

# Introduction to Factorial Design

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# Study Two or More factors

- Suppose that we have two factors A and B, each at two levels. We denote the levels of the factors by A–, A+, B–, and B+. How would you design the experiment?
- Possible solution: **one-factor-at-a-time design**. That it, we fix B at a level (say B–) and study the effect of the two levels of A using the design we've studied for a single factor. After that, we fixed A at a level and study B.
  - OFAT requires more runs for the same precision in effect estimation
  - OFAT cannot estimate interactions

# Factorial Design

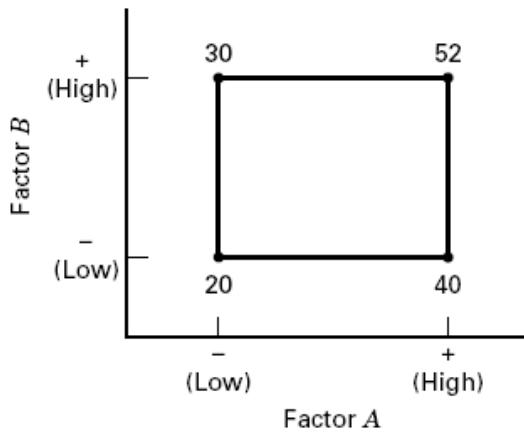
- For experiments that involve the study of the effects of two or more factors.
- In a factorial design, **all possible combinations of the levels of the factors** are investigated. For example, if there are  $a$  levels of factor A and  $b$  levels of factor B, each replicate contains all  $ab$  treatment combinations.
- Study how each factor affects the response
- Let's start with a two-factor, two-level factorial design.

# Two-level Factorial Design

Factor		Treatment Combination	Replicate		
A	B		I	II	III
-	-	A low, B low	28	25	27
+	-	A high, B low	36	32	32
-	+	A low, B high	18	19	23
+	+	A high, B high	31	30	29

