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MAGNETIC BRAKING OF STELLAR CORES IN RED GIANTS AND SUPERGIANTS

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ABSTRACT

Magnetic configurations, stable on the long term, appear to exist in various evolutionary phases, from main-sequence stars to white dwarfs and neutron stars. The large-scale ordered nature of these fields, often approximately dipolar, and their scaling according to the flux conservation scenario favor a fossil field model. We make some first estimates of the magnetic coupling between the stellar cores and the outer layers in red giants and supergiants. Analytical expressions of the truncation radius of the field coupling are established for a convective envelope and for a rotating radiative zone with horizontal turbulence. The timescales of the internal exchanges of angular momentum are considered. Numerical estimates are made on the basis of recent model grids. The direct magnetic coupling of the core to the extended convective envelope of red giants and supergiants appears unlikely. However, we find that the intermediate radiative zone is fully coupled to the core during the He-burning and later phases. This coupling is able to produce a strong spin down of the core of red giants and supergiants, also leading to relatively slowly rotating stellar remnants such as white dwarfs and pulsars. Some angular momentum is also transferred to the outer convective envelope of red giants and supergiants during the He-burning phase and later.

Key words: stars: magnetic field – stars: rotation

1. INTRODUCTION

The degenerate objects in the final stages of stellar evolution have strong magnetic fields. Conservation of the magnetic flux of main-sequence stars with fields in the range of 10 G to 1 kG would lead to neutron stars with magnetic fields in the range of 10^{11}–10^{13} G and to white dwarfs (WDs) with fields in the range of 10^5–10^7 G (Chanmugam 1992). This is generally consistent with the field of classical pulsars of the order of 10^{12} G. Recent results (Landstreet et al. 2012) indicate that about 10% of the WD are hosting magnetic fields in the range of 10^5–10^8 G. At present, it is not clear whether all WD have a magnetic field at a lower level.

The action of magnetic fields is not necessarily limited to the early and final evolutionary stages. The fields may be acting in various ways during star evolution, for example, by disk locking during the accretion phase in star formation (Hartmann 1998), by magnetic braking due to the stellar winds of solar type stars (Kawaler 1988) as well as of massive stars (Meynet et al. 2011), by influencing the distribution of internal rotation and mixing of the chemical elements (Maeder & Meynet 2003; Heger et al. 2005), by their interaction with meridional circulation, turbulence and shears (Mathis & Zahn 2005), by the viscous and magnetic coupling of the core and envelope (Spada et al. 2011), by producing the pulsar emission and spin down, etc.

The long-term stability of non-force-free magnetic topologies has been shown by Duez et al. (2010). According to these authors, this might help to understand some stable magnetic configurations existing in various evolutionary phases, from main-sequence stars to WDs and neutron stars. Duez et al. point out that the large-scale ordered nature of these fields, often approximately dipolar, and their scaling according to the flux conservation scenario favor a fossil field. Although pure poloidal or toroidal fields are unstable, an axisymmetric mixed configuration may provide a stable equilibrium topology (Tayler 1973, 1980; Markey & Tayler 1973).

The object of the present study is to study the effects of a dipolar magnetic field attached to stellar cores of relatively high density, as in the stellar stages from and further the He-burning phase. We examine whether a field with a scaling as a function of its host core density is able to exert some significant torque on the outer layers and to produce some coupling of the central region with the above radiative layers and parts of the extended convective envelope. Such a coupling may lead to an acceleration of the rotation of the outer layers and a braking of the central core. Among the consequences of this effect, the contrast of angular velocity between the core and envelope of the red giants and supergiants may become smaller, and the final remnants may have a slower rotation than predicted by current models that do not account for such effects.

The inclusion of the above effect in models is particularly interesting and topical, since the evolutionary models leading to pulsars always rotate much too fast compared to the observed rotation rates of pulsars (Heger et al. 2004; Hirschi et al. 2005). This is even the case when the magnetic field of the Tayler–Spruit dynamo (Spruit 2002), which could act in rotating radiative regions, is accounted for: some internal coupling is introduced, but it is insufficient to slow down the pulsars into the range of the observed rotation rates. A similar problem appears for WDs, which generally show rotation velocities much lower than predicted by standard models (Berger et al. 2005). The inclusion of the magnetic field which could result from the Tayler–Spruit dynamo within radiative regions improves the comparison, but not entirely since a difference of about one order of a magnitude remains for initial masses in the range of 1–3 $M_\odot$ (Langer 2007; Suijs et al. 2008). Here, we must note that the existence of the Tayler–Spruit dynamo is still strongly debated. MHD simulations of instabilities in the radiative zone of a differentially rotating star by Zahn et al. (2007) well show the presence of the Pitts and Tayler instability (Pitts & Tayler 1985), but find no sign of dynamo action. An analysis with a linear theory of the Tayler instability in the radiative zone of hot stars by Rüdiger et al. (2012) also confirms the absence of a dynamo action resulting from this instability.

