Tarea 4

Rafael Casas 202122121
Juan Felipe Camelo 202122062

5.1)

$$c = \frac{\sum_i x_i^2 \sum_i y_i - \sum_i x_i \sum_i x_i y_i}{\Delta}$$

$$m = \frac{N\sum_i x_i y_i - \sum_i x_i \sum_i y_i}{\Delta}$$

$$\Delta = N\sum_i x_i^2 - \left(\sum_i x_i\right)^2$$

$$y = mx + c$$

Para mostrar que pertenecen podemos reemplazar $\overline{x}$ y resolver:

$$\therefore y = \left(\frac{N\sum_i x_i y_i - \sum_i x_i \sum_i y_i}{N\sum_i x_i^2 - (\sum_i x_i)^2}\right)\frac{1}{N}\sum_i x_i + \frac{\sum_i x_i^2 \sum_i y_i - \sum_i x_i \sum x_i y_i}{N\sum_i x_i^2 - (\sum_i x_i)^2}$$

$$\Rightarrow y = \frac{N\sum_i x_i y_i \sum_i x_i - (\sum_i x_i)^2 \sum_i y_i}{N^2\sum_i x_i^2 - N(\sum_i x_i)^2} + \frac{N\sum_i x_i^2 \sum_i y_i - N\sum_i x_i \sum_i x_i y_i}{N^2\sum_i x_i^2 - N(\sum_i x_i)^2}$$

$$\Rightarrow y = \frac{\cancel{N\sum_i x_i y_i \sum_i x_i} - (\sum_i x_i)^2 \sum_i y_i + N\sum_i x_i^2 \sum_i y_i \cancel{- N\sum_i x_i y_i \sum_i x_i}}{N^2\sum_i x_i^2 - N(\sum_i x_i)^2}$$

$$\Rightarrow y = \frac{N\sum_i x_i^2 \sum_i y_i - (\sum_i x_i)^2 \sum_i y_i}{N(N\sum_i x_i^2 - (\sum_i x_i)^2)}, \text{ sacamos factor } \sum_i y_i$$

$$\Rightarrow y = \left(\cancel{\frac{N\sum x_i^2 - (\sum x_i)^2}{N\sum x_i^2 - (\sum_i x_i)^2}}^{\,1}\right)\frac{1}{N}\sum_i y_i \Rightarrow y = \frac{1}{N}\sum_i y_i$$

Y por definición, entonces $y = \overline{y}$

Luego el Fit pasa por $(\overline{x}, \overline{y})$.

5.2)

i) $V = aU^2 \Rightarrow \ln(V) = \ln(aU^2) \Rightarrow \ln(V) = \ln(a) + \ln(U^2)$

$\Rightarrow \underline{\ln(V) = 2\ln(U) + \ln(a)} \rightarrow$ lineal

Luego $x = \ln(U)$ y $y = \ln(V)$, luego

$y = 2x + \ln(a) \rightarrow$ lineal

Donde $m = 2$ y $c = \ln(a)$.

ii) $V = a\sqrt{U} \Rightarrow V = aU^{1/2} \Rightarrow \ln(V) = \ln(aU^{1/2})$

$\Rightarrow \ln(V) = \ln(a) + \ln(U^{1/2}) \Rightarrow \underline{\ln(V) = \frac{1}{2}\ln(U) + \ln(a)} \rightarrow$ lineal

Luego $x = \ln(U)$ y $y = \ln(V)$, luego

$y = \frac{1}{2}x + \ln(a) \rightarrow$ lineal

Donde $m = \frac{1}{2}$ y $c = \ln(a)$.

