1 Introduction

The big picture in stellar evolution is well known, but there are still many details that remain unclear or even unknown at all. If the evolution is complicated by interaction with a binary companion, the level of complexity increases further. Many binary-interaction mechanisms are known to impact stellar evolution, but their exact workings remain elusive. This thesis aims to uncover some of the interactions in wide binaries by studying long-period hot-subdwarf binaries. In this introduction the basics of stellar and binary evolution are described, followed by a section on hot subdwarfs.

1.1 Stellar evolution in a nutshell

In this section I will briefly describe the evolution of single stars, after which the impact of a binary companion on stellar evolution will be discussed. As this thesis concerns hot subdwarf stars, the main focus is on low mass stars which ignite helium under degenerate conditions. The evolution of an intermediate-mass star with helium ignition under non-degenerate conditions is briefly summarized as a comparison. All figures describing the evolution of single stars are created with the stellar evolution code Modules for Experiments in Stellar Astrophysics (MESA, Paxton et al. 2011, 2013), based on my own calculations. Example inlists for MESA are given in Appendix A. The sections on single star evolution are based on Iben & Renzini (1983); Kippenhahn et al. (2012) and Hansen et al. (2004), while those on binary evolution are based on Iben & Livio (1993); Hilditch (2001); Tauris & van den Heuvel (2006) and Podsiadlowski (2008).

1.1.1 Single stars

In this section the evolution of low- and intermediate-mass stars from the main sequence to the white-dwarf stage will be illustrated. As an example of a low-mass star we use a 1 $M_{\odot}$ model with a quasi-solar composition ($X = 0.7$, $Z = 0.02$). The evolution of the solar-mass star in the Hertzsprung-Russel (HR) diagram is plotted in Fig. 1.1 and in Fig. 1.2 a so-called "Kippenhahn" diagram is shown together with the evolution of the luminosity, radius and central abundances. In the Kippenhahn diagram the hydrogen-burning zones are shown in red, while the helium-burning zone is plotted in blue. Convective zones are shown in green. The letters on the HR and Kippenhahn diagrams indicate stages in the evolution and are referred to in the text.

Main sequence evolution

Stars spend most of their life on the main sequence (A-B), slowly fusing hydrogen into helium in their core. Evolution on the main sequence happens slowly, and depending on their mass, stars spend between 10^6 to 10^9 years in this phase. During the main sequence, the star evolves away from the zero-age main sequence (ZAMS) towards higher luminosities and larger radii. Low-mass stars evolve towards higher effective temperature ($T_{\rm eff}$) while higher-mass stars evolve to lower $T_{\rm eff}$, but increase more strongly in radius.

The star is in hydrostatic and thermal equilibrium, as the gass pressure from the energy produced in the hydrogen fusion balances the gravitational pull. The high-temperature sensitivity of the nuclear-energy-generation rates (ϵ) make the nuclear reactions act like a thermostat in the central regions. If the central temperature drops, the energy generation diminishes, which decreases the outwards pressure. The outer layers of the star will fall inwards, increasing the pressure in the core, and thus increasing the temperature again. If the central temperature increases, nuclear burning will increase, increasing the outwards pressure. The layers outside the core will produce less pressure on the core, and the temperature will decrease again, resulting in an equilibrium in the center.

Hydrogen fusion into helium happens through two cycles: the proton-proton (p-p) chain, and the carbon-nitrogen-oxygen (CNO) cycle. In low-mass stars ($\lesssim 1.3\ M_{\odot}$), the central temperature is too low to ignite the CNO cycle. Their main energy production originates in the p-p chain, where four protons are, in several steps, fused into a helium ion. In higher-mass stars with a higher central temperature, the CNO cycle dominates the energy production. Due to the high temperature sensitivity of this process ($\epsilon_{\rm CNO} \propto T^{18}$ while $\epsilon_{\rm pp} \propto T^4$), the more massive stars expand considerably during the MS evolution.

