

State-Averaged Orbital-Optimized VQE: A quantum algorithm for the democratic description of ground and excited electronic states

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Summary

The electronic structure problem is one of the main problems in modern theoretical chemistry. While there are many already-established methods both for the problem itself and its applications like semi-classical or quantum dynamics, it remains a computationally demanding task, effectively limiting the size of solved problems. Fortunately, it seems, that offloading some parts of the computation to *Quantum Processing Units (QPUs)* may offer significant speed-up, often referred to as *quantum supremacy* or *quantum advantage*. Together with the potential advantage, this approach simultaneously presents several problems, most notably naturally occurring quantum decoherence, hereafter denoted as *quantum noise* and lack of large-scale quantum computers, making it necessary to focus on Noisy-Intermediate Scale Quantum computers when developing algorithms aspiring to near-term applications. SA-OO-VQE package aims to answer both these problems with its hybrid quantum-classical conception based on a typical Variational Quantum Eigensolver approach, as only a part of the algorithm utilizes offload to QPUs and the rest is performed on a classical computer, thus partially avoiding both quantum noise and the lack of *quantum bits (qubits)*. The SA-OO-VQE has the ability to treat degenerate (or quasi-degenerate) states on the same footing, thus avoiding known numerical optimization problems arising in state-specific approaches around avoided crossings or conical intersections.

Statement of need

Recently, quantum chemistry is one of the main areas-of-interest in *Quantum Computing (QC)* ([Bauer et al., 2020](#); [McArdle et al., 2020](#); [Reiher et al., 2017](#)). That said, in many real-life applications, there is the necessity of treating both the ground and excited states accurately and in an equal footing. The problem is magnified when the Born-Oppenheimer approximation breaks down due to a strong coupling among degenerate or quasi-degenerate states, most notably the ground and the first excited state, for which the accurate description requires (computationally demanding) multi-configurational approaches. A good example of such case is a photoisomerization mechanism of the rhodopsin chromophore, which progresses from the initial photoexcitation of the *cis* isomer over the relaxation in the first excited state towards a conical intersection, where the population is transferred back to the ground state of the *trans* isomer. In order to describe such a process thoroughly, one must compute not only relevant *potential energy surfaces (PESs)*, but also their gradients w.r.t. nuclear displacements, utilized

42 further in molecular dynamics simulations. Finally, description of the conical intersection can
43 be done via obtaining *non-adiabatic couplings* (NACs).

44 Formally, the approaches describing PES topology, topography and the non-adiabatic couplings
45 require Hamiltonian diagonalization, which represents the most significant bottleneck. Consider-
46 ing classical methods like State-Averaged Multi-Configurational Self-Consistent Field (Helgaker
47 et al., 2013), only small complete active spaces have to be used for the large computational
48 overhead inherently present. However, such an approximation brings missing dynamical correla-
49 tion treatment, inducing the need to recover it ex-post, usually via some of the quasi-degenerate
50 perturbation techniques (Granovsky, 2011; Park, 2019). On the other hand, QC brings the
51 possibility of large complete active spaces back, thus retaining a substantial part of dynamical
52 correlation. Moreover, the dynamical correlation can be also retrieved a posteriori utilizing
53 QPUs only at the expense of more measurements, with no additional demands on hardware
54 infrastructure (Takeshita et al., 2020).

55 *State-Averaged Orbital-Optimization Variational Quantum Eigensolver* (SA-OO-VQE) method
56 addresses the above-mentioned problems and provides a way to compute both PES gradients
57 and NACs analytically (Yalouz et al., 2021; Yalouz, Koridon, Senjean, Lasorne, et al., 2022).
58 Authored by Yalouz et al., there is an exemplary implementation (Yalouz, Koridon, & Senjean,
59 2022) focusing on the pedagogical aspect and relying on matrix-vector multiplications rather
60 than actual measurements, avoiding the utilization of real QC infrastructure. Our implemen-
61 tation differs in a way that it aims to be a production-ready solver utilizing both QCs and
62 high-performance computing infrastructure in an efficient way, being able to run with different
63 backgrounds, utilizing Qiskit toolbox interface. The whole code is written in Python3, with
64 YAML scripts enabling its fast installation and usage.

65 The results are illustrated on molecule of formalimine, which can be seen in Figure 1.
66 Their comparison with the ones obtained via Molcas (Li Manni et al., 2023) implementation
67 of Complete Active-Space Self-Consistent Field (Malmqvist & Roos, 1989) are shown in
68 Figure 2, Figure 3 and Figure 4. All the computations were computed with 3 active orbitals
69 containing 4 electrons and with *sto-3g* basis set.

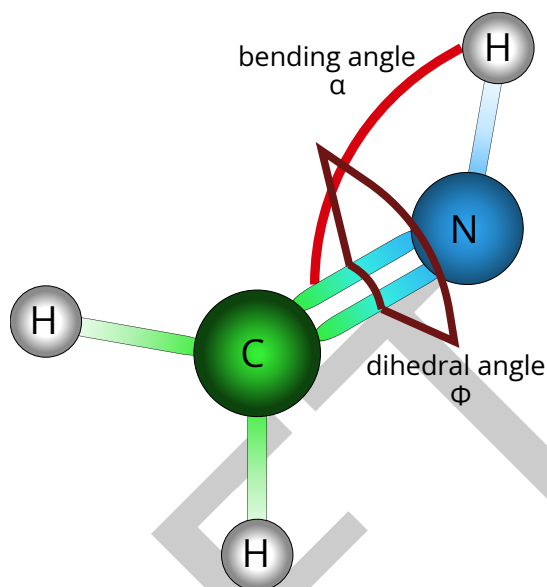


Figure 1: Molecule of formalimine being described with bending and dihedral angles denoted α and ϕ , respectively.