Recent asteroseismic observations also indicate similar trends about the too slow rotation of stellar cores in red giants compared to model predictions. The precise determinations from *Kepler* of the rotational frequency splittings of mixed modes provide information on the internal rotation of red giants. Beck et al. (2012) showed that in the case of the red giant KIC 8366239 the core must rotate about 10 times faster than the surface. Deheuvels et al. (2012, 2014) analyzed the rotation profiles of six red giants and found that the cores rotate between about 3–20 times faster than the envelope. Moreover, they found that the rotation contrast between the core and envelope increases during the subgiant branch, indicating that the cores spin up with time, while the envelopes spin down. The observed differences of rotation between cores and envelopes are much smaller than predicted by evolutionary models of rotating stars. Analyses with stellar modeling by several groups (Eggenberger et al. 2012; Marques et al. 2013; Ceillier et al. 2013; Tayar & Pinsonneault 2013) clearly demonstrate that some generally unaccounted physical process is at work producing an important internal viscosity. Thus, there is an ensemble of observations pointing in favor of some additional internal coupling in star evolution.

In Section 2, we examine up to what level (the truncation radius) the magnetic field of central regions may produce some coupling of a convective envelope. On the basis of standard models of rotating stars, we do some numerical estimates. In Section 3, we do the same but for the intermediate radiative layers subject to horizontal turbulence. In Section 4, we examine the timescale of the magnetic coupling and calculate the change of the rotation for the core and for the external layers. In each case, we make some numerical estimates. Section 5 gives the discussion and conclusions.

2. THE EXTENSION OF THE MAGNETIC BRAKING IN A ROTATING CONVECTIVE ENVELOPE

We consider a rotating stellar core, partly degenerate, of a red giant or red supergiant in the stage of central He-burning or in subsequent stages. We assume that a magnetic field of fossil origin is attached to this dense core and that it is extending in the external regions where dissipation occurs. At the surface of the core, the magnetic field has an intensity B_c. The cases of a dipolar field is considered in this work. Indeed, a dipolar geometry is the preferred one for a magnetic field of fossil origin as demonstrated analytically by Duez & Mathis (2010), who examined the possible magnetic equilibrium configurations of initial fossil fields. This conclusion is in agreement with numerical simulations performed by Braithwaite & Spruit (2004).

Here, we consider the intensity B_c of the fossil field in the core as a free parameter of the models, although it is confined within certain limits according to the density scaling as discussed below. Numerical models will explore the consequences of various field intensities. This may allow us to estimate the range of field intensities necessary to reproduce the differences of angular velocities between the core and envelope of red giants, as obtained from asteroseismic observations. We are guided in our estimate of the field intensity by the rough scaling of the pulsar magnetic field of about 10^{12} G for a density of about $10^{14.6}$ g cm^{-3}. Since the pulsar fields are consistent with the flux conservation of main-sequence stars (Chanmugam 1992), we may hope that this hypothesis is somehow verified in the intermediate stages between main-sequence and neutron stars. With flux conservation in the stellar mass during evolutionary stages, we would get for densities of 10^3, 10^4, 10^5, and 10^6 g cm^{-3} magnetic fields of 2×10^4, 9×10^4, 4×10^5, and 2×10^6 G, respectively. This is uncertain, but nevertheless indicates the kind of field intensities which could be expected in stellar cores. Thus, we have some guidelines about the range of possible fields in the core of red giants and supergiants, which may be further constrained by asteroseismic observations. In case the new field estimates would be very different, this might reveal other processes, such as efficient dynamos, at work for coupling the cores and envelopes.

The field outside the core behaves like

$$B(r) = B_c \left(\frac{R_c}{r}\right)^n . \tag{1}$$

As a result of the flux conservation, $n = 3$ for a dipolar field (Kawaler 1988; Hartmann 1998, while n would be equal to two in a radial spherical case). We note that for low mass stars, a value intermediate between $n = 2$ and $n = 3$ corresponds to the Skumanich law for the long-term decrease of the rotational velocities of solar type stars with time (Skumanich 1972). For Ap stars, the observations suggest a variety of geometries, with a dominant large-scale dipolar component and strong deviations of axisymmetry, with often some indications of multipolar components (Bagnulo 2001) and even strong small-scale components (Silvester et al. 2011). A dipolar field is likely to be preferred on large scales, as mentioned in Section 1. Thus, below we will consider the case $n = 3$ for the field extending in radiative shells outside the core and possibly through a part of the extended convective envelope of red giants or supergiants. For simplification, we assume full magnetic coupling up to a certain truncation radius r_t in the convective envelope and no coupling outside this level.

