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Stellar Rotation II

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1 Diffuse this!

While the interiors of stars are unstable to various MHD phenomena, detailed calculations of such processes during the long timescale of stellar evolution are currently impossible. One has to greatly simplify the problem and derive approximate scalings that can be included in the 1D stellar evolution calculations. A common approach is to use a diffusion approximation, where the transport of quantities across the vertical direction inside the star is calculated using a diffusion equation of the kind:

$$\left(\frac{\partial X_n}{\partial t}\right)_m = \left(\frac{\partial}{\partial m}\right)_t \left[(4\pi r^2 \rho)^2 D \left(\frac{\partial X_n}{\partial m}\right)_t\right] + \left(\frac{\mathrm{d}X_n}{\mathrm{d}t}\right)_{\mathrm{nuc}},$$

where D is the diffusion coefficient constructed from the sum of individual mixing processes (e.g. convection or rotational instabilities) and X_n the mass fraction of species n. The second term on the right hand side accounts for nuclear reactions. The diffusion equation is solved at each time step.

Similarly the transport of angular momentum can also be treated in a diffusion scheme (even if a more complex advection-diffusion scheme has also been adopted by certain groups, e.g. Zahn 1992, Meynet & Maeder 2000). A similar diffusion equation for the angular momentum is solved at each timestep. See for example the pioneering work of Endal & Sofia 1978 and Pinsonneault et al. 1989 and the more recent works of Heger et al. 2000 and Potter et al. 2012.

MESA Star, like other stellar evolution codes (e.g. KEPLER and STERN), is solving the transport in the diffusion approximation and you already had a look at some of the calculated diffusion coefficients in previous problems. Now, imagine you just had an amazing insight on the internal physics of stars: You came out with some new theory for the transport of angular momentum and/or chemical species. You wanna see what is the impact on stellar evolution and observable properties to verify, and eventually calibrate, your theory. MESA provides a great way to solve problems like this, allowing to access and modify internally calculated quantities just adding code to your local work directory in the `run_star_extras` file.

In this exercise you will add an extra diffusion of chemical species in a star. This is quite simple:

1. Locate and inspect the `run_star_extras.f` file in your work directory
2. Locate and inspect the `other_am_mixing.f` file in `star/public/`

Follow the instructions in `other_am_mixing` to modify your `run_star_extras` file in order to include, at every timestep, a diffusion coefficient for chemical mixing with constant value :

```
nu_additional = 3d4 ! cm^2/s
```

You can do this for your favorite star, or you can just use the $15 M_\odot$ star you evolved yesterday. Compile your local work directory (./mk) and run your star from ZAMS for a few timesteps. Plot the relevant quantities showing your `run_star_extras.f` is doing what you want.

2 Magnetic Braking

In this exercise you will try to implement a new piece of physics in MESA: Magnetic braking. In the presence of mass-loss, angular momentum is removed by the material in the stellar wind, resulting in a spin-down effect. This is quite important for example in massive stars, that are known to be rapidly rotating and have strong stellar winds. Some of these stars have also been found to have large scale magnetic fields, with amplitudes between hundreds up to thousands of Gauss.

In stars with surface magnetic fields, the presence of such field will interplay with the mass-loss. In particular, if the kinetic energy density in the wind is smaller than the magnetic energy density at the surface of the star, then the flow will have to follow the magnetic field. As a consequence the material will leave the stellar surface with a higher specific angular momentum, as the co-rotation radius has increased (bigger lever arm). As you can imagine such co-rotation radius will roughly correspond to the Alfvén radius R_A, that in this problem corresponds to the location where the radial components of the field and flow have equal energy density.

The angular momentum loss was first calculated, under simplified assumptions, by Weber & Davis (1967) to study the angular momentum loss of the solar wind. In their seminal work they found that the angular momentum lost by a magnetic rotator is:

$$\dot{J} = \frac{2}{3}\dot{M}\Omega R_A^2 \,, \qquad (2.1)$$

where Ω is the surface angular velocity and $\dot{M}$ is the mass-loss rate. Detailed MHD calculations of this process have been performed, leading to more precise formulations (see e.g. ud-Doula & Owocki 2002, ud-Doula et al. (2008,2009)). However the scaling shown in Eq. 2.1 appears quite robust, and is therefore a good starting point to implement magnetic-braking in MESA.

