Comparative Valuation Dynamics in Models with Financial Frictions

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Motivation and Goal

Recent dynamic stochastic general equilibrium models commonly feature:

- Two types of agents with different levels of productivity and preferences
- Non-trivial financial sectors with frictions impeding risk allocation
- Non-linear behaviors

but differ in

- Assumptions on agents' behavior (such as risk aversion)
- Constraints and assumptions on use of financial instruments to allocate risk

Solution: Build a general nesting framework that allows us to compare and contrast model dynamics and asset pricing implications

Areas of Interest

In our framework, we solve models **globally** to compare

- Wealth distribution, consumption, investment rate, capital price
- Interest rate, risk prices (and their term structure), equity issuance, capital holdings, and leverage

Challenge: All the variables above are functions of the value functions ($\zeta_{e,h}$), which are the solution to a set of two second-order, non-linear elliptic PDEs. The PDEs are connected via their coefficients (which depend on the variables above, which, in turn, depend on both value functions).

Solution Strategy

Using an iterative approach, we make a guess on $\zeta_i = \zeta_i^{(0)}$ before the iterations start. Then starting with iteration $j = 1$,

1. Compute all the variables that depend on $\zeta_i^{(j-1)}$. Replace all the nonlinear terms with $\zeta_i^{(j-1)}$. We then have two linear PDEs.
2. Solve the linear PDEs to get ζ_i^j through the *implicit* finite difference method.
3. Repeat the process until $||\zeta_i^{(j)} - \zeta_i^{(j-1)}|| < \epsilon$.

Note that:

- We use the upwinding scheme when approximating first derivatives so that our scheme is close to achieving the monotonicity property for convergence (Barles/Souganidis criteria).
- The coefficients of the PDEs are themselves functions of the value functions. Thus there is always an intermediate step where we refresh the coefficients of the PDEs using our economic model's equilibrium conditions (and hence we have a new linear system to decompose in each iteration).

Computational Challenge & Implementation

Curse of dimensionality

The solution ζ_i, in turn, is a function of state variables (w, g, s). The size of the linear system, which we need to decompose, derived from the implicit FD scheme varies with the number of state variables exponentially.

Number of iterations

In each iteration, we have a new linear system to solve, as the linear system depends on the solution to the previous linear system.

Implementation

- C++: Fast speed and better access to HPC tools
- Pardiso (Parallel Sparse Direct Solver): Matrix decomposition

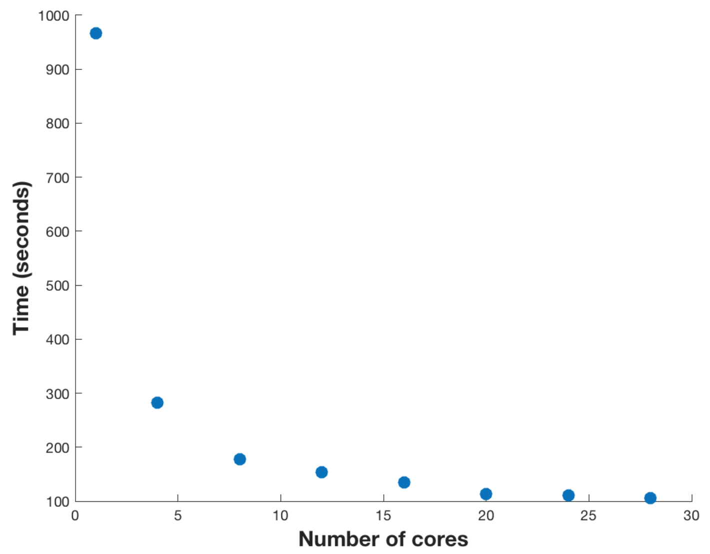
Performance: Without Parallel Processing

Suppose we initialize 100 grid points in each dimension and it takes 500 iterations to converge:

Dimensions	Matrix Size	Time (seconds) per iteration	Total Time (Minutes)
1	$100^1 \times 100^1$	0.02	0.14
2	$100^2 \times 100^2$	0.12	1.03
3	$100^3 \times 100^3$	966.27	8052 (134 hours)