Hydrogen shell burning

When low-mass stars leave the main sequence, they have dense cores that are close to being degenerate. At point B core hydrogen is practically exhausted ($\mathrm{H_c} < 0.001$). Hydrogen fusion moves slowly to a thick shell surrounding the helium core. During phase B-C (subgiant branch), the core slowly grows in mass and contracts further, while the envelope expands, and the H-burning shell gradually becomes thinner. The outer envelope expands, cools down, the opacity rises, and, starting at the surface, becomes convective. At point C, the helium core becomes electron degenerate, and the star is now at the base of the red giant branch (RGB). At point D, the convective envelope is at its deepest point in mass and reaches into layers that were processed by H burning during the main sequence (first dredge-up). As the star continues to climb the RGB, the envelope becomes more loosely bound, and stellar-wind-mass loss increases. Between point D and the tip of the RGB (point E), the example star of 1 $M_\odot$ loses roughly 0.15 $M_\odot$ in a stellar wind. The amount of mass lost in a stellar wind depends on the used prescription, which is not well known. In this case a Reimers wind with $\eta_{\rm reimers} = 0.5$ (Reimers 1975) was used.

He core flash

Helium burning in low-mass stars differs from intermediate mass stars in two main ways. First, in low mass stars, the helium core ignites under electron-degenerate circumstances, giving rise to a helium flash. Second, the mass of the He core at ignition is almost the same for all low-mass stars: $M_{\rm C} \approx 0.47 M_\odot$. The luminosity of low-mass helium-core-burning stars is therefore independent of their mass, a property that results in the horizontal branch and the red clump (point F).

He burning in a degenerate core is unstable. The 3α reaction causes an increase in temperature. Because in a degenerate core the temperature is decoupled from the pressure, the latter does not increase, and thus the He ignition initiates a thermonuclear runaway. This thermonuclear runaway leads to an overproduction of energy. At its maximum, the luminosity in the core can be up to $10^{10} L_\odot$ for a few seconds, comparable with the luminosity of a small galaxy. All this nuclear energy is absorbed by expanding the outer layers of the core and the non-degenerate layers around the core, hiding most of the energy from an outside observer. This is called the helium flash (point E in Figs. 1.1, 1.2 and 1.11).

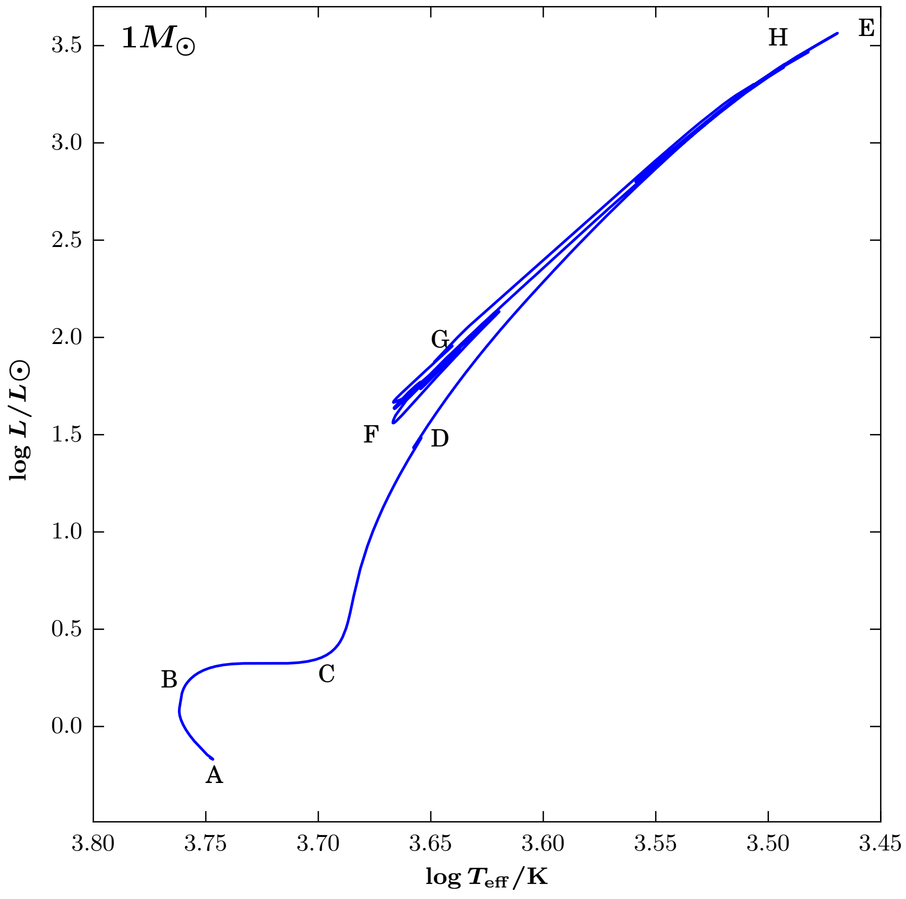

Figure 1.1: Hertzsprung-Russell diagram of the evolution of a 1 $M_{\odot}$ star from the ZAMS to the end of shell helium burning. The letters on the plot indicate events discussed in the text.

