




# UltraDark.jl: A Julia package for simulation of cosmological scalar fields

Nathan Musoke <sup>1</sup>

<sup>1</sup> Department of Physics and Astronomy, University of New Hampshire, USA

DOI: [10.xxxxxx/draft](https://doi.org/10.xxxxxx/draft)

## Software

- [Review](#) 
- [Repository](#) 
- [Archive](#) 

Editor: [Juanjo Bazán](#)  

## Reviewers:

- [@szabo137](#)
- [@kiranshila](#)

Submitted: 16 September 2023

Published: unpublished

## License

Authors of papers retain copyright and release the work under a Creative Commons Attribution 4.0 International License ([CC BY 4.0](#)).

## Summary

UltraDark.jl is a Julia package for the simulation of cosmological scalar fields. Scalar fields are proposed solutions to two of the fundamental questions in cosmology: the nature of dark matter and the universe's initial conditions. Modeling their dynamics requires solving the Gross-Pitaevskii-Poisson equations, which is analytically challenging. This makes simulations essential to understanding the dynamics of cosmological scalar fields. UltraDark.jl is an open, performant and user friendly option for solving these equations numerically.

## Statement of need

Scalar fields are ubiquitous in physics, as solutions to partial differential equations describing the spatial variation of physical quantities. As dark matter candidates, scalar fields including axion-like particles (ALPs) would explain the nature of the missing 85% of the universe's matter ([Adams et al., 2022](#); [Ade & others, 2016](#); [Hu et al., 2000](#); [Matos et al., 2000](#)). As inflaton candidates, scalar fields are proposed to cause a phase of accelerated expansion that sets the stage for big bang nucleosynthesis ([Albrecht & Steinhardt, 1982](#); [Amin et al., 2012](#); [Guth, 1981](#); [Linde, 1982](#)). In each case, a scalar field  $\psi(t, \mathbf{x})$  represents the density of particles as a function of space and time.

Subject to reasonable conditions, a cosmological scalar field  $\psi$  whose particles have mass  $m$  obeys the Gross-Pitaevskii equation for the scalar field,

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2ma(t)^2} \nabla^2 \psi + m\Phi \psi, \quad (1)$$

coupled to the Poisson equation for the gravitational potential  $\Phi$

$$\nabla^2 \Phi = \frac{4\pi G}{a(t)} m |\psi|^2, \quad (2)$$

where  $a(t)$  is the scale factor characterising the expansion of the universe.

These equations are difficult to solve analytically – even static equilibrium solutions do not have a closed form – and necessitate the use of computer simulations. There are codes which solving [Equation 1](#) and [Equation 2](#) with different methods and in different domains, including PyUltraLight ([Edwards et al., 2018](#)), a code written in Chapel ([Padmanabhan et al., 2020](#)), AxioNyx ([Schwabe et al., 2020](#)), SCALAR ([Mina et al., 2020](#)), i-SPin 2 ([Jain et al., 2023](#); [Jain & Amin, 2023](#)); see Zhang et al. ([2018](#)) for an overview.

UltraDark.jl solves [Equation 1](#) and [Equation 2](#) with a pseudo-spectral symmetrized split-step

method, in which each time step consists of four sub-steps:

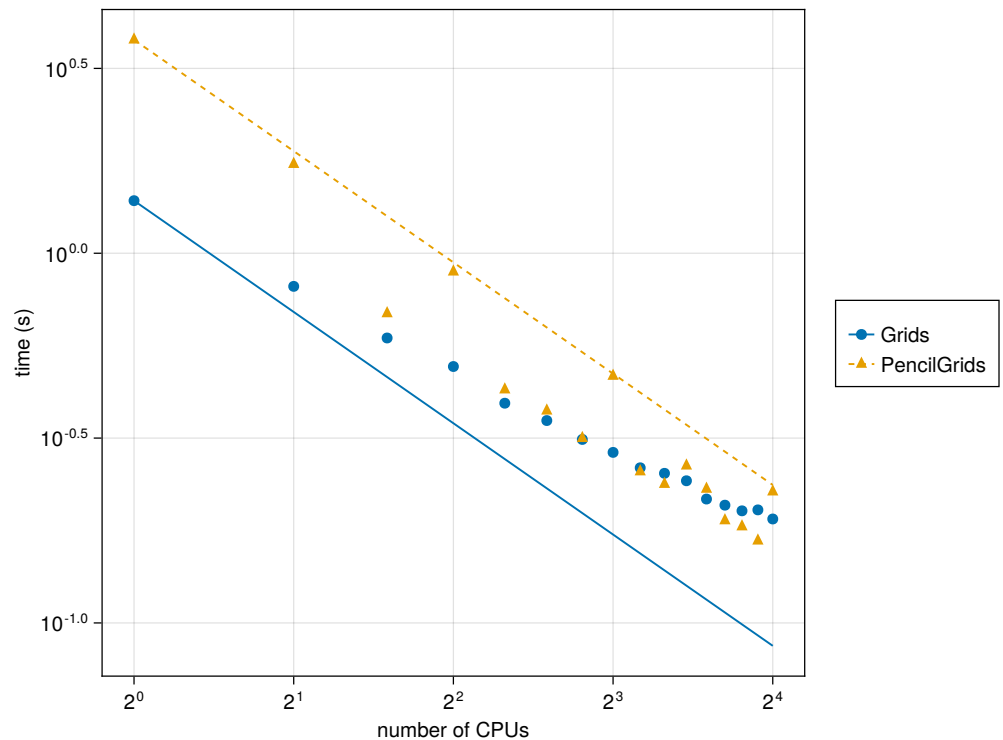
$$\psi \rightarrow \exp\left(-i\frac{h}{2}\Phi\right)\psi \quad (3)$$