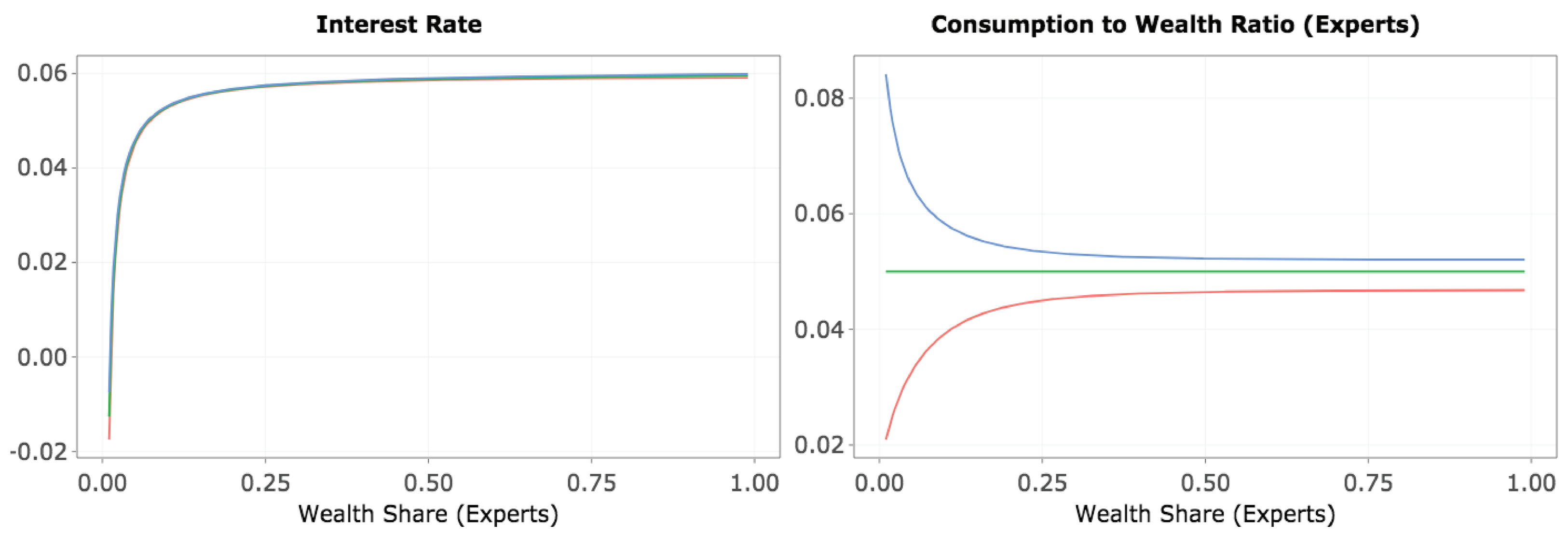
Using RCC's resources, we can employ parallel processors.

We show the time needed to solve two $100^3 \times 100^3$ linear systems with multiple cores:



