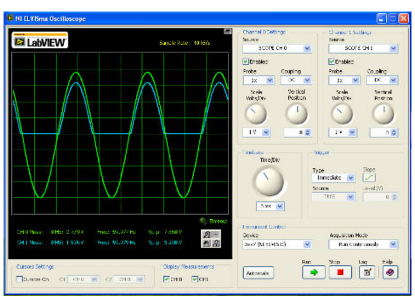
ABSTRACT

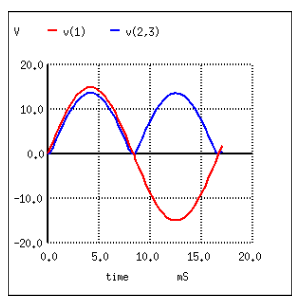
Standard half and full wave rectification circuits were built with filter capacitors to simulate the operation of a typical DC power supply. The capacitance of the filter and resistance of the load were varied independently in each circuit to determine the effect of each on the circuit's output waveform. Capacitance values used ranged from 0 µF to 47 µF and load resistances used ranged from 470 Ω to 100 kΩ. For every combination of capacitance and load resistance, V_m and V_{min} of the output waveform were measured and used to calculate V_r, V_r (rms), V_{dc}, and the Ripple Factor of the waveform. These values were then compared with predicted values obtained from B^2 SPICE simulations and were found to line up with expected results. It was then concluded that the higher the filter capacitance or load resistance became, the closer the graph of the output voltage became to a straight line, verifying the design equation for the ripple voltage of a filter circuit.

INTRODUCTION

As stated above, two of the most important ideas used to convert a sinusoidal AC voltage into a constant DC voltage are rectification and filtration. These ideas are important because devices expecting DC voltages often do not react well to receiving varying AC voltages, and can be destroyed if the signal they are expecting is not constant. Couple this with the fact that the voltages sent out by power plants and through all wall outlets in the United States are AC voltages (specifically sine waves with frequency sixty hertz), and it isn't hard to see why DC power supplies and converters are so fundamental in the proper operation of DC electronics.

DC power supplies operate by first rectifying a signal and then filtering that signal to a smoother value. Rectification is the concept of converting a bi-directional, AC input waveform into a one-directional output waveform. There are two forms of rectifiers: Half wave rectifiers, which only let the positive peaks of the input waveform out to the load, and full wave rectifiers, which let the positive and negative peaks of the input waveform out to the load, but rectify the negative peaks into the positive axis. Both types of rectifiers use diodes to accomplish their purposes. Half wave rectifiers are typically built from one diode and a load resistor and full wave rectifiers are typically built from four diodes and a load resistor. The effects of each rectifier on a standard sinusoidal input voltage are seen below in Figures 2.1 and 2.2. Note that because diodes are used, the some voltage is lost off the amplitude of the input in both circuits.





Figures 2.1 and 2.2: Typical Half Wave Rectifier Output (Left) and Typical Full Wave Rectifier Output (Right) [2]

While rectification brings an AC voltage closer to a constant value, it does not smooth it out completely. In fact, it is very noticeable that the output voltage of both rectification circuits vary quite substantially. Filtration addresses this issue by introducing a filter capacitor in parallel with the load resistor of each rectifier, which essentially creates a small RC circuit at the output and takes advantage of the time constant and other properties of the circuit to smooth the waveform: As the input voltage increases, the capacitor at the output charges itself up and the circuit behaves like it would normally. However, when the input voltage starts to decay, the capacitor begins to slowly discharge, which causes the load to be held at the relatively constant but slowly decaying voltage of the capacitor, since it is in parallel with the load resistor. When the capacitor is fully discharged and the input is increasing again, it begins charging again and the process is repeated (This is no different from a typical RC circuit). Figure 2.3 shows the effect of filtration on a half wave rectification circuit.