$$\psi \rightarrow \mathcal{F}^{-1}\left\{\exp\left(-ih\frac{k^2}{2}\right)\mathcal{F}\{\psi\}\right\} \quad (4)$$

$$\Phi = \mathcal{F}^{-1}\left\{-\frac{4\pi}{ak^2}\mathcal{F}\{|\psi|^2\}\right\} \quad (5)$$

$$\psi \rightarrow \exp\left(-i\frac{h}{2}\Phi\right)\psi, \quad (6)$$

where  $\mathcal{F}$  is a Fourier transform,  $k$  are the corresponding frequencies, and  $h$  is the time step. UltraDark.jl has adaptive time steps which allow it to accelerate simulations while preserving numerical convergence. This is particularly useful in an expanding universe, where the time step is roughly  $h \propto a^2$ . Such time steps result in orders of magnitude speedups when simulating collapse of an inflaton field in the early universe (Musoke et al., 2020).



**Figure 1:** Wall time for a single time step, as a function of number of CPUs. The points represent measured times and the lines represent theoretical  $1/\#\text{CPU}$  scalings. The circles and solid line are for grids constructed from Arrays and the triangles and dashed lines are for MPI-distributed PencilArrays.

Julia (Bezanson et al., 2017) has seen increasing use in scientific computing; see for example Eschle & others (2023) and Roesch et al. (2021) for overviews of its use in high energy physics and biology. The use of Julia is one of the choices that separates UltraDark.jl from similar codes. UltraDark.jl uses Julia's rich parallelism capabilities. The Threads.@threads macro provides simple parallelisation of for loops. Folds.jl enables simple parallelisation of reduction operations (Arakaki, 2020). In a cluster environment, PencilArrays.jl and PencilFFTs.jl enable straightforward cross-node parallelism, a capability that is challenging to reproduce in Python (Polanco, 2021a, 2021b). The scaling of these two approaches to parallelism is demonstrated in Figure 1.

The features described above have allowed collaborators and I to produce results presented in 4 publications, each exploring the small scale structure of ultralight dark matter. We have used UltraDark.jl to explore tidal disruption in dark matter halos comprised of self-interacting ALPs (Glennon et al., 2022), perform the first simulations of multi-species ALPs with intra- and inter-species interactions (Glennon, Musoke, & Prescod-Weinstein, 2023b) and discover a novel mechanism for vortex stabilisation in scalar dark matter (Glennon, Mirasola, Musoke, et al., 2023a). Work in preparation examines the effect of self-interactions on dynamical friction (Glennon, Musoke, Nadler, et al., 2023). More sample output can be found in Glennon, Mirasola, Musoke, et al. (2023b) and Glennon, Musoke, & Prescod-Weinstein (2023a).

## Acknowledgements

I thank Arka Banerjee, Ben Chandran, Richard Easther, Mateja Gosenca, Shaun Hotchkiss, Emily Kendall, Anthony Mirasola, Ethan Nadler, Mark Neyrinck, Chanda Prescod-Weinstein, Yourong Frank Wang, Risa Wechsler, Luna Zagorac for discussion and comments on various stages of the development of UltraDark.jl. Particular thanks to Noah Glennon as the first user of UltraDark.jl.