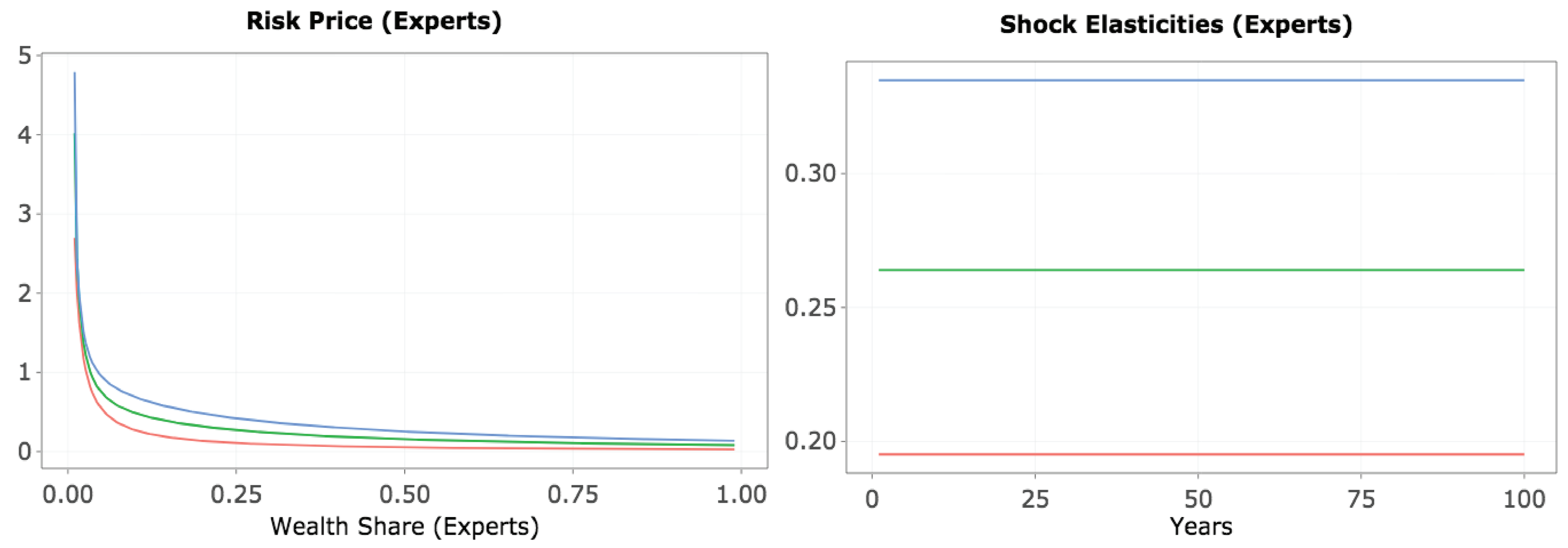
Case 1: Varying Elasticity of Intertemporal Substitution

We hold everything equal except for the elasticity of intertemporal substitution of the more productive agent ("experts"). Note that (left) the interest rate rises with wealth in all three cases. However, on the right, in one model, the agent reduces consumption as a rise in the interest rate incentivizes the agent to save, whereas in another model, we observe more consumption as the agent has more wealth.



