




# Automatic Computation for Robot Design (ACRoD): A Python package for numerical calculation of Jacobian of a robot at a given configuration around a specified end-effector point

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## Summary

The Jacobian of a robot refers to the matrix that linearly maps the velocity components of the end-effector and the velocities at the actuated joints. The Jacobian is extensively used in dimensional synthesis for Jacobian-based optimal performances of robotic manipulators, in which the optimal dimensional parameters of robots are computed. Determination of accurate mobility ([Yang et al., 2008](#)) of planar and spatial mechanisms can also be performed by using Jacobian in cases where Chebychev–Grübler–Kutzbach criterion cannot accurately determine the mobility ([Gogu, 2005](#)). As a result, Jacobian is a significant part for both kinematic analysis, dimensional synthesis and mobility determination of a mechanism. Hence, the formulation of Jacobian has its key importance in the literature and in the application of performance optimisation along with mobility computation. Formulation of Jacobian for serial manipulators can be computed easily, however, it is increasingly complicated to formulate Jacobian for parallel manipulators due to the existence of passive joint velocities and the nature in which these are related to active joint velocities. Several studies ([Altuzarra et al., 2006](#); [Dutre et al., 1997](#); [D. Kim et al., 2000](#); [S.-G. Kim & Ryu, 2003](#)) exist for formulation of parallel manipulators but all these studies are dependent on human inspection at some level. Several open source softwares are also available for Jacobian formulation ([Baumgärtner & Miller, 2022](#); [Lee et al., 2018](#); [Nadeau, 2019](#); [Owan et al., 2018](#)), but either their application is limited to serial manipulators, they require human intervention, or they are part of computationally expensive simulations. For example, TriP ([Baumgärtner & Miller, 2022](#)) does facilitate Jacobian computation with closed-loop chains in the manipulator structure, however, as obvious from the triped robot example in the documentary, human inspection is apparently required to appropriately join the legs of the robot. To alleviate the drawback of the requirement of human inspection, the present research aims to formulate Jacobian that is required for dimensional synthesis for optimal performance around a single point that can be used for any non-redundant manipulator without any dependency on human inspection. Jacob and Dasgupta ([Jacob & Dasgupta, 2022](#)) used a systematic method as a tool to formulate Jacobian matrices for several manipulators in bulk for performance optimisation around a given task point. However, several steps in that algorithm are not totally computerised but rather human-intervention-dependent. Moreover, it can be applicable only with four types of joints. The current paper extends their method to present a fully computerisable Jacobian formulation algorithm that is applicable for general non-redundant planar and spatial manipulators of any topological structure of seven types of joints, namely revolute, prismatic, cylindrical, spherical, universal, helical and plane joints. Based on this extended method, the Python package Automatic Computation for Robot Design (ACRoD) is developed by the authors. ACRoD provides a Python-based package for generating functions required to compute the Jacobian at a given configuration for a

44 given end-effector point, merely from the simple topological information of the robot in a  
45 fully automated manner. This can be directly used in optimisation process to derive optimal  
46 dimensions of the robot for optimal performance around a given end-effector point, thereby  
47 avoiding many tedious steps in manual formulation, especially when a comparison study is  
48 performed on multiple manipulators in bulk ([Jacob & Dasgupta, 2022](#)). ACROD uses NumPy  
49 ([Harris et al., 2020](#)) and SymPy ([Meurer et al., 2017](#)) packages to generate the functions for  
50 Jacobian, which can be directly used in optimisation process to find the optimal dimensional  
51 parameters of the robot.