Computations were performed on Marvin, a Cray CS500 supercomputer at UNH supported by the NSF MRI program under grant AGS-1919310. The author wishes to acknowledge the use of New Zealand eScience Infrastructure (NeSI) high performance computing facilities, consulting support and/or training services as part of this research. New Zealand's national facilities are provided by NeSI and funded jointly by NeSI's collaborator institutions and through the Ministry of Business, Innovation & Employment's Research Infrastructure programme. This work was performed in part at Aspen Center for Physics, which is supported by National Science Foundation under Grant No. PHY-1607611. This work was partially supported by a grant from the Sloan Foundation.

## References

- Adams, C. B., Aggarwal, N., Agrawal, A., Balafendiev, R., Bartram, C., Baryakhtar, M., Bekker, H., Belov, P., Berggren, K. K., Berlin, A., Boutan, C., Bowring, D., Budker, D., Caldwell, A., Carenza, P., Carosi, G., Cervantes, R., Chakrabarty, S. S., Chaudhuri, S., ... Zhou, T. (2022, March). Axion dark matter. *Snowmass 2021*. <https://doi.org/10.48550/ARXIV.2203.14923>
- Ade, P. A. R., & others. (2016). Planck 2015 results. XIII. Cosmological parameters. *Astron. Astrophys.*, 594, A13. <https://doi.org/10.1051/0004-6361/201525830>
- Albrecht, A., & Steinhardt, P. J. (1982). Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking. *Phys. Rev. Lett.*, 48, 1220–1223. <https://doi.org/10.1103/PhysRevLett.48.1220>
- Amin, M. A., Easther, R., Finkel, H., Flauger, R., & Hertzberg, M. P. (2012). Oscillons After Inflation. *Phys. Rev. Lett.*, 108, 241302. <https://doi.org/10.1103/PhysRevLett.108.241302>
- Arakaki, T. (2020). Folds: Sequential, threaded, and distributed fold interface for julia. In *GitHub repository*. GitHub. <https://github.com/JuliaFolds/Folds.jl>
- Bezanson, J., Edelman, A., Karpinski, S., & Shah, V. B. (2017). Julia: A fresh approach to numerical computing. *SIAM Review*, 59(1), 65–98. <https://doi.org/10.1137/141000671>
- Edwards, F., Kendall, E., Hotchkiss, S., & Easther, R. (2018). PyUltraLight: A pseudo-spectral solver for ultralight dark matter dynamics. *JCAP*, 10, 027. <https://doi.org/10.1088/1475-7516/2018/10/027>

