

Introduction to Factorial Design

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Study Two or More factors

- Suppose that we have two factors A and B, each at two levels. We denote the levels of the factors by A⁻, A⁺, B⁻, and B⁺. How would you design the experiment?
- Possible solution: **one-factor-at-a-time design**. That is, we fix B at a level (say B⁻) and study the effect of the two levels of A using the design we've studied for a single factor. After that, we fix A at a level and study B.
 - OFAT requires more runs for the same precision in effect estimation
 - OFAT cannot estimate interactions

Factorial Design

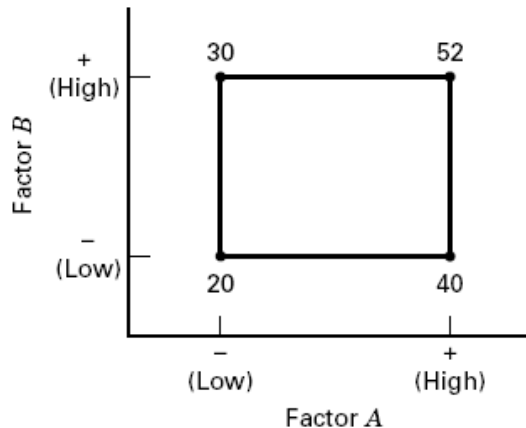
- For experiments that involve the study of the effects of two or more factors.
- In a factorial design, **all possible combinations of the levels of the factors** are investigated. For example, if there are a levels of factor A and b levels of factor B, each replicate contains all ab treatment combinations.
- Study how each factor affects the response
- Let's start with a two-factor, two-level factorial design.

Two-level Factorial Design

| Factor | | Replicate | | | |
|-----------------|-----------------|------------------------------|----------|-----------|------------|
| <i>A</i> | <i>B</i> | Treatment Combination | I | II | III |
| – | – | <i>A</i> low, <i>B</i> low | 28 | 25 | 27 |
| + | – | <i>A</i> high, <i>B</i> low | 36 | 32 | 32 |
| – | + | <i>A</i> low, <i>B</i> high | 18 | 19 | 23 |
| + | + | <i>A</i> high, <i>B</i> high | 31 | 30 | 29 |

Main Effect

Definition of a factor's main effect: The average response increase when the factor is changed from low to high

