

$$D\chi^2(a_0, a_1) = \left[-\frac{\partial \chi^2}{\partial a_0}, \frac{\partial \chi^2}{\partial a_1} \right]$$

$$= \left[2 \sum_{i=1}^n (y_i - (a_0 + a_1 x_i))(-1), 2 \sum_{i=1}^n (y_i - (a_0 + a_1 x_i))(-x_i) \right] = [0, 0]$$

$$\Rightarrow -2 \sum_{i=1}^n y_i - a_0 n - a_1 \sum_{i=1}^n x_i = 0 \quad ; \quad -2 \sum_{i=1}^n y_i x_i - a_0 \sum_{i=1}^n x_i - a_1 \sum_{i=1}^n x_i^2 = 0$$

$$\sum_{i=1}^n y_i - a_0 n - \sum_{i=1}^n a_1 x_i = 0$$

$$a_0 = \frac{1}{n} \sum_{i=1}^n y_i - \frac{a_1}{n} \sum_{i=1}^n x_i = \bar{y} - a_1 \bar{x}$$

$$\sum_{i=1}^n y_i x_i - a_0 \sum_{i=1}^n x_i - a_1 \sum_{i=1}^n x_i^2 = 0$$

$$a_1 \sum_{i=1}^n x_i^2 = \sum_{i=1}^n y_i x_i + \left[\frac{1}{n} \sum_{i=1}^n y_i + a_1 \left(\frac{1}{n} \sum_{i=1}^n x_i \right) \right] \left(\sum_{i=1}^n x_i \right)$$

$$a_1 \left[\sum_{i=1}^n x_i^2 - \frac{1}{n} \left(\sum_{i=1}^n x_i \right)^2 \right] = \sum_{i=1}^n y_i x_i - \frac{1}{n} \left(\sum_{i=1}^n y_i \right) \left(\sum_{i=1}^n x_i \right)$$

$$a_1 = \frac{\sum_{i=1}^n y_i x_i - \frac{1}{n} \left(\sum_{i=1}^n y_i \right) \left(\sum_{i=1}^n x_i \right)}{\sum_{i=1}^n x_i^2 - \frac{1}{n} \left(\sum_{i=1}^n x_i \right)^2}$$

$$D\chi^2(a_0, a_1, a_2) = \left[\frac{\partial \chi^2}{\partial a_0}, \frac{\partial \chi^2}{\partial a_1}, \frac{\partial \chi^2}{\partial a_2} \right] = [0, 0, 0]$$

$$= \left[-2 \sum_{i=1}^n (y_i - a_0 - a_1 x_i - a_2 x_i^2), -2 \sum_{i=1}^n (y_i - a_0 - a_1 x_i - a_2 x_i^2) x_i, -2 \sum_{i=1}^n (y_i - a_0 - a_1 x_i - a_2 x_i^2) (x_i^2) \right] = [0, 0, 0]$$

$$-2 \left(\sum_{i=1}^n y_i - a_0 - a_1 x_i - a_2 x_i^2 \right) = 0$$

$$-2 \left(\sum_{i=1}^n y_i x_i - a_0 x_i - a_1 x_i^2 - a_2 x_i^3 \right) = 0$$

$$-2 \left(\sum_{i=1}^n y_i x_i^2 - a_0 x_i^2 - a_1 x_i^3 - a_2 x_i^4 \right) = 0$$

$$\begin{cases} \sum_{i=1}^n y_i = \sum_{i=1}^n a_0 + a_1 x_i + a_2 x_i^2 \\ \sum_{i=1}^n y_i x_i = \sum_{i=1}^n a_0 x_i + a_1 x_i^2 + a_2 x_i^3 \\ \sum_{i=1}^n y_i x_i^2 = \sum_{i=1}^n a_0 x_i^2 + a_1 x_i^3 + a_2 x_i^4 \end{cases}$$