Introduction to Factorial Design

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Study Two or More factors

- Suppose that we have two factors A and B, each at two levels. We denote the levels of the factors by A−, A+, B−, and B+. How would you design the experiment?
- Possible solution: one-factor-at-a-time design. That it, we fix B at a level (say B−) and study the effect of the two levels of A using the design we've studied for a single factor. After that, we fixed A at a level and study B.
  - OFAT requires more runs for the same precision in effect estimation
  - OFAT cannot estimate interactions

Factorial Design

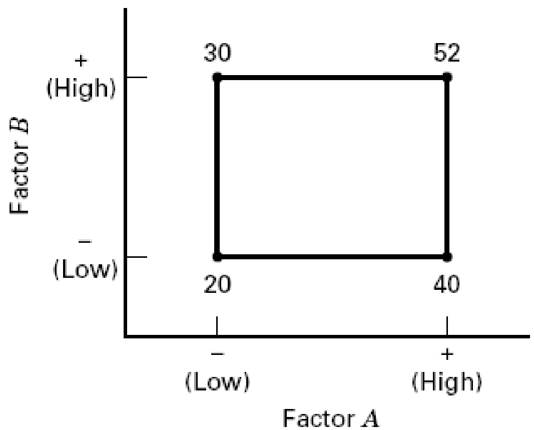
- For experiments that involve the study of the effects of two or more factors.
- In a factorial design, all possible combinations of the levels of the factors are investigated. For example, if there are *a* levels of factor A and *b* levels of factor B, each replicate contains all *ab* treatment combinations.
- Study how each factor affects the response
- Let's start with a two-factor, two-level factorial design.

Two-level Factorial Design

Factor		Replicate			
A	***B***	**Treatment Combination**	**I**	**II**	**III**
–	–	*A* low, *B* low	28	25	27
+	–	*A* high, *B* low	36	32	32
–	+	*A* low, *B* high	18	19	23
+	+	*A* high, *B* high	31	30	29

Main Effect

Definition of a factor's main effect: The average response increase when the factor is changed from low to high

