Tuning Auxetic Properties of Networks by the Bending Resistance

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Introduction

Materials with a negative Poisson's ratio, ν, are called auxetic materials. Over the years, various geometrical origins for auxetic behaviors are identified, providing guidelines for the smart design of auxetic metamaterials with periodic structures. In recent years, the mechanical origins for disordered auxetic materials attract a lot of interests. Disordered materials are promising to generate isotropic mechanical properties. And Poisson's ratio, $\nu = \frac{d - \frac{2G}{B}}{d(d-1) + \frac{2G}{B}}$, where B is the bulk modulus, G is the shear modulus, and d is the dimension.

Here we study the effects of bending resistance, k_θ, on the auxetic properties of disordered networks. We understand the effects by decomposing the moduli into contributions from bond compressing/streching (central force) and bending resistances. We also observe a common ratio of the bending resistance to the central force resistance that leads to a zero Poisson's ratio, more or less regardless of the coordination number. We use a minimum theory to explain this so-called "pinning point" effect.

Methods

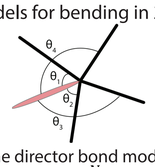
The Poisson's ratio is measured from the elastic constants calculated as the secondary derivative of energy w. r. t. strain tensor components.

The potential energy of the system:

$$E_{pot} = \sum_{l}^{bonds} \frac{k_{bj}}{2l_{bj0}^2}(l_{bj} - l_{bj0})^2 + \sum_{l}^{angles} \frac{1}{2} k_{\theta l}(\theta_l - \theta_{l0})^2$$

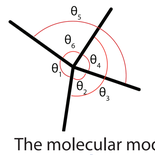
Central force — Bending

Two models for bending in 2D:



The director bond model

$$k_{\theta l} = \frac{N_l}{N_l - 1} k_\theta$$



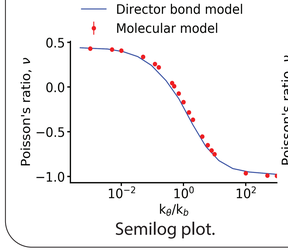
The molecular model

$$k_{\theta l} = \frac{k_\theta}{N_l - 1}$$

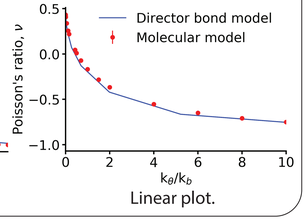
The two models have one-on-one correspondence w. r. t. k_θ.

Model comparison

The two models agree well in 2D networks.



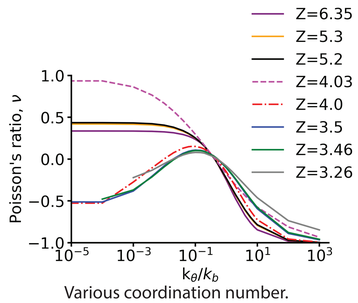
Semilog plot.



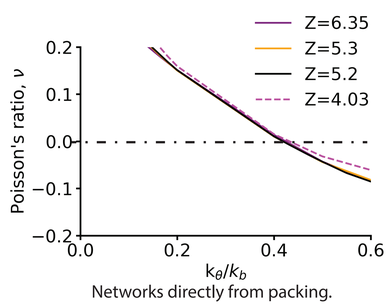
Linear plot.

The observation of pinning point effects

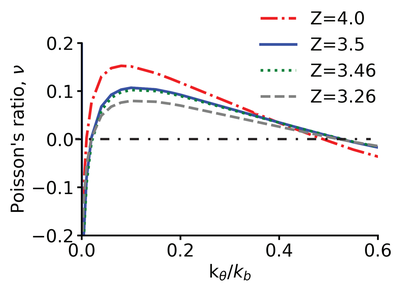
1. For networks directly from packing, there is a common value of k_θ/k_b for Poisson's ratio to be zero.
2. For pruned networks, the value of k_θ/k_b for Poisson's ratio to be zero is shifted to the right.



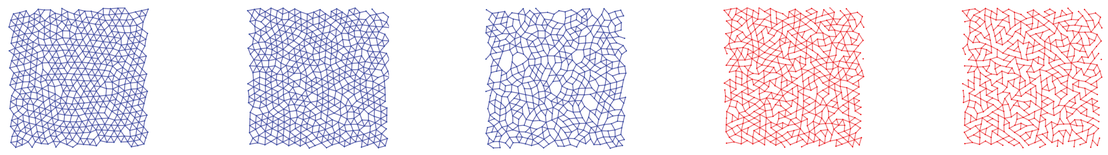
Various coordination number.



Networks directly from packing.