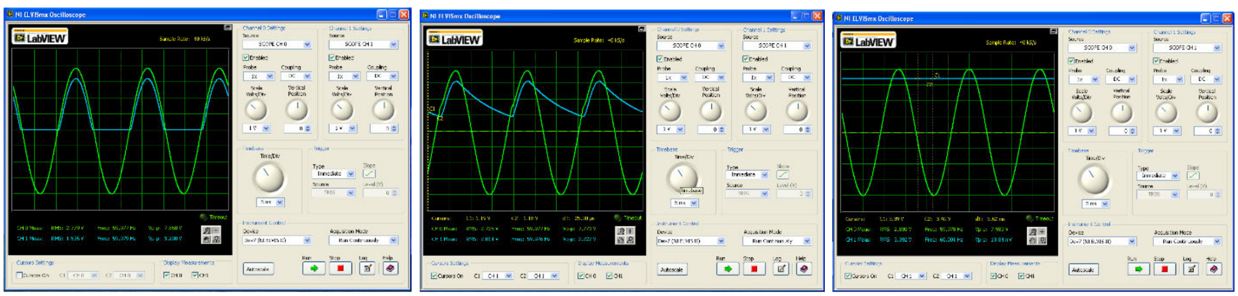
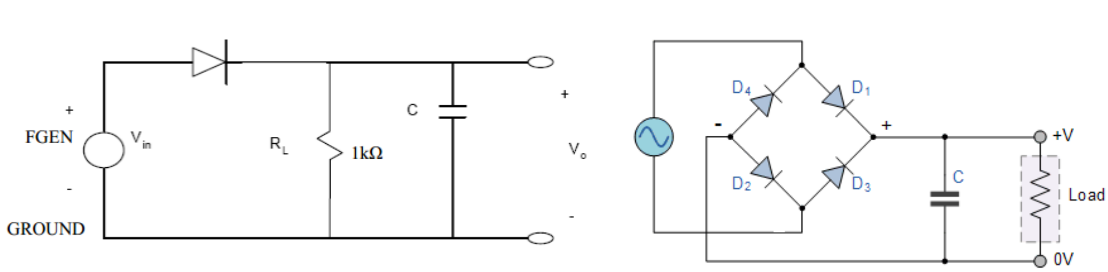


Figure 2.3: Effects of Rectification on Half Wave Rectifier
(Left = Normal Output, Center = Slight Filtration, Right = Heavy Filtration)

As seen in Figure 2.3, the ability of a filter to bring a rectified waveform to a DC voltage varies and is specifically dependent on the values of R and C used in the rectifier's output load. The experiment detailed in this report was undertaken to prove the relationship between the output voltage and these values. The capacitance of the filter and resistance of the load in the filtered half and full wave rectification circuits shown in Figures 2.4 and 2.5 were varied independently. This allowed for the effects of each component on the output wave form to be observed and calculated using an oscilloscope. Results showed that the ripple voltage, V_R, of a filtered rectifier's output followed the trends of the equation $V_r = \frac{V_m}{f \cdot R_L \cdot C}$.



Figures 2.4 and 2.5: Filtered Half Wave Rectification Circuit (Left) and Filtered Full Wave Rectification Circuit (Right) [3]

BACKGROUND

Note that several pieces of information from the lab manual [1] were referenced when writing this section.

There are several characteristics that can define and differentiate the output of a filtered half or full wave rectification circuit for different load resistance and filter capacitance values. These are all centered around V_m and V_{min}, which are simply the highest voltage (or amplitude) and the lowest voltage the filtered output achieves, respectively. They can best be seen in the center of Figure 2.3: V_m is the voltage just before the capacitor in the associated rectification circuit begins discharging and V_{min} is the voltage just before it begins charging.

The ripple voltage (V_r), which can simply be thought of as the variance of voltage of the filtered waveform, can then be defined relatively easily as:

$$V_r = V_m - V_{min}$$

However, this value turns out to be more useful as a root-mean-squared value. Because the filtered waveform's shape can be complicated, it is assumed as a triangular waveform when calculating RMS values. Recall that the instantaneous RMS value for any triangular wave can be obtained by dividing by $2\sqrt{3}$. This means the RMS ripple voltage is:

$$V_r(rms) = \frac{V_m - V_{min}}{2\sqrt{3}} = \frac{V_r}{2\sqrt{3}}$$

It is useful to have a measure of what DC voltage will be provided by the output of a half or full wave rectifier before filtration, even if it is pulsating. This, by logical definition, should be the average value of the output curve, which can be given for any function, f(x), as:

$$f_{avg} = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)dt$$

Where T is the period of f(t). The expression for V_{dc} for a half-wave rectifier is then given by applying this definition to the output waveform. Note that the output signal is piecewise, and that the first piece will evaluate to area zero.

