# Matrius

(1) $A^2 - A - 2I = 0$ $\qquad A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

1. Fem $A^2$

$$A \cdot A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

2. $2 \cdot I$

$$2 \cdot I = 2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

3. $A^2 - A$

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

4. $(A^2 - A) - 2I$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \text{Matriu nul·la}$$

(2) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

1. $A \cdot A \cdot A$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{A^n} \text{ si } n \subset \mathbb{N} = \begin{pmatrix} 1 & 0 & n \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

③

a) 1. Multipliquem $A \cdot A$

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ a & \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ a & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{a+1}{4} & \frac{2}{8} \\ a & \frac{a+1}{4} \end{pmatrix}$$

2. Per a que es compleixi $A^2 = A$, tots els termes han de ser iguals

$$\left.\begin{aligned} &\frac{1}{2} = \frac{1+a}{4} \Rightarrow \boxed{a=1} \\ &\frac{1}{2} = \frac{1+a}{4} \Rightarrow \boxed{a=1} \end{aligned}\right\} a = 1 \rightarrow a = 1 \rightarrow \begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ 1 & \frac{1}{2} \end{pmatrix}$$

~~Donat que~~ Si "a" val 1 es compleix que $A^2 = A$

3. Trobem el det. de A per saber si es invertible

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ 1 & \frac{1}{2} \end{pmatrix} \rightarrow |A| = \frac{1}{2} \cdot \frac{1}{2} - \frac{1}{4} \cdot 1 = \frac{1}{4} - \frac{1}{4} = 0 \rightarrow \text{No és invertible}$$

4. Det. de $A - I$

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ 1 & \frac{1}{2} \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -0{,}5 & \frac{1}{4} \\ 1 & -0{,}5 \end{pmatrix} \rightarrow |A - I| = -\frac{1}{2} \cdot \left(-\frac{1}{2}\right) - 1 \cdot \frac{1}{4} = 0 \rightarrow \text{No és invertible}$$

$$A^2 = A \Rightarrow A^2 - A = 0 \Rightarrow A(A - I) = 0$$

b) 1. Si A fos invertible...

$$A^{-1} \cdot A(A - I) = 0 \Rightarrow A - I = 0 \Rightarrow A = I \rightarrow \text{Impossible}$$

2. Si $A - I$ fos invertible

$$(A - I)^{-1} \cdot (A - I) \cdot A = 0 \Rightarrow A = 0 \rightarrow \text{Impossible}$$

(4)

a) $A = \begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix} \rightarrow |A| = 31 - 31 = 0$

b) $B = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 2 & 3 \end{pmatrix} \rightarrow$ Donat que no és quadrada, no té determinant

c) $\begin{pmatrix} 1 & 2 \\ -1 & 4 \\ 7 & -11 \end{pmatrix} \rightarrow$ Donat que no és quadrada, no té determinant

d) $D = \begin{pmatrix} 4 & 3 \\ -1 & 1 \end{pmatrix} \rightarrow |D| = 4 \cdot 1 - 3 \cdot (-1) = 4 + 3 = 7$

e) $E = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 2 & -1 \\ 0 & 2 & -1 \end{pmatrix} \rightarrow |E| = [-2-4] - [-4-2] = -6 - (-6) = -6 + 6 = 0$

f) $F = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -4 & -1 \\ 0 & 3 & 1 \end{pmatrix} \rightarrow (-4+6] - [-3] = 2 + 3 = 5$

g) $G = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix} \sim \begin{matrix} \cancel{(f_1: f_1 - f_2)} \\ f_2: f_2 - f_1 \\ f_3: f_3 - f_2 \\ f_4: f_4 + f_3 \end{matrix} \begin{pmatrix} -1 & 1 & 1 & 1 \\ 2 & -2 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 2 & 2 & 0 & 0 \end{pmatrix}$

$\boxed{|G|} = a_{14} \cdot A_{14} \cancel{(\ldots)} = 1 \cdot (-1)^5 \cdot \alpha_{14} = -1 \cdot \left(\det \begin{pmatrix} 2 & -2 & 0 \\ 0 & 2 & -2 \\ 2 & 2 & 0 \end{pmatrix}\right) = -1 \cdot \left([8] - (-8)\right) = \boxed{-16}$

h) $H = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \sim \begin{matrix} f_2: f_2 - f_1 \\ f_3: f_3 - f_2 \end{matrix} \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix}$

$\boxed{|H|} = a_{14} \cdot A_{14} = a_{14} \cdot (-1)^{1+4} \cdot \alpha_{14} = 1 \cdot (-1)^5 \cdot \det \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix} =$

$-1 \cdot [(1+1) - (-1)] = \boxed{-3}$

I) $I = \begin{pmatrix} 2 & -5 & 1 & 2 \\ -3 & 7 & -1 & 4 \\ 6 & -9 & 2 & 7 \\ 4 & -6 & 1 & 2 \end{pmatrix} \sim \begin{array}{l} f_1: f_1 - f_4 \\ f_2: f_2 - 2f_1 \\ \\ f_4: f_4 - f_1 \end{array} \begin{pmatrix} -2 & 1 & 0 & 0 \\ -7 & 17 & -3 & 0 \\ 5 & -9 & 2 & 7 \\ 2 & -1 & 0 & 0 \end{pmatrix}$

$\boxed{|I| = a_{34} \cdot A_{34} = 7 \cdot (-1)^{3+4} \cdot \det \begin{pmatrix} -2 & 1 & 0 \\ -7 & 17 & -3 \\ -2 & -1 & 0 \end{pmatrix} = -7 \cdot (\boxed{6} - (-6)) = -7 \cdot 0 = 0}$

j) $J = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 5 \\ 1 & 3 & 1 \end{pmatrix}$ $\boxed{|J| = (1+5) - (2+15) = 6 - 17 = -11}$

5

a) $A = \begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix} \rightarrow \det \begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix} = 3 \cdot 1 - 3 \cdot 1 = 0 \Rightarrow$ No té inversa

$\left(\begin{array}{cc|cc} 3 & 3 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right) \sim f_2: 3f_2 - f_1 \left(\begin{array}{cc|cc} 3 & 3 & 1 & 0 \\ 0 & & & \end{array}\right)$

b) $B = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 2 & 3 \end{pmatrix} \rightarrow$ Per tenir inversa ha de ser quadrada

c) $C = \begin{pmatrix} 1 & 2 \\ -1 & 4 \\ 7 & -11 \end{pmatrix} \rightarrow$ Per tenir inversa ha de ser quadrada

d) $D = \begin{pmatrix} 4 & 3 \\ -1 & 1 \end{pmatrix}$ $|D| = 4 \cdot 1 - 3 \cdot (-1) = 4 + 3 = 7$ $\quad adj(D) = \begin{pmatrix} 1 & 1 \\ -3 & 4 \end{pmatrix}$

