

```
def calcular_c(err,x,y):  
  
    #calculo w  
    w = 1/np.square(err)  
  
    #terminos suma  
    a = np.sum(w*np.square(x))  
    b = np.sum(w*y)  
    c = np.sum(w*x)  
    d = np.sum(w*x*y)  
  
    #terminos delta  
    e = np.sum(w)  
    f = np.sum(w*np.square(x))  
    g = np.square(np.sum(w*x))  
    delta = f*e - g  
  
    #calculo c  
    c = (a*b - c*d)/delta  
  
    return c
```

```
def calcular_m(err,x,y):  
  
    #calculo w  
    w = 1/np.square(err)  
  
    #terminos suma  
    a = np.sum(w)  
    b = np.sum(w*x*y)  
    c = np.sum(w*x)  
    d = np.sum(w*y)  
  
    #terminos delta  
    e = np.sum(w)  
    f = np.sum(w*np.square(x))  
    g = np.square(np.sum(w*x))  
    delta = f*e - g  
  
    #calculo m  
    m = (a*b - c*d)/delta  
  
    return m
```

```
def calcular_ac(err,x):  
  
    #calculo w  
    w = 1/np.square(err)  
  
    #terminos suma  
    a = np.sum(w*np.square(x))  
  
    #terminos delta  
    e = np.sum(w)  
    f = np.sum(w*np.square(x))  
    g = np.square(np.sum(w*x))  
    delta = f*e - g  
  
    #calculo ac  
    ac = np.sqrt(a/delta)  
  
    return ac
```

```
def calcular_am(err,x):  
  
    #calculo w  
    w = 1/np.square(err)  
  
    #terminos suma  
    a = np.sum(w)  
  
    #terminos delta  
    e = np.sum(w)  
    f = np.sum(w*np.square(x))  
    g = np.square(np.sum(w*x))  
    delta = f*e - g  
  
    #calculo am  
    am = np.sqrt(a/delta)  
  
    return am
```

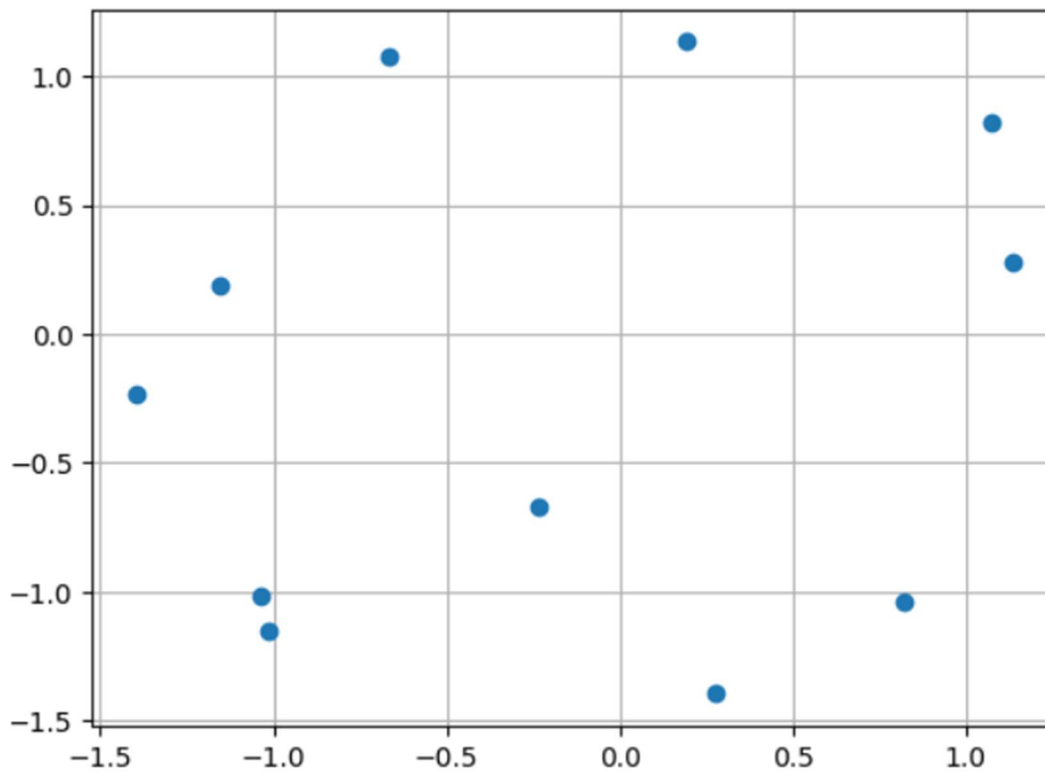
c
✓ 0.0s
-0.9474964662767381
m
✓ 0.0s
2.0284648216102745
ac
✓ 0.0s
3.386858673521736
am
✓ 0.0s
0.05197830827721802

```

model = c + m*frecuencia
residuals = voltaje-model
normalized_residuals = residuals/error
lagged = np.roll(normalized_residuals, -1)
plt.plot(normalized_residuals, lagged, 'o')
plt.grid()
plt.show()

