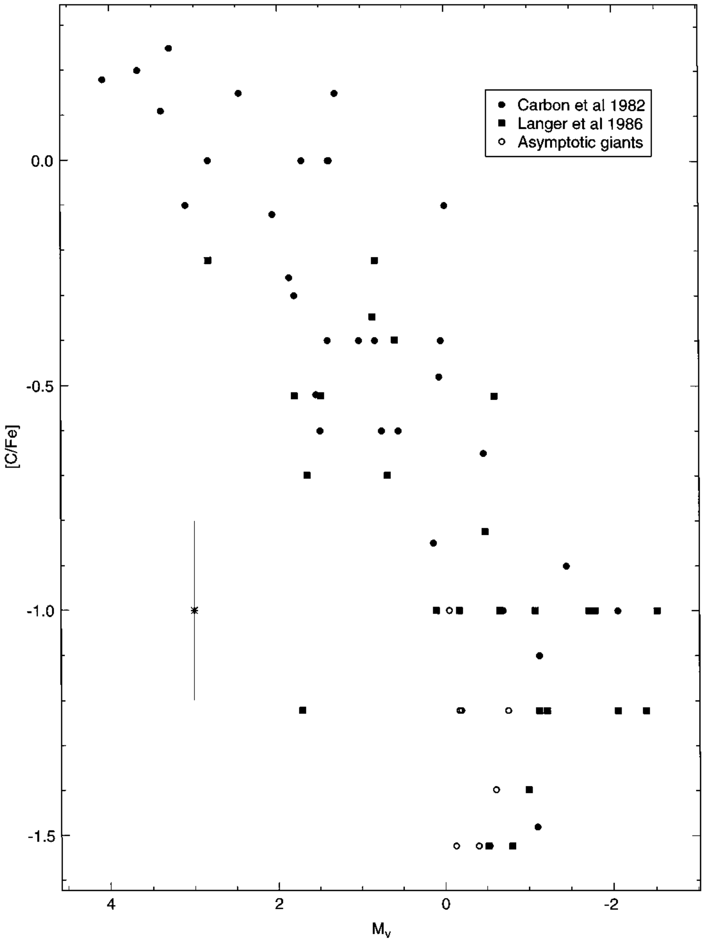

Figure 5 Carbon abundance [C/Fe] as a function of absolute visual magnitude (Mv) in the globular cluster M92. Data from Carbon et al 1982, Langer et al 1986. *Typical error bars* from Carbon et al 1982 are illustrated at the *lower left*.

- Abundances in hot stars are generally compatible with the meteoritic value; there is some evidence for dispersion (Burkhart & Couprey 1989). The absence of deep mixing for spectral types earlier than the dip, along with the existence of chemically peculiar stars, places interesting constraints on the theory.

Beryllium abundances have been obtained for Hyades lithium dip stars by Boesgaard & Budge (1989), showing depletion of factors of 2–4 relative to the

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solar meteoritic value (Anders & Grevesse 1989). Cool Hyades stars have also been measured by Garcia Lopez et al (1995). There appears to be decreased surface beryllium with decreased T_{eff}, but this trend could also be caused by systematic errors (Garcia Lopez 1996).

OLD OPEN CLUSTERS, THE SUN, AND α CEN Old disk stars are an essential component of the study of lithium. The well-known discrepancy between the solar photospheric and meteoritic abundances (Anders & Grevesse 1989 and references therein) is one data point. Data also exist for field stars (Balachandran 1990, Lambert et al 1991, Favata et al 1996). Field star data is more difficult to interpret than cluster data because of the mixture of ages and compositions. There does exist, however, a population of highly overdepleted field objects cooler than the mid-F star region (Lambert et al 1991) and a large dispersion in abundance among post-turnoff F stars (Balachandran 1990).

Data for the old open cluster NGC 752 (1.7 Gyr) was originally obtained by Hobbs & Pilachowski (1986a) and Pilachowski & Hobbs (1988); it was reanalyzed by Balachandran (1995). Hobbs & Pilachowski (1988) obtained data for NGC 188, the oldest open cluster to be studied (6 Gyr). There is a more extensive and varied data set for M67, which at 4 Gyr is comparable in age to the Sun (Hobbs & Pilachowski 1986a, Spite et al 1987, Garcia Lopez et al 1988); abundances for tidally locked binaries in M67 were investigated by Ryan & Deliyannis (1995). The Sun and α Cen A and B also provide well-studied examples of both lithium and beryllium abundances in old disk stars. The data for the clusters NGC 752, M67, the Sun, and α Cen A and B are compared with standard models in the bottom two panels of Figure 4. Important features include the following:

- Significant depletion exists relative to younger systems, but an inferred rate of depletion decreases with increased age. Despite their differences in age, the old open clusters have abundance patterns much more similar to one another than to the intermediate-aged systems (Hobbs & Pilachowski 1988, Soderblom 1995).

- A large dispersion in abundance exists. The old open clusters are fainter than the younger systems, so the errors are significantly higher than for nearby systems such as the Pleiades (Hobbs & Pilachowski 1988). However, the observed magnitude of the dispersion is in excess of the errors as confirmed by recent data (Deliyannis et al 1997).

- The F star lithium dip is clearly seen in NGC 752. The stars in the dip region have evolved off the MS in older systems (see Section 4.3 for discussion of subgiants).

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- The Sun relative to other stars. As an interesting aside, Pilachowski & Hobbs (1988) noted that the solar lithium abundance might be anomalously low relative to that of old open cluster stars. Because some classes of theoretical models are calibrated on the solar lithium, an unusual abundance for the Sun could affect theoretical predictions.

- Tidally locked binaries in M67 have abundances higher than other stars at the same T_{eff} (Deliyannis et al 1994).

- Beryllium and boron. Boesgaard & King (1993) examined the observational database for beryllium in low mass stars, and concluded that extra mixing or a range in intrinsic abundance was needed to explain disk beryllium abundances. A population of beryllium-deficient field stars has long been known to exists in the same general temperature range as the lithium dip (Boesgaard 1976). Thorburn et al (1994) have reported that boron is depleted by a factor of 2 in 110 Her, a subgiant that came from the lithium dip (lithium-depleted by a factor of 100–200) region, and beryllium is depleted by a factor of 10 in this star. This combination of properties indicates mixing with a depth-dependent time scale.

Beryllium is depleted in the solar photosphere with respect to meteorites by 0.3 dex (Chmielewsky et al 1975, Anders & Grevesse 1989). In a recent analysis, King et al (1996) found that a range of solar depletion factors from 0.1–0.5 dex was consistent with the data, but that some beryllium depletion was needed to explain the difference between the photospheric and meteoritic abundances. α Cen A has a surface beryllium comparable to that of the solar photosphere, whereas α Cen B is depleted in beryllium with respect to both A and the meteoritic value. This could provide evidence for extra mixing in cool stars where lithium is unobservable, but standard models predict no beryllium depletion (Primas et al 1996, King et al 1996).

HALO STARS Spite & Spite (1982) discovered that hot metal-poor stars had a abundances of $\sim$2.1, on the order of 10 times greater than the solar photospheric value but 20 times lower than the meteoritic value. Furthermore, the abundances of hot stars appeared to show little dependence on T_{eff}. This lithium plateau, extending from 5600–6300 K, is now firmly established (Spite et al 1984, Hobbs & Duncan 1987, Rebolo et al 1988); the most comprehensive recent observational study has been performed by Thorburn (1994). Key features of the halo star lithium pattern include the following:

- Cool star depletion similar in morphology to that seen for the open clusters.

- A plateau with nearly uniform abundance across a wide range of metal abundances. A slight increase in lithium abundance with increased T_{eff}

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and decreased lithium abundance with decreased metal abundance may be present in the data (Thorburn 1994, Norris et al 1994, Ryan et al 1996), on the order of 0.04 dex/100 K and 0.1 dex per order of magnitude in [Fe/H]. This claim has been disputed (Molaro et al 1995); the difference is largely caused by different temperature determination techniques. A decrease in abundance towards higher T_{eff} is expected in some classes of theoretical models, and its absence is clear and significant.

- Relative to the old open clusters any dispersion at fixed T_{eff} is small for the majority of stars, although some anomalous stars with only limits for Li are present (Thorburn 1994). For the overall halo system, a small intrinsic dispersion has been claimed by some authors, whereas others find the observed range to be consistent with observational errors (Deliyannis et al 1993, Thorburn 1994, Molaro et al 1995, Spite et al 1996). It is clear that any successful model must not predict a large scatter in abundance for the majority of halo stars. By contrast, subgiant stars in the globular cluster M92 (Deliyannis et al 1995) and NGC 6397 (Pasquini & Molaro 1996) appear to show a wide range in abundance at fixed T_{eff}.