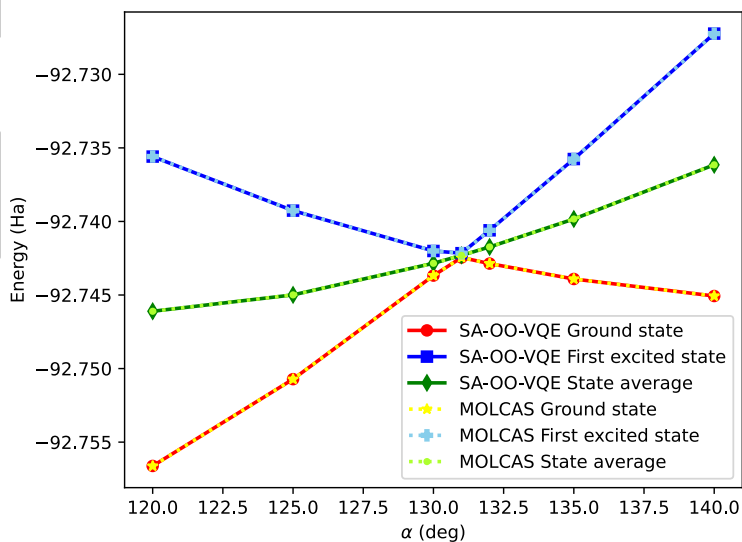


Figure 2: Comparison of potential energy depending on bending angle α in formalimine molecule with dihedral angle $\phi = 90^\circ$.

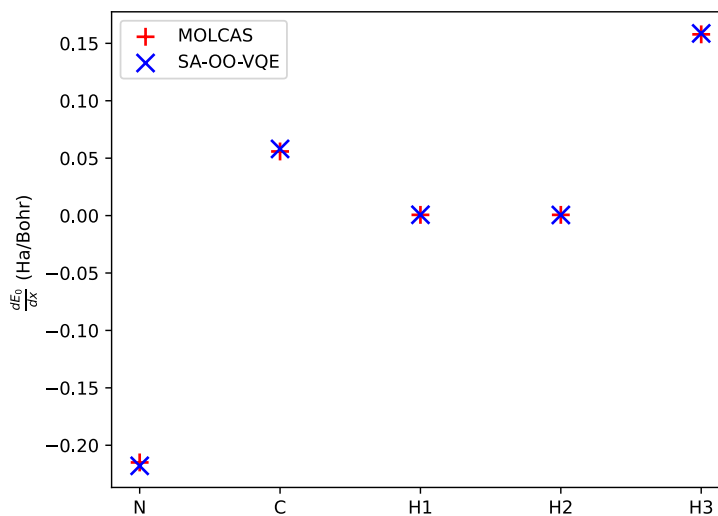


Figure 3: Comparison of ground-state gradients with bending angle $\alpha = 130^\circ$ and dihedral angle $\phi = 90^\circ$ in formalimine molecule.

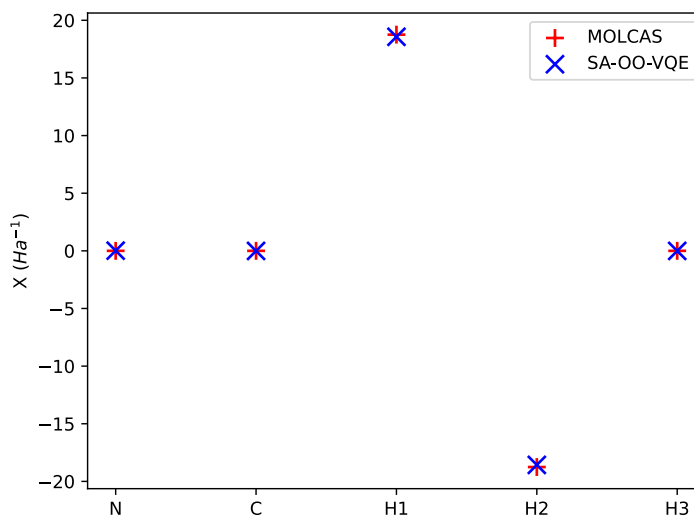


Figure 4: Comparison of total non-adiabatic couplings on bending angle $\alpha = 130^\circ$ and dihedral angle $\phi = 90^\circ$ in formalimine molecule.

Features

With SA-OO-VQE you can obtain following quantitties:

- Potential energy surfaces
- Circuit (or Ansatz) gradients
- Orbital gradients

- 75 ▪ Gradients of potential energy surfaces
- 76 ▪ Non-adiabatic couplings
- 77 Also, for numerical optimization you can use any of the optimizers supported by Qiskit¹ and
- 78 our own implementation of
- 79 ▪ Particle Swarm Optimization

80 Getting Started

81 The package is prepared with a priority of being very simple to use and the concise documenta-
82 tion can be found at sa-oo-vqe-qiskit.rtdf.io. To simplify the installation part, we recommend
83 utilizing Conda management system² together with prepared `environment.yml` file.

84 At first, users should clone the repository.

```
85   git clone git@gitlab.com:MartinBeseda/sa-oo-vqe-qiskit.git
```

86 And install all the dependencies.

```
87   $ conda env create -f environment.yml
88   $ conda init bash
89   $ source ~/.bashrc
90   $ conda activate saooovqe-env
91   $ pip install .
```

92 These commands run in a terminal will download and install all the necessary packages. The
93 package availability can be tested afterward simply by importing the package and looking at
94 its version.

```
95   $ python3
96
97   >>> import saooovqe
98   >>> saooovqe.__version__
```

99 Finally, usage examples are located both in *examples* folder and in the documentation.

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