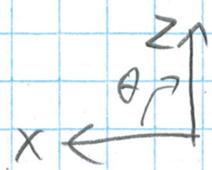
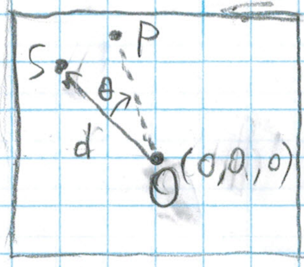
Rotation on the Spot

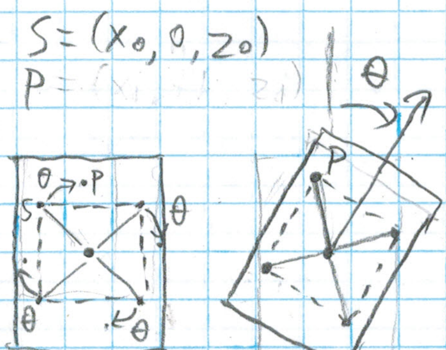


Y up. (out of page)



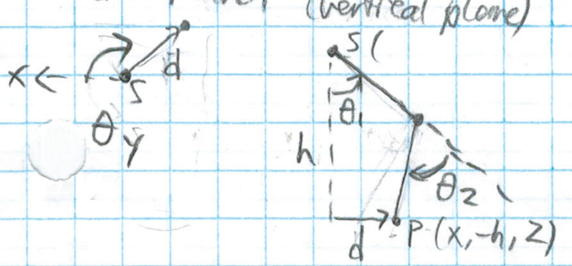
Let $S = (x_0, 0, z_0)$ be the position of the shoulder relative to the center of the robot $(0,0,0)$. At rest position, the foot will be directly under the shoulder. So it starts at $(x_0, -h, z_0)$ where h is the height of the shoulder above the ground, which should be kept constant.

$S = (x_0, 0, z_0)$
$P =$



The goal is to rotate the position of each foot by the angle of rotation θ around $(0,0,0)$, keeping it at a constant distance $d = \sqrt{x_0^2 + z_0^2}$ (in the xz plane). When the robot straightens itself out it will have rotated itself θ.

(xz plane) (vertical plane)



$$P = (d\cos\theta, -h, d\sin\theta)$$
$$= \left(\sqrt{x_0^2+z_0^2}\cos\theta, -h, \sqrt{x_0^2+z_0^2}\sin\theta\right)$$

For rest of problem, do with respect to shoulder position. Rebase coordinates:

$$\overline{OP} = \overline{OS} + \overline{SP} \rightarrow \overline{SP} = \overline{OP} - \overline{OS}$$

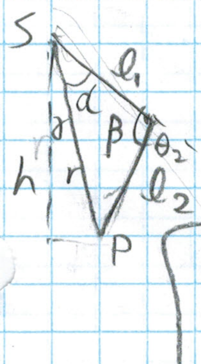
So P becomes: $\left(\sqrt{x_0^2+z_0^2}\cos\theta - x_0, -h, \sqrt{x_0^2+z_0^2}\sin\theta - z_0\right)$

Let's call it $(x, -h, z)$

Motor Angles:

rotation in vertical plane: $\boxed{\theta_Y = \arctan\left(\frac{z}{x}\right) \text{ adjusted to quadrant I, II, III, or IV}}$

In vertical plane: legs and direct path to P form a triangle with 3 known sides, solvable by cosine law.



$$l_2^2 = l_1^2 + r^2 - 2 l_1 r \cos\alpha \quad \text{(solve for } \alpha\text{)}$$
$$r^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos\beta \quad \text{(solve for } \beta\text{)}$$

$$\boxed{\begin{aligned}\theta_1 &= \gamma + \alpha \\ \theta_2 &= 180^\circ - \beta\end{aligned}}$$

$\therefore$ The required motor angles (positions) are found.