- Detections of the isotope ^{6}Li have been claimed in some halo stars (Smith et al 1993, Hobbs & Thorburn 1994). The detection of both isotopes of lithium is based on subtle changes in line profiles, which can be produced by other effects such as convection (Gray 1989); some caution in interpreting the data may therefore be in order. Because ^{6}Li is fragile, its survival constrains the magnitude of deep mixing (Smith et al 1993, Steigman et al 1993); other possibilities such as flare production of ^{6}Li have also been discussed (Deliyannis & Malaney 1995).

- Cool tidally locked binaries have a higher than normal abundance, which is consistent with the pattern seen in open clusters (Deliyannis et al 1994, Spite et al 1994). The situation for binaries in the plateau is more complex (Spite et al 1994, Deliyannis et al 1996); the abundances are similar to single stars, but it is unclear if the specific observed systems would be expected to have unusual abundances.

4.2 *Mixing on the Main Sequence: High Mass Stars*

Recent evidence exists for mixing in high mass stars that is not expected from standard stellar models. Lithium and beryllium are not observable in hot stars, so studies of mixing have focused on helium and CNO. Deep mixing produces enhanced surface helium and nitrogen, lowered surface carbon, and in the case of very deep mixing, lowered surface oxygen. A summary of the observational evidence for mixing can be found in Maeder & Conti (1994); see also Venn

(1995). The MS anomalies are not expected to be produced for masses less than about 40–50 solar masses, even in models that incorporate mass loss. The predictions of standard models for evolved stars depend on their evolutionary state. A and B stars leaving the MS are expected to have normal abundances (i.e. without extensive CNO processing), whereas stars on blue loops have undergone the first dredge-up. Either evolutionary state is possible in principle for A and B supergiants (Venn 1995).

The observational situation depends strongly on spectral type and thus on mass. Slowly rotating MS OB stars (Herrero et al 1992) and MS B stars (Gies & Lambert 1992) have essentially normal abundances. However, rapidly rotating OB stars show large surface helium abundances correlated with the rotation rate. Observational evidence is consistent with a lack of mixing in intermediate mass MS stars (see Section 4.3) from $\sim$1.8–7 $M_\odot$. The correlation between rotation and mixing found by Herrero et al (1992) is important, and it may indicate that such stars follow a completely different evolutionary path than traditional stellar evolution.

Evolved O stars, in general, show surface abundances indicating surface CN processing and helium enrichment (Walborn 1976, 1988, Howarth & Prinja 1989). Gies & Lambert (1992) found evidence for anomalous surface abundances in some B supergiants (see also Barbuy et al 1996); Venn (1995, 1996) examined A supergiants and, surprisingly, found N/C ratios intermediate between solar and that expected from the first dredge-up. The depletion pattern does not appear to be consistent with the pattern expected from the first dredge-up, which may force a revision of the usual evolutionary scenario for A supergiants. The observational analysis of supergiant abundances is challenging, so some caution regarding the interpretation of the data is in order (as stressed by the observers).

Recently, boron has been measured in B stars (Venn et al 1996), and boron may serve as a powerful discriminant between different physical scenarios. The measured boron abundance in the sample is well below solar, and boron depletion is correlated with nitrogen excess. Theoretical models of the mixing in massive stars are discussed in Section 6.2.

4.3 *Mixing in Evolved Low Mass Stars*

THE FIRST DREDGE-UP IN OPEN CLUSTERS The abundances of giants after the first dredge-up provide a test of the internal abundance profile of their MS precursors. Open clusters provide a sample with a range in mass, which is otherwise difficult to constrain for field giants. Lithium and $^{12}C/^{13}C$ data for open cluster giants with a range of masses were obtained by Gilroy (1989). The observed $^{12}C/^{13}C$ was in good agreement with theory for $M > 1.8\ M_\odot$ and was systematically lower than predicted in lower mass giants. $^{12}C/^{13}C$ and C/N abundances were consistent with the first dredge-up in low luminosity giants in the old open cluster M67, whereas lower $^{12}C/^{13}C$ was found for more luminous giants (Gilroy & Brown 1991). This suggests that mixing on the giant branch, and not internal MS mixing, is the cause of the deviation from standard models seen in the low mass giants. The absence of mixing can also be seen in the beryllium and boron abundances of the Hyades giants, which are also consistent with the predictions of standard models (Duncan et al 1994).

Lithium in giants presents a more complex picture. The maximum abundance seen in giants is less than predicted by standard models, which is probably caused by non-LTE effects (Carlsson et al 1994, Duncan et al 1994). There are also giants with anomalously low and high abundances (Gilroy 1989, Brown et al 1989, Fekel & Balachandran 1993). The active RS Cvn stars also tend to have higher than normal lithium abundances, similar to the MS tidally locked binaries (see Pallavichini 1994 for a review). The low lithium stars could have depleted lithium on the MS. The high lithium stars, however, present much more of a theoretical puzzle (even leaving aside the extremely high abundances seen in some asymptotic giants).

Subgiant lithium abundances can serve as a diagnostic of different MS models. Subgiants in the old open cluster M67 came from precursors in the lithium dip, and they show low abundances that indicate that lithium is destroyed and not hidden below the surface in the dip (Balachandran 1995, Deliyannis et al 1997). Intermediate mass stars in the Hertzsprung gap have also been studied for lithium, and they show a complex rotation-dependent abundance pattern (Balachandran 1990, Wallerstein et al 1994, Hiltgen 1996). There is a well-defined halo subgiant lithium pattern, followed by strong depletion in metal-poor giants (Pilachowski et al 1993).

ABUNDANCE ANOMALIES ON THE GIANT BRANCH A rich database exists of observations in metal-poor giants that require extra mixing (see Smith 1987, Suntzeff 1993, Kraft 1994 for reviews). The issue is usually raised in terms of either primordial abundance variations or mixing; this dichotomy is unnecessary, as evidence exists for both. Lithium abundances in nitrogen-rich halo dwarfs are normal (Spite & Spite 1986), which rules out internal MS mixing as the origin of the nitrogen anomalies. There is also evidence for abundance anomalies close to the MS (Bell et al 1983, Suntzeff 1989, Briley et al 1991, Briley et al 1996) and giants for which C + N is not the same in the same cluster (Carbon et al 1982, Trefzger et al 1983). This is difficult to understand from the point of view of the mixing hypothesis because deep mixing is not in general present in models that are consistent with the lithium data for metal-poor MS stars (Deliyannis et al 1989). $^{12}C/^{13}C$, C/N, and oxygen provide tests of mixing at increasingly high temperatures. In addition to the CNO isotopes, Na and Al variations may be produced by proton capture reactions on Ne and Mg, respectively, in the vicinity of the hydrogen-burning shell in giants. The trends inferred from the different indicators are summarized below.

Low $^{12}C/^{13}C$ ratios ^{12}C/^{13}C ratios in globular cluster giants are low, close to the nuclear equilibrium value of 4 in some cases. The observed values are far from the values seen in dwarfs and the predictions of the first dredge-up in standard models (Sneden et al 1986, Brown & Wallerstein 1989, Suntzeff & Smith 1991, Briley et al 1994, Shetrone 1996, Briley et al 1997; see Charbonnel 1994 for a discussion). The more-or-less normal dilution of lithium in halo subgiants suggests that mixing on the red giant branch (RGB), rather than internal MS processing, is responsible for the low carbon isotope ratios (Pilachowski et al 1993; see also Sneden et al 1986). Modest but significant changes in ^{12}C/^{13}C along the giant branch were also found in the old open cluster M67 (Gilroy & Brown 1991).

C/N Carbon is converted to nitrogen in the CN cycle; the globular clusters M92, M3, M4, NGC 6752, M13, and M15 all show evidence for decreased carbon with increased luminosity on the giant branch (Carbon et al 1982, Trefzger et al 1983, Briley et al 1990, Smith et al 1996). The carbon depletion as a function of luminosity on the giant branch is shown in Figure 5. There is a scatter in the mean C/Fe at a given luminosity that is real. C + N + O appears to be conserved in M3 and M13 (Smith et al 1996).