$A_{11} = (-1)^2 \cdot 1 = 1$ $\quad A_{12} = (-1)^3 \cdot (-1) = 1$ $\quad A_{21} = (-1)^3 \cdot 3 = -3$ $\quad A_{22} = (-1)^4 \cdot 4 = 4$

$D^{-1} = \frac{1}{|D|} \cdot (adj(D))^t = \frac{1}{7} \cdot \begin{pmatrix} 1 & -3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{7} & -\frac{3}{7} \\ \frac{1}{7} & \frac{4}{7} \end{pmatrix}$

e) $E = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 2 & -1 \\ 0 & 2 & -1 \end{pmatrix}$ $|E| = (-2-4)-(-4-2) = -6-(-6) = 0$

Donat $|E|=0$, sabem que no té inversa

f) $F = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -4 & -1 \\ 0 & 3 & 1 \end{pmatrix}$ $|F| = (-4+6)-(-3) = 2+3 = 5 \Rightarrow$ Té inversa

$F^{-1} \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & -4 & -1 & 0 & 1 & 0 \\ 0 & 3 & 1 & 0 & 0 & 1 \end{array}\right) \sim f_2: 2f_1 - f_2 \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 4 & 3 & 2 & -1 & 0 \\ 0 & 3 & 1 & 0 & 0 & 1 \end{array}\right) \sim$

$\sim f_3: 3f_2 - 4f_3 \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 4 & 3 & 2 & -1 & 0 \\ 0 & 0 & 5 & 6 & -3 & -4 \end{array}\right) \sim f_2: 3f_3 - 5f_2 \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -20 & 0 & 8 & -4 & -12 \\ 0 & 0 & 5 & 6 & -3 & -4 \end{array}\right)$

$\sim f_1: f_3 - 5f_1 \left(\begin{array}{ccc|ccc} -5 & 0 & 0 & 1 & -3 & -4 \\ 0 & -20 & 0 & 8 & -4 & -12 \\ 0 & 0 & 5 & 6 & -3 & -4 \end{array}\right) \sim \begin{array}{l} f_1: \frac{1}{-5} f_1 \\ f_2: \frac{1}{-20} f_2 \\ f_3: \frac{1}{5} f_3 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{5} & +\frac{3}{5} & \frac{4}{5} \\ 0 & 1 & 0 & -\frac{2}{5} & \frac{1}{5} & \frac{3}{5} \\ 0 & 0 & 1 & \frac{6}{5} & -\frac{3}{5} & -\frac{4}{5} \end{array}\right)$

$F^{-1} = \begin{pmatrix} -\frac{1}{5} & \frac{3}{5} & \frac{4}{5} \\ -\frac{2}{5} & \frac{1}{5} & \frac{3}{5} \\ \frac{6}{5} & -\frac{3}{5} & -\frac{4}{5} \end{pmatrix}$

(6)

$$\begin{pmatrix} 3 & x & x \\ 1 & -1 & 0 \\ 3 & -2 & 0 \end{pmatrix}$$

1. Trobem det

$$\det\begin{pmatrix} 3 & x & x \\ 1 & -1 & 0 \\ 3 & -2 & 0 \end{pmatrix} = (-3-2x)-(-3x) = -3+x$$

2. Per a que una matriu no tingui inversa el det. ha de ser $=0$, per tant l'igualem a 0 i trobem per quins valors de $x$ aquest és el resultat

$-3+x=0 \Rightarrow x=3$

$\rightarrow$ Si $x=3 \Rightarrow$ No té inversa

$\rightarrow$ Si $x \neq 3 \Rightarrow$ Té inversa

(5)

$$A = \begin{pmatrix} 0 & 1 \\ p & q \end{pmatrix}$$

1. Calculem A·A

$$A^2 = \begin{pmatrix} 0 & 1 \\ p & q \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ p & q \end{pmatrix} = \begin{pmatrix} p & q \\ pq & p+q^2 \end{pmatrix}$$

2. Si $A^2 = A$...

$$\begin{pmatrix} 0 & 1 \\ p & q \end{pmatrix} = \begin{matrix} p & q \\ pq & p+q^2 \end{matrix} \left.\right\} \begin{matrix} p=0 \\ q=1 \end{matrix}$$

~~2. Per trobar $A^n$ fem $A^2 \cdot A$~~

$$\begin{pmatrix} p & q \\ pq & p+q^2 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ p & q \end{pmatrix} = \begin{pmatrix} pq & p+q^2 \\ p^2+pq^2 & pq+pq+q^3 \end{pmatrix}$$

~~3. $A^3 \cdot A$~~

$$\begin{pmatrix} pq & p+q^2 \\ p^2+pq^2 & pq+pq+q^3 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ p & q \end{pmatrix} = \begin{matrix} p^2+pq^2 & pq+pq+q^3 \\ \cancel{pq+pq+q^3} \; p^2+pq^2+pq^2+ & \\ p^3+pq^3 & pq^2+q^4 \end{matrix}$$

~~4. $A^n$~~

3. $A^3$

$$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

Observem per inducció que $A^n = A$. Per tant, $A^{10} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$

8

$$A=\begin{pmatrix}1 & 0 & -1\\ 0 & a & 3\\ 4 & 1 & -a\end{pmatrix}$$

1. Calculem det(A)

$|A| = -a^2 - (-4a+3) = -a^2+4a-3$

2. Per tenir inversa hem d'observa que $|A| \neq 0$,

$-a^2+4a-3=0 \begin{cases} \to a_1 = 3 \\ \to a_2 = 1 \end{cases}$ Si $a = 3$ o $1 \Rightarrow$ No tē inversa