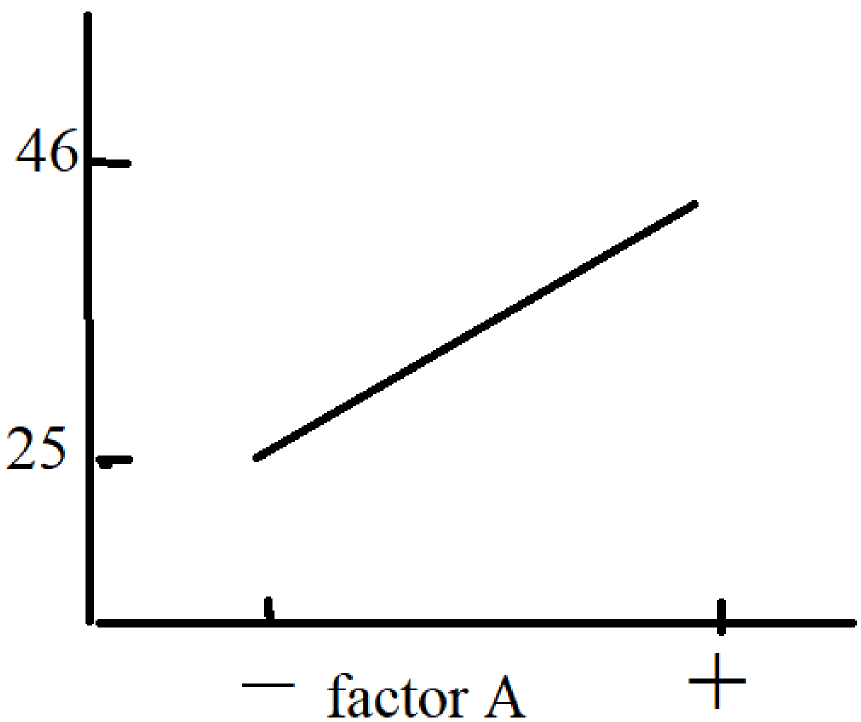


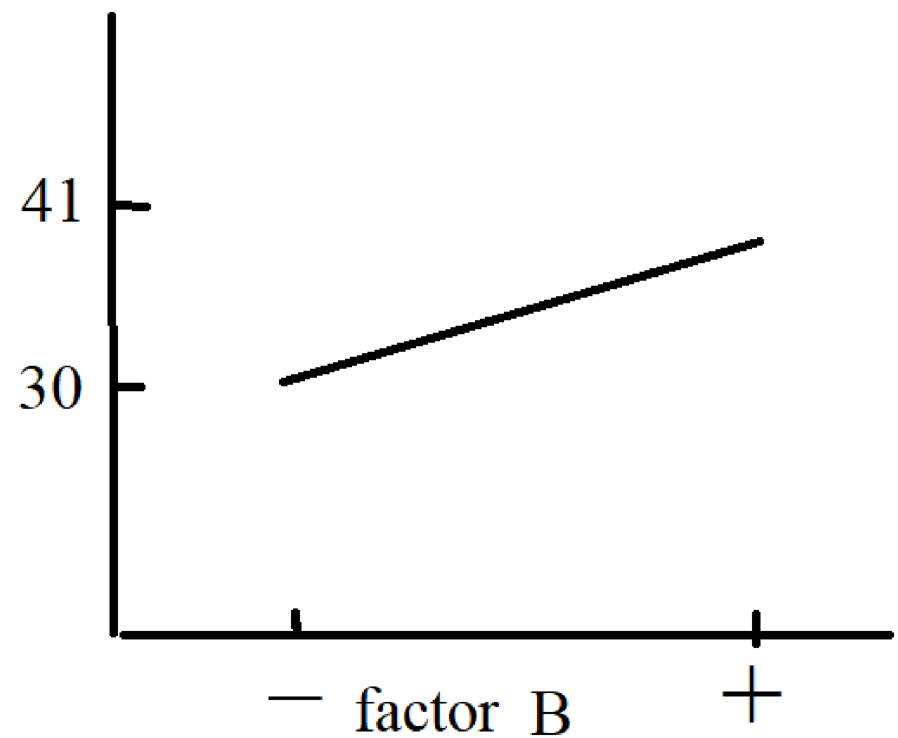
■ FIGURE 5.1 A two-factor factorial experiment, with the response (y) shown at the corners

$$A = \bar{y}_{A^+} - \bar{y}_{A^-}$$
$$= \frac{40+52}{2} - \frac{20+30}{2} = 21$$

$$B = \bar{y}_{B^+} - \bar{y}_{B^-}$$
$$= \frac{30+52}{2} - \frac{20+40}{2} = 11$$

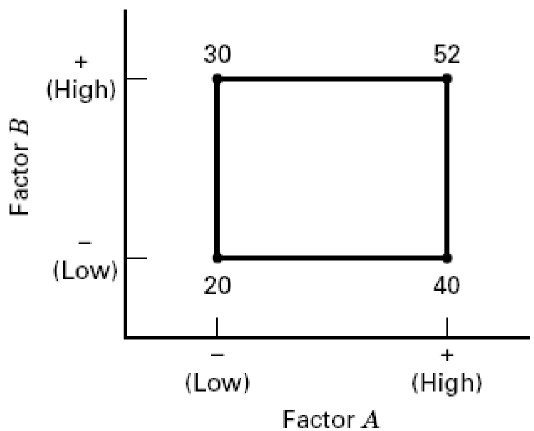
Main Effect Plot





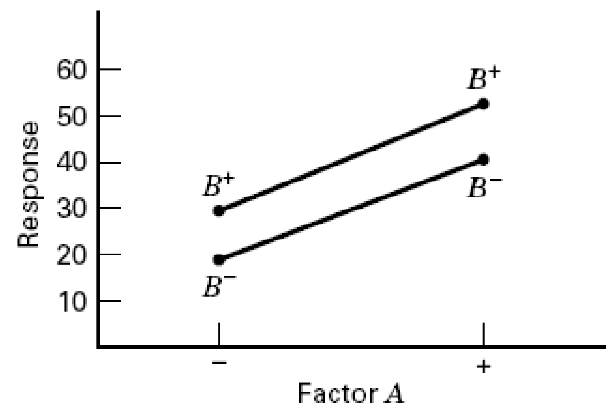
Interaction Between Factors

In some experiments, we may find that the difference in response between the levels of one factor is not the same at all levels of the other factors. When this occurs, there is an interaction between the factors.