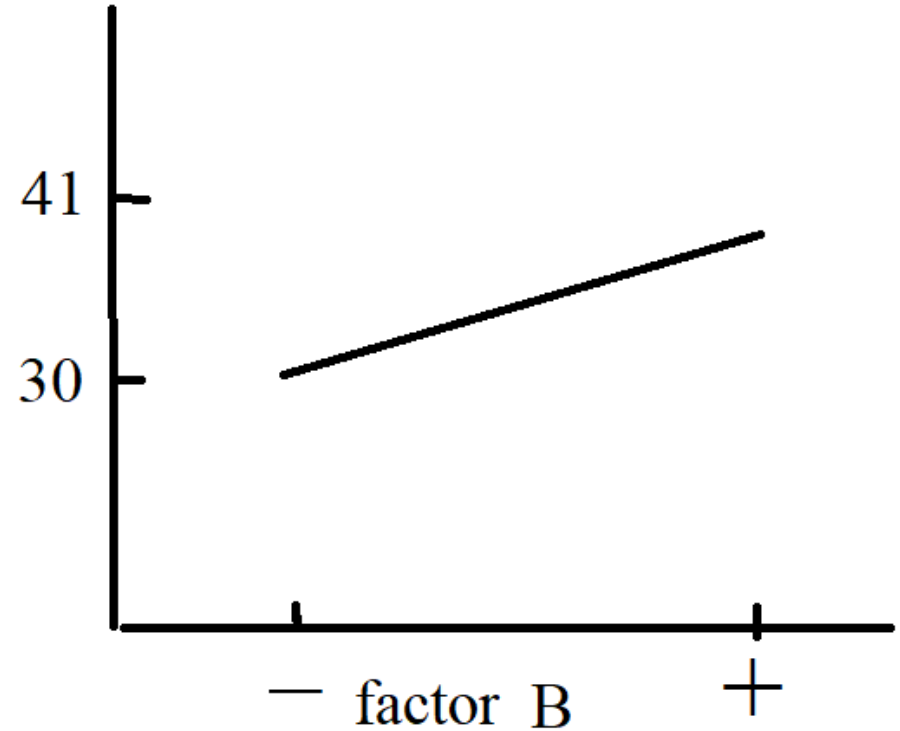
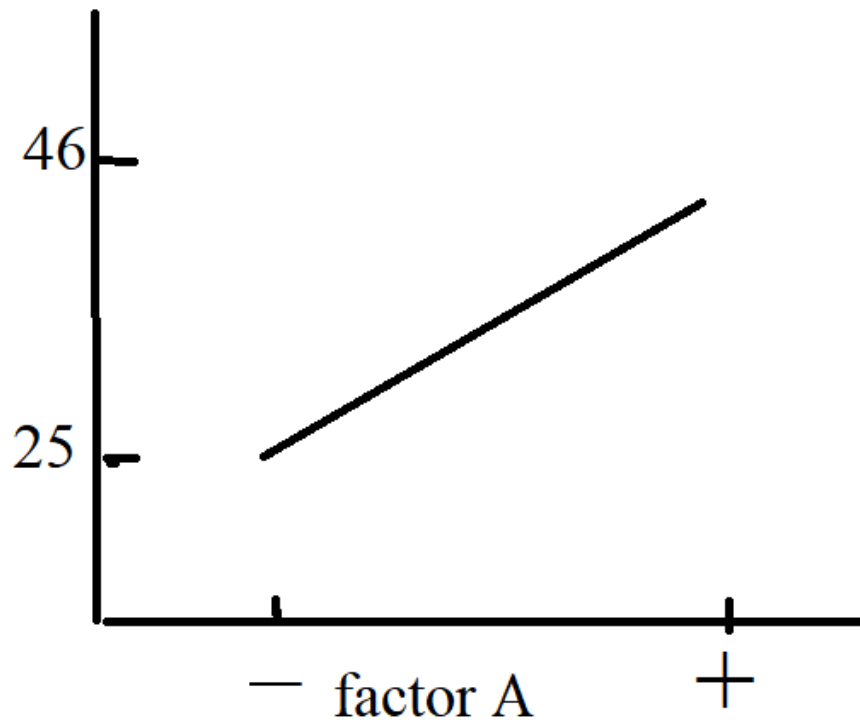


■ **FIGURE 5.1** A two-factor factorial experiment, with the response (y) shown at the corners

$$\begin{aligned} A &= \bar{y}_{A^+} - \bar{y}_{A^-} \\ &= \frac{40 + 52}{2} - \frac{20 + 30}{2} = 21 \end{aligned}$$

$$\begin{aligned} B &= \bar{y}_{B^+} - \bar{y}_{B^-} \\ &= \frac{30 + 52}{2} - \frac{20 + 40}{2} = 11 \end{aligned}$$

Main Effect Plot



Interaction Between Factors

It measures how much the joint effect of A and B departs from what would be expected if their effects were purely additive.

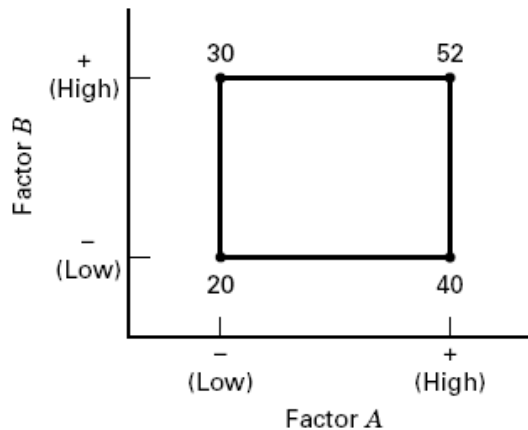
Interaction occurs when the influence of one factor depends on the level of the other factor.