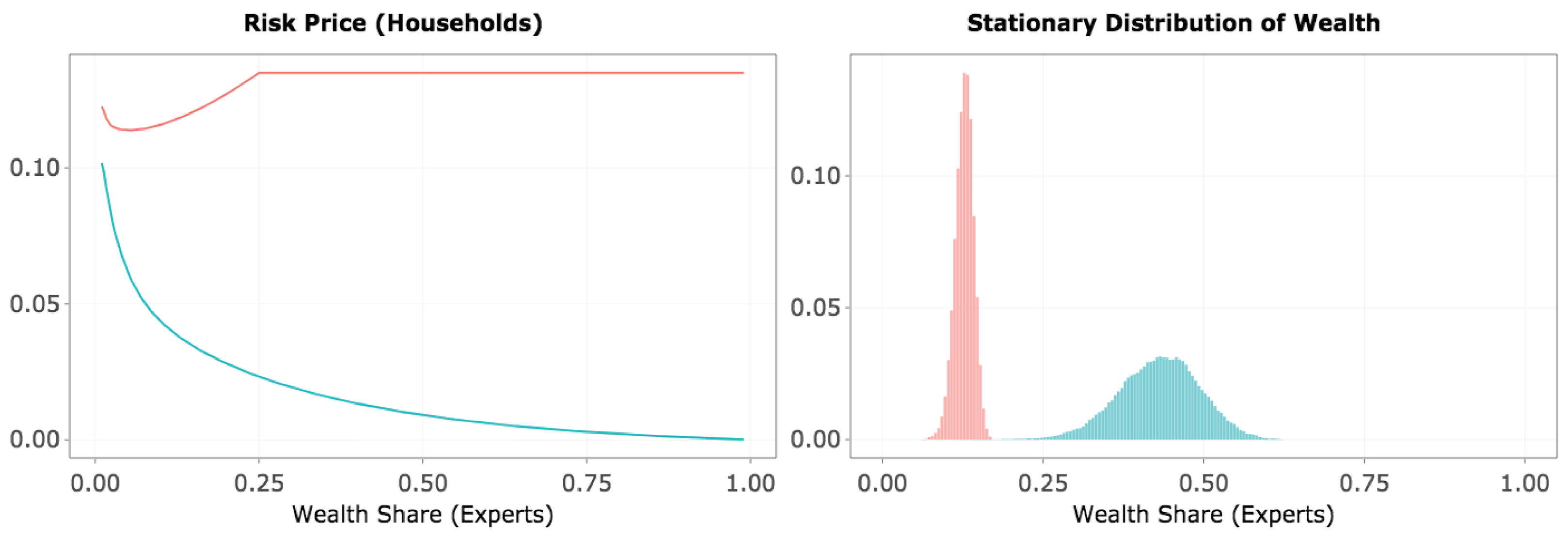
Case 2: Varying Risk Aversion

We vary the risk aversion of experts. The blue model corresponds to the highest degree of risk aversion whereas the red model the lowest. Both risk prices (left), i.e. compensation required to hold risk, and the shock elasticities (right), interpreted as the % increase in return per % increase in risk for a fixed period of time, increase.



Case 3: Introducing Financial Frictions

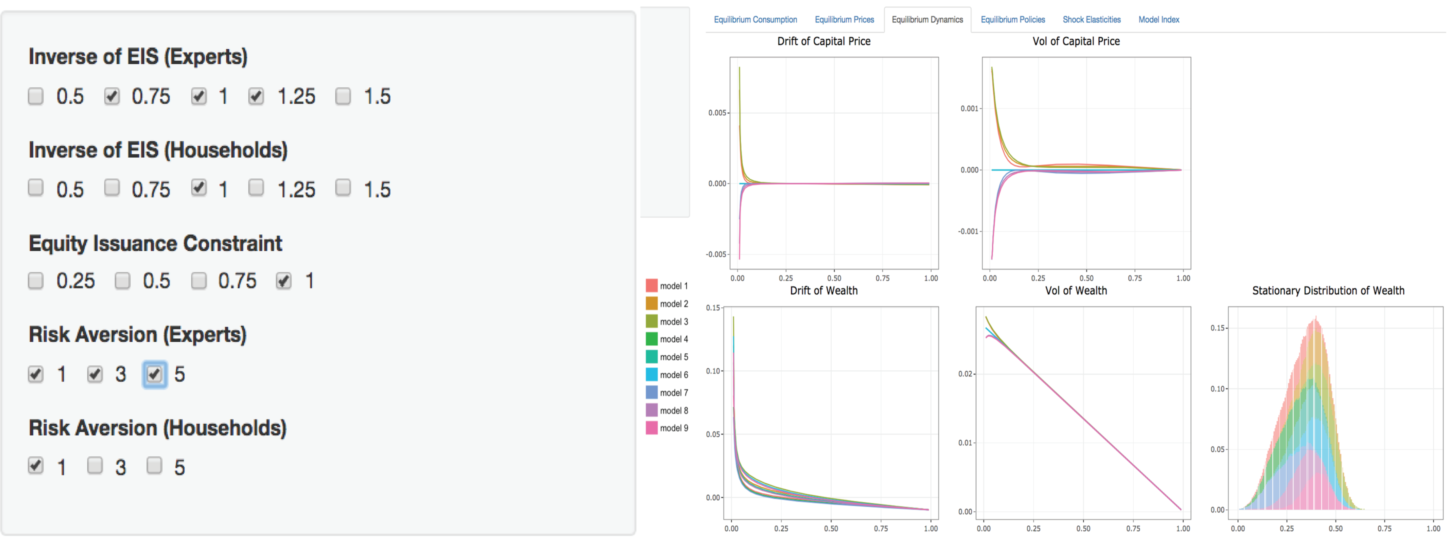
Ideally, experts would sell equity to diversify risk. However, in reality, experts are required to hold equity of their firm up to a certain level. Therefore, we put various levels of required equity retention on experts.



The green model does not allow equity issuance (experts must retain 100% of equity), whereas the red model only requires a 25% retention. When experts sell equity, households bear the risk and thus demand a higher compensation (left). However, when experts are not allowed to sell equity, they must bear the risk but get compensated more and are wealthier (right).

Web Interface (In Progress)

We will release a website where the user can easily compare models by changing parameters and constraints. No coding required, only point and click.



The total permutations of all the parameter choices are huge, but they are embarrassingly parallel. Using RCC's computational resources, we can solve all the model configurations simultaneously.