Oxygen Surface oxygen depletion requires that material be O $\rightarrow$ N processed, which requires higher temperatures than C $\rightarrow$ N processing. The first evidence for oxygen depletion in metal-poor cluster giants was found in M92 by Pilachowski (1988) and confirmed both in M92 and in M15 (Sneden et al 1991). Evidence for strong oxygen depletion exists for luminous giants in M13 (Brown et al 1991, Kraft et al 1992) and in ω Cen (Norris & DaCosta 1995). M3 and M10 show more normal oxygen abundances as a function of luminosity on the upper RGB (Kraft et al 1993, 1995); this implies that whatever governs the oxygen depletion varies from cluster. Because both M10 and M13 have blue horizontal branches, oxygen depletion is not a unique function of horizontal branch morphology. The observed depletion pattern is also metallicity dependent; the more metal-rich cluster M71 shows much-reduced evidence for oxygen depletion at the tip of the giant branch (Sneden et al 1994, Briley et al 1997). More normal oxygen abundances are seen in field giants (see Pilachowski et al 1996 for a discussion of this point). Kraft et al (1997) have traced O-depleted stars well down the giant branch in M13; the field and cluster data appear to be different.

NaMgAl A correlation between the strength of the CN band and the strength of the sodium and aluminum line strengths was initially found by Peterson (1980). Because changes in the abundance of Na and Al were thought to require neutron capture, this was taken as evidence for primordial variations in abundance. The observational evidence for Na and Al variations has accumulated (see Kraft 1994 for a discussion), but the interpretation has shifted. Proton capture on Ne and Mg can also occur under conditions typical for the hydrogen-burning shell in giants (Denissenkov & Denissenkova 1990, Langer et al 1993, Cavallo et al 1996). In an extensive study, Pilachowski et al (1996) found evidence for changes in the mean sodium abundance as a function of luminosity on the upper giant branch of M13. There was an abrupt change in behavior at log g = 1, possibly indicative of the onset of mixing at this point on the RGB. Simultaneous Mg and Mg isotope data (Shetrone 1996a,b) observations reveal a more complex pattern, and the abundance pattern among second ascent giants appears different from that in luminous first ascent giants.

There are significant trends with metal abundance in the data, in the sense that metal-poor stars have more marked composition anomalies (Shetrone 1996a,b, Kraft et al 1997). The open cluster data shows that the deviations from the standard model are strongly mass dependent, becoming pronounced only for the lowest-mass open cluster giants. By extension, mixing in metal-poor stars could also be mass (or equivalently age) dependent. Variations in abundance at a given luminosity are also present. The observed mixing pattern appears to depend on luminosity as well: Deeper mixing (as probed by O, Na, and Al) appears strongest in the brightest giants, whereas milder C $\rightarrow$ N mixing appears to be possible for less luminous giants. The status of theoretical models of RGB mixing is discussed in Section 6.3.

5. MIXING AND SEPARATION MECHANISMS

5.1 *Rotational Mixing*

Rotation has long been known to be capable of inducing mixing in the radiative interior of stars. Eddington (1925) found that meridional circulation must necessarily arise in rotating stars as a consequence of the von Zeipel paradox: namely the impossibility of maintaining hydrostatic equilibrium in a rotating star because the effective gravity on an equipotential varies with latitude. There is an extensive catalogue of other instabilities that arise in the presence of rotation (Zahn 1974, Tassoul 1978, Knobloch & Spruit 1982), especially in the presence of differential rotation as a function of depth. However, rotation has not in general been an ingredient of the standard stellar model. This can be attributed largely to the increase in both complexity and uncertainty of models that include rotation and to the availability of arguments that rotation is at most a perturbation to the structure and evolution of stars. In the present-day Sun, for example, the departure from spherical symmetry is on the order of 10^{-5}, and the time scale for meridional circulation is on the order of 10^{12} years, which is far in excess of the nuclear burning time scale.

The observational data, however, requires a mixing mechanism not present in standard stellar models. For all of the problems of interest in this review, mixing driven by rotation is at least a potential candidate.

The problem of stellar evolution with rotation can be broken into a series of ingredients that must be included. The best overall approach remains that advocated by Endal & Sofia (1978), in their pioneering study of stellar models with rotation. The standard equations of stellar structure must be supplemented with appropriate corrections for the direct impact of rotation on the structure (Kippenhahn & Thomas 1970). The initial angular momentum distribution must be specified, a proper surface boundary condition is needed (including angular momentum and mass loss), and a prescription for the internal transport of angular momentum is required for a fully self-consistent model. This yields a solution of the angular momentum distribution within a stellar model as a function of time. A prescription for the mixing associated with a given angular momentum distribution is obviously necessary to compute rotational mixing; if vigorous enough, such mixing could also materially affect the structure and evolution of the star.

Broadly speaking, the study of rotational mixing therefore requires both an understanding of the angular momentum evolution of stars and an understanding of the degree of mixing for a given angular momentum distribution. The former subject is outside the scope of this review; the important features of angular momentum evolution are summarized at the beginning of the discussion of rotational mixing in Section 6.1. The remainder of the section is devoted to the degree of mixing and angular momentum transport for a given angular momentum distribution.

OVERALL TREATMENT OF ROTATIONAL MIXING AND ANGULAR MOMENTUM TRANSPORT Because internal angular momentum transport is the crucial ingredient for determining the extent of rotationally induced mixing, I describe the diffusion coefficient estimates that have been proposed. Following Endal & Sofia (1978), the transport of angular momentum and the associated material transport can be described by coupled diffusion equations

$$\frac{d\omega}{dt} = \frac{1}{4\pi\rho r^2 \tilde{I}} \frac{d}{dr} \left(4\pi\rho r^2 \tilde{I} D \frac{d\omega}{dr} \right), \tag{1}$$

and

$$\frac{dX_i}{dt} = \frac{1}{4\pi\rho r^2}\frac{d}{dr}\left(4\pi\rho r^2 f_c D\frac{dX_i}{dr}\right), \tag{2}$$

where $\tilde{I}$ is the moment of inertia per unit mass, D is the diffusion coefficient, and f_c is an efficiency factor relating the diffusion coefficient for material transport to that for mixing. The local angular velocity and its derivative allow us to determine the diffusion coefficient for angular momentum transport subject to considerations of stability. The efficiency factor f_c can be calibrated empirically; in general, the diffusion coefficients for mixing must be about 30 times smaller than those for angular momentum transport (Pinsonneault et al 1989).

Chaboyer & Zahn (1992) derived a physical basis for this factor based upon the inhibition of vertical mixing by horizontal turbulence. Angular momentum transport is much less affected because it varies on a level surface if $\omega = \omega(r)$, and thus it will occur even in the presence of horizontal turbulence. Based on this reasoning, the same equation should be solved for material mixing, but the angular momentum transport equation is replaced by

$$\frac{d\omega}{dt} = \frac{1}{4\pi\rho r^2\tilde{I}}\left[\frac{d}{dr}(4\pi\rho r^2\tilde{I}v\omega) + \frac{d}{dr}\left(4\pi\rho r^2\tilde{I} f_c D\frac{d\omega}{dr}\right)\right], \tag{3}$$

where v is the diffusion velocity, and for meridional circulation $D = vR$ ($R =$ radius). The efficiency factor f_c can be computed from the velocity field (Zahn 1992; see below).

MERIDIONAL CIRCULATION The best-known hydrodynamic angular momentum transport and mixing mechanism is meridional circulation (Eddington 1925, Sweet 1950, Tassoul & Tassoul 1982, Zahn 1992). It is caused by the inability of a rotating star to maintain hydrostatic equilibrium in radiative regions because of variations in gravity between pole and equator on isobars. For rotation on cylinders, the potential is conservative and surfaces of constant thermodynamic variables (P, T, ρ) coincide and lie on equipotential surfaces. In this case, the circulation velocity can be estimated reliably; Kippenhahn & Möllenhof (1974) give

$$v_{es} = \frac{-1}{g\delta}\frac{\nabla_{ad}}{\nabla_{ad} - \nabla}\left\{2\left(\frac{L}{M} - \varepsilon\right)\frac{\tilde{g}}{g} + \frac{L}{4\pi GM\rho r}\frac{d}{dr}(\omega^2 r^2)\right\}, \tag{4}$$

where δ is $(d \ln \rho)/(d \ln T)$, ε is the energy generation rate, and $\tilde{g}/g$ is the departure from spherical symmetry.