Interaction Effect: the difference between the effect of A when B is at its high level and the effect of A when B is at its low level.

$$\begin{aligned} AB &= \frac{1}{2} (\bar{y}_{A^+B^+} - \bar{y}_{A^-B^+}) - \frac{1}{2} (\bar{y}_{A^+B^-} - \bar{y}_{A^-B^-}) \\ &= \frac{1}{2} (\bar{y}_{A^+B^+} + \bar{y}_{A^-B^-}) - \frac{1}{2} (\bar{y}_{A^-B^+} + \bar{y}_{A^+B^-}) \end{aligned}$$

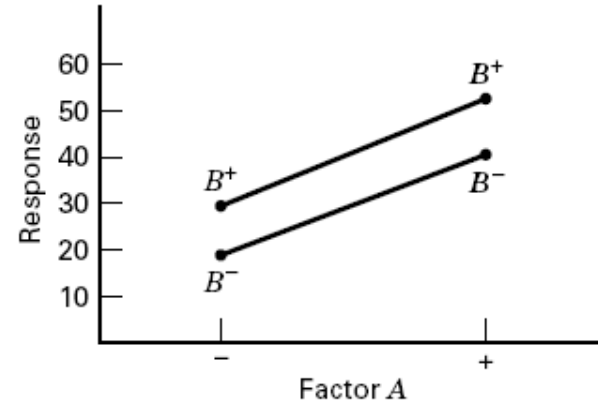
Interaction Plot

An **interaction plot** is a graphical way to detect whether interaction is present by visualizing conditional means.



■ **FIGURE 5.1** A two-factor factorial experiment, with the response (y) shown at the corners

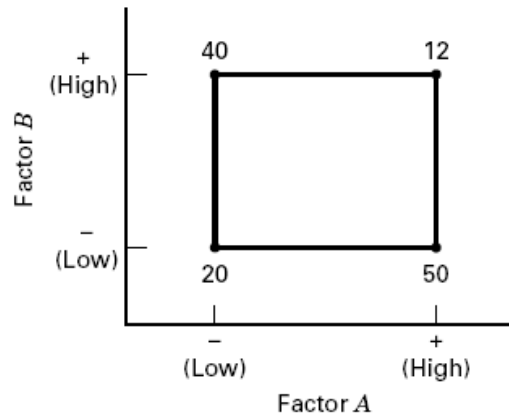
Interaction Plot



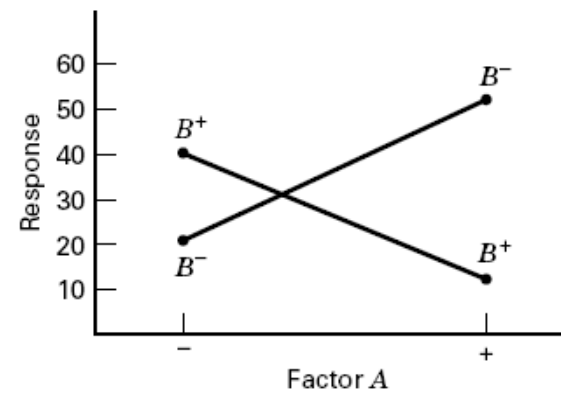
■ **FIGURE 5.3** A factorial experiment without interaction

$$AB = \frac{52 - 30}{2} - \frac{40 - 20}{2} = \frac{52 + 20}{2} - \frac{40 + 30}{2} = 1$$