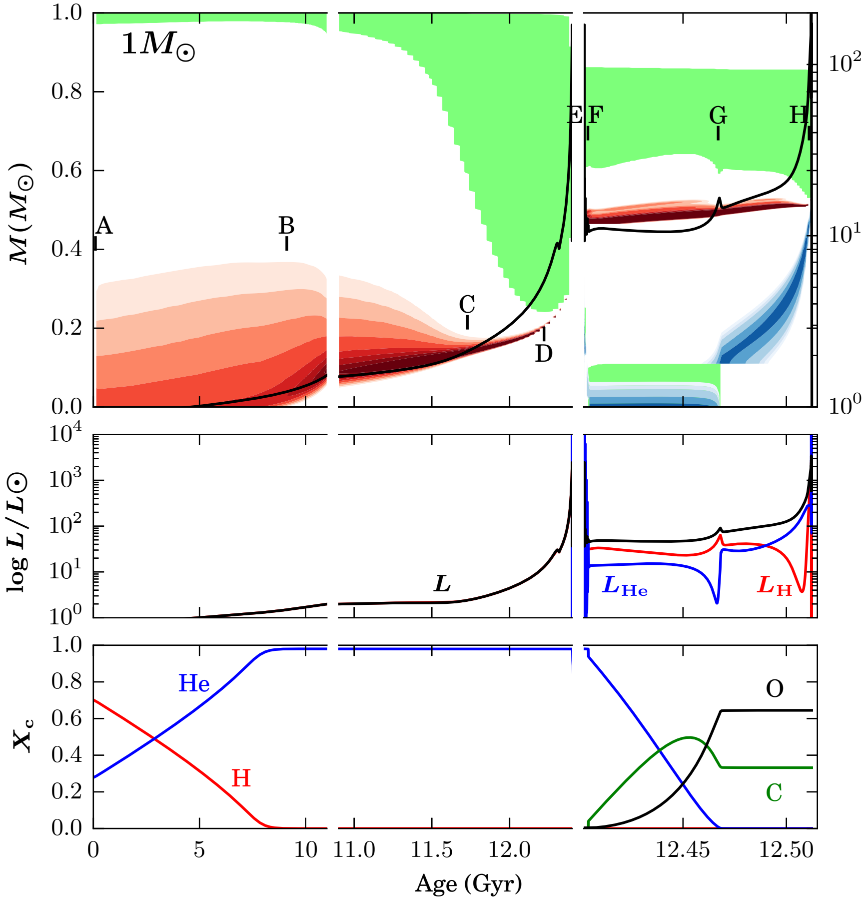

Figure 1.2: Evolution of a 1 $M_\odot$ star from the ZAMS to the end of shell helium burning. The letters on the plot indicate events discussed in the text. Top panel: Kippenhahn diagram. red colors indicate hydrogen burning, blue indicates helium burning and convective zones are plotted in green. The radius evolution is plotted on the right axes. The x-axes is subdivided in three parts with different time scales to improve clarity. Middle panel: Evolution of the luminocity. Bottom panel: Evolution of the central abundances.