Si $a \neq 3$ o $1 \Rightarrow$ Tē inversa

3. $A^{-1}$ si $a=-1$

$$A^{-1} \to \left(\begin{array}{ccc|ccc}1 & 0 & -1 & 1 & 0 & 0\\ 0 & -1 & 3 & 0 & 1 & 0\\ 4 & 1 & 1 & 0 & 0 & 1\end{array}\right) \sim \underset{f_3: 4f_1 - f_3}{} \left(\begin{array}{ccc|ccc}1 & 0 & -1 & 1 & 0 & 0\\ 0 & -1 & 3 & 0 & 1 & 0\\ 0 & -1 & -5 & 4 & 0 & -1\end{array}\right) \sim$$

$$\sim \underset{f_3: f_3 - f_2}{} \left(\begin{array}{ccc|ccc}1 & 0 & -1 & 1 & 0 & 0\\ 0 & -1 & 3 & 0 & 1 & 0\\ 0 & 0 & -8 & 4 & -1 & -1\end{array}\right) \sim f_2: 3f_3 + 8f_2 \longrightarrow$$

$$\to \left(\begin{array}{ccc|ccc}1 & 0 & -1 & 1 & 0 & 0\\ 0 & -8 & 0 & 12 & 5 & -3\\ 0 & 0 & -8 & 4 & -1 & -1\end{array}\right) \sim \underset{f_1: 8f_1 - f_3}{} \left(\begin{array}{ccc|ccc}8 & 0 & 0 & 4 & 1 & 1\\ 0 & -8 & 0 & 12 & 5 & -3\\ 0 & 0 & -8 & 4 & -1 & -1\end{array}\right) \sim$$

$$\sim \begin{array}{l} f_1: \frac{1}{8} f_1 \\ f_2: \frac{1}{-8} f_2 \\ f_3: -\frac{1}{-8} f_3 \end{array} \left(\begin{array}{ccc|ccc}1 & 0 & 0 & \frac{1}{2} & \frac{1}{8} & \frac{1}{8}\\ 0 & 1 & 0 & -\frac{3}{2} & -\frac{5}{8} & \frac{3}{8}\\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{8} & \frac{1}{8}\end{array}\right)$$

$$A^{-1} = \begin{pmatrix}\frac{1}{2} & \frac{1}{8} & \frac{1}{8}\\ -\frac{3}{2} & -\frac{5}{8} & \frac{3}{8}\\ -\frac{1}{2} & \frac{1}{8} & \frac{1}{8}\end{pmatrix}$$

9

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & -1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & -3 & 1 \\ 0 & -2 & 4 \\ 2 & 1 & 3 \end{pmatrix}$$

$$Ax + 2B = 3A + B \Rightarrow Ax = 3A - B \Rightarrow A^{-1} \cdot A \cdot x = A^{-1}(3A - B) \Rightarrow x = A^{-1}(3A-B)$$

1. Calculem $A^{-1}$

1.1. Calculem adj(A)

$$\left.\begin{aligned}
A_{11} &= (-1)^2 \cdot \det\begin{pmatrix} -2 & 1 \\ -1 & 1 \end{pmatrix} = 1 \cdot (-2 \cdot 1 - 1 \cdot (-1)) = -2 + 1 = -1 \\
A_{12} &= (-1)^3 \cdot \det\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = -1 \cdot (1 \cdot 1 - 1 \cdot 1) = -1 \cdot 0 = 0 \\
A_{13} &= (-1)^4 \cdot \det\begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix} = -1 - 1 \cdot (-2) = -1 + 2 = 1 \\
A_{21} &= (-1)^3 \cdot \det\begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix} = -1 \cdot (-1 - 2 \cdot (-1)) = -1 \cdot 1 = -1 \\
A_{22} &= (-1)^4 \cdot \det\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = 1 - 2 = -1 \\
A_{23} &= (-1)^5 \cdot \det\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} = 0 \\
A_{31} &= (-1)^4 \cdot \det\begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix} = -1 - 2 \cdot (-2) = -1 + 4 = 3 \\
A_{32} &= (-1)^5 \cdot \det\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = -1 \cdot (1 - 2) = -1 \cdot (-1) = 1 \\
A_{33} &= (-1)^6 \cdot \det\begin{pmatrix} 1 & -1 \\ 1 & -2 \end{pmatrix} = -2 + 1 = -1
\end{aligned}\right\} \begin{pmatrix} -1 & 0 & 1 \\ -1 & -1 & 0 \\ 3 & 1 & -1 \end{pmatrix}$$

1.2. Calculem |A|

$$\begin{vmatrix} 1 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & -1 & 1 \end{vmatrix} = (-2 - 2 - 1) - (-4 - 1 - 1) = -5 - (-6) = 1$$

1.3. Calculem $A^{-1}$

$$A^{-1} = \frac{1}{|A|} \cdot (adj(A))^t = \frac{1}{1} \cdot (adj(A))^t = A^{-1} = (adj(A))^t = \cancel{\begin{pmatrix} -1 & 0 & 1 \\ -1 & -1 & 0 \\ 1 & 1 & -1 \end{pmatrix}} \begin{pmatrix} -1 & -1 & 3 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$$

Següent

2. Calculem x

2.1 Calculem 3A

$$3\begin{pmatrix}1 & -1 & 2\\ 1 & -2 & 1\\ 1 & -1 & 1\end{pmatrix} = \begin{pmatrix}3 & -3 & 6\\ 3 & -6 & 3\\ 3 & -3 & 3\end{pmatrix}$$

2.2 Calculem 3A - B

$$\begin{pmatrix}3 & -3 & 6\\ 3 & -6 & 3\\ 3 & -3 & 3\end{pmatrix} - \begin{pmatrix}2 & -3 & 1\\ 0 & -2 & 4\\ 2 & 1 & 3\end{pmatrix} = \begin{pmatrix}1 & 0 & 5\\ 3 & -4 & -1\\ 1 & -4 & 0\end{pmatrix}$$

2.1 $A^{-1} \cdot (3A-B)$

$$\begin{pmatrix}-1 & -1 & 3\\ 0 & -1 & 1\\ 1 & 0 & 1\end{pmatrix} \cdot \begin{pmatrix}1 & 0 & 5\\ 3 & -4 & -1\\ 1 & -4 & 0\end{pmatrix} = \begin{pmatrix}-1 & -8 & 0\\ -2 & 0 & 1\\ 2 & -4 & 5\end{pmatrix}$$

2.1 $x = A^{-1} \cdot (3A-B)$

$$x = \begin{pmatrix}-1 & -8 & 0\\ -2 & 0 & 1\\ 2 & -4 & 5\end{pmatrix}$$

10

a) $\begin{pmatrix}1 & 1\\ 2 & 1\\ -1 & -1\\ 3 & 2\\ 0 & -1\end{pmatrix}_{5\times 2}$

1. Sabent que rang màxim = 2, agafem menors 2x2 per veure si algún dona $\neq 0$

$\det\begin{pmatrix}1 & 1\\ 2 & 1\end{pmatrix} = 1-2 = -1 \neq 0 \rightarrow$ Rang 2

b) $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & -1 & 2 \\ 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}_{4\times 4}$

1. Agafem menors d'ordre 2

$\det\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = 2-1 = 1 \Rightarrow Rang \geq 2$

