




# WAVI.jl: Ice Sheet Modelling in Julia

David T. Bett Alexander T. Bradley Robert Arthern <sup>1</sup>

<sup>1</sup> British Antarctic Survey, Cambridge, UK

DOI: [10.xxxxxx/draft](https://doi.org/10.xxxxxx/draft)

## Software

- [Review](#) 
- [Repository](#) 
- [Archive](#) 

Editor: [Hauke Schulz](#) 

## Reviewers:

- [@PennyHow](#)
- [@daniel-cheng](#)

Submitted: 05 May 2023

Published: unpublished

## License

Authors of papers retain copyright  
and release the work under a  
Creative Commons Attribution 4.0  
International License ([CC BY 4.0](#)).

## Summary

Ice sheet models are used to improve our understanding of the past, present, and future evolution of ice sheets. To do so, they solve the equations describing the flow of ice when forced by other climate elements, particularly the atmosphere and oceans. We present WAVI.jl, an ice sheet model written in Julia. WAVI.jl is designed to lower the bar to ice sheet modelling, whilst including sufficient detail to be used for addressing cutting-edge research questions.

## Statement of Need

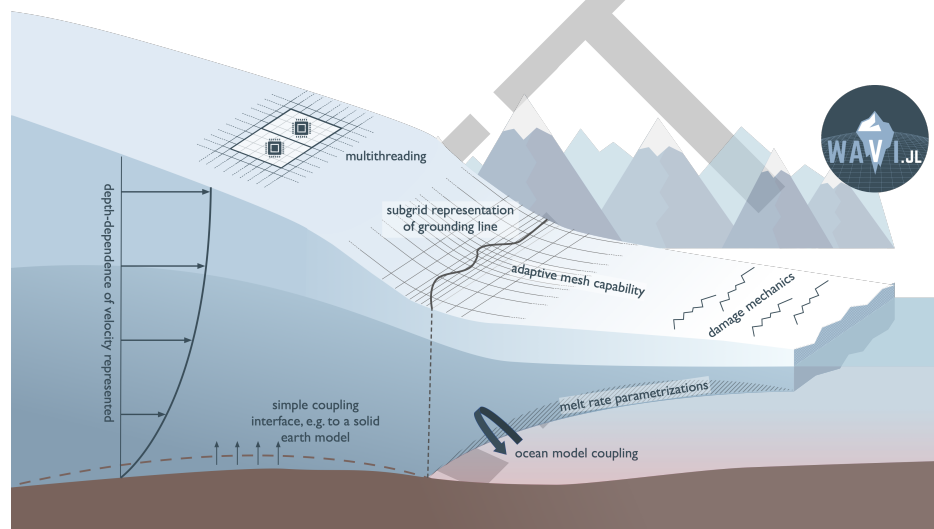
Ice sheet models allow us to simulate the behaviour and evolution of ice sheets, which are large masses of glacial land and marine ice. There are two main uses of ice sheet models: firstly, prognostic use, which involves making predictions about the future of ice sheets. Prognostic predictions often relate to sea level rise contributions: the world's two largest ice sheets, located in Antarctica and Greenland, hold enough ice to raise sea levels by approximately 58 and 7 meters, respectively ([Bamber et al., 2018](#)); prognostic modelling of these ice sheets enables us to make predictions on (for example) how much of this ice will be lost ([Edwards et al., 2021](#)), to investigate the possibility of runaway ice loss ([DeConto et al., 2021](#)), and analyze whether or not such instabilities have already been initiated ([Favier et al., 2014](#)). Secondly, ice sheet models are also used diagnostically, which involves using a model to investigate processes controlling the behaviour of an ice sheet, such as how loss of ice shelves – the floating extensions of ice sheets – influences ice flow speed ([Joughin et al., 2021](#)), how different bed conditions affect ice sliding ([De Rydt et al., 2021](#)), and probing the conditions under which so-called ‘tipping-points’ might be passed ([Schoof, 2007](#)).

On the long [O(1000s km)] lengthscales that are relevant to ice sheets, ice behaves approximately as a highly viscous fluid with a shear-thinning rheology (meaning that as the ice deforms, it becomes thinner and flows more easily). WAVI.jl (Wavelet-based Adaptive-grid Vertically-integrated Ice-sheet-model) is a Julia package for the numerical solution of an accurate approximation to the Stokes equations, which describe conservation of mass and momentum in such a fluid. This approximation, which is appropriate for fluid flows with a high aspect ratio (as is the case for the vast majority of ice sheets), treats longitudinal and lateral stresses as depth independent, but accounts for vertical velocity gradients in the nonlinear viscosity and in the treatment of basal stress ([Goldberg, 2011](#)).