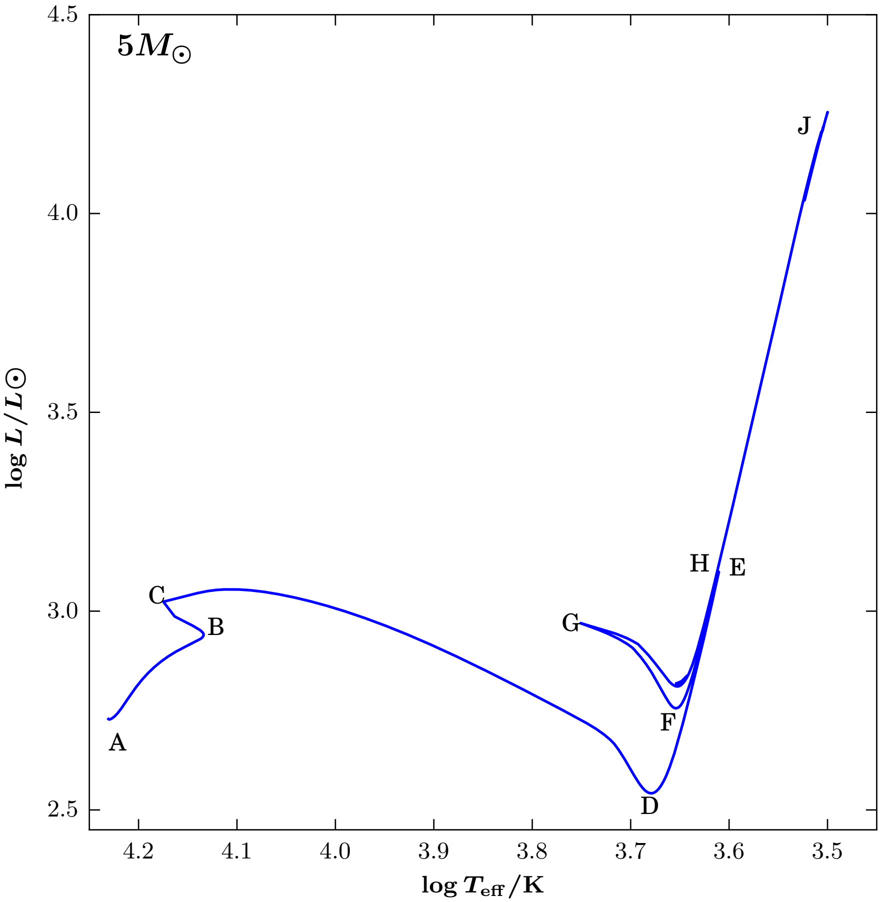

Figure 1.3: Hertzsprung-Russell diagram of the evolution of a 5 $M_\odot$ star from the main sequence to the end of shell helium burning. The letters on the plot indicate events discussed in the text.

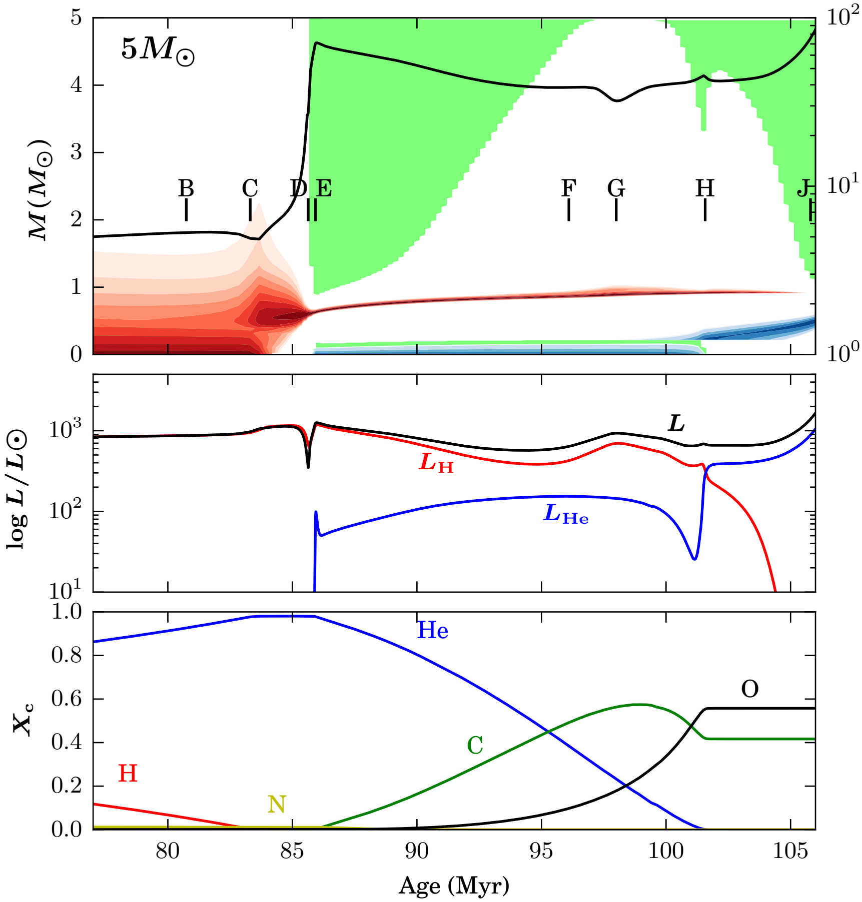

Figure 1.4: Evolution of a 5 $M_\odot$ star from the main sequence to the end of shell helium burning. The letters on the plot indicate events discussed in the text. Top panel: Kippenhahn diagram. red colors indicate hydrogen burning, blue indicates helium burning and convective zones are plotted in green. The radius evolution is plotted on the right axes. Middle panel: Evolution of the luminocity. Bottom panel: Evolution of the central abundances.