2. Agafem menors d'ordre 3

$\det\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{pmatrix} = (2-2)-(1+1) = -2 \Rightarrow Rang \geq 3$

3. Menors d'ordre 4

$\det\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & -1 & 2 \\ 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix} \sim F_2 : F_2 - F_4 \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & -2 & 0 \\ 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

$\det\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & -2 & 0 \\ 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix} = 2\cdot(-1)^8\cdot\det\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & -2 \\ 2 & -1 & 1 \end{pmatrix} = 2\cdot([2-4]-[1+2]) = 2\cdot(-5) = -10 \Rightarrow \boxed{Rang = 4}$

c) $\begin{pmatrix} 1 & -1 & 2 & 4 & -3 \\ 2 & -2 & 4 & 8 & -6 \\ 1 & -1 & 2 & 4 & -4 \end{pmatrix}_{3\times 5}$

1. Menors d'ordre 2

$\det\begin{pmatrix} 8 & -6 \\ 4 & -4 \end{pmatrix} = (8\cdot(-4)) - (4\cdot(-6)) = -32+24 = -8 \Rightarrow Rang \geq 2$

2. Menors d'ordre 3

$\det\begin{pmatrix} 2 & 4 & -3 \\ 4 & 8 & -6 \\ 2 & 4 & -4 \end{pmatrix} = [-64-48-48]-[-48-64-48] = 0$

Donat que $C_4$ és proporcional a $C_1$, $C_2$ i $C_3$ sabem que els determinants sempre seran $=0$, loqual $\Rightarrow Rang = 2$

d) $\begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & 2 & 2 \\ 1 & 0 & 0 & -1 \\ 1 & 2 & 1 & 2 \end{pmatrix}$

1. Menors d'ordre 3

$$\det\begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 2 \\ 1 & 0 & 0 \end{pmatrix} = 4-(-1) = 4+1 = 5 \Rightarrow \text{Rang} \geq 3$$

2. ~~Menors~~ Det de la matriu per comprovar rang 4 o 3

$$\begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & 2 & 2 \\ 1 & 0 & 0 & -1 \\ 1 & 2 & 1 & 2 \end{pmatrix} \sim \begin{array}{l} \\ \\ C_3 : C_4 \\ C_4 : C_3 \end{array} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & 2 & 2 \\ 1 & 2 & 1 & 2 \\ 1 & 0 & 0 & -1 \end{pmatrix} \sim C_4 : C_4 + C_1 \begin{pmatrix} 1 & 2 & 1 & 3 \\ 1 & -1 & 2 & 3 \\ 1 & 2 & 1 & 3 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\det\begin{pmatrix} 1 & 2 & 1 & 3 \\ 1 & -1 & 2 & 3 \\ 1 & 2 & 1 & 3 \\ 1 & 0 & 0 & 0 \end{pmatrix} = 1 \cdot (-1)^5 \cdot \det\begin{pmatrix} 2 & 1 & 3 \\ -1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} = -1 \cdot 0 = 0 \Rightarrow \boxed{\text{Rang} = 3}$$

e) $\begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \\ 5 & 10 \end{pmatrix}$

1. Menors d'ordre 2

$$\det\begin{pmatrix} 3 & 6 \\ 5 & 10 \end{pmatrix} = 3 \cdot 10 - 5 \cdot 6 = 30 - 30 = 0$$

~~Totes les files són~~ (~~Tots els menors d'ordre 2 són~~)
Donat que totes les files són proporcionals, tots els menors d'ordre 2 tindran det $= 0 \Rightarrow$ Rang $= 1$

8) $\begin{pmatrix} 2 & 1 & 0 \\ -2 & 3 & 1 \\ 0 & -1 & 2 \end{pmatrix}$

1. Det. de la matriu

$$\begin{vmatrix} 2 & 1 & 0 \\ -2 & 3 & 1 \\ 0 & -1 & 2 \end{vmatrix} = 12 - (-4-2) = 12 - (-6) = 12+6 = 18 \Rightarrow \boxed{\cancel{Rang \leq 3}} \quad Rang \leq 2$$

2. Det. de menors d'ordre 2

$$\det\begin{pmatrix} 2 & 1 \\ -2 & 3 \end{pmatrix} = 6 - (-2\cdot 1) = 6+2 = 8 \Rightarrow \boxed{Rang = 2}$$

11)

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 4 & 1 & -m \\ 0 & m & 3 \end{pmatrix}$$

a) Valors de m per rang $A<3$

Per rang $<3$, el determinant de la matriu d'ordre 3 ha de ser $=0$

1. Calculem $\det\begin{pmatrix} 1 & 0 & -1 \\ 4 & 1 & -m \\ 0 & m & 3 \end{pmatrix}$

$$\begin{vmatrix} 1 & 0 & -1 \\ 4 & 1 & -m \\ 0 & m & 3 \end{vmatrix} = (3-4m) - (-m^2) = m^2 - 4m + 3$$

2. L'igualem a 0

$$m^2 - 4m + 3 = 0 \begin{cases} m_1 = 3 \\ m_2 = 1 \end{cases}$$

Si $m = 3$ o $1 \Rightarrow Rang < 3$

Si $m \neq 3$ o $1 \Rightarrow Rang = 3$

b) Pot rang=1 per algun valor de m?

Si algun ~~(det)~~ menor d'ordre 2 té det $\neq 0$, implica que $\text{rang} A \geq 2$

1. Calculem det. menors d'ordre 2 per trobar algun $\neq 0$

$$\det\begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix} = 1-4=-3 \Rightarrow \text{Rang} A \geq 2$$

Com no depèn de m, cap valor de m farà que $\text{rang} A = 1$

(12)

a) $\begin{pmatrix} 1 & -a \\ a & 1 \end{pmatrix}$

1. Det. de la matriu

$$\det\begin{pmatrix} 1 & -a \\ a & 1 \end{pmatrix} = 1-(-a\cdot a) = 1-(-a^2) = 1+a^2$$

2. Comprovem per a quins valors de "a" hi ha rang=1 igualant el det a 0

$1+a^2=0 \Rightarrow a^2=-1 \Rightarrow a=\sqrt{-1} \notin \mathbb{R}$.

