

NEoST: A Python package for nested sampling of the neutron star equation of state

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Software

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Summary

The Nested Equation of State Sampling (NEoST) package is an open-source code that allows users to infer the parameters of the dense matter Equation of State (EoS) in neutron stars via nested sampling. It provides a Bayesian inference framework that compares pre-existing EoS models (parameterized or tabulated, for both crust and core) to a variety of user-defined astrophysical input data (real or synthetic), namely mass-radius samples, mass-tidal deformability samples, and mass samples. NEoST can also be used to provide a fast solver for the Tolman-Oppenheimer-Volkoff equations for neutron star structure (Oppenheimer & Volkoff, 1939; Tolman, 1939).

Statement of need

Matter in the cores of neutron stars can reach several times the nuclear saturation density. The EoS of matter under such circumstances is not well understood: in addition to extreme levels of neutron-richness there could also exist stable states of strange matter, in the form of either hyperons or deconfined quarks (Hebeler et al., 2015; Lattimer & Prakash, 2016; Tolos & Fabbietti, 2020). Neutron star properties like mass, radius and tidal deformability depend on the EoS, so measurement of these quantities provides insight into the properties of ultradense nuclear matter.

Astrophysical data sets that can be used to constrain the EoS take the form of posterior distributions that are derived from separate inference analyses. Examples include: mass posteriors from pulsar timing analysis of radio pulsars in binary systems (Fonseca et al., 2021), joint mass-radius posteriors from Pulse Profile Modeling using NICER data (Miller et al., 2021; Riley et al., 2021); and joint mass-tidal deformability posters from gravitational wave observations of neutron star binary mergers (Abbott et al., 2019). NEoST provides a framework for EoS inference that couples these various different types of astrophysical data to either parameterized or tabulated EoS models.

The NEoST package and science use

NEoST is an open source Python package for Bayesian inference of EoS parameters (for parameterized models) and/or evidence computation (for parameterized and tabulated models), given astrophysical data sets in the form of posterior distributions. NEoST samples from the prior distribution of the EoS model parameters and central densities, computes the corresponding

mass and radius/tidal deformability and then evaluates the likelihood by applying a kernel density estimation to the posterior distributions of the astrophysical data sets using the nested sampling software MultiNest (Buchner et al., 2014; Feroz et al., 2009), Figure 1. The full Bayesian inference framework, including notes relating to prior distributions, is described in detail in (Raaijmakers et al., 2020). It includes a library of existing EoS models for crust and core, and users can easily define their own models.

NEoST also offers various options for post-processing, including generating plots showing the inferred EoS credible regions in pressure-energy density space, and the associated inferred mass-radius relation credible intervals.

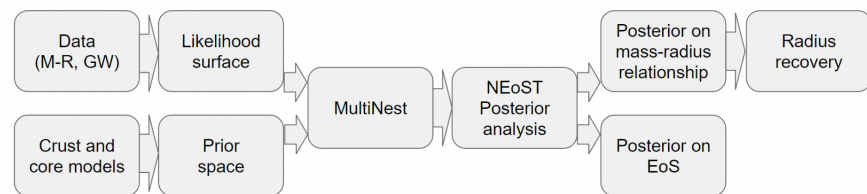


Figure 1: A schematic representation of the inference process using NEoS. It shows how the track for physical measurements and the track for theoretical models are fed through the framework, and what the main steps of analysis are after inference is complete.

NEoS is being used by the NICER collaboration for EoS inference using mass-radius posteriors generated from pulse profile modeling (Raaijmakers et al., 2019, 2020, 2021), specifically those generated using the X-PSI package (Riley et al., 2019, 2021, 2023). It has also been used to study EoS prior sensitivities using synthetic mass-radius posteriors (Greif et al., 2019) and to study the consequences of a potential dark matter component in neutron stars (Rutherford et al., 2023).

The core routines of NEoS are written in Cython (Behnel et al., 2011), and are dependent on the GNU Scientific Library (GSL, Gough, 2009). In case the user does not wish to use cythonised code, there is also an alternative set of routines written purely in Python. High-level object-oriented model construction is performed by a user in the Python language.

Release versions of NEoS are freely available on GitHub under the GNU General Public License. Extensive documentation, step-by-step tutorials, and reproduction code for existing data analyses, are available via the GitHub repository, along with a suite of unit tests. Future plans include tutorials documenting different types of astrophysical data sets, EoS models that include a dark matter component, and options for coupling to different samplers.

Software: Python/C language (Oliphant, 2007), GNU Scientific Library (GSL, Gough, 2009), NumPy (van der Walt et al., 2011), Cython (Behnel et al., 2011), OpenMP (Dagum & Menon, 1998), MPI for Python (Dalcín et al., 2008), Matplotlib (Droettboom et al., 2018; Hunter, 2007), IPython (Perez & Granger, 2007), Jupyter (Kluyver et al., 2016), MultiNest (Feroz et al., 2009), PyMultiNest (Buchner et al., 2014), GetDist (Lewis, 2019), SciPy (Jones et al., 2001–), Seaborn (Waskom, 2021), corner.py (Foreman-Mackey, 2016), alive-progress (Sampaio de Almeida, 2019 –).

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