This is evidently a simplified way of treating the complex process of winding up the field lines and magnetic coupling in the limiting region. Such magnetic interactions between a radiative core and a convective envelope have been studied by several authors, particularly in the case of the solar spin down. By assuming a time independent poloidal field, Charbonneau & MacGregor (1993) identify essentially three different phases of the interactions: a rapid buildup of a toroidal field component in the radiative zone, then phase mixing (interaction of Alfvén waves on neighbor field lines) leading to the damping of the large-scale toroidal oscillations and coupling of the rotation profile. The third stage is a quasi-stationary evolution with a toroidal field such that its stress, added to the viscous one, compensates for the torque at the solar surface, while the rotation rate becomes almost constant on the surfaces of constant poloidal field (Ferraro's state of iso-rotation). These three different phases of magnetic coupling between core and envelope were also well confirmed by the detailed analysis of Spada et al. (2011): (1) linear buildup of toroidal field; (2) torsional Alfvén waves; and (3) quasi-stationary evolution, where the first two phases may typically last for less than 1 Myr. They emphasize the need of a viscosity enhancement by a factor of 10^4 with respect to molecular viscosity and also pointed out that the transport of angular momentum by the Tayler instability appears negligible.

Numerical three-dimensional MHD simulations of the magnetic coupling between the solar core and envelope were also performed by Strugarek et al. (2011a, 2011b) with the aim to examine whether the thinness of the solar tachocline may result from magnetic confinement (this is the transition region from the convective envelope where the rotation varies with latitude to the radiative core where the rotation is almost uniform). They found that the magnetic confinement fails to explain the thinness of the tachocline. Most interestingly, they demonstrated that the interior magnetic field does not stay confined in the radiative core, but expands into the convective zone. This leads to an efficient outward transport of angular momentum through the tachocline, imposing Ferraro's law of iso-rotation over some distance. However, subsequent numerical models by Acevedo-Arreguin et al. (2013) reconsider the magnetic confinement by a large-scale meridional circulation associated to the convective envelope first proposed by Gough & McIntyre (1998) and support the results of these authors.

2.1. Estimate of the Truncation Radius in a Convective Zone

The dipolar magnetic field extends in the outer convective zone, which is subject to vertical motions and strong turbulence, which may interfere with the effects of the magnetic coupling. As the velocity of the convective motions increases from the deep interior to reach sonic velocities in the very external regions with partial hydrogen ionization, the effects associated to convection increase outward. There are several effects which may influence the penetration of the fossil field in the convective envelope and the extension of the magnetic coupling. The two-dimensional geometry of the axisymmetric field is important in the global description of the field as shown by Charbonneau & MacGregor (1993), who also account for the interaction of the field components with differential rotation and of the damping of magnetic oscillations. Also, we may mention such effects as the initial conditions, the Ohmic diffusion, the interaction of the field with the meridional circulations both in the radiative and convective regions, the interactions with rotational shears, and convective motions and turbulence—an ensemble of effects that were often studied in relation with the problem of the thinness of the solar tachocline.

In the present work, we assume that the truncation radius r_t lies at the place where the energy density u_B of the magnetic field B is equal to some factor *f* multiplying the energy density of the convective motions $u_{\rm conv}$. We introduce this factor *f* with values 0.1, 1.0, and 10 to account for the uncertainties associated to the multiple processes which may influence the magnetic coupling of a dipolar field to an external convective zone. The use of such a factor may also allow us to test the robustness of the conclusions we obtain about the coupling.

Below r_t, magnetic energy dominates and coupling is present, however the coupling is not instantaneous and in Section 4 we account for the timescale of the process. Above r_t convection dominates and there is no magnetic coupling. One has

$$u_B(r) = \frac{B^2(r)}{8\pi} \quad \text{and } u_{\rm conv} = \frac{1}{2}\,\varrho\, v_{\rm conv}^2. \tag{2}$$

The cgs system of units is used here with 1 G = 1 g$^{1/2}$ cm$^{-1/2}$ s^{-1}. For the convective velocity, one takes the usual expression appropriate to adiabatic interior stellar regions in the mixing-length theory:

$$v_{\rm conv}^2 = g\,\delta(\nabla - \nabla_{\rm ad})\,\frac{\ell^2}{8H_P}, \tag{3}$$

where the usual notations of stellar structure have been used. The mixing-length is usually given by $\ell = \alpha\, H_P$, where $\alpha \cong 1.6$ for the Sun. The energy density of the convective motions is

$$u_{\rm conv} = \frac{1}{16}\alpha^2\, g\,\varrho\,\delta(\nabla - \nabla_{\rm ad})\,H_P. \tag{4}$$