Per a qualsevol valor de "a" tenim $\text{rang} A = 2$

b) $\begin{pmatrix} 3 & 1 \\ 1 & -3 \\ 1 & a \end{pmatrix}$

1. Comprovar det. dels menors d'ordre 2, si donen diferent de $0 \Rightarrow$ rang=2, si $\neq 0 \Rightarrow$ rang 1

$$\begin{pmatrix} 3 & 1 \\ 1 & -3 \end{pmatrix} = -9-1=-10 \Rightarrow \text{Rang } 2$$

(13)

$\rightarrow$

c) $\begin{pmatrix} 2 & -1 & 5 \\ 1 & 1 & a \\ 3 & 2 & 4 \end{pmatrix}$

1. Comprovem si el det. d'algun menor d'ordre 2 és ≠ de 0

$$\det\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2-(-1) = 2+1 = 3 \Rightarrow Rang \geq 2$$

2. Det. de la matriu d'ordre 3

$$\det\begin{vmatrix} 2 & -1 & 5 \\ 1 & 1 & a \\ 3 & 2 & 4 \end{vmatrix} = (8+10-3a)-(15-4+4a) \Rightarrow 8+10-3a-15+4-4a \Rightarrow -7a+7$$

3. Igualem a 0 per veure el rang segons "a"

$$-7a+7=0 \Rightarrow 7=7a \Rightarrow a=1 \begin{cases} \text{Si } a=1 \Rightarrow Rang=2 \\ \text{Si } a\neq 1 \Rightarrow Rang=3 \end{cases}$$

d) $\cancel{\begin{pmatrix} 0 & 3 & -2 \\ 0 & a-2 & 5 \\ 0 & & \end{pmatrix}}$ $\begin{pmatrix} 0 & 1 & 1 \\ -2 & 1 & 1 \\ 2-2a & 0 & 2a-2 \end{pmatrix}$

1. Comprovem si el det d'algun menor d'ordre 2 és ≠ 0

$$\det\begin{vmatrix} 0 & 1 \\ -2 & 1 \end{vmatrix} \Rightarrow 0-(-2) = 2 \Rightarrow Rang \geq 2$$

2. Trobem el det de la matriu

$$\det\begin{vmatrix} 0 & 1 & 1 \\ -2 & 1 & 1 \\ 2-2a & 0 & 2a-2 \end{vmatrix} = (2-2a)-(2-2a-4a+4) = 4a-4$$

3. Igualem a 0 per trobar el rang en funció de "a"

$$4a-4=0 \Rightarrow a=1 \begin{cases} \text{Si } a=1 \Rightarrow Rang=2 \\ \text{Si } a\neq 1 \Rightarrow Rang=3 \end{cases}$$

e) $\begin{pmatrix} 1 & 3 & -2 \\ 0 & a-2 & 5 \\ 0 & 0 & a-1 \end{pmatrix}$

1. Veure si, ~~en funció~~ hi ha un valor de "a" que faci valer 0 a tots els dets. de matrius d'ordre 2

$\det\begin{pmatrix} 1 & 3 \\ 0 & a-2 \end{pmatrix} = a-2 \to a-2=0 \Rightarrow a=2$

$\det\begin{pmatrix} 3 & -2 \\ a-2 & 5 \end{pmatrix} = 15-(-2a+4) = 11+2a \to 11+2a=0 \Rightarrow a=-\frac{11}{2}$

com no hi ha cap valor de "a" que faci valer 0 tots els dets. de menors d'ordre 2, rang $\geq 2$

2. calculem el det de la matriu d'ordre 3

$\begin{pmatrix} 1 & 3 & -2 \\ 0 & a-2 & 5 \\ 0 & 0 & a-1 \end{pmatrix} = \cancel{a^2-a-2a+2} \; a^2-a-2a+2 = a^2-3a+2$

3. Igualem a 0 per veure el rang en funció de "a"

$a^2-3a+2=0 \begin{cases} a_1=2 \\ a_2=1 \end{cases}$ Si $a=2$ o $1 \Rightarrow$ rang $=2$; Si $a\neq 2$ o $1 \Rightarrow$ Rang $=3$

f) $\begin{pmatrix} a & 1 & 1 \\ 2 & -1 & a \\ 1 & -1 & 1 \end{pmatrix}$

1. Veure si algun menor d'ordre 2 té det $=0$

$\det\begin{pmatrix} 2 & -1 \\ 1 & -1 \end{pmatrix} = -2-(-1) = -2+1 = -1 \Rightarrow$ Rang $\geq 1$

2. Rang en funció de "a"

$\det\begin{pmatrix} a & 1 & 1 \\ 2 & -1 & a \\ 1 & -1 & 1 \end{pmatrix} = (-a-2+a)-(-1+2-a^2) = a^2-3$

$a^2-3=0 \Rightarrow a=\sqrt{3}$ Si $a=\sqrt{3} \Rightarrow$ Rang $=2$; Si $a\neq\sqrt{3} \Rightarrow$ Rang $=3$

g) $\begin{pmatrix} 1 & 2 & a+2 \\ 1 & 2a & 3 \\ 2 & 0 & -1 \end{pmatrix}$

1. Trobem si hi ha un valor de "a" que faci valer 0 el det de tots els menors d'ordre 2

$\rightarrow \det \begin{vmatrix} 1 & 2 \\ 1 & 2a \end{vmatrix} = 2a-2 = 0 \Rightarrow 2a = 2 \Rightarrow a = 1$

$\rightarrow \det \begin{vmatrix} 1 & 2a \\ 2 & 0 \end{vmatrix} = -4a = 0 \Rightarrow a = 0$

No hi ha cap valor de "a" que faci valer 0 els dets de tots els menors d'ordre $2 \Rightarrow \text{Rang}\,A \geq 2$

2. Det de la matriu d'ordre 3

$\det \begin{vmatrix} 1 & 2 & a+2 \\ 1 & 2a & 3 \\ 2 & 0 & -1 \end{vmatrix} = (2a+12) - (4a^2+8a-2) = -4a^2-10a+14$

3. $=0$

$-4a^2-10a+14=0 \begin{cases} a_1 = 1 \\ a_2 = -\frac{7}{2} \end{cases}$ Si $a = 1$ o $-\frac{7}{2} \Rightarrow \text{Rang} = 2$; Si $a \neq 1$ o $-\frac{7}{2} \Rightarrow \text{Rang} = 3$

h) $\begin{pmatrix} 1 & 2 & 5 \\ 2 & 1 & 2 \\ 5 & a & 9 \end{pmatrix}$

1. Comprovem si algún menor $\xrightarrow{2\times 2}$ val $\neq 0$

$A \begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix} = 4-5 = -1 \Rightarrow \text{Rang} \geq 2$

2. Det de la matriu d'ordre 3

$\det \begin{vmatrix} 1 & 2 & 5 \\ 2 & 1 & 2 \\ 5 & a & 9 \end{vmatrix} = 9+10a+20-25-2a-36 = 8a-32$