In practice, the He flash does not take place in the core. In Fig. 1.11 a close up of the He burning phases of a 1 $M_\odot$ star is shown. From this figure, it is clear that the main He flash takes place in a shell around mass coordinate 0.20 $M_\odot$. The reason for this is that the core itself cools down due to energy loss by neutrinos in the preceding red-giant phase. These neutrinos are created by the plasmon-neutrino process (Yakovlev et al. 2001; Odrzywołek 2007), which increases in efficiency with increasing density. The most dense part of the core will thus be cooler than the surrounding layers. This main helium flash is followed by a series of smaller flashes that reach closer and closer to the stellar center. As the temperature of the core keeps increasing, the degeneracy is eventually lifted, re-coupling temperature to pressure, and further nuclear burning is stable. The number of flashes before the core starts stable He burning depends on the total mass of the star.

The core now startes stable He burning, and the star emerges on the horizontal branch (point F). The luminosity and radius of the star have decreased by more than a magnitude compared to before the He flash. The core has expanded, and the outer shell above the H-burning shell has contracted. The life time during stable He burning is roughly 10^8 yr (phase F-G), and is rather independent of stellar mass. The luminosity during stable He burning, roughly 50 $L_\odot$, is mainly determined by the core mass, which is ≈ 0.47 $M_\odot$ for all low-mass stars.

In a population of stars with a given composition, for example in a cluster, only the envelope mass of the He-burning stars will vary. At solar metallicity, all such stars cluster together in the HR diagram, giving rise to the red clump, which is observed in low-mass populations. The radius and effective temperature of stars on the He main sequence depend on envelope mass. Stars with a lower envelope mass can be substantially hotter than those with higher envelope masses. He-burning stars with different envelope masses will form the horizontal branch observed in old stellar populations. He-burning stars with no hydrogen envelope are found at the left end of the horizontal branch.

After central He is exhausted, the carbon-oxygen (CO) core contracts, He shell burning starts, and the star enters the asymptotic-giant branch (AGB). The star consists of a CO core, a He-burning shell, a H-burning shell and a large H envelope. The star increases in radius and moves up in the HR diagram along the AGB (phase G-H). The ashes of the H-burning shell cause the He-burning shell to build up. This can, in time, cause a thermal pulse. As the mass of the CO core increases, the star will increase in radius and luminosity.

When the star climbs the AGB, it develops a stellar wind outside its envelope, which will blow the outer layers into space. In combination with strong stellar pulsations, this wind can cause mass loss up to 10^{-4} $M_\odot \mathrm{yr}^{-1}$. In roughly 10 000 yr, the envelope is removed, leaving only a hot degenerate CO core, a CO white dwarf, surrounded by a planetary nebula. This WD radiates energy and cools on the WD cooling track.

Evolution of intermediate mass stars

To illustrate the evolution of intermediate-mass stars, we use a 5 $M_\odot$ model with quasi-solar composition ($X = 0.7$, $Z = 0.02$). The evolution of the 5 $M_\odot$ star in the Hertzsprung-Russel (HR) diagram is plotted in Fig. 1.3 and in Fig. 1.4 a Kippenhahn diagram is shown together with the evolution of the luminosity, radius and central abundances. In the Kippenhahn diagram the H-burning zones are shown in red, while He-burning is plotted in blue. Convective zones are shown in green.

The main-sequence evolution of an intermediate-mass star is similar to that of a low-mass star, with the difference that the CNO cycle is the most important source of H fusion. Intermediate-mass stars will thus expand more during their evolution along the MS.

Once central hydrogen is getting depleted, hydrogen fusion in the core diminishes ($\mathrm{H_c} = 0.03$ at point B). As the outward pressure of hydrogen fusion is declining, the stellar core contracts. At point C, central hydrogen is completely exhausted, and H fusion in the core ceases. In intermediate-mass stars the shell burning can be divided into two phases, thick- and thin-shell burning. After the core contraction (point C), there is a quick transition from core H burning to shell H burning. Directly after the core contraction the temperature and density gradients between the core and envelope are small (phase C-D). The burning shell then occupies a large mass region, hence the name thick-shell burning, and the burning is relatively slow. During this H-shell burning the core increases in mass until the Schönberg-Chandrasekhar limit is reached and the core contraction speeds up. The envelope of the star will expand due to the increased radiation pressure. This leads to stronger temperature and density gradients, thus the H-burning shell will become thinner as the star reaches point D. As point D is approached, the stellar envelope cools down, and its opacity rises. Starting from the surface the envelope becomes convective. During phase D-E, the star is an expanding red giant with a deep convective envelope.