Expressing the equality

$$u_B(r_t) = f\; u_{\rm conv}(r_t), \tag{5}$$

we get for the ratio of truncation radius to the core radius in the case of a dipolar field:

$$\left(\frac{r_t}{R_c}\right) = \left(\frac{2\,B_c^2}{f\;\pi\alpha^2\,g\varrho\delta(\nabla - \nabla_{\rm ad})H_P}\right)^{\frac{1}{6}}. \tag{6}$$

We notice the very weak dependence on the factor *f* and also see that the truncation radius in the convective envelope increases slowly with the magnetic field of the core; for lower densities and gravities, it extends further out.

2.2. Numerical Estimates During the Red Supergiants

Let us consider the case of a model of a 15 $M_\odot$ star with solar metallicity $Z = 0.014$ and an initial rotation velocity equal to 40% of the critical value (Ekström et al. 2012). We first examine the situation during the helium-burning phase when the central helium mass fraction is $Y_c = 0.30$ (age = 14.467535×10^6 yr), with a central density of 1.26×10^3 g cm^{-3}. The star with an actual mass of 12.991 $M_\odot$ is in the stage of red supergiant with $\log(L/L_\odot) = 4.868$ and $\log T_{\rm eff} = 3.566$. The core totally deprived of hydrogen ($X_c < 10^{-10}$) extends up to $M_r/M = 0.328$ with a radius of $r = 2.164 \times 10^{10}$ cm, while the base of the convective envelope lies at $M_r/M = 0.687$, with $r = 3.954 \times 10^{12}$ cm, i.e., at a distance equal to 182.7 times the size of the core. At such a large distance, the dipolar field has so much decreased that it is not likely to exert some coupling of the convective envelope. Only an extreme dipolar core field with a very unrealistic intensity of 10^{11} G or more could build an efficient coupling of the convective envelope at this stage, while a field of about 2.2×10^4 G would be expected according to the central density and the pulsar scaling.

The same mass model at the end of the central C-burning stage at an age of 15.065866×10^6 yr with a central density of 4.37×10^6 g cm^{-3} looks slightly less unfavorable. The core totally deprived of hydrogen extends further out up to $M_r/M = 0.4260$ with a radius of $r = 2.357 \times 10^{10}$ cm, while the base of the convective envelope goes deeper than during the He-burning phase down to $M_r/M = 0.469$, with $r = 2.565\times 10^{11}$ cm, i.e., at a distance equal to only 10.88 times the size of the core. The estimate of the ratio (r_t/R_c) from Equation (6) in the region at the base of the convective envelope indicates that due to the fast decrease of the dipolar field, a field in the core of 2.5×10^8 G (for $f = 1$) would be necessary to produce the coupling of the core up to the base of the convective envelope. For *f* values of 0.1 or 10, the necessary core field would be 8×10^7 G and 8×10^8 G, respectively. Now, adopting the rough scaling of pulsar fields mentioned in Section 2, we see that the central density would imply a magnetic field of 5×10^6 G. Thus, we see that, even if *f* is equal to 0.1, it seems very unlikely that a coupling of the regions at the base of the convective envelope is possible, and this by a relatively wide margin.

We note that the envelope is extremely extended, the star radius is 886.2 $R_\odot$, which represents 2625 times the core radius. Near the stellar surface, the magnetic dipolar field would be as low as 8.4×10^{-4} G and totally unable to produce some coupling of the outer layers subject to nearly sonic velocities. We note that this value is very different from the value of a few Gauss found at the surface of red giants (Konstantinova-Antova et al. 2013). We take this as an indication that the fossil field does not permeate the huge convective envelope in which the interaction of convective motions and rotation may produce some dynamo like in the solar case. The observed field may thus be a result of the stellar convective dynamo rather than a signature of central fossil field.

From these numerical estimates, we reach the following conclusion. Magnetic fields consistent with the pulsar field-density scaling (Section 2) in the core of stars in the H-burning or later evolutionary phases appear unable to produce a magnetic coupling of the large convective envelopes present in red giants and supergiants.

3. THE TRUNCATION RADIUS IN A ROTATING RADIATIVE ZONE

We have seen above that in the stages of helium burning, there may be a large zone between the core and the outer convective envelope. The intermediate region may be totally or partly occupied by a radiative zone. Indeed, depending on mass loss, there may exist some small convective zones in this intermediate region. If so, within such convective zones, the appropriate expression (Equation (6)) of the truncation radius also applies.

