

PyPEEC: A 3D Quasi-Magnetostatic Solver using an FFT-Accelerated PEEC Method with Voxelization

Thomas Guillod¹ and Charles R. Sullivan¹

¹ Dartmouth College, NH, USA ¶ Corresponding author

DOI: [10.xxxxxx/draft](https://doi.org/10.xxxxxx/draft)

Software

- [Review](#)
- [Repository](#)
- [Archive](#)

Editor: [↗](#)

Submitted: 20 December 2023

Published: unpublished

License

Authors of papers retain copyright and release the work under a Creative Commons Attribution 4.0 International License ([CC BY 4.0](#)).

Summary

The partial element equivalent circuit method (PEEC) is a particularly interesting integral-equation method for electromagnetic problems as it can be easily combined with circuit simulations and does not require a discretization of the free space. This paper introduces PyPEEC, an open-source Python solver targeting 3D quasi-magnetostatic problems such as inductors, transformers, chokes, IPT coils, or busbars. The geometry is described with voxels and the solver uses an FFT-accelerated variant of the PEEC method. The FFT acceleration drastically reduces the computational costs and the memory footprint.

Statement of Need

Quasi-magnetostatic field simulations are widely used for the design and optimization of electrical components (e.g., power electronics, packaging, IC design). Among the existing numerical methods (e.g., FEM, FDTD, PEEC, BEM), the PEEC method features several advantages ([Jithesh & Pande, 2003](#); [Ruehli, 1974](#); [Torchio, 2019](#)):

- Only the active materials are discretized (no need to mesh the free space).
- Intuitive understanding of the equation discretization process.
- Straightforward connection of external circuit elements.

The fundamental drawback of the classical PEEC method is that the matrix describing the equation system is not sparse. This means that the computational cost and the memory requirement become problematic for large problems.

This problem can be mitigated if the geometry is represented with voxels ([Torchio, Lucchini, et al., 2022](#); [A. C. Yucel et al., 2018](#)). The first advantage of such geometries is that the Green's functions (i.e. the inductance and potential matrices) have analytical solutions, avoiding computationally intensive numerical integrals ([Hoer & Love, 1965](#); [Piatek & Baron, 2012](#)). The second advantage is that, due to the regular voxel discretization, many coefficients of the inductance and potential matrices are repeated. The dense matrices are block-Toeplitz Toeplitz-block matrices and feature the following key properties ([Lee, 1986](#); [Polimeridis et al., 2014](#)):

- The block-Toeplitz Toeplitz-block matrices can be embedded into circulant tensors reducing the memory requirements from $O(n^2)$ to $O(n)$.
- The matrix-vector multiplications can be done with Fourier transforms. With an FFT algorithm, the computational complexity of matrix-vector multiplications is reduced from $O(n^2)$ to $O(n \log(n))$.

Different variants of the FFT-accelerated PEEC method have been shown to be extremely fast for a large variety of electromagnetic problems such as high-frequency transformers ([Torchio, Lucchini, et al., 2022](#)), power inductors ([Marconato & Lucchini, 2021](#)), human body field

40 exposure (Torchio, Arduino, et al., 2022), nuclear fusion devices (Bettini et al., 2020), and
41 PCB layouts (Romano et al., 2023).

42 An open-source implementation (“FFT-PEEC”) of the FFT-accelerated PEEC method already
43 exists (Torchio & Lucchini, 2020, 2021). However, this code can only handle magnetic
44 materials for static simulations (and not in the frequency domain). Moreover, depending on
45 the considered quasi-static problem (e.g. geometries with multiple conductors), the source code
46 requires some adaptations. The open-source tools “VoxHenry” and “MARIE” are also used
47 FFT-accelerated methods but are specialized for full-wave solutions without magnetic materials
48 (Villena et al., 2015; A. Yucel, 2018). Finally, it should be noted that all the aforementioned
49 implementations depend on proprietary programming languages and libraries.