# Main Effect

**Definition of a factor's main effect:** The average response increase when the factor is changed from low to high

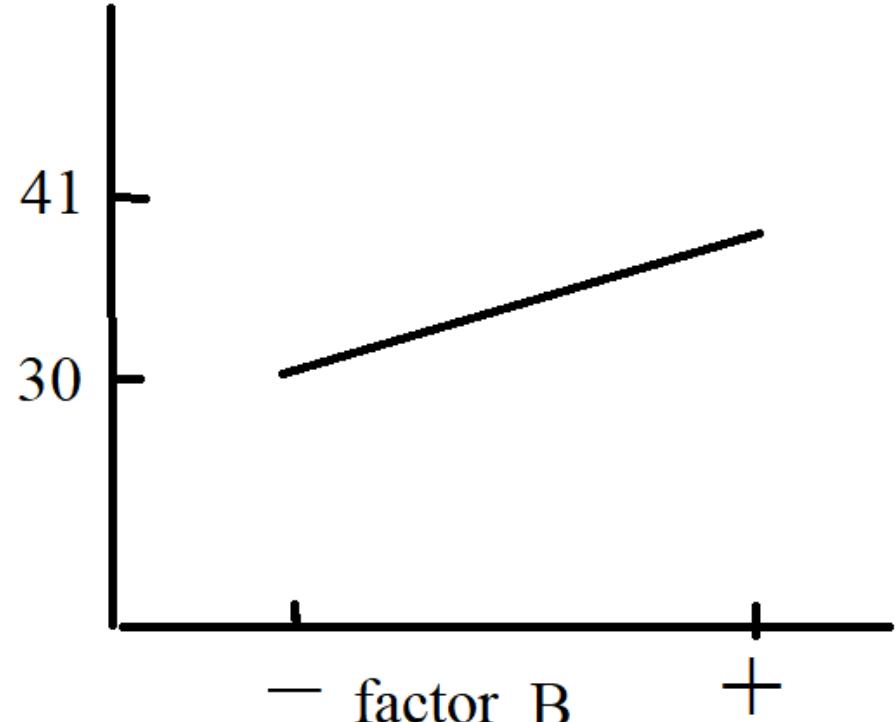
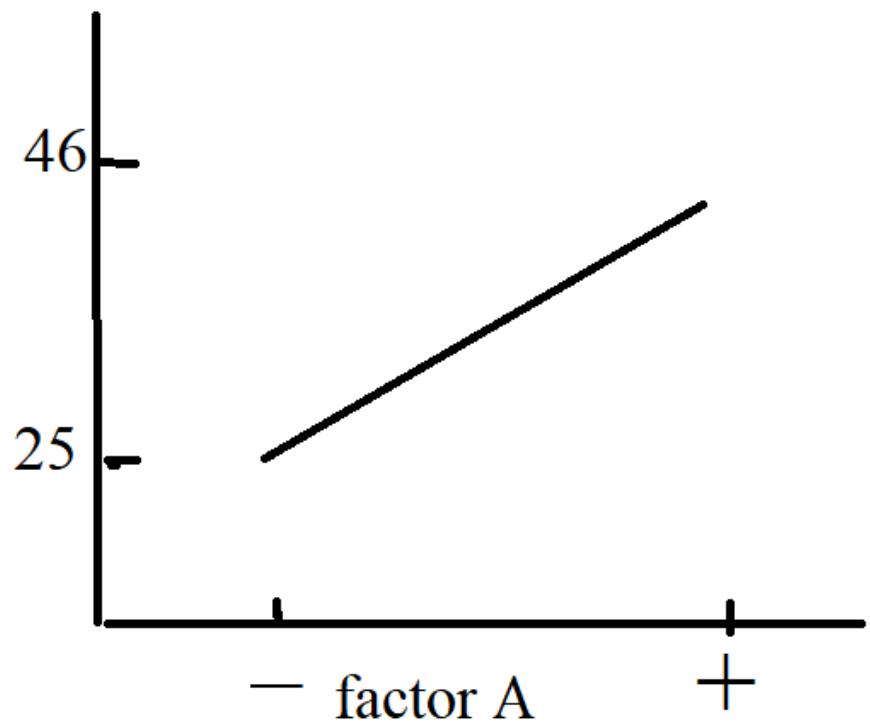


■ FIGURE 5.1 A two-factor factorial experiment, with the response ( $y$ ) shown at the corners

$$A = \bar{y}_{A^+} - \bar{y}_{A^-} \\ = \frac{40 + 52}{2} - \frac{20 + 30}{2} = 21$$

$$B = \bar{y}_{B^+} - \bar{y}_{B^-} \\ = \frac{30 + 52}{2} - \frac{20 + 40}{2} = 11$$

# Main Effect Plot



# Interaction Between Factors

It measures how much the joint effect of A and B departs from what would be expected if their effects were purely additive.

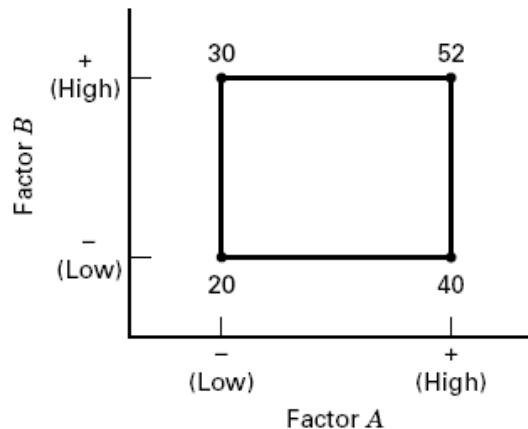
Interaction occurs when the influence of one factor depends on the level of the other factor.

