3.2 i) $\int_{\bar{x}-a/2}^{\bar{x}+a/2} \frac{1}{a}\, dx$

$$\frac{1}{a} x \Big|_{\bar{x}-a/2}^{\bar{x}+a/2}$$

$$\frac{1}{a}\left(\bar{x} + a/2 - (\bar{x} - a/2)\right) = 1 \checkmark$$ está normalizado

ii) $\int_{\bar{x}-a/2}^{\bar{x}+a/2} \frac{1}{a} x \, dx$

$$\frac{1}{2a} x^2 \Big|_{\bar{x}-a/2}^{\bar{x}+a/2}$$

$$\frac{1}{2a}\left[\left(\bar{x}^2 + \bar{x}a + \frac{a^2}{4}\right) - \left(\bar{x}^2 - \bar{x}a + \frac{a^2}{4}\right) \right]$$

$$\frac{1}{2a}\left[2\bar{x}a \right]$$

$$\bar{x} \quad \checkmark$$

iii) $\sigma = \sqrt{Var}$

$$var = E(x^2) - E(x)^2$$

$$E(x^2) = \int_{\bar{x}-a/2}^{\bar{x}+a/2} \frac{1}{a} x^2 \, dx$$

$$\frac{1}{3a} x^3 \Big|_{\bar{x}-a/2}^{\bar{x}+a/2}$$

$$\frac{1}{3a}\left[\left(\bar{x}^3 + \frac{3}{2}\bar{x}^2 a + \frac{3}{4}\bar{x}a^2 + \frac{a^3}{8}\right) - \left(\bar{x}^3 - \frac{3}{2}\bar{x}^2 a + \frac{3}{4}\bar{x}a^2 - \frac{a^3}{8}\right)\right]$$

$$\frac{1}{3a}\left[3\bar{x}^2 a + \frac{a^3}{4} \right]$$

$$E(x^2) = \bar{x}^2 + \frac{1}{12}a^2$$

Del punto ii tenemos que $E(x) = \bar{x}$, entonces $E(x)^2 = \bar{x}^2$

$$E(x^2) - E(x)^2 = \frac{1}{12}a^2 \longrightarrow \sigma = \frac{a}{\sqrt{12}}$$

3.4) Usando las funciones de excel DISTR.NORM.ESTAND & " " " .INV.

Tabla 3.1:

Centred on mean	$\pm\sigma$	$\pm 1.65\sigma$	$\pm 2\sigma$	$\pm 2.58\sigma$	$\pm 3\sigma$
Measurements within range	68%	90%	95%	99.0%	99.7%
Measurements outside range	32%	10%	5%	1.0%	0.3%
	1 in 3	1 in 10	1 in 20	1 in 100	1 in 400

a)

i) $P(-4 \leq x \leq 4) = \int_{-4\sigma}^{4\sigma} \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\bar{x})^2}{2\sigma^2}\right\} dx$

$$1 - 2\int_{0}^{4\sigma} \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\bar{x})^2}{2\sigma^2}\right\} dx = 1 - 0.999937 = \underline{\underline{0.000063}}$$

$\Rightarrow 0.000063 = \frac{1}{15873} \Rightarrow$ outside range $\pm 4\sigma$ 0.0063%

ii) $1 - 2\int_{0}^{5\sigma} Pdf(x)\,dx = 1 - 0.99999943 = \underline{\underline{0.00000057}}$

$\Rightarrow 0.00000057 = \frac{1}{1754386} \Rightarrow$ outside range $\pm 5\sigma$ 0.000057%

b)

i) $\int_{0}^{z\sigma} Pdf(x)\,dx = 0.5 \Rightarrow z \simeq 0.67 \Rightarrow \underline{\underline{\pm 0.67\sigma}}$

ii) $\int_{0}^{z\sigma} Pdf(x)\,dx = 1 - 0.999 \Rightarrow z \approx 3.29 \Rightarrow \underline{\underline{\pm 3.29\sigma}}$

3.5

i)

$\bar{x} = 502\,g$ $\qquad P(x < 500) = ?$

$\sigma = 14\,g$

$$P(x < 500) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{500} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}\, dx$$

$$P(x < 500) = 0.4432 \rightarrow 44.32\%$$

ii)

$$P(530 \leq x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{530}^{\infty} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}\, dx$$

$$P(530 \leq x) = 0.02275 \rightarrow 2.28\%$$

$$1000 \cdot 0.02275 = 22.75$$

En 1000 maletas se espera que 22 tengan minimo 530g

3.7) $N_{ex} = 58$

N	1	3	4	5	6	7	8	9	10	11	12	13
Ocurrence	1	2	3	6	9	11	8	8	6	2	1	1

i) $N_{total} = (1\cdot 1) + (2\cdot 3) + (3\cdot 4) + (5\cdot 6) + (9\cdot 6) + (11\cdot 7) + (8\cdot 8) + (8\cdot 9) + (6\cdot 10) + (2\cdot 11) + 12 + 13$

$\underline{N_{total} = 423}$

ii) $\bar{X} = \dfrac{N}{N_{ex}} \Rightarrow \bar{X} = \dfrac{423}{58} \Rightarrow \underline{\bar{X} = 7.29}$

iii) $P(N;\bar{N}) = \dfrac{\exp(-\bar{N})\bar{N}^N}{N!}$, $\bar{N} = \bar{x} = 7.29$

$P(N \leqslant 5) = P(0) + P(1) + P(2) + P(3) + P(4)$

$P(N \leqslant 5) = 0.00068 + 0.0049 + 0.018 + 0.044 + 0.0803 + 0.12$

$P(N \leqslant 5) = 0.26788$

$\# \text{esperado} = 0.26788 \cdot 58 = \underline{15.53704}$

iv)

$P(N \geqslant 20) = 1 - P(N \leqslant 19) \Rightarrow P(N \leqslant 19) = P(0) + P(1) + \cdots + P(19)$

$P(N \leqslant 19) = 0.9994$

$P(N \geqslant 20) = 1 - 0.9994 = 0.0006.$

$\# \text{esperado} = 0.0006 \cdot 58 = \underline{0.035}$

3.10 $\sigma = \frac{a}{\sqrt{12}} = \frac{49}{\sqrt{12}} = 14.145$

$\bar{x} = \frac{1}{49} \sum_{i=1}^{49} i = 25$

Estos valores se ajustan bastante bien a los obtenidos en el 2000

Para el $\bar{x}'$ y σ' del $\bar{x}$ tenemos:

$\sigma' = \frac{\sigma}{\sqrt{6}} = 5.77$

Como hacemos una distribución estandar centrada en $\bar{X}$, esperaríamos que $\bar{x}'$ sea igual a $\bar{X}$

$\bar{X}' = 25$

Estos resultados son bastante similares a los resultados del 2000