50 PyPEEC, the tool introduced in this paper, addresses these needs by providing a 3D quasi-
51 magnetostatic implementation of the FFT-accelerated PEEC method (Torchio, Lucchini, et
52 al., 2022). PyPEEC is a fully open-source (MPL 2.0 licence and implemented in Python) and
53 offers a general-purpose solver by allowing the description of arbitrary problems without having
54 to modify the source code.

55 Capabilities and Workflow

56 In this paper, the release 4.8.0 of PyPEEC is considered. PyPEEC solves 3D magnetostatic
57 and quasi-magnetostatic problems with voxel geometries. An arbitrary number of conductive
58 domains, magnetic domains (ideal and/or lossy), and sources (voltage and/or current) can be
59 used. The current density, flux density, potential, loss density, and energy density are computed.
60 The free-space magnetic field (near-field) can also be computed on a point cloud. Additionally,
61 the voltage, current, complex power, and impedance of the different terminals are extracted.

62 PyPEEC is implemented in pure Python using NumPy, SciPy, Joblib, Rasterio, Shapely,
63 Pillow, Matplotlib, and PyVista. The solver is able to leverage multi-core CPUs and GPUs.
64 Optional HPC libraries are available for accelerating the sparse preconditioner factorization
65 (MKL/PARDISO, UMFPACK, and PyAMG) and the FFT operations (FFTW and CuPy/CUDA).
66 PyPEEC can be used through an API, a command-line tool, or Jupyter notebooks.

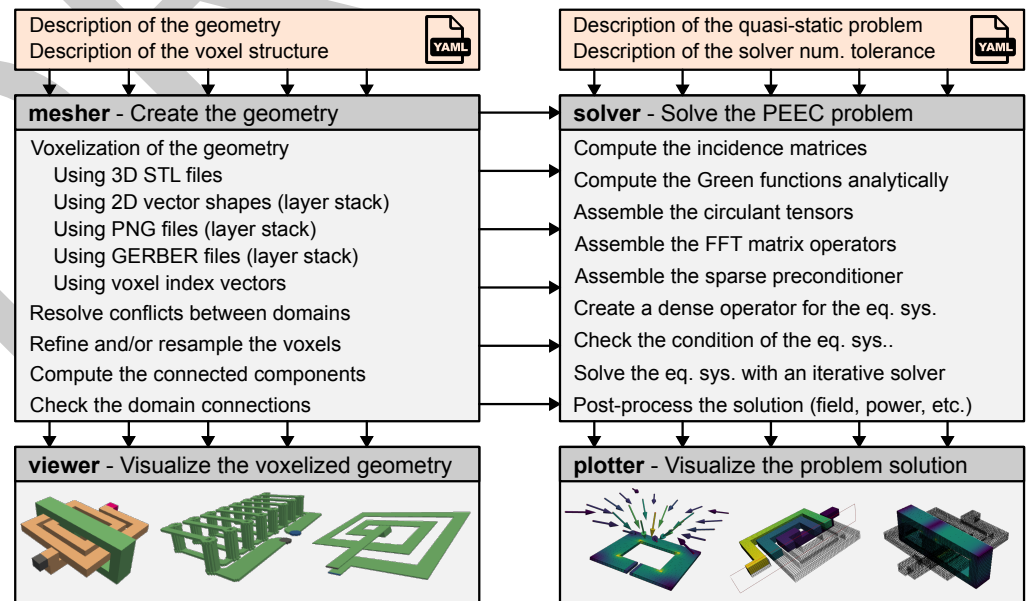


Figure 1: PyPEEC workflow consisting of the *mesher*, *solver*, *viewer*, and *plotter*

Figure 1 depicts the PyPEEC workflow. First the *mesher* builds the geometry (from vector shapes, STL files, PNG files, or GERBER files), performs the voxelization, and checks the validity of the discretization. Afterward, the *solver* creates the FFT-multiplication operators, assembles the equation systems, solves the problem, and post-processes the solution. The *viewer* and the *plotter* are used to visualize the results.