In general, as stars evolve, differential rotation with depth is generated, both by structural evolution and angular momentum loss. Angular velocity gradients on surfaces of constant density are relatively easy to remove by shear instabilities (see Zahn 1992). The angular velocity is therefore likely to be constant on surfaces of constant pressure, but it can be a function of radius. In this case, the potential is nonconservative, and surfaces of constant (P, T, ρ) do not coincide. This gives rise to additional terms in the circulation velocity estimate (Sakuri 1991, Zahn 1992). Zahn (1992) estimates the circulation velocity as

$$\begin{aligned} v_{es} = {} & \frac{-\nabla_{ad} r^2}{\frac{d\ln\rho}{d\ln T}(\nabla_{ad} - \nabla) G M} \\ & \times \left\{ 2\left[\frac{L}{M}\left(1 - \frac{\omega^2}{2\pi G\rho}\right) - \varepsilon\right]\frac{\tilde{g}}{g} \right. \\ & \quad + \left[\left(\frac{H_T}{r} - \frac{1}{3}\right)\frac{L\omega r}{\pi G M\rho} - \frac{2\varepsilon\omega r^4}{3GM}(\varepsilon_T - \chi_T)\right]\frac{d\omega}{dr} \\ & \quad \left. + \frac{L}{4\pi\rho r^2}\frac{d}{dr}\left(H_T\frac{d\theta}{dr} - \chi_T\theta\right) - \varepsilon H_T\frac{d\theta}{dr}\right\}, \end{aligned} \tag{5}$$

where $\chi = [(4ac)/(3\kappa\rho)]T^3$, ε is the energy generation rate, H_T is the temperature scale height, ω is the angular velocity. The departure from spherical symmetry $\tilde{g}/g$ is

$$\frac{\tilde{g}}{g} = \frac{\omega^2}{3}\frac{d}{dr}\left(\frac{r^4}{GM}\right) - \frac{d}{dr}\left(\frac{\hat{\Phi} r^2}{GM}\right), \tag{6}$$

where $\hat{\Phi}$ is the quadrupole moment. θ is a function of the derivative of the angular velocity. The diffusion coefficient for meridional circulation is taken as $|v_{es} r|$. I note that the classical expression for the departure from symmetry, $(4\delta^2 r^3)/(3GM)$, is in significant error both in stellar cores (where *dM/dr* is large) and for strong differential rotation (where the quadrupole is important). If the quadrupole dominates, the sign of $\tilde{g}/g$ will be reversed; this can also occur in some of the other terms in the circulation velocity estimate. The most important difference from the classic picture of meridional circulation is that a circulation-free state can be established in the Zahn (1992) framework, which may help greatly in explaining some of the observational data.

The Kippenhahn and Zahn prescriptions have been energetically challenged by Tassoul & Tassoul in a series of papers (e.g. Tassoul & Tassoul 1995 and references therein). The essential question is whether or not viscous stresses can prevent the large increase in circulation velocity implied by the above equations in the outer envelopes of stars with thin surface convection zones. There is an overall similarity between the philosophy of the latest works by Zahn and the Tassouls, namely the explicit recognition that the final meridional circulation velocity results from the balance of a series of effects. The different techniques reduce to similar results for stars with thick surface convection zones.

The above expression does not include the inhibition of meridional circulation by gradients in mean molecular weight μ. Mean molecular weight gradients can inhibit circulation in two ways. A latitude-dependent μ distribution can choke off the circulation (Mestel 1953). Lifting material in the presence of a μ gradient also requires energy [see Roxburgh 1991 for a nice discussion of this point in the context of the Goldreich-Schubert-Fricke (GSF) instability]; this would dampen rather than suppress meridional circulation.

The generation of a latitude-dependent μ distribution is much more difficult to quantify. Because meridional circulation relies on small imbalances between pole and equator to drive large-scale circulation currents, even small latitude-dependent effects could stabilize a region with a μ gradient. Because horizontal turbulence is relatively easy to produce in stars (Zahn 1992), and it will tend to erase horizontal composition gradients, the net effect will be the balance between two (uncertain) processes, namely the rate at which latitude-dependent μ gradients can be produced (by vertical mixing in the presence of a μ barrier or by nuclear reactions occurring at slightly different T) and the rate at which they can be removed (by horizontal mixing). To further complicate matters, other instabilities [i.e. shear, GSF, axisymmetric baroclinic diffusive (ABCD)] can also be present, and such mechanisms will in general require a different latitude-dependent profile to suppress mixing; in fact, as discussed by Knobloch & Spruit (1983), variations in abundance on a level surface can themselves trigger instabilities and cause mixing (the "haloclinic" instabilities).

Empirical data, however, favors mixing in the presence of μ gradients in at least some cases. There is evidence in the Sun for both helium diffusion, which would create a μ gradient, and mixing sufficient to deplete the solar lithium and beryllium (Michaud & Proffitt 1993, Chaboyer et al 1995a). Data in massive stars and evolved low mass stars also requires that mixing be capable of penetrating μ gradients. The degree to which composition gradients can reduce mixing, however, is still important and cannot be regarded as a solved theoretical issue.

OTHER INSTABILITIES A variety of other rotational instabilities have been discussed in the literature. In most cases, the time scale estimates are significantly more uncertain than the time scale for meridional circulation. They can be divided into two general classes: dynamical instabilities, which operate on time scales much shorter than the evolutionary time scale, and secular instabilities, which operate on time scales comparable to or greater than the evolutionary time scale. See Tassoul (1978), Endal & Sofia (1978), and Knobloch & Spruit (1982) for a discussion of dynamical instabilities, which in general require larger angular velocity gradients than are expected in stars during most evolutionary phases. The major other instabilities that have been discussed in the astrophysical literature are as follows.

Secular Shear Differential rotation can survive in the presence of a density gradient in stars; the Richardson criterion expresses the condition for marginal stability (e.g. Shu 1992). Radiative heat losses can reduce the stabilizing effect of a density gradient (Townsend 1958, Zahn 1974); recent estimate of the stability conditions and vertical diffusion coefficient can be found in Zahn (1992), Maeder (1995), and Maeder & Meynet (1996). Gradients in μ can prevent the onset of shear instabilities, but recent work indicates that mixing by the shear instability may be important in semiconvective regions (Maeder 1995).

GSF and ABCD instabilities Stars are unstable against axisymmetric perturbation in the case where rotation is not on cylinders (Goldreich & Schubert 1967, Fricke 1968). Horizontal turbulence suppresses latitude dependence of the rotation profile, and evolutionay effects generate differential rotation as a function of radius. The net effect of the GSF instability will be to reduce radial angular velocity gradients. The time scale for the instability was initially thought to be dynamical, but more recent estimates indicate that it operates over a time scale comparable to meridional circulation (Kippenhahn et al 1980). The GSF instability is strongly damped by μ gradients, and even viscosity, but the ABCD instability is not inhibited by μ gradients (Knobloch & Spruit 1983, Spruit et al 1983).

5.2 *Internal Waves and Magnetic Fields*

Internal waves have recently received attention both as a potentially efficient mechanism for the internal transport of angular momentum and as a possible means of mixing in the envelopes of low mass stars. Magnetic fields can also be remarkably efficient at internal angular momentum transport. Because the internal angular momentum distribution is crucial for computing the degree of rotational mixing, magnetic fields may have an important indirect impact on mixing in stars. An excellent summary of the various mechanisms for angular momentum transport can be found in Charbonneau et al (1995). I begin with waves and then discuss magnetic fields.

WAVES AS ANGULAR MOMENTUM TRANSPORT AGENTS Gravity waves can be generated in stars, possibly by turbulence in convective regions. Waves have long been appreciated to be important for angular momentum transport in the Earth's atmosphere and ocean (see for example Lighthill 1978, Phillips 1977). Press (1981) demonstrated that significant internal gravity waves could exist in stars, and Goldreich & Nicholson (1989) investigated the behavior of gravity waves in stellar interiors in the context of tidal braking (see also Goldreich et al 1994). Two recent papers (Kumar & Quataert 1997, Zahn et al 1996) have raised the provocative prospect that gravity waves could enforce nearly rigid rotation in solar mass stars with a time scale on the order of 10^7 years. If confirmed, this would reduce significantly the complexity of the problem of rotational mixing: After a brief period in the early MS, low mass MS stars could be treated as solid body rotators. An extension of the theory to higher mass stars (with convective cores) and evolved giants (with deep convective envelopes) would be highly desirable. Given the relatively short lifetimes of both massive stars and giants, it is possible that solid body rotation would not be enforced by waves in either case.

WAVES AND MIXING Gravity waves may theoretically produce mixing directly; in the astrophysical context, this issue was initially explored by Press & Rybicki (1981), who found only a small effect. Enhanced mixing may be produced with difficulty via waves in stellar interiors (Garcia Lopez & Spruit 1991, Schatzmann 1991, Montalban 1994, Montalban & Schatzmann 1996). The properties of wave-driven mixing are compared with the data in Section 6.1.

