1. $\Omega = ((a, f(a)), (b, f(b)))$

$$f(x) \approx p(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$$

$$\int_a^b f(x)\,dx \approx \int_a^b p(x)\,dx$$

$$\int_a^b p(x)\,dx = \int_a^b \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)\,dx$$

$$= \frac{f(a)}{a-b} \int_a^b x-b\,dx + \frac{f(b)}{b-a} \int_a^b x-a\,dx$$

$$= \frac{f(a)}{a-b} \left(\frac{1}{2}x^2 - bx \Big|_a^b \right) + \frac{f(b)}{b-a} \left(\frac{1}{2}x^2 - ax \Big|_a^b \right)$$

$$= \frac{f(a)}{a-b} \left(\frac{1}{2}b^2 - b^2 - \frac{1}{2}a^2 + ab \right) + \frac{f(b)}{b-a} \left(\frac{1}{2}b^2 - ab - \frac{1}{2}a^2 + a^2 \right)$$

$$= \frac{f(a)}{a-b} \left(-\frac{1}{2}b^2 - \frac{1}{2}a^2 + ab \right) + \frac{f(b)}{b-a} \left(\frac{1}{2}a^2 + \frac{1}{2}b^2 - ab \right)$$

$$= \left(\frac{1}{2}b^2 - ab + \frac{1}{2}a^2 \right) \left(\frac{f(b)}{b-a} - \frac{f(a)}{a-b} \right)$$

$$= \frac{1}{2}(b^2 - 2ab + a^2) \left(\frac{f(b) + f(a)}{b-a} \right)$$

$$= \frac{(b-a)^2}{2(b-a)} (f(b) + f(a)) = \frac{b-a}{2} (f(b) + f(a))$$

2.

$$f(x) = P(x) + \epsilon(x)$$

$$\epsilon(x) = \frac{f''(\xi)}{2}(x-a)(x-b) \quad ; a \leq \xi \leq b \qquad h = b - a$$

$$\int_a^b f(x) = \int_a^b P(x) + \epsilon(x)\, dx$$

$$\int_a^b \epsilon(x)\, dx = \int_a^b \frac{f''(\xi)}{2}(x-a)(x-b)\, dx$$

$$= \frac{f''(\xi)}{2}\int_a^b (x-a)(x-b)\, dx$$

	D	I
+	$x-a$	$x-b$
−	1	$\frac{1}{2}x^2 - bx$
+	0	$\frac{1}{6}x^3 - \frac{b}{2}x^2$

$$= \frac{f''(\xi)}{2}\left(\left.(x-a)\left(\frac{1}{2}x^2 - bx\right) + \frac{b}{2}x^2 - \frac{1}{6}x^3\right|_a^b\right)$$

$$= \frac{f''(\xi)}{2}\left[(b-a)\left(\frac{1}{2}b^2 - b^2\right) + \frac{b^3}{2} - \frac{1}{6}b^3 - \frac{ba^2}{2} + \frac{a^3}{6}\right]$$

$$= \frac{f''(\xi)}{2}\left((b-a)\left(-\frac{1}{2}b^2\right) + \frac{2b^3}{6} - \frac{ba^2}{2} + \frac{a^3}{6}\right)$$

$$= \frac{f''(\xi)}{2}\left(-\frac{1}{2}b^3 + \frac{1}{2}b^2 a + \frac{1}{3}b^3 - \frac{1}{2}ba^2 + \frac{a^3}{6}\right)$$

$$= \frac{f''(\xi)}{2}\left(-\frac{1}{6}b^3 + \frac{a^3}{6} + \frac{1}{2}b^2 a - \frac{1}{2}ba^2\right) = \frac{f''(\xi)}{2}\left(\frac{a^3 - b^3}{6} + \frac{b^2 a - ba^2}{2}\right)$$

$$= \frac{f''(\xi)}{2}\left(\frac{a^3 - 3a^2 b + 3b^2 a - b^3}{6}\right) = \frac{f''(\xi)}{12}(a-b)^3$$

$$\epsilon(x) = \frac{-f''(\xi)}{12}h^3$$

acotado por arriba : $|E| \leq \frac{h^3}{12}\, \text{máx}\,|f''(\xi)|, \; a \leq \xi \leq b$

3. $\Omega = \{(a, f(a)), (x_m, f(x_m)), (b, f(b))\}$

$x_m = \frac{a+b}{2}$ $\qquad h = x_m - a = b - x_m$

$2h = b - a$

$L_0 = \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)}$ $\qquad L_1 = \frac{(x-b)(x-a)}{(x_m-b)(x_m-a)}$

$L_2 = \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}$

$p(x) = f(a)L_0 + f(x_m)L_1 + f(b)L_2$

$$\int_a^b f(x)\,dx \approx \int_a^b p(x)\,dx = \int_a^b f(a)L_0 + f(b)L_2 + f(x_m)L_1\,dx$$

$$= \int_a^b f(a)\frac{(x-b)(x-x_m)}{(a-b)(a-x_m)}dx + \int_a^b f(x_m)\frac{(x-b)(x-a)}{(x_m-b)(x_m-a)}dx$$

$$+ \int_a^b f(b)\frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}dx$$

$$= \frac{f(a)}{(a-b)(a-x_m)}\int_a^b (x-b)(x-x_m)\,dx + \frac{f(x_m)}{(x_m-b)(x_m-a)}\int_a^b (x-b)(x-a)\,dx$$

$$+ \frac{f(b)}{(b-a)(b-x_m)}\int_a^b (x-a)(x-x_m)\,dx$$

$$= \frac{f(x_m)(a-b)^3}{(x_m-b)(x_m-a)6} - \frac{f(a)(a-b)^2(2a+b-3x_m)}{6(a-b)(a-x_m)} + \frac{f(b)(a-b)^2(a+2b-3x_m)}{6(b-a)(b-x_m)}$$

$$= \frac{f(x_m)(-2h)^3}{6(-h)(h)} - \frac{f(a)(-2h)^2(h-2(h))}{6(-2h)(-h)} + \frac{f(b)(-2h)^2(-h+2(h))}{6(2h)(h)}$$

$$= \frac{f(x_m)-8h^3}{-6h^2} - \frac{f(a)4h^2(-h)}{6(2h^2)} + \frac{f(b)(4h^2)(h)}{6(2h^2)}$$

$$= \frac{4f(a)h}{12} + \frac{8f(x_m)h}{6} + \frac{4f(b)h}{12}$$

$$= \frac{h}{3}\left(f(a) + 4f(x_m) + f(b)\right)$$