If the He core reaches a temperature of 10^8 K, the star starts helium burning in the $3\alpha \rightarrow \mathrm{C}$ reaction. In intermediate- and high-mass stars with a non-degenerate core, the nuclear burning is thermally stable and the ignition proceeds quietly. The He-burning region is highly concentrated towards the center due to the strong temperature sensitivity of the 3α process. Surrounding the He-burning core are a H-burning shell, and a very extended outer envelope. The luminosity of the star slightly increases, even though the energy yield of He

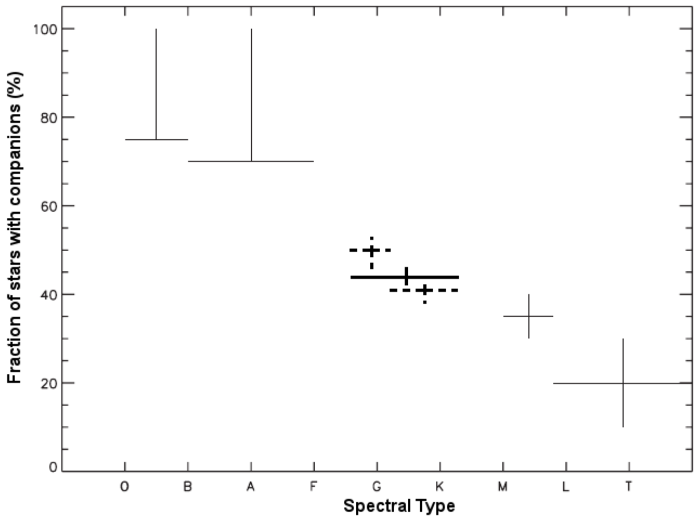

Figure 1.5: Binary statistics by spectral type. The thick solid and dashed lines are binary statistics calculated by Raghavan et al. (2010), while thin solid lines show results from other studies summaried by them. Figure 12 from Raghavan et al. (2010).

burning is only 10% of that of H burning. The reason why the star can maintain its luminosity is because the majority of the energy is produced by H-shell burning. This is shown clearly in the middle panel of Fig. 1.4.

Intermediate-mass stars evolve through a loop in the HR diagram when burning He in their core (E-H), also called the blue loop. The evolution from the He shell burning through the AGB, post-AGB and planetary-nebula phase to the WD cooling track is similar as with the low-mass stars described before.

1.1.2 Binary stars

In the previous sections only single-star evolution was considered. However, a large part of the stellar population resides in binary or multiple systems. Raghavan et al. (2010) shows that depending on the spectral type 20 to 70 % of all stars reside in binaries (see Fig. 1.5). Binary interaction can lead to many interesting types of systems, including but certainly not limited to: Cataclysmic variables, type Ia supernovae, pulsars, hot subdwarfs.

Roche lobes

The interaction between two stars in a binary system is described using their gravitational wells, also referred to as their Roche lobes[1]. A Roche lobe is the equipotential surface within which matter is gravitationally bound to one star in the binary system. The Roche lobes of the two components of a binary system meet at the inner Lagrange point (L1), a saddle point where mass can be transferred from one star to its companion (see Fig. 1.6). The classical way to derive the Roche lobes is using the assumtion of point masses on synchronized circular orbits.

Binary evolution can be described based on the radii of the components with respect to their Roche-lobe radii. As long as both stars are well within their Roche lobes, they evolve as if they are single stars. These detached binaries can be excellent sources for probing stellar parameters, especially when they are eclipsing. However, when one of the stars evolves onto the RGB or further, its radius increases significantly. Depending on the ratio between the stellar radii and the orbit we differentiate between different channels (see also Fig. 1.6). 1) If the binary is very wide, the primary stays contained in its Roche lobe and further evolution continues as if it were a single star. 2) The primary starts to fill its Roche lobe, and it starts to lose mass, which can be accreted by its companion star or lost entirely from the system. The binary becomes a semi-detached system. 3) If the primary increases so much in size that it engulfs its companion, a contact binary is formed. The companion star then orbits within the atmosphere of the primary, and spirals in. This results in a common-envelope ejection or, if not enough energy is available, a merger.