$$V_{dc,half\ wave} = \frac{1}{T}\int_{-\frac{T}{2}}^{0} 0\ dt + \frac{1}{T}\int_{0}^{\frac{T}{2}} V_m \sin\left(\frac{2\pi t}{T}\right) dt = \frac{V_m}{\pi}$$

The expression for V_{dc} of a full wave rectifier can be obtained either using the integral definition again, or by recognizing that since a full wave rectifier lets twice the amount of signal through, V_{dc} will be twice as large. It should also be noted because of this, it is easier to perform rectification with a full wave rectifier than it is with a half wave rectifier.

$$V_{dc,full\ wave} = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left| V_m \sin\left(\frac{2\pi t}{T}\right) dt \right| = \frac{2V_m}{\pi}$$

Finally, the Ripple Factor of a filtered waveform is simply the ratio of V_r (rms) to V_{dc} of the waveform. Think of the ripple factor as a rating of how much the voltage produced by the filter varied around the expected output, V_{dc}. The closer this value to zero, the better of a job performed by the filter.

$$Ripple\ Factor = \frac{V_r(rms)}{V_{dc}}$$

By substituting the above equations, the fact that $V_m = V_r \cdot f_p \cdot R_L \cdot C$, and assuming the output waveform is triangular, the ripple factor can alternatively be given by the below.

$$Ripple\ Factor = \frac{1}{\sqrt{3}} \cdot \frac{1}{2f_pR_LC - 1}$$

EXPERIMENTAL PROCEDURE

The circuit in Figure 2.4 was constructed with the function generator set to 4 volts peak to peak. The load resistance was initially set to 10 kΩ and the capacitance was varied from 0 µF to 10 µF to 22 µF to 47 µF. For each capacitance value, V_m and V_{min} were measured using an oscilloscope. V_r, V_r (rms), V_{dc}, and the Ripple Factor for each case were then calculated. Each output waveform was sketched.

The filter capacitance was then fixed to 10 µF and the load resistance was varied from 470 Ω to 10 kΩ to 100 kΩ. V_m and V_{min} were measured with an oscilloscope and V_r, V_r (rms), V_{dc}, and the Ripple Factor for each case were calculated. Each output waveform was sketch.

Finally, the entire process was then repeated for the circuit in Figure 2.5 with the function generator set at 4 volts peak to peak.

DISCUSSION OF EXPERIMENTAL RESULTS

The experimental output data is shown below in Figures 2.6 and 2.7.

R_L = 1 kΩ	V_m	V_{min}	V_r	V_r (rms)	V_{dc}	Ripple Factor	Calculated Ripple Factor
No Capacitor	3.2 V	-499.72 µV	3.2005 V	0.924 V	1.019 V	0.9068	-0.5774
10 µF	3.11 V	895.94 mV	2.214 V	0.639 V	0.9899 V	0.6405	2.887
22 µF	2.92 V	1.64 V	1.28 V	0.3695 V	0.9295 V	0.3975	0.3520
47 µF	2.72 V	2.05 V	0.67 V	0.1934 V	0.8658 V	0.2234	0.1244

C = 10	V_m	V_{min}	V_r	V_r (rms)	V_{dc}	Ripple	Calculated

μF						Factor	Ripple Factor
470 Ω	2.91 V	267.37 mV	2.643 V	0.763 V	0.925 V	0.8244	-1.324
10 kΩ	3.3 V	2.86 V	0.44 V	0.127 V	1.049 V	0.1210	0.05249
100 kΩ	3.42 V	3.36 V	0.06 V	0.017 V	1.088 V	0.0159	0.004852

Figure 2.5: Half Wave Rectifier Experimental Data

R_L = 1 kΩ	V_m	V_{min}	V_r	V_r (rms)	V_{dc}	Ripple Factor	Calculated Ripple Factor
No Capacitor	1.38 V	326.56 μV	1.053 V	303.97 mV	0.879 V	0.3458	-0.5774
10 μF	962.01 mV	699.79 mV	272.22 mV	78.58 mV	0.619 V	0.1270	0.4124
22 μF	905.36 mV	779.96 mV	125.4 mV	36.20 mV	0.576 V	0.0628	0.1349
47 μF	874.22 mV	812.9 mV	61.32 mV	17.702 mV	0.557 V	0.0318	0.05616