iii) $V = a\exp(-bU) \Rightarrow \ln(V) = \ln(a\exp(-bU))$

$\Rightarrow \ln(V) = \ln(a) + \ln(\exp(-bU))$

$\Rightarrow \underline{\ln(V) = \ln(a) - bU} \rightarrow$ lineal

Luego: $y = \ln(V)$ y $x = U$

$\therefore \underline{y = -bx + \ln(a)}$

Donde $m = -b$ y $c = \ln(a)$

iv) $\frac{1}{U} + \frac{1}{V} = \frac{1}{a} \Rightarrow \frac{1}{V} = -\frac{1}{U} + \frac{1}{a} \rightarrow$ lineal

Luego $y = \frac{1}{V}$ y $x = \frac{1}{U}$, $m = -1$ y $c = \frac{1}{a}$

5.3

i) Del 5.2.1 tenemos que $y = 2x + \ln(a)$; $y = \ln(V)$; $x = \ln(u)$

Entonces c:

$$c = \frac{\sum_i \ln(u_i)^2 \sum_i \ln(V_i) - \sum_i \ln(u_i) \sum_i \ln(u_i)\ln(V_i)}{9\sum_i \ln(u_i)^2 - (\sum_i \ln(u_i))^2} = -2.32602$$

$$m = \frac{9 \sum_i \ln(u_i)\ln(V_i) - \sum_i \ln(u_i) \sum_i \ln(V_i)}{9 \sum_i x_i^2 - (\sum_i x_i)^2} = 1.00355$$

$$\alpha_{cu} = \sqrt{\frac{1}{7} \sum_i (\ln(V_i) - m\ln(u_i) - c)^2}$$

$$\alpha_c = \alpha_{cu} \sqrt{\frac{\sum_i \ln(u_i)^2}{9 \sum_i x_i^2 - (\sum_i x_i)^2}} = 0.05996$$

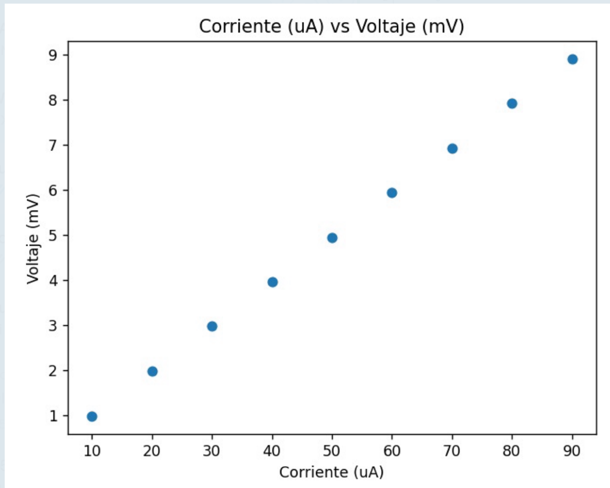
$$\alpha_m = \alpha_{cu} \sqrt{\frac{9}{9 \sum_i x_i^2 - (\sum_i x_i)^2}} = 0.00158$$