Roche-lobe overflow

In the case of stable Roche lobe overflow (RLOF), the donor star (or primary) has to continue filling its Roche lobe. We discriminate between two ways. The donor continues expanding because of its own internal evolution, or the system loses angular momentum, causing the orbit and hence the Roche lobes to shrink. Mass loss can continue as long as there is sufficient mass left in the envelope of the donor star. At the end of RLOF, the donor star is left with only a small envelope.

Depending on the evolutionary phase of the donor star, three different types of mass loss are distinguished.

[1] Named after the French astronomer Edouard Roche

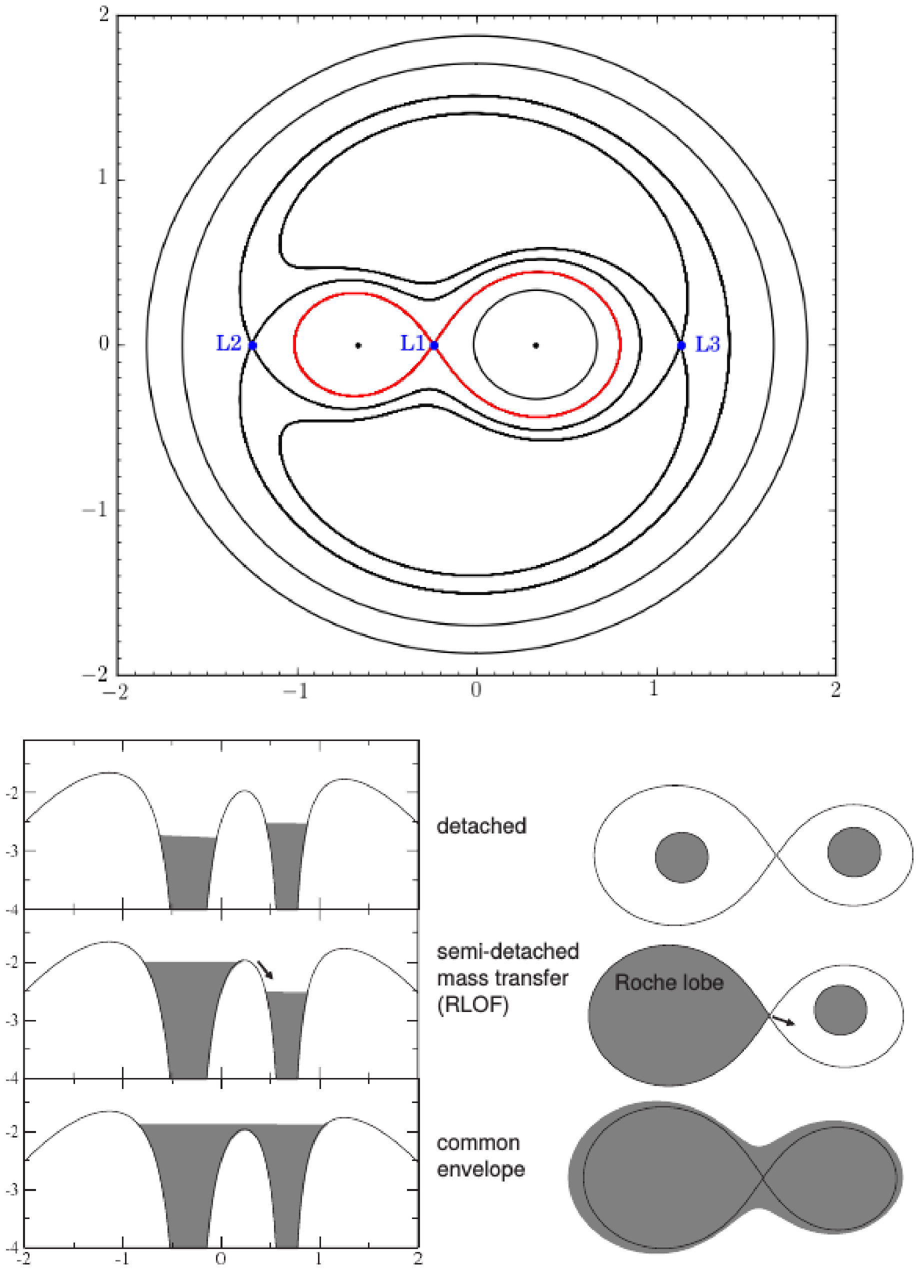

Figure 1.6: Top: sections in the orbital plane of Roche equipotentials for a system with q = 0.5. The Roche lobes are shown in red, the Lagrange points in blue and the location of both stars as black dots. Bottom: Possible binary configurations; detached, semi-detached with RLOF and common envelope. For each a schematic representation of the equipotential well and the filling of the Roche lobes is shown. Figure adapted from Jorissen (2003).