**Interaction Effect:** the difference between the effect of A when B is at its high level and the effect of A when B is at its low level.

$$\begin{aligned} AB &= \frac{1}{2} (\bar{y}_{A^+B^+} - \bar{y}_{A^-B^+}) - \frac{1}{2} (\bar{y}_{A^+B^-} - \bar{y}_{A^-B^-}) \\ &= \frac{1}{2} (\bar{y}_{A^+B^+} + \bar{y}_{A^-B^-}) - \frac{1}{2} (\bar{y}_{A^-B^+} + \bar{y}_{A^+B^-}) \end{aligned}$$

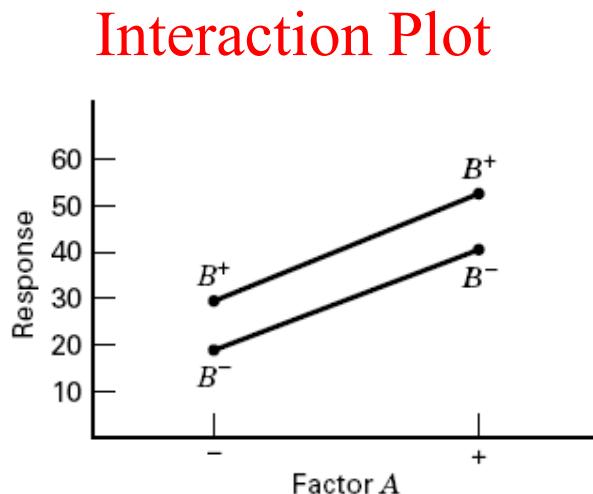
# Interaction Plot

An **interaction plot** is a graphical way to detect whether interaction is present by visualizing conditional means.



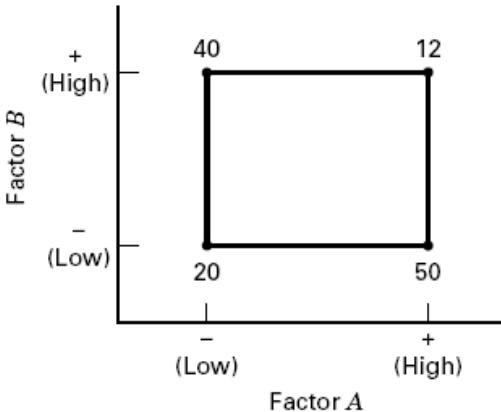
■ **FIGURE 5.1** A two-factor factorial experiment, with the response ( $y$ ) shown at the corners

$$AB = \frac{52 - 30}{2} - \frac{40 - 20}{2} = \frac{52 + 20}{2} - \frac{40 + 30}{2} = 1$$

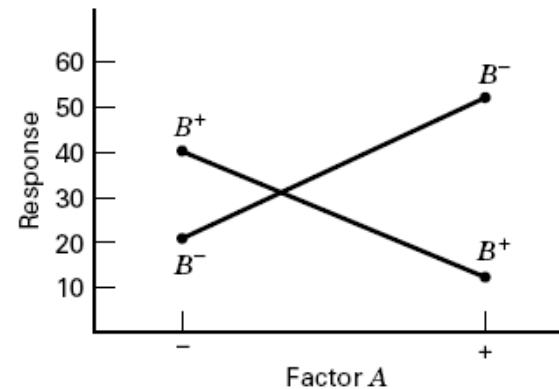


■ **FIGURE 5.3** A factorial experiment without interaction

# Another Example



■ FIGURE 5.2 A two-factor factorial experiment with interaction



■ FIGURE 5.4 A factorial experiment with interaction

$$A = \bar{y}_{A^+} - \bar{y}_{A^-} = \frac{50 + 12}{2} - \frac{20 + 40}{2} = 1$$

$$B = \bar{y}_{B^+} - \bar{y}_{B^-} = \frac{40 + 12}{2} - \frac{20 + 50}{2} = -9$$