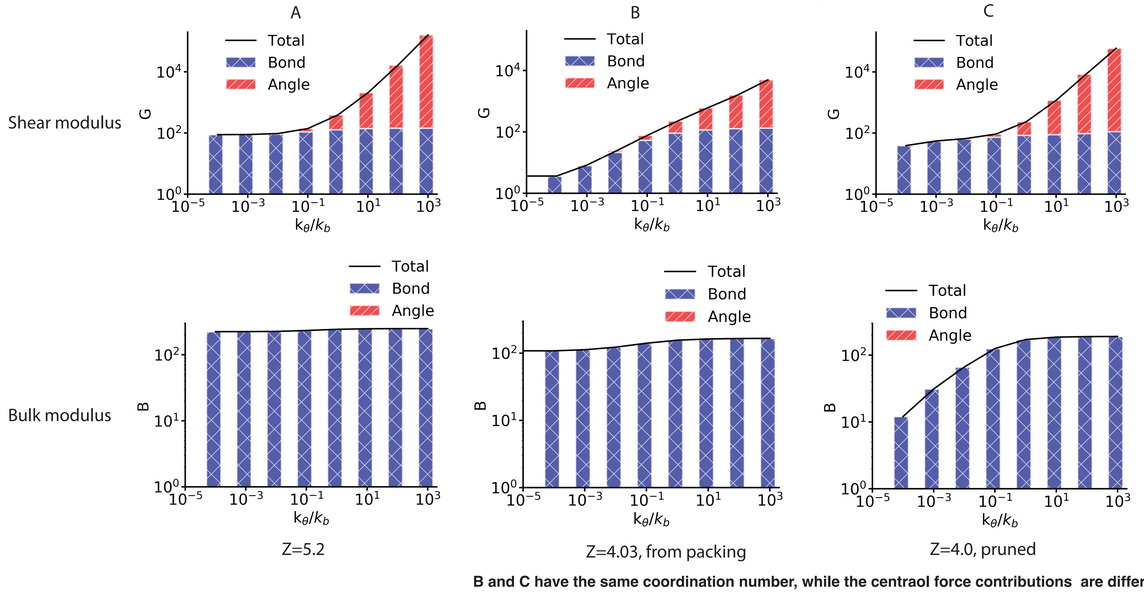
Pruned networks.



Z=5.3 | Z=5.2 | Z=4.03 | Z=4.0, pruned | Z=3.26, pruned

Decomposition of moduli

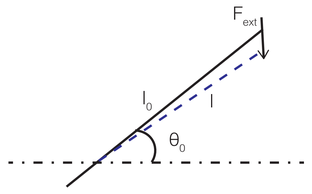
1. For all networks, the increase of k_θ/k_b leads to the increase of shear modulus contributed from both central force and bending resistance.
2. For networks generated from packing near the isostatic point, the initial increase of shear modulus is mainly contributed by the central force.
3. For all networks, the bending resistance does not directly contribute to the bulk modulus. However, the bulk modulus from central force resistance increases with k_θ/k_b.
4. For pruned networks, the bulk modulus from central force resistance increases by an order of magnitude with the increase of k_θ/k_b.



Z=5.2 | Z=4.03, from packing | Z=4.0, pruned

B and C have the same coordination number, while the centraol force contributions are different in the low k_θ/k_b regime, indicating the Poisson's ratio can be tuned by bond swapping.

Minimum theory



$$\begin{cases} F_c = -\frac{k_b}{l_0^2} dl \\ F_\theta = -\frac{k_\theta d\theta}{l_0} \end{cases}$$

In equilibrium: $F_c \cos\theta = F_\theta \sin\theta$

$\nu = 0$: $dl \cos\theta = l_0 d\theta \sin\theta$

For small strains → $k_\theta \cong k_b$ when $\nu = 0$

In a network, $k_\theta \cong 0.5 k_b$, due to 2 nodes per bond

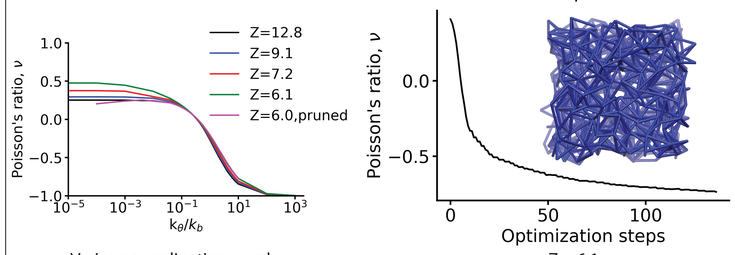
15-s summary

Question: Can we design auxetic networks by varying the bending stiffness?

Results: 1. Networks become auxetic when k_θ/k_b exceeds the pinning point.
2. The auxetic properties can be tuned by bond pruning/swapping when $k_\theta \ll k_b$.

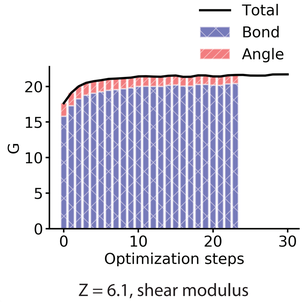
Discussion: The decomposition of moduli enables us to get insights about the mechanical origins of bending resistance dependence of Poisson's ratio.

Bending resistance and 3D design

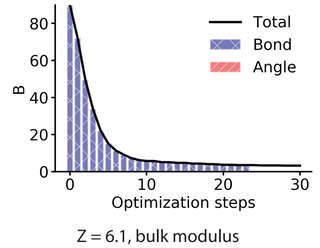


Various coordination number

Z = 6.1



Z = 6.1, shear modulus



Z = 6.1, bulk modulus

References/acknowledgement

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