- 92 Eschle, J., & others. (2023). *Potential of the Julia programming language for high energy*  
 93 *physics computing*. <https://arxiv.org/abs/2306.03675>
- 94 Glennon, N., Mirasola, A. E., Musoke, N., Neyrinck, M. C., & Prescod-Weinstein, C. (2023a).  
 95 Scalar dark matter vortex stabilization with black holes. *Journal of Cosmology and*  
 96 *Astroparticle Physics*, 2023(07), 004. <https://doi.org/10.1088/1475-7516/2023/07/004>
- 97 Glennon, N., Mirasola, A. E., Musoke, N., Neyrinck, M. C., & Prescod-Weinstein, C. (2023b).  
 98 *Supplementary animations for "Scalar dark matter vortex stabilization with black holes"*.  
 99 Zenodo. <https://doi.org/10.5281/zenodo.7675830>
- 100 Glennon, N., Musoke, N., Nadler, E. O., Prescod-Weinstein, C., & Wechsler, R. H. (2023).  
 101 Dynamical friction in self-interacting ultralight dark matter. *Under Review*.
- 102 Glennon, N., Musoke, N., & Prescod-Weinstein, C. (2023a). *Supplementary animations for*  
 103 *"Simulations of multi-field ultralight axion-like dark matter"*. Zenodo. [https://doi.org/10.](https://doi.org/10.5281/zenodo.7675775)  
 104 [5281/zenodo.7675775](https://doi.org/10.5281/zenodo.7675775)
- 105 Glennon, N., Musoke, N., & Prescod-Weinstein, C. (2023b). Simulations of multi-field ultralight  
 106 axion-like dark matter. *Physical Review D*, 107(6), 063520. [https://doi.org/10.1103/](https://doi.org/10.1103/physrevd.107.063520)  
 107 [physrevd.107.063520](https://doi.org/10.1103/physrevd.107.063520)
- 108 Glennon, N., Nadler, E. O., Musoke, N., Banerjee, A., Prescod-Weinstein, C., & Wechsler,  
 109 R. H. (2022). Tidal disruption of solitons in self-interacting ultralight axion dark matter.  
 110 *Physical Review D*, 105(12), 123540. <https://doi.org/10.1103/physrevd.105.123540>
- 111 Guth, A. H. (1981). The Inflationary Universe: A Possible Solution to the Horizon and Flatness  
 112 Problems. *Phys. Rev. D*, 23, 347–356. <https://doi.org/10.1103/PhysRevD.23.347>
- 113 Hu, W., Barkana, R., & Gruzinov, A. (2000). Cold and fuzzy dark matter. *Phys. Rev. Lett.*,  
 114 85, 1158–1161. <https://doi.org/10.1103/PhysRevLett.85.1158>
- 115 Jain, M., & Amin, M. A. (2023). i-SPin: An integrator for multicomponent schrödinger-poisson  
 116 systems with self-interactions. *JCAP*, 04, 053. [https://doi.org/10.1088/1475-7516/2023/](https://doi.org/10.1088/1475-7516/2023/04/053)  
 117 [04/053](https://doi.org/10.1088/1475-7516/2023/04/053)
- 118 Jain, M., Amin, M. A., & Pu, H. (2023). *I-SPin 2: An integrator for general spin-s gross-*  
 119 *pitaevskii systems*. <https://arxiv.org/abs/2305.01675>
- 120 Linde, A. D. (1982). A New Inflationary Universe Scenario: A Possible Solution of the Horizon,  
 121 Flatness, Homogeneity, Isotropy and Primordial Monopole Problems. *Phys. Lett. B*, 108,  
 122 389–393. [https://doi.org/10.1016/0370-2693\(82\)91219-9](https://doi.org/10.1016/0370-2693(82)91219-9)
- 123 Matos, T., Guzman, F. S., & Urena-Lopez, L. A. (2000). Scalar field as dark matter in the  
 124 universe. *Class. Quant. Grav.*, 17, 1707–1712. [https://doi.org/10.1088/0264-9381/17/7/](https://doi.org/10.1088/0264-9381/17/7/309)  
 125 [309](https://doi.org/10.1088/0264-9381/17/7/309)
- 126 Mina, M., Mota, D. F., & Winther, H. A. (2020). SCALAR: An AMR code to simulate  
 127 axion-like dark matter models. *Astron. Astrophys.*, 641, A107. [https://doi.org/10.1051/](https://doi.org/10.1051/0004-6361/201936272)  
 128 [0004-6361/201936272](https://doi.org/10.1051/0004-6361/201936272)
- 129 Musoke, N., Hotchkiss, S., & Easter, R. (2020). Lighting the dark: Evolution of the  
 130 postinflationary universe. *Phys. Rev. Lett.*, 124(6), 061301. [https://doi.org/10.1103/](https://doi.org/10.1103/PhysRevLett.124.061301)  
 131 [PhysRevLett.124.061301](https://doi.org/10.1103/PhysRevLett.124.061301)
- 132 Padmanabhan, N., Ronaghan, E., Zagorac, J. L., & Easter, R. (2020). Simulating ultralight  
 133 dark matter in chapel. *2020 IEEE International Parallel and Distributed Processing*  
 134 *Symposium Workshops (IPDPSW)*, 678–678. [https://doi.org/10.1109/ipdpsw50202.2020.](https://doi.org/10.1109/ipdpsw50202.2020.00120)  
 135 [00120](https://doi.org/10.1109/ipdpsw50202.2020.00120)
- 136 Polanco, J. I. (2021a). *PencilArrays.jl: Distributed Julia arrays using the MPI protocol* (Version  
 137 0.9.9). <https://doi.org/10.5281/zenodo.5148035>

- 138 Polanco, J. I. (2021b). *PencilFFTs.jl: FFTs of MPI-distributed Julia arrays* (Version 0.12.4).  
139 <https://doi.org/10.5281/zenodo.3618781>
- 140 Roesch, E., Greener, J. G., MacLean, A. L., Nassar, H., Rackauckas, C., Holy, T. E., & Stumpf,  
141 M. P. H. (2021). *Julia for biologists*. <https://doi.org/10.48550/ARXIV.2109.09973>
- 142 Schwabe, B., Gosenca, M., Behrens, C., Niemeyer, J. C., & Easther, R. (2020). Simulating  
143 mixed fuzzy and cold dark matter. *Phys. Rev. D*, 102(8), 083518. [https://doi.org/10.](https://doi.org/10.1103/PhysRevD.102.083518)  
144 [1103/PhysRevD.102.083518](https://doi.org/10.1103/PhysRevD.102.083518)
- 145 Zhang, J., Liu, H., & Chu, M.-C. (2018). Cosmological simulation for fuzzy dark matter model.  
146 *Frontiers in Astronomy and Space Sciences*, 5. <https://doi.org/10.3389/fspas.2018.00048>

DRAFT