MAGNETIC FIELDS AND ANGULAR MOMENTUM TRANSPORT Even small internal stellar magnetic fields could potentially be remarkably effective agents for angular momentum transport (Spruit 1987). There have been vigorous debates in the literature about whether or not magnetic fields enforce rigid rotation in stars (for example, Mestel & Weiss 1987, Tassoul & Tassoul 1989). Charbonneau & MacGregor (1992, 1993) investigated the full magnetohydrodynamic problem for a variety of field strengths and configurations. They found a weak dependence on field strength but a strong dependence on the field morphology. Some configurations produced uniform rotation on a short time scale, whereas others permitted strong differential rotation with depth to survive to the age of the Sun. Some difficulties are present in reconciling the strong coupling case with the helioseismic data on the internal rotation of the Sun: The flatness of the solar rotation curve is naturally explained, but the latitude dependence present in the solar convection zone does not appear to be present in the radiative interior (Zahn et al 1996). The rich behavior of full-scale simulations, and the potential importance of magnetic instabilities (Balbus & Hawley 1994), is an indication that further study of magnetic angular momentum transport is needed. Given the difficulty in constraining the strength or the morphology of internal stellar magnetic fields, observational tests to distinguish the imprint of magnetic angular momentum transport on surface properties of stars would be highly desirable.

5.3 *Gravitational Settling, Thermal Diffusion, and Radiative Levitation*

Strictly speaking, the processes discussed in this section are element separation processes rather than mixing processes. They are germane to the discussion of mixing in stars because they are a competing explanation for some of the surface abundance anomalies I discuss above and because the interaction between atomic diffusion and mixing processes could be important. Heavier elements tend to sink relative to lighter elements because of both gravitational settling and thermal diffusion; radiative levitation can drive neutral or partially ionized species upwards (Burgers 1969, Chapman & Cowling 1970, Michaud et al 1976; see Vauclair 1983 for a review of the physics of microscopic diffusion). In general, radiative levitation is important only in stars with thin surface convection zones, particularly the chemically peculiar A stars. Recent calculations of the coefficients in a form convenient for astrophysical use can be found in Thoul et al (1994) and Proffitt (1994); see also Paquette et al (1986), Michaud & Proffitt (1993), and Richer & Michaud (1993).

Microscopic diffusion is a strong function of depth, with the time scale increasing rapidly with depth within a given model. The time scale for diffusion is also far shorter for stellar models with shallow convection zones than for stellar models with deep surface convection zones. Diffusion of helium will produce a μ gradient at the base of the surface convection zone, which provides an observational test of the ability of mixing to cross a barrier in mean molecular weight. Diffusion is expected to be a small effect for giants, MS stars with deep surface convection zones, and stars with strong mass loss. It could play a role in the origin of the mid-F lithium dip.

6. THEORETICAL RESULTS FROM NONSTANDARD MODELS

All of the phenomena discussed in Section 4 have been investigated with theoretical models in varying degrees of detail. Such models can be roughly divided into three classes. In initial studies, arbitrary mixing is induced to constrain the relevant depth and time scale needed to explain the data. Such models represent the state of the art for the treatment of mixing in giants. Mixing can also be tied to specific physical mechanisms, such as the time scale estimates for rotational mixing or wave-driven mixing discussed in Section 5. At this stage, a detailed comparison between the observational data and theoretical models is not generally carried out, but the observed systematics are tied to underlying physical causes. Models for extra mixing in massive stars fall into this category. In low mass stars, detailed models have been constructed that involve both mixing and angular momentum evolution, and a number of independent tests of the theory can be applied. After presenting the theoretical studies of mixing in low mass stars, I discuss mixing in massive and evolved stars.

6.1 *Mixing on the Main Sequence: Low Mass Stars*

Mild envelope mixing is the most likely explanation of the light element depletion pattern seen in disk MS stars. Both lithium and beryllium must be destroyed, lithium more efficiently than beryllium. The normal abundances seen on the RGB after the first dredge-up (Gilroy 1989) severely constrain deeper mixing, as does the excellent agreement between the thermal structure of unmixed solar models and the actual solar sound speed as a function of depth inferred from helioseismology (Bahcall et al 1997). Lithium must be preserved in intermediate mass MS stars and mixing cannot suppress the development of chemical peculiarities linked with microscopic diffusion and radiative levitation (Michaud & Charbonneau 1991, Charbonneau et al 1989). The absence of mixing in these cases places strong constraints on the theory. Several major classes of physical processes have been claimed as explanations for the light element abundance pattern seen in low mass stars:

- microscopic diffusion and radiative levitation,
- MS mass loss,
- mixing driven by waves, and
- mixing driven by rotation.

Some authors have explored models containing more than one of the above ingredients. Mass loss and microscopic diffusion have difficulty in reproducing the observed abundance pattern in open clusters (see below), so most theoretical work on nonstandard models has involved mixing. For the halo stars, standard models are competitive with nonstandard models. To establish that standard models can accurately predict the initial abundances of halo stars, however, simple demonstration of agreement between the halo star data and standard models is insufficient. The origin of anomalous abundances in open clusters must be identified, and a reason why it is not operating in the halo stars must be presented.

Many of the overall features will be important for the other cases for nonstandard mixing (massive and evolved stars) because the rotational properties of low mass MS stars are far better studied than the rotational properties of higher mass and evolved stars.

MICROSCOPIC DIFFUSION AND RADIATIVE LEVITATION Microscopic diffusion is an elegant explanation for the mid-F star lithium dip (Michaud 1986, Richer & Michaud 1993); the time scale is too long for it to contribute significantly to surface lithium depletion in cool stars. The time scale for downward diffusion decreases as the surface convection zone becomes shallower; this leads to a strong decrease in surface abundance with increased T_{eff}. Once lithium becomes partially ionized below the surface convection zone, radiative levitation can cause a rise in the surface abundance for hotter stars. A smaller beryllium dip, offset in temperature, is also expected from models with diffusion. The lithium is stored below the convection zone, and not destroyed, by diffusion. The surface lithium abundances of stars coming from the dip region would therefore be expected to increase dramatically as their surface convection zones become deeper and dredge up lithium-rich material. This is inconsistent with data for subgiants in M67 (Balachandran 1995). A dispersion in abundance is not expected for models with diffusion and is present in the data. Diffusion is a physical effect that should be included in models, but it is probably not the sole agent responsible for the observed light element depletion pattern, either in the dip or for cooler open cluster stars (Pinsonneault 1994, Balachandran 1995).

Another application of diffusion is in the lithium abundance pattern seen in halo stars (Proffitt & Michaud 1991, Chaboyer & Demarque 1994, Vauclair & Charbonnel 1995, Swenson 1995). Helium diffusion occurs over a comparable time scale and can have implications for the ages inferred for globular clusters (Stringfellow et al 1983, Proffitt & Vandenberg 1991, Chaboyer et al 1992). Models with diffusion predict little dispersion, which is compatible with the data, and a mean trend of decreased abundance with increased T_{eff}, which is in conflict with the observed flatness of the Li-T_{eff} relationship for halo stars. Vauclair & Charbonnel (1995) and Swenson (1995) explored a combination of enhanced MS mass loss and diffusion as a possible solution; note that initial abundances higher than the observed value are needed in general. A combination of mixing and diffusion is more likely (Chaboyer & Demarque 1994) because the open cluster data is not consistent either with mass loss or diffusion alone. However, none of the models examined by Chaboyer & Demarque were entirely consistent with the halo star data, a rather troubling state of affairs for an important issue. The lithium data does constrain globular cluster age changes induced by helium diffusion (Chaboyer et al 1992).

MAIN SEQUENCE MASS LOSS Lithium survives only in the outer $\sim$0.02 $M_\odot$ of stars. MS mass loss could therefore potentially cause surface lithium depletion, provided the rate is far in excess of the solar wind rate on the order of 10^{-14} $M_\odot$ year (Weymann & Sears 1965, Hobbs et al 1989, Swenson & Faulkner 1992). Swenson & Faulkner demonstrated that mass loss cannot produce the observed abundance pattern for cool stars with deep convective envelopes, but they left it as a possibility for the mid-F star lithium dip (see also Dearborn et al 1992). The measurement of normal lithium abundances in δ Scuti stars (Russell 1995), which has been postulated as the phase where extensive MS mass loss could be generated, and the lithium dilution pattern seen in subgiants (Pilachowski et al 1993, Balachandran 1995, Hiltgen 1996, Deliyannis et al 1997) argue against MS mass loss as an explanation for lithium depletion even in the mid-F star regime.