Type-A Mass transfer starts when the donor star is on the MS. This mass transfer will occur on the very long nuclear time scale. As this mass transfer is very slow, the donor can adapt to the changes in its Roche-lobe radius without deviating from thermal equilibrium.

Type-B Mass loss starts when the donor is in its H-shell-burning phase, and is ascending on the RGB. This mass loss occurs on a shorter time scale. Whether the donor can adapt to the changing Roche lobe depends on the system.

Type-C In this case the donor star has already ignited He, for example on the AGB. The mass transfer is so extreme that the donor cannot stay within its Roche lobe.

Effect of mass transfer on stellar structure

When mass loss has been initiated, and the mass loss is conservative, the Roche lobe of the donor star starts to shrink. As the stellar radius cannot be much larger than the Roche-lobe radius, the donor itself also has to shrink. The evolution of the donor then depends on whether it can let its radius evolve with the changing Roche lobe, and still remain in equilibrium. The hydrostatic equilibrium of the donor star is not disrupted by mass loss, as the time scale for restoring the equilibrium is very short compared to the mass-loss time scale. The thermal equilibrium on the other hand can be disturbed by mass loss.

If the donor star has a radiative envelope during mass loss, the envelope will shrink while the core expands. Due to the expanding core the nuclear reaction rate decreases and the donor will remain in thermal equilibrium. Donor stars with a convective envelope will expand when losing mass, and will therefore not be able to remain in thermal equilibrium. Depending on how the orbit evolves, the mass loss can occur on a short dynamical time scale, and can lead to a CE. If the orbit expands, the mass loss can still be stable.

The evolution of the accreting star is more extreme than for the donor. The mass stream of the donor will form an accretion disc from which the mass is accreted onto the companion. The accretor will not only accrete mass, but also gain momentum, causing it to spin up. In certain cases it is possible that the companion reaches its break-up velocity.

If mass transfer is close to conservative, the mass gain of the secondary is significant. This has two main effects on its further evolution. Firstly, the convective core will grow, mixing in new fuel from the surrounding layers. This process, also known as rejuvenation, causes the lifetime on the MS to extend. A second effect is that due to the increased mass, nuclear fusion speeds up and the lifetime on the MS decreases. If the second process is stronger than the first, the secondary can overtake the evolution of the primary.

Common-envelope evolution

When the donor star is losing mass from a convective envelope, this envelope might respond to the mass loss by increasing even further in radius. This results in runaway mass loss, and a common-envelope situation. There are currently two different mechanisms to model a common-envelope ejection.

α **formalism** The classical look at CE evolution is that of a companion orbiting inside the envelope of a giant (Paczynski 1976). The companion experiences a drag force due to the friction when moving inside the CE. Due to this drag, it will spiral in towards the core of the giant. In this process, orbital energy is transferred to heat and motion of the gas in the CE, and eventually into kinetic energy that causes the CE to expel. This envelope ejection effectively ends the spiral-in phase, and the system is now a much closer binary than before the CE phase. This CE evolution is considered the main mechanism to convert wide binaries into very close binaries. General consensus is that the companion will not accrete much matter during the very short CE phase, and will thus not change significantly.

Current models of the α formalism are based on the energy balance of the system, assuming angular-momentum conservation. The orbital energy of the binary, assuming some unknown efficiency factor is used to expel the common envelope. However, it is found from some observations that the efficiency of this mechanism has to be higher than unity, indicating that there is another energy source at play.

γ **formalism** Nelemans et al. (2000, 2001) proposed an alternative CE-ejection channel. This γ formalism assumes that a CE caused by runaway mass loss will be in co-rotation with the orbit. There is thus very little drag on the components, and no or very little spiral-in. The energy necessary to expel the envelope can be supplied by the luminosity of the giant or tidal heating.

The energy balance in the γ formalism is created by linking the change in angular momentum to the total amount of mass that is lost when the CE is ejected.

A more extreme version of the CE evolution occurs when the energy released during the CE phase is not sufficient to eject the common envelope. The