```

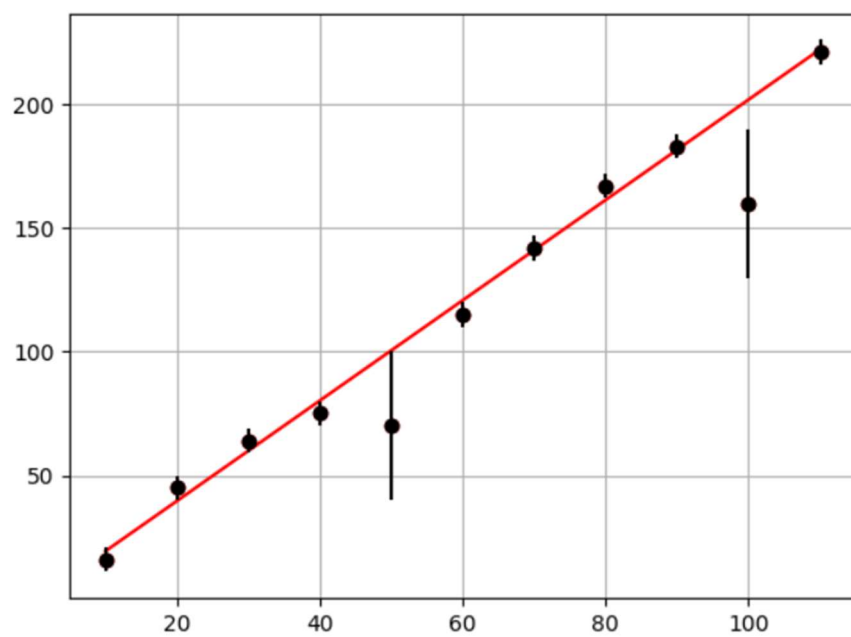
Lagged plot



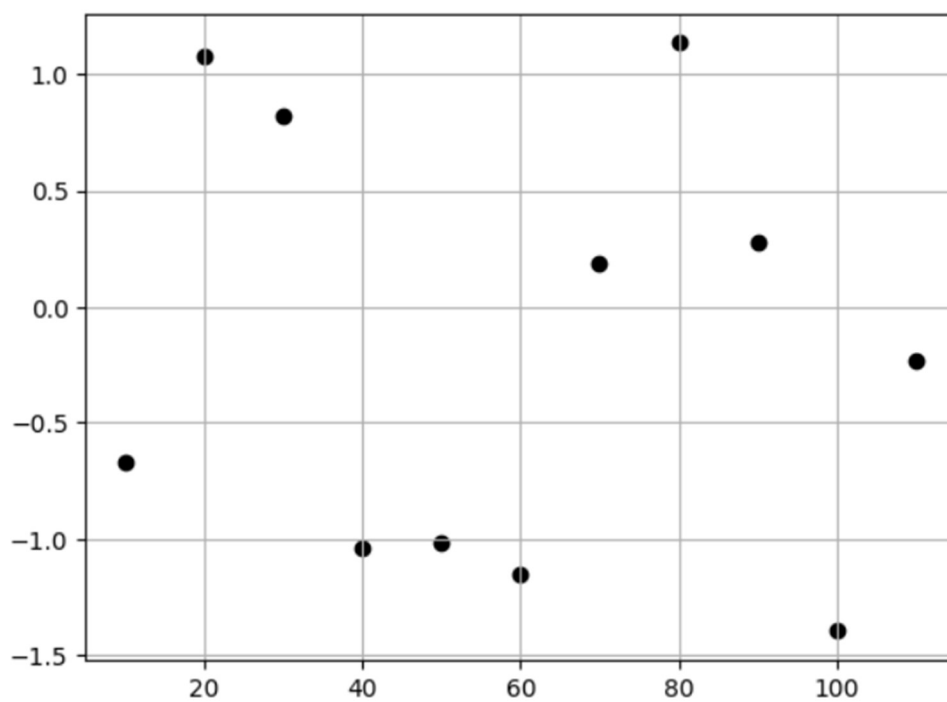
```
def Durbin_Watson(residuals):  
    return np.sum(np.square(normalized_residuals-lagged))/np.sum(normalized_residuals**2)  
46] ✓ 0.0s  
  
D = Durbin_Watson(residuals)  
D  
47] ✓ 0.0s  
1.572140044519231
```