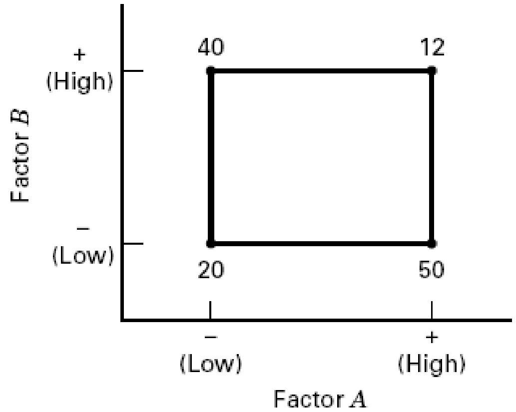
■ **FIGURE 5.1** A two-factor factorial experiment, with the response (y) shown at the corners

Interaction Plot

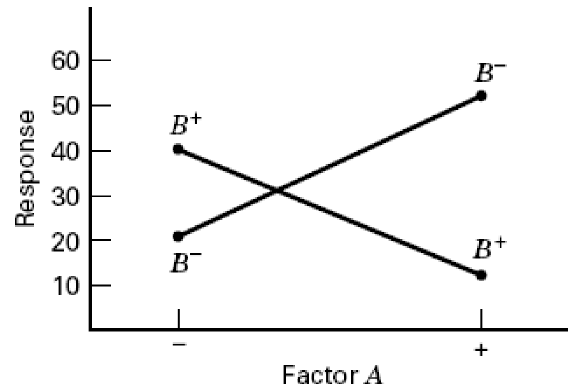


■ **FIGURE 5.3** A factorial experiment without interaction

The Case of Interaction



■ **FIGURE 5.2** A two-factor factorial experiment with interaction



■ **FIGURE 5.4** A factorial experiment with interaction

$$A = \bar{y}_{A^+} - \bar{y}_{A^-} = \frac{50+12}{2} - \frac{20+40}{2} = 1$$

$$B = \bar{y}_{B^+} - \bar{y}_{B^-} = \frac{40+12}{2} - \frac{20+50}{2} = -9$$

At the low level of factor B (or B−), the A effect is 50−20=30

At the high level of factor B (or B+), the A effect is 12−40=−28

Interaction Effect

$$AB = \frac{1}{2}(\bar{y}_{A^+B^+} - \bar{y}_{A^-B^+}) - \frac{1}{2}(\bar{y}_{A^+B^-} - \bar{y}_{A^-B^-})$$

$$= \frac{1}{2}(\bar{y}_{A^+B^+} + \bar{y}_{A^-B^-}) - \frac{1}{2}(\bar{y}_{A^-B^+} + \bar{y}_{A^+B^-})$$

$$AB = \frac{12 + 20}{2} - \frac{40 + 50}{2} = -29$$

Interaction plot:

parallel: no evidence of interaction

cross: interaction exists

This is like we create a new factor (column) named AB, which is the element product of A and B

Three-level Factorial Design

A	B
1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

Examine all nine combinations of levels

How to draw the main effect plot?

Life (in hours) Data for the Battery Design Example						
	Temperature (°F)					
Material Type	**15**		**70**		**125**	
1	130	155	34	40	20	70
	74	180	80	75	82	58
2	150	188	136	122	25	70
	159	126	106	115	58	45
3	138	110	174	120	96	104
	168	160	150	139	82	60

A = Material type; B = Temperature What **effects** do material type & temperature have on life?

Is there a choice of material that would give long life ***regardless of temperature*** (a **robust** product)?

Interaction Plot