In the intermediate radiative region, many physical variables are generally changing rapidly, such as the density, the composition, and also the angular velocity Ω. The layers in differential rotation are subject to turbulence both vertical and horizontal. The horizontal turbulence is the largest since it is not damped by the vertical density stratification (Zahn 1992). This turbulence is characterized by a diffusion coefficient $D_{\rm h}$, which is rather uncertain and various expressions have been proposed for it (Zahn 1992; Maeder 2003; Mathis et al. 2004). Moreover, the horizontal turbulence can be modified by the presence of the field.

3.1. Estimate of the Truncation Radius in a Horizontal Turbulent Zone

The velocity $v_{\rm h}$ of the horizontal turbulence can be written

$$v_{\rm h} = \frac{D_{\rm h}}{r}, \tag{7}$$

since the turbulence is highly anisotropic there is no factor of one-third here. The corresponding energy density $u_{\rm h}$ is

$$u_{\rm h} = \frac{1}{2}\,\varrho\,\frac{D_{\rm h}^2}{r^2}. \tag{8}$$

As in Section 2.1, we define the truncation by the location where the energy densities are equal and also consider an f factor equal to 0.1 or 10 in order to account the many uncertainties intervening in the coupling,

$$u_B(r_t) = f\,u_{\rm h}(r_t)\,. \tag{9}$$

With Equations (1) and (2), this gives

$$\frac{B_{\rm c}^2}{4\pi}\,\frac{R_{\rm c}^6}{r_t^4} = f\,\varrho\,D_{\rm h}^2. \tag{10}$$

We obtain for the ratio of the truncation to the core radius:

$$\left(\frac{r_t}{R_c}\right) = \left(\frac{B_c^2\,R_c^2}{4\pi\,f\,\varrho\,D_{\rm h}^2}\right)^{\frac{1}{4}}. \tag{11}$$

Consistently, this ratio increases with the magnetic field and decreases with increasing turbulence. We see that the truncation is more sensitive to the field intensity in the case of horizontal turbulence than in the case of convective motions. We also notice that the truncation radius depends very weakly on the power $-(1/2)$ of the coefficient of horizontal turbulence.

The expression of the coefficient of horizontal turbulence $D_{\rm h}$ currently used in Geneva models in that by Zahn (1992):

$$D_{\rm h} = \frac{1}{c_{\rm h}}\,r|2\,V_2 - \alpha\,U_2|, \tag{12}$$

where V_2 and U_2 are, respectively, the amplitudes of the radial and horizontal components of the velocity of meridional circulation. The coefficient $c_{\rm h}$ is normally $\leqslant 1$, but is generally taken equal to one, while $\alpha = (1/2)[d\ln(r^2\,\Omega)/d\ln r]$, which is equal to one in region of uniform rotation. Below, we mention the cases of the other diffusion coefficients of horizontal turbulence (Mathis et al. 2004; Maeder 2009).

3.2. Numerical Estimates in Radiative Zones of Red Supergiants

We first consider the same model as in Section 2.2 of a 15 $M_\odot$ star at the stage of central He burning with $Y_{\rm c} = 0.30$ in the red supergiant phase. We recall that in this model the base of the convective envelope lies at a distance equal to 182.7 times the size of the convective core. The layers between the core and the convective envelope are fully radiative. (In some cases, there could be some intermediate convective zones, particularly in models with little or no mass loss, but in the present model the whole intermediate zone is radiative).

The horizontal turbulence is $D_{\rm h} = 8.11 \times 10^{10}$ cm^2 s^{-1} and the density $\varrho = 3.85 \times 10^{-6}$ near the top of the intermediate zone (at $M_r/M = 0.673$, this means at 1.4% in mass fraction below the bottom of the outer convective zone). We make such a choice in order that the value of $D_{\rm h}$ is not influenced by some edge effects. We find with these values and expression (11) that a dipolar field in the core equal to about 8.7×10^2 G would be sufficient to fully couple the intermediate zone to the core. Such core fields are much lower than the one approximately suggested (2.2×10^4 G) by the pulsar field scaling on the actual central core density (1.26×10^3 g cm^{-3}), as given at the beginning of Section 2. Thus, we conclude that the intermediate radiative zone is likely to be fully magnetically coupled to the core. This conclusion applies whatever the factor f, since we consider the top of the intermediate radiative zone. From the dependence of (r_t/R_c) on B_c and $D_{\rm h}$ in expression (11), we notice that the above conclusion on the coupling remains valid even if $D_{\rm h}$ is larger by one order of magnitude than given here. However, this would not necessarily be the case if $D_{\rm h}$ would be larger by several orders or magnitude and become close to values of diffusion more typical of convective zones.