$8a-32=0 \Rightarrow a = 4 \rightarrow$ Si $a = 4 \Rightarrow \text{Rang} = 2$

Si $a \neq 4 \Rightarrow \text{Rang} = 3$

1) $\begin{pmatrix} 3 & 1 & 5 \\ 1 & -3 & -5 \\ 1 & a & a \end{pmatrix}$

1. Comprovem si algún menor té det $\neq 0$

$$\det\begin{pmatrix} 1 & 5 \\ -3 & -5 \end{pmatrix} = -5-(5\cdot(-3)) = -5+15 = 10 \Rightarrow Rang \geq 2$$

2. Trobem ~~rang~~ det. de la matriu

$$\det\begin{pmatrix} 3 & 1 & 5 \\ 1 & -3 & -5 \\ 1 & a & a \end{pmatrix} = (-9a+5a-5)-(-15+a-15a) = 10a+10$$

3. Igualem el det de la matriu a 0 per trobar el rang en funció de "a"

$$10a+10=0 \Rightarrow a=-1 \begin{cases} \text{Si } a=-1 \Rightarrow Rang = 2 \\ \text{Si } a\neq -1 \Rightarrow Rang = 3 \end{cases}$$

) $\begin{pmatrix} 1 & 1 & -a \\ 2 & 1 & -8 \\ -1 & -2 & 10 \end{pmatrix}$

1. Trobem un menor $\neq 0$

$$\det\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} = 1-2 = -1 \Rightarrow Rang \geq 2$$

2. Det. de la matriu

$$\det\begin{pmatrix} 1 & 1 & -a \\ 2 & 1 & -8 \\ -1 & -2 & 10 \end{pmatrix} = 10+4a+8-(a+16+20) = 3a-18$$

3. Igualem a 0

$$3a-18=0 \Rightarrow a=6 \begin{cases} \text{Si } a=6 \Rightarrow Rang = 2 \\ \text{Si } a\neq 6 \Rightarrow Rang = 3 \end{cases}$$

(13) $\cancel{B = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}}$ $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $A^{55}$?

1. $A^2$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2. $A^3$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

3. $A^n$

$$A^n = \begin{pmatrix} 1 & 0 \\ 0 & (-1)^n \end{pmatrix}$$

4. $A^{55}$

$$A^{55} = \begin{pmatrix} 1 & 0 \\ 0 & (-1)^{55} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(14)

$B = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$ Si $BxB = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$, $x$?

1. $B^{-1}$

1.1 adj(B)

$\cancel{a_{11} = (-1)^2 \cdot \det\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}}$ $a_{11} = (-1)^2 \cdot 1 = 1$

$a_{12} = (-1)^3 \cdot 1 = -1$ $a_{21} = (-1)^3 \cdot 3 = -3$

$a_{22} = (-1)^4 \cdot 2 = 2$

$\left.\right\}$ $\operatorname{adj}(B) = \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix}$

1.2 $(\operatorname{adj}(B))^t$

$$\begin{pmatrix} 1 & -3 \\ -1 & 2 \end{pmatrix}$$

1.3. $|B|$

$|B| = 2 - 3 = -1$

1.4 $B^{-1}$

$$B^{-1} = \frac{1}{|B|} \cdot (\operatorname{adj}(B))^t = -1 \cdot \begin{pmatrix} 1 & -3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix}$$

2. Plantejament per saber x

$$BxB = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix} \Rightarrow B^{-1}BxBB^{-1} = B^{-1}\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}B^{-1} \Rightarrow x = B^{-1}\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}B^{-1}$$

$\rightarrow$

3. $X$

3.1 $B^{-1} \cdot \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$

$$\begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 8 & 2 \\ -5 & 0 \end{pmatrix}$$

3.1 $\left(B^{-1} \cdot \begin{matrix} 1 & 4 \\ 3 & 2 \end{matrix}\right) \cdot B^{-1}$

$$\begin{pmatrix} 8 & 2 \\ -5 & 0 \end{pmatrix} \cdot \begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} -6 & 20 \\ 5 & -15 \end{pmatrix}$$

3.2 $X$

$$X = \begin{pmatrix} -6 & 20 \\ 5 & -15 \end{pmatrix}$$

15 $A = \begin{pmatrix} a+b & 1 \\ 0 & a-b \end{pmatrix}$

a) 1. $A^2$

$$\begin{pmatrix} a+b & 1 \\ 0 & a-b \end{pmatrix} \cdot \begin{pmatrix} a+b & 1 \\ 0 & a-b \end{pmatrix} = \begin{pmatrix} (a+b)^2 & (a+b)+(a-b) \\ 0 & (a-b)^2 \end{pmatrix}$$

2. $A^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} (a+b)^2 & 2a \\ 0 & (a-b)^2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \left\{ \begin{array}{l} (a+b)^2 = 1 \Rightarrow a^2 + 2ab + b^2 = 1 \\ 2a = 2 \Rightarrow a = 1 \\ (a-b)^2 = 1 \Rightarrow a^2 - 2ab + b^2 = 1 \end{array} \right.$$

Si $a = 1$...

$\cancel{\to 2^2 + 2\cdot 2b + b^2 = 1 \Rightarrow 4 + 4b + b^2 = 1 \Rightarrow b^2 + 4b + 3 = 0}$ $\begin{cases} b_1 = -1 \\ b_2 = -3 \end{cases}$

$\cancel{\to 2^2 - 2\cdot 2b + b^2 = 1 \Rightarrow 4 + 4b + b^2 = 1 \Rightarrow b^2 - 4b + 3 = 0}$ $\begin{cases} b_1 = 3 \\ b_2 = 1 \end{cases}$

Cap valor de b fa complir la igualtat

$\to 1^2 + 2b + b^2 = 1 \Rightarrow b(2+b) = 0 \begin{cases} b_1 = 0 \\ b_2 = -2 \end{cases}$

$1 - 2b + b^2 = 1 \Rightarrow b(-2+b) = 0 \begin{cases} b_1 = 0 \\ b_2 = 2 \end{cases}$

Només $b_1 = 0$ és solució per a tota la igualtat

b) 1. $A^3$

$$A=\begin{pmatrix}1&1\\0&1\end{pmatrix} \longrightarrow A^3 = \overbrace{\begin{pmatrix}1&2\\0&1\end{pmatrix}}^{A^2}\cdot\begin{pmatrix}1&1\\0&1\end{pmatrix}=\begin{pmatrix}1&3\\0&1\end{pmatrix}$$