Solver Performance

To demonstrate the performance of the solver, a planar air-core spiral inductor is considered. The inductor has a footprint of 4 mm^2 and is operated in the 40.68 MHz ISM band. Figure 2 shows the ratio between the AC and DC current densities, the relative error on the extracted impedance, and the computational cost. The number of degrees of freedom represents the number of unknowns for the PEEC problem. The computational cost is evaluated with an AMD EPYC 7543 CPU.

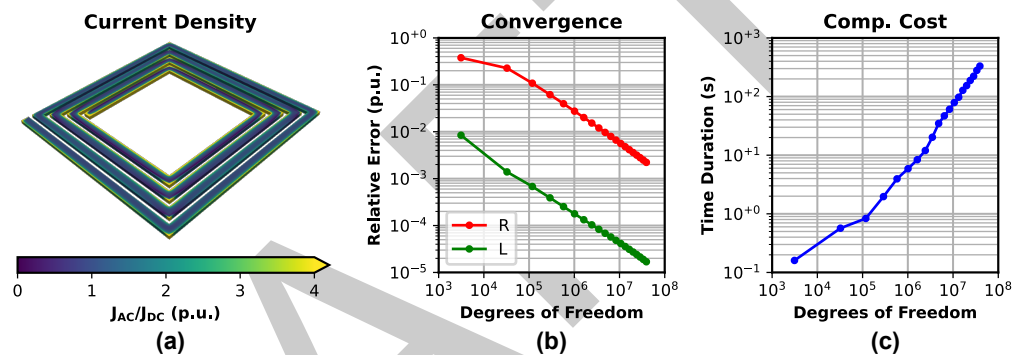


Figure 2: (a) Ratio between the AC and DC current densities. (b) Relative error on the extract equivalent resistance and the inductance. (c) Wall clock time duration for the complete workflow.

As the skin depth is smaller than the dimension of the conductor, the eddy currents are non-negligible. This implies that, with a coarse discretization, the resistance value is less accurate than the inductance value. With a tolerance of 4%, the quasi-static problem is solved in 4 seconds. The FFT acceleration allows dense PEEC problems with 10^7 degrees of freedom to be solved in 80 seconds.

Acknowledgments

This work was supported by the Power Management Integration Center (NSF IUCRC) at Dartmouth College under Grant No. PMIC-062. The authors would like to thank Yue (Will) Wu for discovering and reporting several bugs.

References

- Bettini, P., Torchio, R., Lucchini, F., Voltolina, D., & Alotto, P. (2020). Fast Fourier Transform-Volume Integral: a Smart Approach for the Electromagnetic Design of Complex Systems in Large Fusion Devices. *Plasma Physics and Controlled Fusion*, 63(2), 025010. <https://doi.org/10.1088/1361-6587/abce8f>
- Hoer, C., & Love, Y. (1965). Exact Inductance Equations for Rectangular Conductors with Applications to more Complicated Geometries. *Journal of Research of the National Bureau of Standards*, 69(2), 127–137. https://nvlpubs.nist.gov/nistpubs/jres/69C/jresv69Cn2p127_A1b.pdf