6.3

Datos con best-fit line



Normalized residuals



6.4

```
def chi2(M, C):  
    res = ((voltaje - (M * frecuencia + C)) / error) ** 2  
    return np.sum(res)
```

✓ 0.0s

```
M_values = np.linspace(1, 3, 1000)  
C_values = np.linspace(-1, 1, 1000)  
best_chi = np.inf  
# Calculate chi-squared for each combination of parameters  
chi2_grid = np.zeros((len(M_values), len(C_values)))  
for i, M in enumerate(M_values):  
    for j, C in enumerate(C_values):  
        chi2_grid[i, j] = chi2(M, C)  
        if chi2_grid[i, j] < best_chi:  
            best_chi = chi2_grid[i, j]  
            best_M = M  
            best_C = C
```

✓ 8.0s


```
best_M
252] ✓ 0.0s
... 2.029029029029029

best_C
253] ✓ 0.0s
... -0.97997997997998

best_chi
254] ✓ 0.0s
... 9.116021484725753
```

```
def solve_Espacio_fase_c(m=m_min,c=c_min,x=x,y=y,contorno=1,alpha=dy,gmm=0.0001,infer=-1,tolerancia=1e-7):

    mjAnterior = m
    d = 1
    cj = c

    while d>tolerancia:
        #Loop que minimiza c

        cjAnterior = cj

        cj = nwt(f_c,cj+0.01*infer,args=(mjAnterior,x,y,alpha,contorno)) #Newton rapshon para minimizar c

        termino = 2*gmm*np.sum(x*(y-(mjAnterior)*x-cj)/(alpha)**2)

        mjAnterior += termino

        diff=np.abs(cj-cjAnterior)

        d = diff

    return cj
```

```
def solve_Espacio_fase_m(m=m_min,c=c_min,x=x,y=y,contorno=1,alpha=dy,gmm=0.05,infer=-1,tolerancia=1e-7):

    cjAnterior = c
    d = 1
    mj = m

    while d>tolerancia:
        #Loop que minimiza m

        mjAnterior = mj

        mj = nwt(f_m,mj+0.02*infer,args=(cjAnterior,x,y,alpha,contorno)) #Newton rapshon para minimizar m

        termino = 2*gmm*np.sum((y-(mj)*x-cjAnterior)/(alpha)**2)

        cjAnterior += termino

        diff=np.abs(mj-mjAnterior)

        d = diff

    return mj
```

Errores calculados de m y c:

Contorno 1:

```
Inferior:

m:
-0.017325659748360422

c:
-1.1289533645592529

Superior:

m:
0.017325677963016872

c:
1.1289523737901974
```

Contorno 4:

Inferior:

m:

-0.03465177823317589

c:

-2.2579062566317956

Superior:

m:

0.03465179644876434

c:

2.257905265862669

Contorno 9:

Inferior:

m:

-0.05197786414732053

c:

-3.3868591537285657

Superior:

m:

0.05197788236315951

c:

3.386858162959412

Esto se ajusta bastante bien a lo esperado.

6.5

c
✓ 0.0s
-0.10700883885508011
m
✓ 0.0s
1.2397219665412587
ac
✓ 0.0s
0.03266514103078884
am
✓ 0.0s
0.03925005362824148

$$M = 1.24 \pm 0.04$$

$$C = -0.11 \pm 0.03$$

Calculando la velocidad de la luz tenemos:

$$V_c = 304025095.50870967 \pm 9807260.370254517$$

Ósea

$$V_c = 3.04 \cdot 10^8 \pm 0.10 \cdot 10^8$$

El intercepto no es consistente, pero la velocidad de la luz da razonable.

6.6

i) Errores originales

c
✓ 0.0s
1.7267713877515376
m
✓ 0.0s
49.89790121339644
ac
✓ 0.0s
0.9872895098081177
am
✓ 0.0s
0.31658015706387804

ii) Errores ctes de 4

c
✓ 0.0s
-1.0
m
✓ 0.0s
50.472727272727276
ac
✓ 0.0s
2.7325202042558927
am
✓ 0.0s
0.4403855060505442

iii) errores 1 y final de 1, el resto de 8

0]	c	✓ 0.0s	-0.45340345705480367
1]	m	✓ 0.0s	51.19874715261959
2]	ac	✓ 0.0s	1.0979945499870805
3]	am	✓ 0.0s	0.155877022570768

A pesar de que no cambiamos los datos usados, tan solo cambiar los errores asociados a las mediciones impactó fuertemente el resultado. Esto, claramente se da porque en un cálculo con pesos de los mejores valores para la pendiente y corte, entre más preciso sea el dato, más contribuirá al resultado final. Por esto cambiar dichas precisiones afecta nuestros valores óptimos calculados.