MIXING DRIVEN BY WAVES Garcia Lopez & Spruit (1991) investigated mixing driven by waves as a possible explanation of the F star lithium dip. Montalban (1994) developed a prescription for mixing following the method suggested by Schatzmann (1991) and explored wave-driven mixing in the Sun. These models were extended to low mass MS stars by Montalban & Schatzmann (1996). Wave-driven mixing occurs over a long time scale that can vary within the envelope and as a function of time. It therefore has some of the properties of rotational mixing including simultaneous beryllium and lithium depletion. The light element depletion expected from theoretical models can be adjusted to reproduce the mass dependence of open cluster lithium depletion in the Hyades and Praesepe clusters with a suitable choice of pre-MS depletion added to the MS models. A dispersion in abundance, however, would not be expected from mixing driven by waves. As the existence of a dispersion is one of the central features of the observed depletion pattern, this suggests that mixing driven by waves is not the sole mechanism for light element depletion. Qualitative suggestions have been made that mixing from waves could be modulated by other effects, such as rotation; these ideas should be quantified and compared with other mechanisms.

MIXING DRIVEN BY ROTATION Rotational mixing and light element depletion have been investigated by a series of authors; recent reviews of theoretical models of light element depletion can be found in Michaud & Charbonneau (1991) and Pinsonneault (1994, 1995).

Early theoretical work focused on using empirical constraints on turbulent diffusion coefficients, following the suggestion of Schatzmann (1969). Modest transport coefficients were found to have an appreciable impact on surface abundances, with some authors investigating the possibility of extensive deep mixing sufficient to explain the solar neutrino problem (Schatzmann 1977, Vauclair et al 1978, Schatzmann et al 1981, Baglin et al 1985, Lebreton & Maeder 1987, Schatzmann & Baglin 1991). Lack of data on the angular momentum evolution of stars, along with limited empirical data on stellar abundances, made it difficult to constrain theoretical models.

Since the late 1980s, the focus of theoretical work has changed, with mixing tied to specific physical mechanisms and extensive use of the observed constraints on angular momentum evolution. The method of Endal & Sofia (1978) has proved especially useful in this regard. Endal & Sofia advocated treating rotation as an initial value problem and solving simultaneously for angular momentum loss, the transport of angular momentum, and mixing. Full evolutionary calculations of rotation from the pre-MS have been computed for open cluster stars below the dip and halo stars, including angular momentum loss and assuming angular momentum transport from hydrodynamic mechanisms (Pinsonneault et al 1989, Pinsonneault et al 1990, Pinsonneault et al 1992, Chaboyer & Demarque 1994, Chaboyer et al 1995a,b). Zahn's (1983, 1992) prescriptions for mixing have also been investigated for open cluster and halo star models, which assume solid body rotation in the interior and include angular momentum loss (Vauclair 1988, Charbonnel & Vauclair 1992, Charbonnel et al 1992, Charbonnel et al 1994, Richard et al 1996). The similarity between the results of the two classes of models is indicative of the limited sensitivity of the results to the details of the treatment of angular momentum transport. Models of mixing from meridional circulation have also been computed for stars in the lithium dip and above (Charbonneau & Michaud 1988a,b, 1990, 1991). Zahn (1992, 1994) proposed a new model for meridional circulation and examined the behavior of tidally locked binaries; Martin & Claret (1996) looked at the structural impact of rotation on pre-MS lithium burning. Stars below the mid-F lithium dip suffer extensive angular momentum loss, which dominates their angular momentum evolution; stars in the dip and above have a different rotational history. This has a strong impact on the mixing properties of the models, which are examined separately below.

Stars below the dip A large and growing database of stellar rotation observations has been complemented by a series of theoretical studies of stellar angular momentum evolution. The improvement in our understanding of stellar rotation holds great promise in reducing the most significant source of uncertainty in rotational mixing, so I briefly summarize some of the most important features of stellar angular momentum evolution below (for recent papers with a discussion of angular momentum evolution, see Keppens et al 1996, Charbonneau et al 1997, Krishnamurthi et al 1997).

The natural starting point for stellar models with rotation is the pre-MS. The most widely accepted hypothesis for the origin of the range of rotation rates in low mass stars is that accretion disks enforce constant rotation periods on the order of 10 days, which is typical for classical T Tauri stars (Edwards et al 1993, Bouvier 1994). A range of accretion disk lifetimes will therefore produce a range of MS rotation rates, as stars that detach from their disks early will experience a larger change in their moment of inertia than stars that detach from their disks closer to the MS.

A wide range of rotation rates is seen in young stars. Angular momentum loss is more rapid in fast rotators, and the observed surface rotation rates converge by a few hundred million years for solar analogs (Stauffer 1994). At late ages the surface rotation velocities of stars decline as $v \sim t^{-1/2}$ (Skumanich 1972).

These overall properties lead to some general features of rotational mixing for models that are consistent with the observed stellar rotation data.

Rotational mixing is not effective in the pre-MS, so little dispersion in abundance is expected among young cluster stars (Pinsonneault et al 1990, Chaboyer et al 1995b). The observed dispersion for cool stars in young clusters is therefore not a manifestation of variations in early rotational mixing. The structural impact of rotation on pre-MS depletion has recently been suggested as a possible culprit (Martin & Claret 1996) and deserves further exploration.

Rotational mixing is tied to the absolute rotation rate and its derivative. On the MS, stars with higher rotation rates will therefore experience more rapid mixing than slow rotators at the same mass. The minimum dispersion is set by the time scale for the observed convergence of the surface rotation rates; the time scale for convergence of the internal rotation profile is model dependent but on the order of a few hundred million years in the most weakly coupled plausible models. The existence of a dispersion is thus a prediction of rotational mixing rather than a problem.

Theoretical models of rotational mixing in solar analogs produce a rate of depletion that decreases with age as stars spin down. The time scale for mixing also increases with distance from the surface convection zone; solar models with rotational mixing predict beryllium depletion of a factor of 2–3 for lithium depletion of a factor of 100–200 (Pinsonneault et al 1989, Richard et al 1996).

Tidally locked binaries have a different rotational history, and a period range exists where their rotation period evolves slowly. A circulation-free state is therefore possible for some tidally locked binaries (Zahn 1994, Ryan & Deliyannis 1995), which may explain their high abundances and better agreement with standard stellar models.

Rotational mixing therefore has the qualitative features needed to explain the depletion pattern seen in open cluster stars, with the possible exception of the dispersion in abundance among young cool stars. Detailed examination of the theoretical models, however, raises a series of questions: the validity of the underlying angular momentum and mixing models, the mass dependence of extra mixing, and the expected magnitude of the dispersion.

Models with internal angular momentum transport from hydrodynamic mechanisms in general predict the survival of a rapidly rotating core in the

Sun, which is in conflict with the helioseismic data (Tomczyk et al 1995). In general, the mixing expected from the theoretical models is relatively insensitive to the treatment of internal angular momentum transport in stars with angular momentum loss; the models adjust themselves to a rotation profile for which the diffusion coefficients for angular momentum transport balance the torque applied by the wind. Calibrating the models on the solar lithium depletion can further reduce the sensitivity of the results to errors in the physical model. The solar models of Richard et al (1996) and Pinsonneault et al (1989), for example, use very different treatments for angular momentum transport but have the same relative lithium and beryllium depletion. However, a proper matching of the surface rotation as a function of time can be important, and only recently have the theoretical models been able to reproduce the surface rotation rates of low mass stars as a function of mass and time (Keppens et al 1996, Krishnamurthi et al 1996). The uniqueness of the model is also an issue (Michaud & Charbonneau 1991, Schatzmann & Baglin 1991), although most of the degrees of freedom can be removed through the application of empirical constraints on the angular momentum evolution.

The mass dependence of depletion is an important test; the models of Chaboyer et al (1995b), for example, predict mixing that depends only weakly on mass for open cluster stars; the open cluster data appears to show increased mixing for lower mass stars (Jones et al 1996a). Models of halo stars with mixing show a plateau that is nearly flat, but they do not have the observed increase in lithium for the hottest and most metal-poor stars (Chaboyer & Demarque 1994). Because similar discrepancies exist in the mass dependence of the rotation rates for these models, this indicates the importance of an accurate treatment of the angular momentum evolution.

Rotational mixing predicts a dispersion in abundance, but the magnitude of the dispersion will depend on the distribution of initial conditions. Quantitative tests of the distribution of lithium depletion factors expected from the distribution of stellar rotation rates have not been performed. The majority of young stars are slow rotators, with a small population of fast spinning stars (Stauffer 1994, Allain et al 1996). This will tend to produce a well-defined mean trend with a subpopulation of overdepleted stars, rather than a large scatter in abundance. Consistency between the distribution of initial conditions needed to explain rotation and the distribution needed to explain lithium has been neither demonstrated nor disproven to date.

The rotational models predict a correlation between the rotation history of a star and lithium depletion, rather than a correlation between the current rotation of a star and its lithium abundance. This makes establishing a causal connection between rotation and mixing difficult; systems such as M34, which retain a range of surface rotation and have a dispersion in lithium, will be important in testing the detailed properties of the models. Future models will need to address the above issues, along with quantitative estimates of the degree of light element depletion in tidally locked binaries.