- 97 Jithesh, V., & Pande, D. C. (2003). A Review on Computational EMI Modelling Techniques.
98 *Proc. Of the Int. Conf. On Electromagnetic Interference and Compatibility (INCEMIC)*,
99 159–166. <https://doi.org/10.1109/ICEMIC.2003.1287800>
- 100 Lee, D. (1986). Fast Multiplication of a Recursive Block Toeplitz Matrix by a Vector and its Ap-
101 plication. *Journal of Complexity*, 2(4), 295–305. [https://doi.org/10.1016/0885-064X\(86\)](https://doi.org/10.1016/0885-064X(86)90007-5)
102 [90007-5](https://doi.org/10.1016/0885-064X(86)90007-5)
- 103 Marconato, N., & Lucchini, F. (2021). Application of FFT-PEEC Method for Nonlinear
104 Inductance Extraction. *Proc. Of the Int. Conf. On Electrical, Computer, Communications*
105 *and Mechatronics Engineering (ICECCME)*, 1–6. [https://doi.org/10.1109/ICECCME52200.](https://doi.org/10.1109/ICECCME52200.2021.9590864)
106 [2021.9590864](https://doi.org/10.1109/ICECCME52200.2021.9590864)
- 107 Piatek, Z., & Baron, B. (2012). Exact Closed Form Formula for Self Inductance of Conductor
108 of Rectangular Cross Section. *Progress In Electromagnetics Research*, 26, 225–236. <http://doi.org/10.2528/PIERM12080314>
109
- 110 Polimeridis, A. G., Villena, J. F., Daniel, L., & White, J. K. (2014). Stable FFT-JVIE Solvers
111 for Fast Analysis of Highly Inhomogeneous Dielectric Objects. *Journal of Computational*
112 *Physics*, 269, 280–296. <https://doi.org/10.1016/j.jcp.2014.03.026>
- 113 Romano, D., Kovacevic-Badstuebner, I., Antonini, G., & Grossner, U. (2023). Efficient PEEC
114 Iterative Solver for Power Electronic Applications. *IEEE Transactions on Electromagnetic*
115 *Compatibility*, 65(2), 546–554. <https://doi.org/10.1109/TEM.2023.3238394>
- 116 Ruehli, A. E. (1974). Equivalent Circuit Models for Three-Dimensional Multiconductor
117 Systems. *IEEE Transactions on Microwave Theory and Techniques*, 22(3), 216–221.
118 <https://doi.org/10.1109/TMTT.1974.1128204>
- 119 Torchio, R. (2019). A Volume PEEC Formulation Based on the Cell Method for Electromagnetic
120 Problems From Low to High Frequency. *IEEE Transactions on Antennas and Propagation*,
121 67(12), 7452–7465. <https://doi.org/10.1109/TAP.2019.2927789>
- 122 Torchio, R., Arduino, A., Zilberti, L., & Bottausci, O. (2022). A Fast Tool for the Parametric
123 Analysis of Human Body Exposed to LF Electromagnetic Fields in Biomedical Applications.
124 *Computer Methods and Programs in Biomedicine*, 214, 106543. [https://doi.org/10.1016/j.](https://doi.org/10.1016/j.cmpb.2021.106543)
125 [cmpb.2021.106543](https://doi.org/10.1016/j.cmpb.2021.106543)
- 126 Torchio, R., Lucchini, F., Schanen, J. L., Chadebec, O., & Meunier, G. (2022). FFT-PEEC:
127 A Fast Tool From CAD to Power Electronics Simulations. *IEEE Transactions on Power*
128 *Electronics*, 37(1), 700–713. <https://doi.org/10.1109/TPEL.2021.3092431>
- 129 Torchio, R., & Lucchini, L. (2020). FFT-PEEC. In *GitHub repository*. GitHub. <https://github.com/UniPD-DII-ETCOMP/FFT-PEEC>
130
- 131 Torchio, R., & Lucchini, L. (2021). FFT-NLIE. In *GitHub repository*. GitHub. <https://github.com/UniPD-DII-ETCOMP/FFT-NLIE>
132
- 133 Villena, J. F., Polimeridis, A., & Serralles, J. (2015). MARIE. In *GitHub repository*. GitHub.
134 <https://github.com/thanospol/MARIE>
- 135 Yucel, A. (2018). VoxHenry. In *GitHub repository*. GitHub. [https://github.com/acyucel/](https://github.com/acyucel/VoxHenry)
136 [VoxHenry](https://github.com/acyucel/VoxHenry)
- 137 Yucel, A. C., Georgakis, I. P., Polimeridis, A. G., Bagci, H., & White, J. K. (2018). VoxHenry:
138 FFT-Accelerated Inductance Extraction for Voxelized Geometries. *IEEE Transactions on*
139 *Microwave Theory and Techniques*, 66(4), 1723–1735. [https://doi.org/10.1109/TMTT.](https://doi.org/10.1109/TMTT.2017.2785842)
140 [2017.2785842](https://doi.org/10.1109/TMTT.2017.2785842)