## 52 Statement of need

53 For a manipulator of a given topology, designing the dimensions based on optimising Jacobian-  
54 based performance parameters (such as manipulability index and condition number ([Patel &  
55 Sobh, 2015](#))) around a given end-effector point would require only the topological information  
56 for the formulation of Jacobian, as every other step can be automated. Formulation of Jacobian  
57 for parallel manipulators and serial-parallel hybrid manipulators are non-trivial, although all  
58 the steps of Jacobian formulation even in those cases would have to stem from the mere  
59 information of topology of the robot. ACROD automates the non-trivial formulation of Jacobian  
60 systematically. It uses a matrix-based representation of the topology of the robotic manipulator  
61 (referred to here as the robot-topology matrix, of which more information is provided [here](#))  
62 which is a modified version of the graph adjacency matrix representation ([Jacob et al., 2022](#)) of  
63 robotic manipulators. This Jacobian formulation can be used to generate numerical Jacobian  
64 matrices with a few random configurations, from which the singular values can be calculated  
65 which can confirm the Degree of Freedom (DoF). In other words, the DoF of a given robot  
66 topology for a given base link and a given end-effector link can be verified by using this Jacobian  
67 function even in cases where Chebychev–Grübler–Kutzbach criterion fails to verify. This can be  
68 useful in mechanism synthesis to accurately verify the mobility of a given manipulator directly  
69 from its robot-topology matrix by using the method shown in Yang et al.'s paper ([Yang et al.,  
70 2008](#)).

## 71 Method

72 The topology of a valid robot (with a single base-link and a single end-effector link and without  
73 non-contributing chains) is to be specified using robot-topology matrix in NumPy matrix  
74 format. The jacobian class object takes this robot-topology matrix as input argument and  
75 generates functions that are required to compute Jacobian. As byproducts, the Jacobian  
76 function generation produces the symbolic matrices, the set of independent paths, etc., the  
77 sets of active joint velocities and passive joint velocities, etc., which can be accessed from  
78 the attributes of the jacobian class object. More technical details on formulation of Jacobian  
79 (along with appropriate algorithms) can be found [here](#), and the notations and the nomenclature  
80 are explained [here](#) in detail. The robot-topology matrix representation is explained [here](#) in  
81 detail. Jacobian formulation for three robot examples, namely the [3R planar serial robot](#), a  
82 [4R-4P planar serial-parallel hybrid robot](#) and an [RSSR-SSR spatial parallel robot](#), are explained  
83 in detail in the corresponding hyperlinks.

## 84 Comparison with other Jacobian-computation softwares

85 TriP ([Baumgärtner & Miller, 2022](#)) is a software that is developed to address kinematics of  
86 hybrid linkages. However, it is evident from the triped example that human inspection is  
87 required to develop the model for each leg of triped robot. DART ([Lee et al., 2018](#)) uses biped  
88 robot (which can be seen as involving a closed-loop mechanism) in the examples, however it is  
89 imported from .sktl file rather than modelling a customised closed-loop robot from scratch.  
90 Even though C++ based softwares such as DART ([Lee et al., 2018](#)), CoreRobotics ([Owan et al.,  
91 2018](#)) and pinocchio ([Carpentier et al., 2019](#)) may facilitate closed-loop linkages, apparently no

documentation is provided for modelling closed-loop linkages. Furthermore, all these software (TriP, DART, CoreRobotics, pinocchio) apparently require human intervention to build a robot of a given topology. Even though some of them support importing models from URDF files, URDF has the limitation of “inability to model parallel linkages and closed-chain systems” (Tola & Corke, 2023), and furthermore preparing a URDF file (or similar file) of a robot from its mere topological information also requires human intervention. Pybotics provides automatic modelling of robot from the mere information of DH parameters, however it is apparently limited to serial manipulators. ACROD addresses this issue of human intervention by automatically generating functions required to compute Jacobian for a given end-effector point. A comparison of ACROD with other softwares is shown in the table below.

Soft-ware	Base Lan- guage	Closed- loop Linkages	Automation Level From Mere Topology	Primary focus
TriP (2022)	Python	yes	H*	Kinematics of Hybrid Linkages
Py- botics (2019)	Python	no	A	Kinematics, Dynamics, Trajectory Generations and Calibration of Serial Robots
DART (2018)	C++	X	H*	Kinematic and Dynamic Applications of Robotics
CoreR- obotics (2018)	C++	X	H	Computational Algorithms for Real Time Robot Control
pinoc- chio (2019)	C++	X	H*	Analytical Derivatives for Kinematics and Dynamics, Features for Control, Planning and Simulation
ACROD (2023)	Python	yes	yes	Dimensional Synthesis

- X = possible but neither documentation is provided nor an example is provided.
- \* = Accepts importing models from URDF files, etc.
- H = Human intervention required to build the robot.
- A = Automatically buildable from its DH parameters.

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