The lithium dip and above The effective temperature range where the lithium dip occurs is a transition zone from the point of view of stellar structure and angular momentum. Stars below the dip have deep surface convection zones and suffer angular momentum loss; stars above the dip have shallow surface convection zones and do not experience MS angular momentum loss from a magnetic wind. All of the explanations for the dip rely on these changes in behavior to explain the lithium dip. Important overall rotation properties in the dip are as follows.

The rotation velocities of stars increase across the lithium dip as the convection zones become shallower and angular momentum loss becomes ineffective. Stars earlier than spectral type F8 have rotation velocities uncorrelated with their age (Benz & Mayor 1984).

There is a wide range of surface rotation rates on the MS above the dip. In stars hotter than the dip, peculiar abundances indicative of gravitational settling, thermal diffusion, and radiative levitation are seen in slow rotators (Charbonneau & Michaud 1991), and cluster giants have normal CN abundances, with some anomalous lithium abundances (see Section 6.3).

All of the mixing explanations rely on the increase in rotation with increased T_{eff} to produce the cool side of the lithium dip. There are varying suggestions for the increase in surface abundance on the hot side of the dip, which are described in the next few paragraphs.

For stars with a sufficiently thin surface convection zone, a separation into two distinct mixed zones is possible (Vauclair 1988, Charbonnel et al 1992); the region where such a separation would occur in cool stars is within their surface convection zone. The hot side of the lithium dip is therefore explained by the presence of a "quiet zone" that inhibits mixing to temperatures sufficient to destroy lithium. A complete separation into two distinct zones, however, is not found in more detailed models (Charbonneau & Michaud 1990, Charbonnel & Vauclair 1992, Zahn 1992, Tassoul & Tassoul 1995), although the efficiency of transport could be reduced.

The interaction between mixing and microscopic diffusion is another possible solution for such an increase in surface abundance (Charbonneau & Michaud 1988a,b, Michaud & Charbonneau 1991). Radiative levitation produces the rise on the hot side of the dip. Microscopic diffusion proceeds unimpeded up to some threshold rotation rate, above which it is damped. The threshold rotation rate for producing composition anomalies via diffusion was found to be lower ($v < 15$ km/s) than the observed rotation rates of stars in the middle of the lithium dip (~50 km/s), although the velocity cutoff for producing diffusion in higher mass stars was in good agreement with the data (Charbonneau & Michaud 1991).

A third possibility centers on the possibility of establishing a circulation-free state in stars without angular momentum loss (Zahn 1992). In this picture, stars adjust themselves to a state with zero net circulation in the absence of angular momentum loss (mass loss or rapid structural change could presumably also prevent the establishment of a circulation-free state). This explains the lack of mixing in stars above the dip. The picture advocated by Zahn is an intriguing suggestion that should be verified with full calculations.

6.2 *Mixing on the Main Sequence: High Mass Stars*

Nonstandard models of mixing in massive stars have now been investigated by a variety of authors, in response to the data indicating evidence for departures from the predictions of standard models. Rotational mixing may even prove an alternative to the various theories of convective overshoot and semiconvection that have traditionally been debated in the literature (Deng et al 1996a,b, Talon et al 1996). There are some important differences between the rotation and evolution of massive stars when compared with the low mass case.

- A wide range of rotation rates persists for the MS lifetime; a significant fraction of the stars rotate at or near the critical rotation rate (Fukuda 1982).

- Energy generation is in a convective core in massive stars. If rotational mixing is able to penetrate the core, it may permit fully mixed evolution; otherwise the development of a steep μ barrier during the lifetime will prevent deep mixing. Changes in the internal abundance profiles on the MS can influence subsequent evolution. Some extra mixing appears to be needed to explain the blue progenitor of SN 1987a (Langer 1991).

- The viscosity is far higher in massive stars than in low mass stars, which implies that the time scale for mixing may be comparable to, or less than, the MS lifetime (Maeder 1987).

- Mass loss is a strong effect on the structure and evolution of massive stars; the changes in surface abundance produced by mass loss need to be separated from those produced by mixing.

A successful model should also be able to reproduce the absence of mixing in intermediate mass MS stars and the increased mixing seen with increased mass and rotation. Maeder (1987) showed that a bifurcation in evolution is possible for the most massive stars, with ~15% of the stars experiencing nearly fully mixed evolution and the more traditional evolutionary path for the remainder. The inhibition of mixing by μ gradients was identified as the most important ingredient of the models. More recently, rotational models have been constructed for 9- to 20-$M_\odot$ stars (Langer 1992, Denissenkov 1994, Eryurt et al 1994, Fliegner et al 1996, Urpin et al 1996, Talon et al 1996), and different prescriptions for mixing have been explored. The major conclusions of these studies are summarized below.

Abundance anomalies in OBN stars can be explained within the framework of rotational mixing (Maeder 1987, Langer 1992, Eryurt et al 1994), and the effects of rotational mixing may be important for understanding the progenitor of SN 1987a. Full evolutionary models with rotation have not been computed for the most massive stars because of the complications of such effects as semiconvection, mass loss, and the possibility that a true zero-age MS may not exist for very high masses. The qualitative arguments of Maeder (1987) therefore need to be verified with direct calculations.

Full evolutionary models of a 9-$M_\odot$ model with rotation were computed by Talon et al (1996). Surface CN anomalies correlated with increased rotation were produced, but with little surface helium enrichment. Denissenkov (1994) found that a surface N enrichment that correlated with increased age was present in a 10-$M_\odot$ model. More recently, Fliegner et al (1996) examined boron and nitrogen abundances in 10- to 15-$M_\odot$ stars with rotation. They concluded that the N/B ratio was a diagnostic of deep mixing, with the measured B depletion not expected in models with mass loss but no extra mixing.

An important component in the absence of mixing for intermediate mass stars is the possibility of a circulation-free state. Urpin et al (1996) found that a circulation-free model can be produced within the framework of the Zahn (1992) prescription (see also Talon et al 1996). An extension of their static treatment to evolutionary models with mass loss would be desirable, in order to verify that mixing is indeed present when needed and suppressed when it is not.

The overall picture appears to be consistent with rotational mixing: Mixing is most rapid for the most massive stars and the fastest rotators. There are some important steps that need to be taken, however, to quantify the role of rotation in the evolution of massive stars. Most of the models to date have not evolved the angular momentum distribution from the pre-MS (see Eryurt et al 1994 for an exception); the origin of the range of rotation rates seen is almost certainly in the pre-MS, and the pre-MS evolution could influence the behavior at later ages. The effects of rotational mixing need to be clearly separated from other effects, such as the treatment of convection and mass loss. The distribution of surface abundance anomalies should also be consistent with the distribution of rotation rates, and the impact of mixing on chemical evolution models also needs to be addressed. Finally, it remains to be verified that the same physical model that produces mixing in massive stars does not produce excess mixing in intermediate mass stars.

6.3 *Mixing in Evolved Low Mass Stars*

The abundance pattern seen in giants is as rich as that for lithium in dwarfs and is equally compelling evidence for nonstandard mixing. Despite this, the status of the theory cannot be regarded as well developed, although most work implicitly assumes that mixing is driven by rotation. This can be traced to the greater computational difficulty in constructing giant branch models in part. However, there are also significantly greater physical uncertainties related to the paucity of rotational data for giants. The alternative possibilities of solid body rotation and constant specific angular momentum in the convective envelopes of giants, for example, give radically different prospects for mixing (e.g. Sweigart & Mengel 1979), and they cannot be distinguished observationally. Rotation data on the MS and in evolved stars is now available, and the constraints such data impose on the models are discussed below. Much of the recent interest in giant branch mixing stems from the realization that such mixing could have cosmological significance. ^{3}He can be destroyed in low mass stars, which would affect Big Bang nucleosynthesis (BBN) yields depending on the chemical evolution model (Rood et al 1984, Deliyannis 1995, Hogan 1995, Wasserburg et al 1995, Weiss et al 1996). Mixing of ^{4}He into the envelope could affect a variety of globular cluster properties, including age (Sweigart 1997).

LITHIUM IN INTERMEDIATE MASS GIANTS The presence of low lithium in some open cluster giants of intermediate mass (Gilroy 1989) could be explained if rapid rotators experienced mixing on the MS (Charbonneau et al 1989, Charbonnel & Vauclair 1992). There are strong observational selection effects against measuring lithium in hot rapid rotators, so MS surface depletion does not necessarily contradict the data. The measurement of high lithium in rapidly rotating subgiants (Wallerstein et al 1994) presents some difficulty for the Charbonneau et al (1989) model. Charbonnel & Vauclair (1992) proposed that surface lithium would be intact, but the size of the lithium preservation zone would vary with rotation, leading to differences in giant branch abundance. As noted in Section 6.1, there are some physical difficulties with the separation into two mixed zones that this model requires. The first ascent giants with high lithium abundance are a puzzle, with no compelling theoretical explanation (Fekel & Balachandran 1993); some speculative recent possibilities are discussed by de la Reza et al (1995).