C = 10 μF	V_m	V_{min}	V_r	V_r (rms)	V_{dc}	Ripple Factor	Calculated Ripple Factor
470 Ω	641.03 mV	344.83 mV	0.296 V	0.086 V	0.408 V	0.2095	4.511
10 kΩ	2.17 V	2.07 V	0.1 V	0.029 V	1.381 V	0.0208	0.02510
100 kΩ	2.84 V	2.84 V	0 V	0 V	1.808 V	0	0.002416

Figure 2.6: Full Wave Rectifier Experimental Data

As a reminder, it was stated that the relationship between R_L, C, and V_R is given by:

$$V_r = \frac{V_m}{f \cdot R_L \cdot C}$$

Where f is the frequency of the filtered wave and all other variables are as defined. Note the below limit implies that as R or C increase, V_R should decrease.

$$\lim_{R_L \to \infty} \frac{V_m}{f \cdot R_L \cdot C} = 0 \ \ and \ \ \lim_{C \to \infty} \frac{V_m}{f \cdot R_L \cdot C} = 0$$

If V_R decreases, then V_r (rms) will decrease, causing the ripple factor to become closer to zero. So:

$$\lim_{V_r(rms)\to 0} \frac{V_r(rms)}{V_{dc}} = 0$$

Notice that the data in Figures 2.6 and 2.7 show the numerical trends discussed occurring. The higher R_L or C get, the lower V_R and the closer the Ripple Factor to zero. Also, remember that the closer the ripple factor is to zero, the closer the output should be to a straight line. As it turns out, Figure 2.3 is actually three graphs from the half wave rectification circuit with different R_L and C values. The graph on the left is R_L = 1 kΩ and C = 0 µF, the graph in the center is R_L = 1 kΩ and C = 10 µF, and the graph on the right is R_L = 100 kΩ and C = 47 µF. This means that graphically, the data from the half-wave rectifier also follows the discussed trends. Figure 2.7 shows that the data from the full wave rectifier also follows the trends, as the graph on the left is R_L = 1 kΩ and C = 10 µF, the graph in the center is R_L = 10 kΩ and C = 22 µF, and the graph on the right is R_L = 100 kΩ and C = 47 µF.

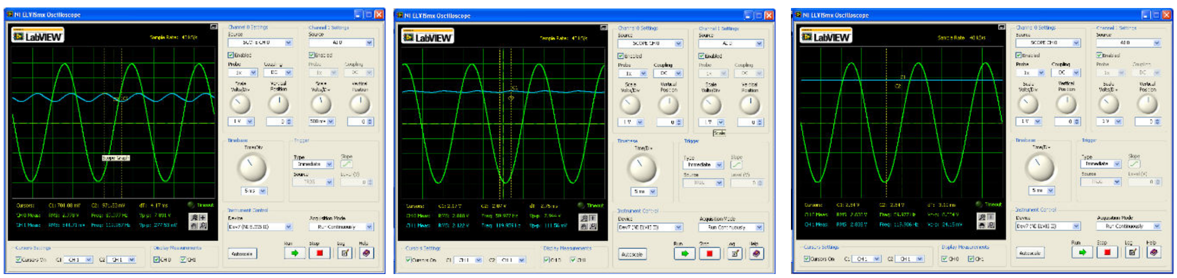


Figure 2.7: Full Wave Rectifier Data Graphically

Also, note even though the function generator is set to four volts peak to peak, V_m is not exactly four volts. This again, was expected, since diodes are performing the rectification. Specifically, V_M should be $V_{p\text{-}p}$ minus Vγ (About 0.7 V at room temperature for the diodes used) in the half-wave rectifier and $V_{p\text{-}p}$ minus two times V_γ in the full wave rectifier. These values check out.

There is great discrepancy between most of the actual ripple factor values and calculated ripple factor values in figures 2.5 and 2.6, however, neither set of values is incorrect. The key to the discrepancy is that the calculated ripple factor formula (The last formula in the Background section) is an estimation of the ripple factor of a wave. It is only valid assuming that the output waveform is triangular and gets more and more accurate with increasing values for the RC time constant of the filter in the rectification circuits. So, for no capacitance at the load, the formula is does not hold and returns negative values. For small values of R_LC, like R_L = 470 Ω and C = 10 µF for both circuits, the margin of error is very large for the estimations, and they are off from the actual values. As R_LC increases, the margin of error decreases for the estimations, and the values get closer and closer to the actual values. The estimations check with the actual values.