It is useful to define the wind-confinement parameter η, the energy density ratio between ra-

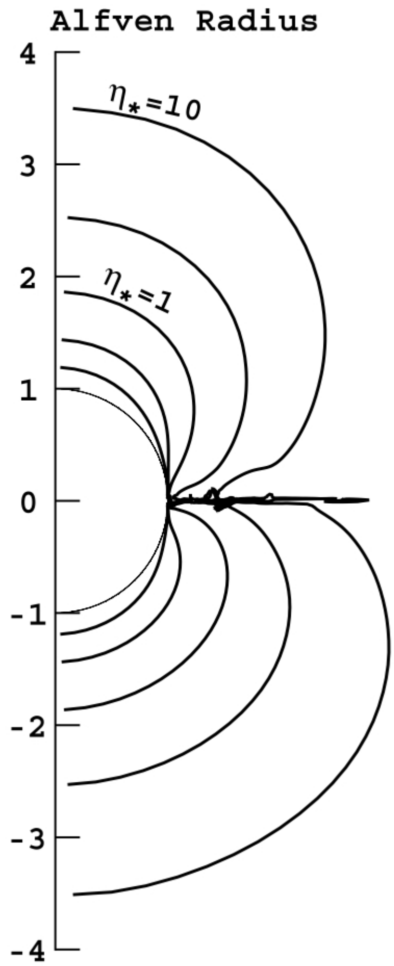

Figure 2.1: Contours of the Alfvén radius as function of η in the MHD models of ud-Doula & Owocki 2002

dial magnetic field and flow

$$\eta(r) \equiv \frac{B_r^2/8\pi}{\rho v_r^2/2}\,. \tag{2.2}$$

This quantity can be rewritten at the stellar surface as $\eta_* = R_*^2\, B^2/\dot{M}\, v_\infty$, where v_∞ is the terminal velocity of the stellar wind. The line-driven winds of massive OB stars have terminal velocities that scale with the photospheric escape velocity ($v_\infty \simeq 1.92 v_{\rm esc}$, see e.g. Lamers & Cassinelli 2000), and are therefore of the order $\sim 1000 - 4000\,{\rm km\,s^{-1}}$ or so. Using the wind confinement parameter, it is possible to re-write Eq. 2.1 in a form that mostly only depends on values already calculated in MESA:

$$\dot{J} \approx \frac{2}{3}\dot{M}\Omega R_*^2 \eta_*\,, \tag{2.3}$$

The quantities Ω, $\dot{M}$, R_* can be directly extracted from the `Star_info` data structure. B is our parameter (we will first assume the magnetic field is constant during the evolution, so this can become an inlist control parameter. Q: Do you think this is a good assumption?). v_∞ can also be calculated self-consistently, using the definition of escape velocity:

$$v_\infty \simeq 1.92\, v_{\rm esc} = 1.92 \times 618 \left(\frac{R_\odot}{R_*}\,\frac{M_*}{M_\odot}\right)^{1/2} {\rm km\,s^{-1}}. \tag{2.4}$$

Improvements on Eq. 2.3 can be found in the series of paper of ud-Doula et al. (2008,2009) or in Matt et al. 2012, but here you have enough to start playing around. You'll see that there are more important / basic things to solve, before implementing the latest formulation of the theory.

Now that you know how to calculate $\dot{J}$ how can you extract the angular momentum from the stellar structure? The answer can be found in `star\public\other_torque.f`. Examine the file and read Bill's comments. You also wanna check how the hooks provided here work in the guts of MESA (e.g. `extra_jdot`). A `grep` should direct you to `solve_`... As in previous exercise you will have to add your new subroutine into `/src/run_star_extras.f` in your work directory.

Once your subroutine has been compiled (./mk), run the main sequence evolution of your rotating $15 M_\odot$ including the magnetic torque for fields [B = 10 , 50, 100, 1000 G]. Plot the time evolution of total angular momentum and surface rotation velocity for the 4 cases. If you are confident your implementation is working, try to compare with results in Meynet et al. 2011 (for example their Fig.3).