DEEP MIXING IN LOW MASS GIANTS Giant branch mixing depends on both the structural evolution and the angular momentum evolution. The most important properties (see Sweigart & Mengel 1979 for a good discussion) are as follows.

A μ gradient exists at the base of the convection zone on the lower giant branch caused by the dredge-up of material that was partially burned by the proton-proton (pp) chain on the MS. This may prevent mixing on the lower giant branch (Sweigart & Mengel 1979, Charbonnel 1994). This μ gradient increases with increased metallicity.

Material falls into the hydrogen-burning shell with a time scale that decreases as the luminosity rises. At the same time, the outer radius expands. Giant branch mixing is therefore a threshold process. The time scale for mixing must also decrease with increased luminosity more rapidly than the infall time scale, given the observational evidence for enhanced mixing in luminous giants.

The ability of mixing to penetrate μ gradients determines how deep the mixing can go into the vicinity of the hydrogen-burning shell. Because some species (^{4}He, Na, O) are affected at higher temperatures than others (C, N), the sensitivity to μ gradients affects the observed mixing pattern. C $\rightarrow$ N processing occurs farther out in metal-poor stars than in more metal-rich stars, which makes mixing easier for lower Z (Sweigart & Mengel 1979).

Structural evolution on the giant branch produces strong differential rotation with depth. Constant angular momentum per unit mass (J/M) in the convection zone requires much lower rotation rates than solid body rotation in the convection zone, but the available angular momentum reservoir for either case is smaller than that required by Sweigart & Mengel (1979) to drive meridional circulation.

MS metal-poor stars rotate slowly, whereas rapid rotation is present in some evolved horizontal branch stars (Peterson 1983, Peterson 1985a,b, Peterson et al 1995). The degree of horizontal branch rotation differs from cluster to cluster; M13 has much higher typical rotation rates than M3 or NGC 288, for example. The combination of slow MS rotation and rapid horizontal branch rotation requires differential rotation with depth, either on the MS or in the convective envelope on the giant branch (Pinsonneault et al 1991). This is comforting, as either case makes it easier to sustain rotational mixing on the giant branch.

Sweigart & Mengel (1979) investigated meridional circulation as the possible agent for giant branch mixing. They assumed that mixing was inhibited by μ gradients, so neither ^{4}He mixing nor mixing on the lower giant branch was expected in their model. They used the classical expression for meridional circulation and assumed no internal angular momentum transport. With these assumptions, they found that mixing could be driven, but with rotation rates that exceeded the surface v sin i limits (and which violate present constraints on the internal angular momentum content of such stars).

Charbonnel (1994) reviewed the open cluster $^{12}C/^{13}C$ and C/N giant data, finding that the mass dependence could be explained if the μ gradient on the lower giant branch inhibited mixing. Note, however, that this gradient is smaller in more metal-poor stars, and the M92 carbon data (see Figure 7) seems to show mixing that begins earlier. Charbonnel (1995) then examined mixing by meridional circulation in giants. Her model assumes solid body rotation, inhibition of mixing by μ gradients, constant specific angular momentum in the convective envelope, and a constant surface rotation velocity of 1 km/s. Mixing can be generated in this model, although the angular momentum evolution it assumes should be verified with more detailed models. The onset of mixing in the data also appears to be more gradual than predicted in the model.

For further progress, self-consistent models that include both angular momentum evolution and mixing will be needed. Differential rotation can generate mixing, as discussed in Section 5, which may lower the rotation rates needed to drive mixing; angular momentum transport will reduce the rotation rates near the hydrogen-burning shell and will increase the rotation rates needed to drive mixing.

Other recent studies that include arbitrary mixing have been performed (Vandenberg & Smith 1988, Denissenkov & Denissenkova 1990, Langer et al 1993, Boothroyd et al 1995, Wasserburg et al 1995, Denissenkov & Weiss 1996, Weiss et al 1996, Sweigart 1997). These studies are consistent in requiring mixing to extend through the envelope essentially down to the hydrogen-burning shell. In particular, changes in surface O and Na require mixing in the vicinity of the hydrogen-burning shell, which implies some ability for the mixing to mix helium (Sweigart 1997). Explaining the details of the Mg, Na, and Al variations appears to present challenges for nuclear reaction rates, stellar evolution, or both (Zaidins & Langer 1997, Langer et al 1997).

Wasserburg et al (1995) also stress the importance of matching the $^{16}O/^{18}O$ ratios in asymptotic giants, and they argue for this as an additional indicator of mixing. In general, indicators that burn at lower temperatures exhibit changes at lower luminosity than indicators that burn at higher temperatures. ^{3}He destruction, rather than production, appears likely for the lowest mass stars.

The assumptions of instantaneous mixing to an arbitrary depth, a constant mixing time scale, or a constant diffusion coefficient are strong ones that have consequences for the physical behavior of the system. Model properties calibrated on one indicator can produce incorrect results for another that burns at a different temperature. For example, Wasserburg et al (1995) find that models that reproduce the $^{16}O/^{18}O$ ratios in asymptotic giants predict $^{12}C/^{13}C$ ratios that are too low. This probably indicates that the mixing is depth dependent (the ^{12}C in AGB stars arises from a deeper layer than the oxygen processing does). Chemical evolution models for ^{3}He depend sensitively on the properties ascribed to intermediate mass giants at low metal abundance (Dearborn et al 1996), and empirical checks on mixing in such stars are not available. A consistent physical model will probably be necessary for the problems of cosmological interest and for explaining all of the observed data.

7. SUMMARY: IMPLICATIONS OF MIXING

The need for mixing in stars rests on solid observational and theoretical ground. Three cases for mixing beyond that present in standard models have been reviewed: light element depletion in low mass stars, abundance anomalies in low mass giants, and deep mixing in massive stars. These departures from the predictions of standard stellar models will have implications for Big Bang nucleosynthesis (BBN) and possibly the ages of globular clusters. It is quite likely that an extensive reevaluation of the evolution of massive stars will also be needed.

Observational data on stellar abundances and stellar rotation have formed the basis for the study of mixing in stars. Nonstandard stellar models are more complicated than standard models, so empirical constraints are even more necessary for models that include such phenomena as rotational mixing than they are for standard models. With the advent of multiobject spectrographs, obtaining large abundance and rotation databases at high S/N is feasible. Lithium abundances in globular cluster MS stars and large samples of lithium abundances in old open clusters are two examples of projects that could place strong constraints on light element depletion in stars. In the case of mixing on the giant branch, establishing the luminosity at which mixing sets in would be an important step, as well as determining how the abundance pattern on the giant branch maps onto the horizontal branch and the AGB. The correlation between mixing on the giant branch, cluster age and composition, and rotation on the horizontal branch will also need to be carefully delineated. In the massive star regime, the existence of anomalies related to rotation has been demonstrated. Some such anomalies can be produced by models. The relationship between abundance anomalies, rotation, and evolutionary state in massive stars now needs to be clarified.

The angular momentum evolution of stars has been one of the greatest uncertainties in rotational mixing. Observational studies of stellar rotation, along with helioseismic studies of the internal rotation of the Sun, now place strong constraints on stellar models that include rotation. A particularly valuable contribution has been the realization of the connection between star formation and stellar rotation on the lower MS. A satisfactory theory of the origin of the observed range in rotation for massive stars does not yet exist. No true zero-age MS appears to exist for the most massive stars, for example. Further studies of rotation in stars will be essential for further progress.

Theoreticians need to take advantage of the work on the underlying physical mechanisms driving mixing. Recent revisions of the treatment of meridional circulation and the role of other hydrodynamic instabilities should be tested in stellar models. The long-standing questions of the roles of internal waves and magnetic fields on angular momentum transport have recently been addressed with quantitative work. The focus of theoretical work will likely shift from demonstrating that mixing is possible to quantifying how much mixing is expected for a given physical model and what the distribution of abundances should be. In addition, models should be checked in different phases of evolution and mass ranges. For examples, models that explain giant branch mixing should not overmix on the MS, and models that mix massive stars should not overmix intermediate mass stars. Hard and careful observational and theoretical work has established the limits of validity of the standard model of stellar structure and evolution. The implications of the deviations from the standard model now can, and should, be explored.

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