2. $A^n$

$$A^n=\begin{pmatrix}1&n\\0&1\end{pmatrix}$$

3. $A^4$

$$A^4=\begin{pmatrix}1&4\\0&1\end{pmatrix}$$

c) 1. $A^n$

$$A^n=\begin{pmatrix}1&n\\0&1\end{pmatrix}$$

16

$$A=\begin{pmatrix}0&1&0\\0&0&1\\1&0&1\end{pmatrix} \qquad B=\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}$$

1. $A^{-1}$

$$\left(\begin{array}{ccc|ccc}0&1&0&1&0&0\\0&0&1&0&1&0\\1&0&1&0&0&1\end{array}\right) \sim \begin{array}{l}f_1: f_3\\f_2: f_1\\f_3: f_2\end{array}\left(\begin{array}{ccc|ccc}1&0&1&0&0&1\\0&1&0&1&0&0\\0&0&1&0&1&0\end{array}\right)\sim$$

$$\sim f_1: f_1-f_3\left(\begin{array}{ccc|ccc}+1&0&0&0&-1&1\\0&1&0&1&0&0\\0&0&1&0&1&0\end{array}\right) \Rightarrow A^{-1}=\begin{pmatrix}0&-1&1\\1&0&0\\0&1&0\end{pmatrix}$$

2. $A^2$

$$\begin{pmatrix}0&1&0\\0&0&1\\1&0&1\end{pmatrix}\cdot\begin{pmatrix}0&1&0\\0&0&1\\1&0&1\end{pmatrix}=\begin{pmatrix}0&0&1\\1&0&1\\1&1&1\end{pmatrix}$$

Enunciat incorrecte, $A^{-1}\neq A^2$

(23) $A=\begin{pmatrix}1 & 1\\ -3 & -4\end{pmatrix}$

3. $A^3$

$$\begin{pmatrix}0 & 0 & 1\\ 1 & 0 & 1\\ 1 & 1 & 1\end{pmatrix}\cdot\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 1\end{pmatrix}=\begin{pmatrix}1 & 0 & 1\\ 1 & 1 & 1\\ 1 & 1 & 2\end{pmatrix}$$

Donat que quan $n\to\infty$ la matriu A pateix un augment de valor dels seus elements, podem preveure que $A^{518}=B$ no es complirà, per lo que $A^{518}\neq B$

(17) $A=\begin{pmatrix}1 & 2\\ a & b\\ b & a^2\end{pmatrix}$ Per a que sigui rang 1 tots els menors d'ordre 2 han de valer 0

1. Calcular el det. dels menors 2x2

$\det\begin{pmatrix}1 & 2\\ b & a^2\end{pmatrix}=a^2-2b$ $\det\begin{pmatrix}1 & 2\\ a & b\end{pmatrix}=b-2a$ $\det\begin{pmatrix}a & b\\ b & a^2\end{pmatrix}=a^3-b^2$

2. Plantejar los igualats a 0

$a^2-2b=0$

$b-2a=0$

$a^3-b^2=0$

3. Resoldre el sistema

$\left.\begin{array}{l}a^2-2b=0\\ -2a+b=0\Rightarrow b=2a\end{array}\right\}a^2-2\cdot2a=0\Rightarrow a^2-4a=0\Rightarrow a(a-4)=0\left\langle\begin{array}{l}a=0\\ a=4\end{array}\right.$

Si $a=0$

$-b^2=0\Rightarrow b=0$

Si $a=4$

$4^3=b^2\Rightarrow \boxed{b}=\sqrt{4^3}=8$

4. Conclusió

Per a que la matriu A tingui rang 1, hi ha dues opcions de valors per a "a" i "b":

$\rightarrow a=0,\ b=0$

$\rightarrow a=4,\ b=8$

(in) (n 0 1\

(18)

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & m & 2m \\ m & 2 & 2+m \end{pmatrix}$$

1. Comprovem si hi ha ~~dos~~ menors de ordre 2 que puguin, segons m, valer tots 0 (el determinant)

$$\det\begin{pmatrix} 1 & 1 \\ 2 & m \end{pmatrix} = m-2 \rightarrow m-2=0 \Rightarrow m=2$$

$$\det\begin{pmatrix} 1 & 2 \\ m & 2m \end{pmatrix} = 2m-2m \rightarrow 0=0 \rightarrow$$ No depén de m, sempre tindrà det. $=0$

$$\det\begin{pmatrix} 2 & m \\ m & 2 \end{pmatrix} = 4-m^2 \rightarrow 4-m^2=0 \Rightarrow m=\pm 2$$

$$\det\begin{pmatrix} 1 & 2 \\ 2 & 2m \end{pmatrix} = 2m-4 \rightarrow m=2$$

Si $m=2$, rang 1 Si $m\neq 2$, rang $\geq 2$

2. Det. de la matriu

$$\det\begin{pmatrix} 1 & 1 & 2 \\ 2 & m & 2m \\ m & 2 & 2+m \end{pmatrix} = 2m+m^2+8+\cancel{2m^2}-\cancel{2m^2}-4-2m-4m =$$

$$\cancel{-3m^2+4 \Rightarrow m \rightarrow -3m^2+4=0 \Rightarrow m=\sqrt{\tfrac{4}{3}}}$$

$$m^2-4m+4=0 \begin{cases} m_1=2 \\ m_2=2 \end{cases}$$

Si $m=2 \Rightarrow$ Rang $=1$

Si $m\neq 2 \Rightarrow$ Rang $=3$

(19)

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$

1. $A^2$

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

2. $A^3$

$$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -I$$

3. $A^{60124}$

60124 | 3
00    20041
01
12
04
1

$$\boxed{A^{60124}} = A^{20041 \cdot 3+1} = (A^3)^{20041} + A = (-I)^{20041} + A =$$

$$-I + A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} = \boxed{A}$$

(20)

$$\begin{pmatrix} 1 & 0 & a \\ a & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