For the model of Section 2.2 at the end of the C-burning phase, the intermediate radiative zone extends up to 10.88 times the radius of the core. The coefficient of horizontal turbulence ($D_{\rm h} = 4.69 \times 10^8$ cm^2 s^{-1}) near the top of the radiative zone is much smaller than the coefficient of convective diffusion of the order of a few 10^{16} cm^2 s^{-1} at the base of the convective zone. This means that the perturbations of the magnetic field by the horizontal turbulence in the radiative region are extremely small compared to the effects of the turbulence in the convective zone. We find that even a dipolar field of 1 G in the core would be sufficient to couple the radiative zone up to its top.

We conclude the following.

1. Radiative rotating stellar zones, subject to horizontal turbulence, are much more easily coupled to the core than convective regions. The reason is the much lower turbulent diffusion coefficient than the one characterizing convection.
2. Even weak dipolar fields attached to the core are sufficient to couple an extended intermediate radiative zone up to the base of the convective envelope. This applies to the phase of He burning and is even more likely in further evolutionary phases. However, the overall results also depend on the timescales of the process as shown in the next section.

4. THE EXCHANGE OF ANGULAR MOMENTUM

The magnetic coupling between the core and the external layers produces a slowing down of the core and a gain of angular momentum by the outer layers. The treatment of the problem is also depending whether these layers are radiative or convective.

4.1. The Reduction of the Angular Momentum in the Core

Let us examine the reduction of the angular momentum J_c of the core of mass M_c, radius R_c, and angular velocity Ω_c due to the coupling with the above layers up to truncation radius r_t. Assuming that the spherical symmetry of the inner layers is not much modified by rotation, we write the angular momentum of the core,

$$J_c = \left(\frac{2}{3}\right) \Omega_c \int_0^{M_c} r^2 dM_r. \tag{13}$$

Whether convective or radiative, the core is supposed to rotate uniformly due to the internal magnetic coupling. The reduction of the angular momentum of the core due to the coupling of the layers from M_c to $M(r_t)$ is

$$\delta J_c = -\left(\frac{2}{3}\right) \int_{M_c}^{M(r_t)} r^2 \left[\Omega_c - \Omega(M_r)\right] dM_r\,, \tag{14}$$

since the core generally rotates faster than the envelope, as a result of core contraction. The rate (dJ_c/dt) of the change of the angular momentum of the core provides the change of the angular velocity of the core:

$$\frac{1}{\Omega_c}\frac{d\Omega_c}{dt} = \frac{1}{J_c}\frac{dJ_c}{dt}. \tag{15}$$

Here, we must not account for the change of the momentum of inertia of the core over a time step because the consequences of stellar expansion or contraction on the transport of angular momentum are already accounted for in the general equation describing the evolution of $\Omega(M_r)$, see, for example, expressions (10.121) and (10.122) in Maeder (2009).

The frequency of a magnetic wave is the Alfvén frequency ω_A:

$$\omega_A(M_r) = \frac{B(M_r)}{r\,(4\pi\,\varrho)^{1/2}}. \tag{16}$$

The characteristic timescale for the exchange of angular momentum between differentially rotating layers is the Alfvén timescale $\omega_A^{-1}(M_r)$, calculated with the poloidal component of the magnetic field (Mestel 1970). The rate of change of the angular momentum thus becomes

$$\frac{dJ_c}{dt} = -\left(\frac{2}{3}\right) \int_{M_c}^{M(r_t)} r^2 \left[\Omega_c - \Omega(M_r)\right] \omega_A(M_r)\, dM_r\,. \tag{17}$$

At each time step, this expression permits (with the parameters of the considered time step) to calculate the change of the rotation of the regions within the radius R_c. Some attention is to be paid to the definition of the core of mass M_c. It is not necessarily the mass of the convective core at the considered evolutionary phase, since in some advanced nuclear phases central convection is even absent. We suggest to consider that the outer limit of the core with strong magnetic field is located in the region of the very steep density gradient, which separates the central region deprived of hydrogen from the envelope where hydrogen is present. The variation of the magnetic field with the density at the power $2/3$ applies when the mass and the magnetic flux remains constant. After the end of core He burning, the core mass has essentially reached its final value, but its radius can change, so that from this point onward the dependence of the field on density is likely very well represented by the $2/3$ exponent. Detailed numerical models will allows us to explore the domain of parameters.