Another Example



■ **FIGURE 5.2** A two-factor factorial experiment with interaction



■ **FIGURE 5.4** A factorial experiment with interaction

$$A = \bar{y}_{A+} - \bar{y}_{A-} = \frac{50 + 12}{2} - \frac{20 + 40}{2} = 1$$

$$B = \bar{y}_{B+} - \bar{y}_{B-} = \frac{40 + 12}{2} - \frac{20 + 50}{2} = -9$$

At the low level of factor B (or B-), the A effect is 50-20=30

At the high level of factor B (or B+), the A effect is 12-40=-28

$$AB = \frac{12 + 20}{2} - \frac{40 + 50}{2} = -29$$

Three-level Factorial Design

| A | B |
|---|---|
| 1 | 1 |
| 1 | 2 |
| 1 | 3 |
| 2 | 1 |
| 2 | 2 |
| 2 | 3 |
| 3 | 1 |
| 3 | 2 |
| 3 | 3 |

Examine all nine combinations of levels

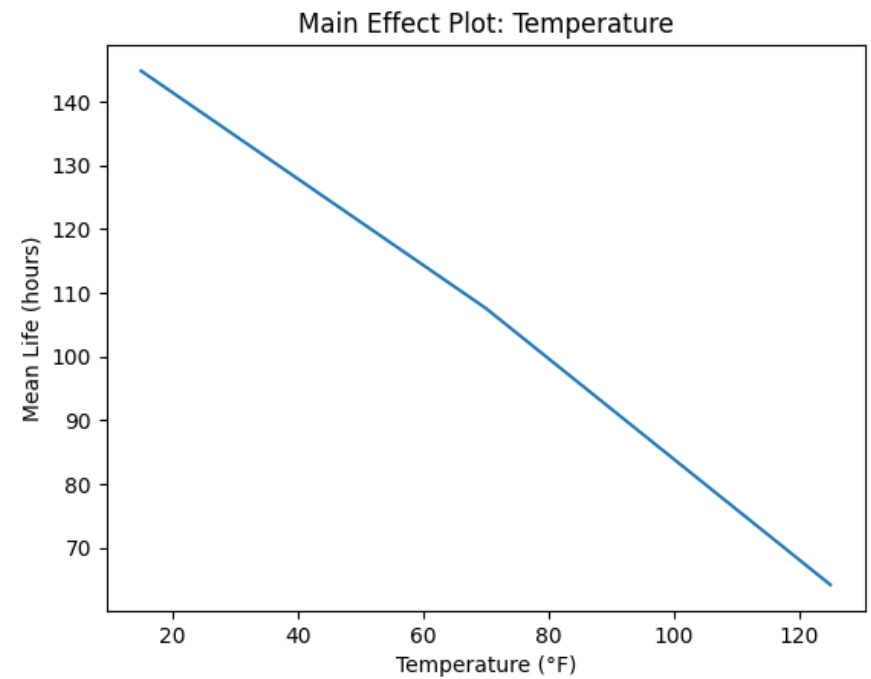
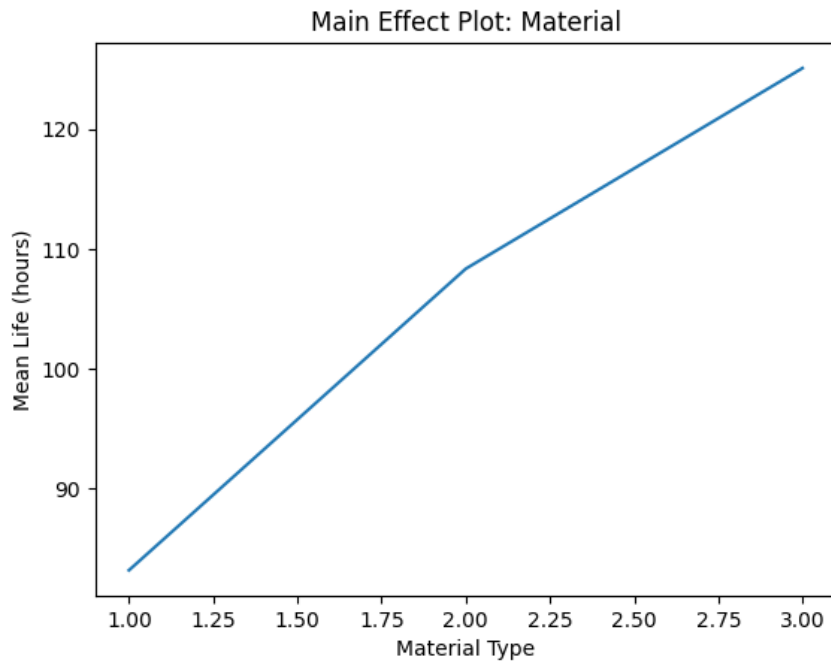
How to draw the main effect plot?

| Life (in hours) Data for the Battery Design Example | | | | | | |
|-----------------------------------------------------|------------------|-----|-----|-----|-----|-----|
| | Temperature (°F) | | | | | |
| Material Type | 15 | | 70 | | 125 | |
| 1 | 130 | 155 | 34 | 40 | 20 | 70 |
| | 74 | 180 | 80 | 75 | 82 | 58 |
| 2 | 150 | 188 | 136 | 122 | 25 | 70 |
| | 159 | 126 | 106 | 115 | 58 | 45 |
| 3 | 138 | 110 | 174 | 120 | 96 | 104 |
| | 168 | 160 | 150 | 139 | 82 | 60 |

A = Material type; B = Temperature

What effects do material type & temperature have on life?

Main Effects



Interaction Plot