Finally, note that all values obtained were within reasonable estimation of the simulation data provided. A screenshot of the simulation data, Figure 2.8, is included for reference.

Half Wave Rectifier Simulations

R_L = 1kΩ							
C (µF)	V_m (V)	V_{min} (V)	V_r	V_r (rms)	V_{dc}	Ripple Factor	Calculated Ripple Factor
No Cap	3.3	0	3.3	0.953	1.05	0.907	-0.577
10	3.3	0.88	2.42	0.699	1.05	0.665	2.887
22	3.3	1.8	1.5	0.433	1.05	0.412	0.352
47	3.3	2.4	0.9	0.260	1.05	0.247	0.124

C = 10µF							
R_L (Ω)	V_m (V)	V_{min} (V)	V_r	V_r (rms)	V_{dc}	Ripple Factor	Calculated Ripple Factor
470	3.3	0.24	3.06	0.883	1.05	0.841	-1.324
10000	3.3	2.9	0.4	0.115	1.05	0.110	0.052
100000	3.4	3.3	0.1	0.029	1.08	0.027	0.005

Full-Wave Rectifier Simulations

R_L = 1kΩ							
C (µF)	V_m (V)	V_{min} (V)	V_r	V_r (rms)	V_{dc}	Ripple Factor	Calculated Ripple Factor
No Cap	1.3	0	1.3	0.375	0.83	0.453	-0.577
10	0.93	0.66	0.27	0.078	0.59	0.132	0.412
22	0.86	0.73	0.13	0.038	0.55	0.069	0.135
47	0.83	0.76	0.07	0.020	0.53	0.038	0.056

C = 10µF							
R_L (Ω)	V_m (V)	V_{min} (V)	V_r	V_r (rms)	V_{dc}	Ripple Factor	Calculated Ripple Factor
470	0.62	0.32	0.3	0.087	0.39	0.219	4.511
10000	2	1.9	0.1	0.029	1.27	0.023	0.025
100000	2.61	2.59	0.02	0.006	1.66	0.003	0.002

Figure 2.8: Simulation Data

CONCLUSIONS

Typical DC power supplies perform AC to DC conversion using the process of filtered rectification. Rectification involves transforming a bi-polar AC signal into a single pole, pulsating DC waveform. Filtration then smooth's this curve into a relatively constant DC waveform. The expected output voltage of a half or full wave rectifier, V_{DC}, can be calculated and compared to the variance of the filtered rectifier, V_r, to determine the filtered rectifier's ripple factor.

Half and full wave rectifiers work using diodes, which only allow current through a circuit in the forward-biased state. A half wave rectifier is built with one diode, while a full wave rectifier is built with either two diodes (less common) or four diodes (more common). Filters work by adding a capacitor in parallel with the rectifier's load resistance, creating an RC circuit at the output. As the input waveform increases, the capacitor charges and nothing happens to the output waveform. However, when the input starts to decrease, the filter capacitor begins discharging and holds the output voltage to a relatively constant but decaying value. The repeating pattern of this process smooth's the output waveform.

The ability and efficiency of a filtered rectifier to transform an AC wave into a DC signal is dependent on the amplitude of the input waveform, the frequency of the input waveform, the capacitance of the filter capacitor used, and the resistance of the load. In general, it is given by $V_r = \frac{V_m}{f \cdot R_L \cdot C}$. This means that as the load resistance or filter capacitance increase, the ripple voltage decreases, causing the Ripple Factor and actual ripple in the wave to decrease.

Acknowledgements: My lab partners were David Saxton and Joshua Blake

References:

1. D J Dumin (Edited by J E Harriss), ECE 311 LABORATORY MANUAL VER 1.5, pg. 17 to 26, Clemson University, Clemson, SC, July 2011.
2. Tony R. Kuphaldt (Formatted by Jason Strack), Lessons In Electric Circuits—Volume II Chapter 7, Other Waveshapes Image, http://www.ibiblio.org/kuphaldt/electricCircuits/AC/AC_7.html, June 2000.
3. Author not specified, Full Wave Rectifier, Full-wave Rectifier with Smoothing Capacitor Image, electronics-tutorials.ws/diode/diode_6.html, Date not specified.