At the low level of factor B (or B-), the A effect is  $50 - 20 = 30$

At the high level of factor B (or B+), the A effect is  $12 - 40 = -28$

$$AB = \frac{12 + 20}{2} - \frac{40 + 50}{2} = -29$$

# Three-level Factorial Design

A	B
1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

Examine all nine combinations of levels

# How to draw the main effect plot?

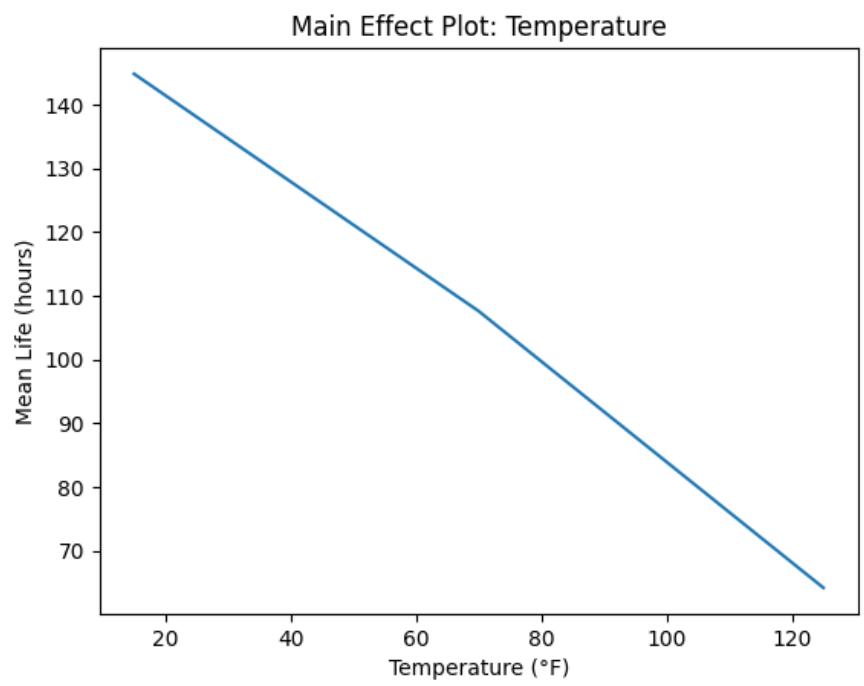
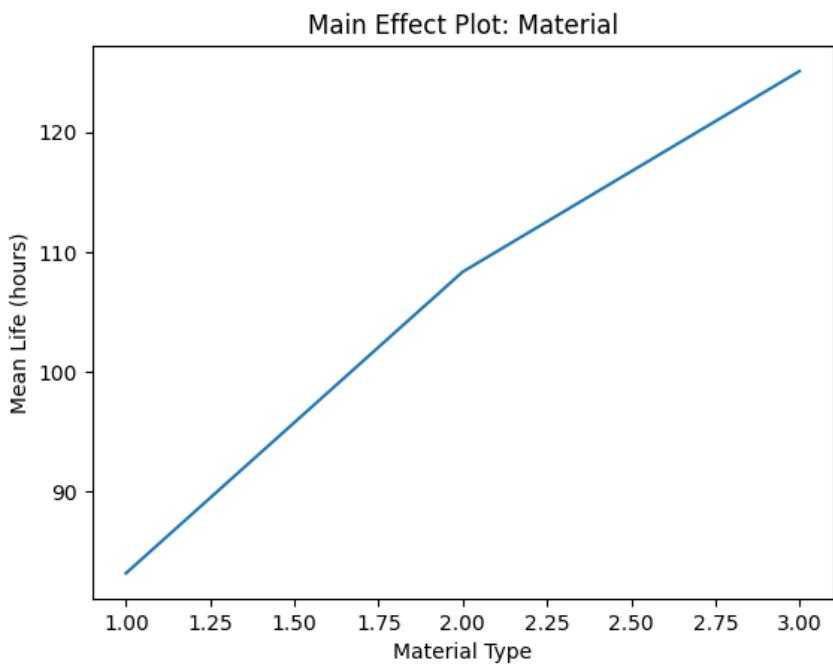
**Life (in hours) Data for the Battery Design Example**

Material Type	Temperature (°F)					
	15	70	125			
1	130	155	34	40	20	70
	74	180	80	75	82	58
2	150	188	136	122	25	70
	159	126	106	115	58	45
3	138	110	174	120	96	104
	168	160	150	139	82	60

$A$  = Material type;  $B$  = Temperature

What effects do material type & temperature have on life?

# Main Effects



# Interaction Plot