4.2. Gain of Angular Momentum by the External Layers

Now, we have also to be concerned by the gains of angular momentum and changes of angular velocity of the magnetically coupled layers external to the core. Let us first examine the case of the radiative layers above the core. Each layer of mass thickness ΔM_r (the width of the mass steps in the models) between radius r_1 and r_2 ($r_2 > r_1$) has a certain momentum of inertia $\Delta I(M_r)$ at the time considered:

$$\Delta I(M_r) = \frac{2}{5}\,\frac{r_2^5 - r_1^5}{r_2^3 - r_1^3}\,\Delta M_r\,. \tag{18}$$

A mass shell is usually characterized by a mean value of the density and thus this assumption applies. The shell has a certain amount of angular momentum $\Delta J(M_r)$,

$$\Delta J(M_r) = \Delta I(M_r)\,\Omega(M_r)\,. \tag{19}$$

The rate of increase of the angular momentum due to the magnetic torque on this shell per unit of time is

$$\frac{d[\Delta J(M_r)]}{dt} = \Delta I(M_r)\left[\Omega_c - \Omega(M_r)\right]\omega_A(M_r)\,. \tag{20}$$

Thus, the corresponding rate of change of the angular velocity $\Omega(M_r)$ is

$$\frac{1}{\Omega(M_r)}\frac{d\Omega(M_r)}{dt} = \frac{1}{\Delta J(M_r)}\frac{d[\Delta J(M_r)]}{dt}. \tag{21}$$

Again, we have not to account for the change of the momentum of inertia for the same reasons as above. Expression (21) can be rewritten with the help of Equations (20) and (19) at the level M_r (which we now generally omit in the equations below):

$$\frac{d\Omega}{dt} = \Omega\frac{1}{\Delta J}\Delta I\left[\Omega_c - \Omega\right]\omega_A = \left[\Omega_c - \Omega\right]\omega_A. \tag{22}$$

This just means that for a shell of a given mass and momentum of inertia the change of Ω due to the magnetic torque is equal to the difference of angular velocity between the core and the considered layer multiplied by the ratio of the interval of time dt to $(1/\omega_A)$, the characteristic time of the coupling. This differential equation is easily integrated, if we consider that ω_A

is constant, which is valid over a small interval of time Δt. The solution is

$$[\Omega_c - \Omega(\Delta t)] = [\Omega_c - \Omega(\Delta t)]_0 \, e^{-(\omega_A \, \Delta t)} \tag{23}$$

at the mass level M_r. It gives the Ω difference after the time interval Δt as a function of the Ω difference at the beginning of this time interval. Since ω_A is not a constant but a slowly varying function of time, the interval Δt must at each level be small with respect to $(1/\omega_A)$. In case the timescale $(1/\omega_A)$ at a certain level M_r would be very small, i.e., negligible with respect to the evolutionary timescale, the solution is simply to assume $\Omega(M_r) = \Omega_c$, i.e., that magnetic equilibrium is instantaneous. The effects of meridional circulation and shears, when present, have also to be included, as is usual in models of rotating stars, for the calculation of the evolution of $\Omega(M_r)$.

For a convective envelope, some further operations are needed. Such envelopes are often considered to the first order as having a solid body rotation due to the very high viscosity associated to convective turbulence. In such case, one also has to estimate by Equation (20) the rates of change $(d[\Delta J(M_r)]/dt)_i$ of the angular momentum for all convective layers $i = 1$ to N up to the truncation radius. In this way, the various changes $\Delta[\Delta J(M_r)]_i$ over a time step Δt are obtained. Then, by summing up all the changes one gets the total change ΔJ_{conv} of angular momentum of the convective envelope due to magnetic coupling during that interval of time:

$$\Delta J_{\mathrm{conv}} = \sum_{i=1,N} \Delta[\Delta J(M_r)]_i . \tag{24}$$

Now, the change $\Delta\Omega_{\mathrm{conv}}$ of angular velocity Ω_{conv} of the convective envelope over the time interval Δt is obtained by

$$\frac{\Delta\Omega_{\mathrm{conv}}}{\Omega_{\mathrm{conv}}} = \frac{\Delta J_{\mathrm{conv}}}{J_{\mathrm{conv}}}. \tag{25}$$

The usual treatment of convective envelopes in our models conserves the angular momentum of the convective envelope with a proper account of the change of the momentum of inertia. Thus, Equation (25) represents only the change of angular velocity due to the magnetic torque.

There the angular momentum J_{conv} is given by

$$J_{\mathrm{conv}} = \left(\frac{2}{3}\right) \Omega_{\mathrm{conv}} \int_{M_{\mathrm{base}}}^{M_{\mathrm{top}}} r^2 \, dM_r \, , \tag{26}$$

where the integration is performed from the base to the top of the convective envelope. In red giants and supergiants, the top of this envelope is the total stellar radius.