Physically, ice sheets do not stand alone, but are forced by other parts of the climate system. For example, the rapid changes that have occurred in the West Antarctic Ice Sheet in the previous decades are understood to have been driven by an increase in oceanic heat content reaching the floating ice shelves which fringe this region ([Pritchard et al., 2012](#)). For basal melting in particular, WAVI.jl includes a broad range of community melt rate parametrizations ([Asay-Davis et al., 2017](#)), as well as a developmental coupling with the ocean general circulation model MITgcm ([Marshall et al., 1997](#)). More generally, WAVI.jl leverages Julia's multiple dispatch paradigm to create a simple, user-friendly interface for embedding models of other

physical processes, such as accumulation, ice damage, ice shelf calving, and solid earth effects, into WAVI.jl.

WAVI.jl is designed to be usable by anyone interested in ice sheet modelling, from students with no programming experience to expert researchers in the field, and everyone in between. To facilitate detailed research, including simulations at high spatial and temporal resolution, WAVI.jl employs a number of tools to improve computational speed, including multithreading capabilities, an adaptive numerical grid and a wavelet-based preconditioner (Arthern & Williams, 2017). To facilitate accessibility, WAVI.jl includes a simple, user friendly API, which is aided by Julia's convenient syntax. In addition, the GitHub repository in which the code is stored includes a number of well-documented examples, which demonstrate the software's capabilities in a wide variety of situations.



**Figure 1:** Schematic diagram of a marine ice sheet-shelf system, whose flow may be simulated using WAVI.jl. Labels and text indicate features of the software.

WAVI.jl is the successor to a similar code, written in the proprietary programming language MATLAB, which was never publicly released. This previous code has been used extensively as research software [e.g. (Arthern & Williams, 2017), (Arthern et al., 2015)], as well as having participated in the most recent ice sheet model intercomparison exercise (Cornford et al., 2020), which acts as a benchmark for ice-flow models. The new version, WAVI.jl, has also been verified independently against these benchmark experiments.

There exists a wide variety of ice sheet models with varying levels of complexity. Examples include (but are certainly not limited to) the Ice Sheet System Model (Larour et al., 2012), the Parallel Ice Sheet Model (Bueler & Brown, 2009), BISICLES (Cornford et al., 2013), Elmer/Ice (Gagliardini et al., 2013), and Úa (Gudmundsson, 2019). Every ice sheet model makes approximations in order to facilitate the numerical solution of the appropriate governing equations; since these equations have no analytic solutions, the *intercomparison* between ice sheet models is of paramount importance when assessing the trustworthiness of models; WAVI.jl contributes to this community of ice sheet models. WAVI.jl is also, to our knowledge, the first ice sheet model written entirely in Julia and, alongside other accessible ice sheet models such as IcePack (Shapiro et al., 2020), helps to lower the barrier to entering ice sheet modelling.

## Acknowledgements

A.T.B and D.T.B are supported by NERC Grant NE/S010475/1. We would like to thank Xy Wang and Daniel Goldberg for useful conversations, which helped to improve both the code and documentation. Thanks also to Bryony Freer, who designed the WAVI.jl logo.