1. Det de la matriu

$det = a^2 - 1$

Si det = 1, la matriu no té inversa

$a^2 - 1 = 0 \Rightarrow a^2 = 1 \Rightarrow a = \sqrt{1} \Rightarrow a = \pm 1$

Si $a = \pm 1 \Rightarrow$ No té inversa

Si $a \neq \pm 1 \Rightarrow$ Té inversa

Escaneado con CamScanner

21) $M = \begin{pmatrix} 1 & a & a^2 \\ 1 & a+1 & (a+1)^2 \\ 1 & a-1 & (a-1)^2 \end{pmatrix}$

a) ~~1. Det. de la matriu~~

$\det \begin{pmatrix} 1 & a & a^2 \\ 1 & a+1 & (a+1)^2 \\ 1 & a-1 & (a-1)^2 \end{pmatrix} = (a+1)(a-1)^2$

a) 1. Gauss

$$\begin{pmatrix} 1 & a & a^2 \\ 1 & a+1 & (a+1)^2 \\ 1 & a-1 & (a-1)^2 \end{pmatrix} \sim f_2 : f_2 - f_1 \begin{pmatrix} 1 & a & a^2 \\ 0 & 1 & (a+1)^2 - a^2 \\ 1 & a-1 & (a-1)^2 \end{pmatrix} \sim f_3 : f_3 - f_1 \begin{pmatrix} 1 & a & a^2 \\ 0 & 1 & (a+1)^2 - a^2 \\ 0 & -1 & (a-1)^2 - a^2 \end{pmatrix} \sim$$

$$\sim f_3 : f_3 + f_2 \begin{pmatrix} 1 & a & a^2 \\ 0 & 1 & (a+1)^2 - a^2 \\ 0 & 0 & (a+1)^2 - a^2 + (a-1)^2 - a^2 \end{pmatrix}$$

2. Si $a_{33} = 0$, rang 2, si $\neq 0$ rang 3

$(a+1)^2 - a^2 + (a-1)^2 - a^2 = 0 = \cancel{a^2} + \cancel{2a} + 1 \cancel{- a^2} + \cancel{a^2} - \cancel{2a} + 1 \cancel{- a^2} = 0 \Rightarrow 2 = 0 \Rightarrow$
Cap valor de ~~a~~ fa que $a_{33} = 0$, per tant rang $= 3$

b) Donada la matriu resultant de l'apartat anterior aplicat el mètode de Gauss, sabem que aquest sistema no té solució possible

(22) $A=\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}$ $A^2x = A-3I$

1. Desenvolupar l'expressió

$A^2x = A-3I \Rightarrow (A^2)^{-1}\cdot A^2\cdot x = (A^2)^{-1}(A-3I) \Rightarrow x = (A^2)^{-1}(A-3I)$

2. Calculem $A^2$

$$\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}\cdot\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}=\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}$$

3. $(A^2)^{-1}$

$$(A^2)^{-1}=\left(\begin{array}{ccc|ccc}0&1&0&1&0&0\\0&0&1&0&1&0\\1&0&0&0&0&1\end{array}\right)\sim \begin{array}{l}F_1:F_3\\F_2:F_1\\F_3:F_2\end{array}\left(\begin{array}{ccc|ccc}1&0&0&0&0&1\\0&1&0&1&0&0\\0&0&1&0&1&0\end{array}\right)\Rightarrow (A^2)^{-1}=\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}$$

4. $A-3I$

$$\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}-\begin{pmatrix}3&0&0\\0&3&0\\0&0&3\end{pmatrix}=\begin{pmatrix}-3&0&1\\1&-3&0\\0&1&-3\end{pmatrix}$$

5. $(A^2)^{-1}\cdot(A-3I)$

$$\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}\cdot\begin{pmatrix}-3&0&1\\1&-3&0\\0&1&-3\end{pmatrix}=\begin{pmatrix}0&1&-3\\-3&0&1\\1&-3&0\end{pmatrix}$$

6. $x=(A^2)^{-1}\cdot(A-3I)$

$$x=\begin{pmatrix}0&1&-3\\-3&0&1\\1&-3&0\end{pmatrix}$$

(E3) $A = \begin{pmatrix} 1 & 1 \\ 3 & -4 \end{pmatrix}$

a)

1. $Ax = I - 3x \Rightarrow$ Desenvolupem l'expressió

$Ax + 3x = I \Rightarrow x(A+3I) = I \Rightarrow x(A+3I)(A+3I)^{-1} = I(A+3I)^{-1} \Rightarrow$
$x = I(A+3I)^{-1} \Rightarrow x = (A+3I)^{-1}$

2. Calculem $A+3I$

$$\begin{pmatrix} 1 & 1 \\ -3 & -4 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ -3 & -1 \end{pmatrix}$$

3. Calculem $(A+3I)^{-1}$

3.1 $\text{adj}(A+3I)$

$$\left.\begin{array}{l} a_{11} = (-1)^2 \cdot (-1) = 1 \cdot (-1) = -1 \\ a_{12} = (-1)^3 \cdot (-3) = -1 \cdot (-3) = 3 \\ a_{21} = (-1)^3 \cdot 1 = -1 \\ a_{22} = (-1)^4 \cdot 4 = 4 \end{array}\right\} \text{adj}(A+3I) = \begin{pmatrix} -1 & 3 \\ -1 & 4 \end{pmatrix}$$

3.2 $\det\begin{pmatrix} 4 & 1 \\ -3 & -1 \end{pmatrix}$

$$\det\begin{pmatrix} 4 & 1 \\ -3 & -1 \end{pmatrix} = -4 - (-3) = -4 + 3 = -1$$

3.3 $(A+3I)^{-1}$

$$(A+3I)^{-1} = \frac{1}{|A+3I|} \cdot (\text{adj}(A+3I))^t = \frac{1}{-1} \cdot \begin{pmatrix} -1 & -1 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -3 & -4 \end{pmatrix}$$

4. $x = (A+3I)^{-1}$

$$x = \begin{pmatrix} 1 & 1 \\ -3 & -4 \end{pmatrix}$$

$\longrightarrow$

b) Si la matriu X té determinant $\neq 0$, es pot calcular la seva inversa

$\begin{vmatrix} 1 & 1 \\ -3 & -4 \end{vmatrix} = -4 - (-3) = -1$

Si $X = \begin{pmatrix} 1 & 1 \\ -3 & -4 \end{pmatrix} = (A+3I)^{-1} \Rightarrow \boxed{X^{-1} = A+3I = \begin{pmatrix} 4 & 1 \\ -3 & -1 \end{pmatrix}}$

24

$A = \begin{pmatrix} 1 & 0 & a \\ a & 0 & -1 \\ 2 & -1 & 1 \end{pmatrix}$

1. Comprovem si algun menor d'ordre 2 té det $\neq 0$

$\det\begin{pmatrix} 0 & -1 \\ -1 & 1 \end{pmatrix} = 0 - (1) = -1 \Rightarrow Rang \geq 2$

2. Calculem el det. de A

$|A| = -a^2 - (1) = -a^2 - 1$

3. Igualem $|A|$ a 0

$-a^2 - 1 = 0 \Rightarrow a^2 = -1 \Rightarrow a = \sqrt{-1} \Rightarrow$ Cap valor de a fa que $|A| = 0 \Rightarrow Rang = 3$

Donat que $|A|$ mai pot ser $= 0$, sabem que cap valor de "a" farà que no tingui inversa.