4.3. Numerical Estimates of the Timescale of the Coupling During the Helium-burning Phase

The timescale of the magnetic coupling is a major characteristic of the process determining the kind of distribution of Ω in the stars at the various phases. Let us first consider the same model of a 15 $M_\odot$ in the stage of central helium burning with $Y_c = 0.30$ as in Sections 2.2 and 3.2. We have seen that at this stage the magnetic coupling is possible for the intermediate zone, while it seems unlikely for the outer convective envelope. At the edge of the core deprived of hydrogen ($M_r/M = 0.328$), the Alfvén frequency for a typical field in the core of 2.2×10^4 G as given by the pulsar scaling is $\omega_{\mathrm{A}} = 4.96 \times 10^{-8}$ s^{-1}. This corresponds to a timescale of 0.64 yr. Thus, in view of the long timescale of the He-burning phase (1.49×10^6 yr), we conclude that the coupling is extremely fast at the edge of the core.

Now, we examine the timescales near the outer edge of the intermediate zone, at $M_r/M = 0.673$ like in Section 3.2. There, for the same typical intensity of the dipolar field in the core of 2.2×10^4 G, we have $\omega_{\mathrm{A}} = 4.07 \times 10^{-13}$ s^{-1}, corresponding to an Alfvén timescale of 7.8×10^4 yr. This timescale is longer than the one at the edge of the core. This value is shorter than the timescale of 1.49×10^6 yr of the He-burning phase of a rotating star of 15 $M_\odot$. Thus, for fields in the range of 10^3–10^5 G compatible with the pulsar scaling (Section 2), the timescale for the magnetic coupling of the dipolar field is short enough that we may expect full coupling up to the truncation radius. We recall that this radius is unlikely to be in the outer convective envelope, but the intermediate radiative zone is likely fully coupled.

Globally, we can say that the red giants and supergiants in the He-burning phase will experience no direct magnetic coupling between the core and the outer convective envelope, but the intermediate radiative zone between the core and the envelope will be magnetically coupled to the core. This intermediate radiative zone at the stage $Y_c = 0.30$ contains a fraction of the stellar mass comparable to that of the core deprived of hydrogen, while the momentum of inertia of the intermediate zone is at least one order of magnitude larger than that of the core. Thus, one may expect a strong braking of the core rotation during the He-burning phase. Through the other transport mechanisms such as shear, meridional circulation, and convection, some transport of angular momentum to the outer convective envelope may also result.

4.4. The Timescale at the End of the Central C-burning Phase

We now examine the timescale at the edge of the core deprived of hydrogen ($M_r/M = 0.426$). We consider a field of 5×10^6 G at the surface of the core, consistent with the pulsar scaling. The Alfvén frequency is 1.90×10^{-5} s^{-1}, which corresponds to a timescale of 14.6 hr. The coupling is thus instantaneous at the edge of the core.

We have seen that the intermediate zone may experience the magnetic torque, but not the convective envelope. Thus, we consider the situation near the top of the intermediate radiative zone as before. The same dipolar field of 5×10^6 G at the surface of the core leads here to a field 4.41×10^3 G. The corresponding Alfvén frequencies are 2.97×10^{-7} s^{-1}, which corresponds to a timescale of 0.11 yr. Thus, the coupling of the core with the entire intermediate radiative zone is very fast in the C-burning phase and full coupling up to the truncation radius is expected.

5. CONCLUSIONS

These are first estimates of the possible magnetic coupling between dense stellar cores and external layers based on a simple modeling. Despite this fact, the first estimates of the order of magnitude of the main intervening effects support some clear conclusions.

1. The direct magnetic coupling of the convective envelopes with the stellar cores is very unlikely throughout the advanced evolution.
2. However, the magnetic coupling between the core and the intermediate radiative zone appears very effective during the He-burning phase in red supergiants and even more in later phases.

3. This coupling of the core and intermediate radiative zone is able to produce a strong spin down of the core of red giants and supergiants, leading also to relatively slowly rotating stellar remnants such as WDs and pulsars.
4. The slowly rotating envelopes of supergiants will receive some additional angular momentum from the He-burning phase and later for two reasons. First, because the other processes of angular momentum transport, such as shear and circulation, transmit anyway some angular momentum between the radiative and convective envelope; and second, because as evolution proceeds the convective envelope gets much deeper and encompasses layers which have been spun up previously.

As mentioned in the Introduction, magnetic braking may intervene at all stages of stellar evolution. The properties of the magnetic field needs to be further studied by detailed numerical simulations through the various evolutionary stages. The first results presented here, which seem rather robust in view of the orders of magnitude obtained, are encouraging for future study concerning the magnetic coupling between stellar cores and the adjacent radiative layers.

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