## References

- Arthern, R. J., Hindmarsh, R. C., & Williams, C. R. (2015). Flow speed within the antarctic ice sheet and its controls inferred from satellite observations. *Journal of Geophysical Research: Earth Surface*, 120(7), 1171–1188. <https://doi.org/10.1002/2014JF003239>
- Arthern, R. J., & Williams, C. R. (2017). The sensitivity of west antarctica to the submarine melting feedback. *Geophysical Research Letters*, 44(5), 2352–2359. <https://doi.org/10.1002/2017GL072514>
- Asay-Davis, X. S., Jourdain, N. C., & Nakayama, Y. (2017). Developments in simulating and parameterizing interactions between the southern ocean and the antarctic ice sheet. *Current Climate Change Reports*, 3(4), 316–329. <https://doi.org/10.1007/s40641-017-0071-0>
- Bamber, J. L., Westaway, R. M., Marzeion, B., & Wouters, B. (2018). The land ice contribution to sea level during the satellite era. *Environmental Research Letters*, 13(6), 063008. <https://doi.org/10.1088/1748-9326/aac2f0>
- Bueler, E., & Brown, J. (2009). Shallow shelf approximation as a “sliding law” in a thermo-mechanically coupled ice sheet model. *Journal of Geophysical Research: Earth Surface*, 114(F3). <https://doi.org/10.1029/2008JF001179>
- Cornford, S. L., Martin, D. F., Graves, D. T., Ranken, D. F., Le Brocq, A. M., Gladstone, R. M., Payne, A. J., Ng, E. G., & Lipscomb, W. H. (2013). Adaptive mesh, finite volume modeling of marine ice sheets. *Journal of Computational Physics*, 232(1), 529–549. <https://doi.org/10.1016/j.jcp.2012.08.037>
- Cornford, S. L., Seroussi, H., Asay-Davis, X. S., Gudmundsson, G. H., Arthern, R., Borstad, C., Christmann, J., Dias dos Santos, T., Feldmann, J., Goldberg, D., & others. (2020). Results of the third marine ice sheet model intercomparison project (MISMIP+). *The Cryosphere*, 14(7), 2283–2301. <https://doi.org/10.5194/tc-14-2283-2020>
- De Rydt, J., Reese, R., Paolo, F. S., & Gudmundsson, G. H. (2021). Drivers of pine island glacier speed-up between 1996 and 2016. *The Cryosphere*, 15(1), 113–132. <https://doi.org/10.5194/tc-15-113-2021>
- DeConto, R. M., Pollard, D., Alley, R. B., Velicogna, I., Gasson, E., Gomez, N., Sadai, S., Condrón, A., Gilford, D. M., Ashe, E. L., & others. (2021). The paris climate agreement and future sea-level rise from antarctica. *Nature*, 593(7857), 83–89. <https://doi.org/10.1038/s41586-021-03427-0>
- Edwards, T. L., Nowicki, S., Marzeion, B., Hock, R., Goelzer, H., Seroussi, H., Jourdain, N. C., Slater, D. A., Turner, F. E., Smith, C. J., & others. (2021). Projected land ice contributions to twenty-first-century sea level rise. *Nature*, 593(7857), 74–82. <https://doi.org/10.1038/s41586-021-03302-y>
- Favier, L., Durand, G., Cornford, S. L., Gudmundsson, G. H., Gagliardini, O., Gillet-Chaulet, F., Zwinger, T., Payne, A., & Le Brocq, A. M. (2014). Retreat of pine island glacier controlled by marine ice-sheet instability. *Nature Climate Change*, 4(2), 117–121. <https://doi.org/10.1038/nclimate2094>
- Gagliardini, O., Zwinger, T., Gillet-Chaulet, F., Durand, G., Favier, L., Fleurian, B. de, Greve, R., Malinen, M., Martín, C., Råback, P., & others. (2013). Capabilities and performance

- 115 of elmer/ice, a new-generation ice sheet model. *Geoscientific Model Development*, 6(4),  
116 1299–1318.
- 117 Goldberg, D. N. (2011). A variationally derived, depth-integrated approximation to a higher-  
118 order glaciological flow model. *Journal of Glaciology*, 57(201), 157–170. [https://doi.org/](https://doi.org/10.3189/002214311795306763)  
119 [10.3189/002214311795306763](https://doi.org/10.3189/002214311795306763)
- 120 Gudmundsson, H. G. (2019). *Úa: A large-scale ice-flow model*. [https://github.com/GHilmarG/](https://github.com/GHilmarG/UaSource)  
121 [UaSource](https://github.com/GHilmarG/UaSource).
- 122 Joughin, I., Shapero, D., Smith, B., Dutrieux, P., & Barham, M. (2021). Ice-shelf retreat  
123 drives recent pine island glacier speedup. *Science Advances*, 7(24), eabg3080. [https://](https://doi.org/10.1126/sciadv.abg3080)  
124 [doi.org/10.1126/sciadv.abg3080](https://doi.org/10.1126/sciadv.abg3080)
- 125 Larour, E., Seroussi, H., Morlighem, M., & Rignot, E. (2012). Continental scale, high order,  
126 high spatial resolution, ice sheet modeling using the ice sheet system model (ISSM). *Journal*  
127 *of Geophysical Research: Earth Surface*, 117(F1). <https://doi.org/10.1029/2011JF002140>
- 128 Marshall, J., Hill, C., Perelman, L., & Adcroft, A. (1997). Hydrostatic, quasi-hydrostatic,  
129 and nonhydrostatic ocean modeling. *Journal of Geophysical Research: Oceans*, 102(C3),  
130 5733–5752. <https://doi.org/10.1029/96JC02776>
- 131 Pritchard, H. D., Ligtenberg, S. R. M., Fricker, H. A., Vaughan, D. G., Broeke, M. R. van  
132 den, & Padman, L. (2012). Antarctic ice-sheet loss driven by basal melting of ice shelves.  
133 *Nature*, 484(7395), 502–505. <https://doi.org/10.1038/nature10968>
- 134 Schoof, C. (2007). Ice sheet grounding line dynamics: Steady states, stability, and hysteresis.  
135 *Journal of Geophysical Research: Earth Surface*, 112(F3). [https://doi.org/10.1029/](https://doi.org/10.1029/2006JF000664)  
136 [2006JF000664](https://doi.org/10.1029/2006JF000664)
- 137 Shapero, D., Badgeley, J., Ham, D. A., Lilien, D., & Hoffman, A. (2020). *icepack/icepack:*  
138 *icepack: glacier flow modeling with the finite element method in Python* (Version v1.0.0).  
139 Zenodo. <https://doi.org/10.5281/zenodo.4318147>