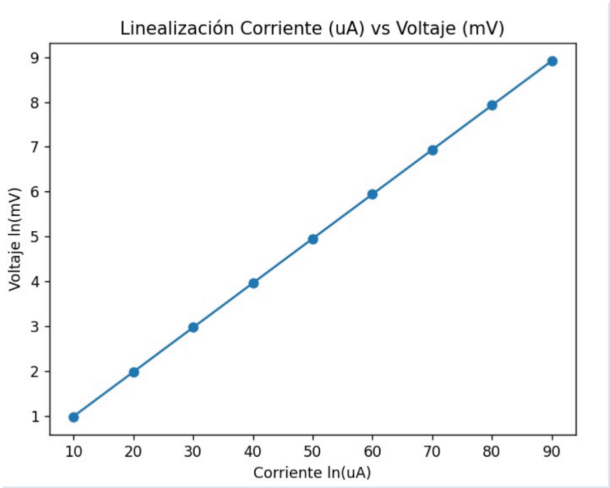
$C = -0.0008 \pm 0.0052$

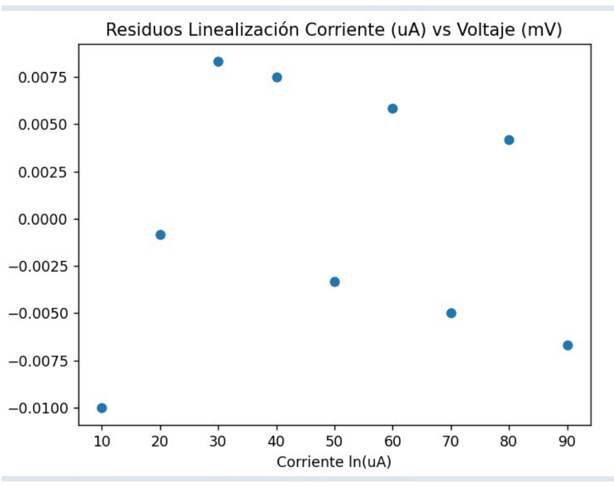
$M = 0.1 \pm 9.2$

ii) De acuerdo con lo ya escrito

$\alpha_{cu} = 0.0072$







Como podemos ver, los residuos son bastante pequeños en magnitud en comparación con nuestros datos linealizados, por lo cual este metodo resulta ser una muy buena aproximación.

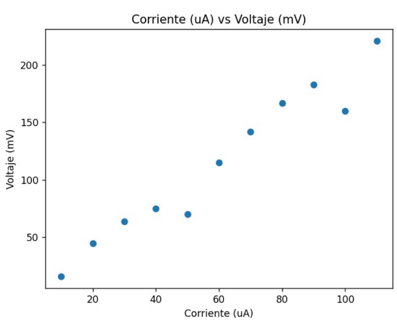
```python
v = np.array([0.98,1.98,2.98,3.97,4.95,5.95,6.93,7.93,8.91])
u = np.array([(i*10) for i in range(1,10)])

def aproximacion_c_m(u,v):
    a = np.log(u)**2
    b = np.log(v)
    c = np.log(u)
    d = b*c

    C = (np.sum(a)*np.sum(b) - np.sum(c)*np.sum(d))/(len(u)*np.sum(a) - np.sum(c)**2)
    M = (len(u)*np.sum(d) - np.sum(c)*np.sum(b))/(len(u)*np.sum(a) - np.sum(c)**2)

    aCU = np.sqrt((1/(len(u)-2))*np.sum((b - C - M*c)**2))

    aC = aCU*np.sqrt(np.sum(a)/(len(u)*np.sum(a) - np.sum(c)**2))
    aM = aCU*np.sqrt(len(u)/(len(u)*np.sum(a) - np.sum(c)**2))

    return C, M, aC, aM, aCU
```

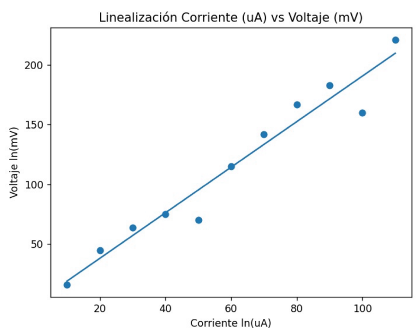
Este es el script que hicimos para resolver esta parte

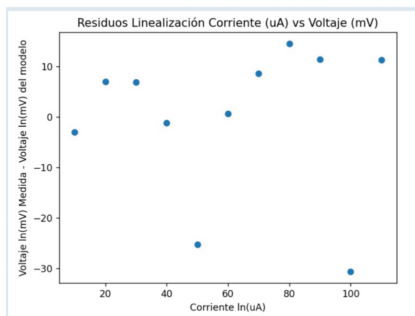
6.1

Usando un proceso similar a antes y cambiando un poco el codigo obtenemos:

$C = 0 \pm 10$ $\quad$ $M = 1.9 \pm 0.2$







Que se ajusta a lo dado anteriormente (6.1 d)