Quiz II

① $\ln a = \int_1^a \frac{1}{x}\,dx$ $\qquad\qquad$ $\int \frac{1}{x}\,dy = \ln|x| + C$

② $\frac{d}{dx} \ln v = \frac{1}{v}\frac{dv}{dy}$

③ $\frac{d}{dy} e^v = e^v \frac{dv}{dy}$ $\qquad\qquad$ $\int e^v\,dv = e^v + C$

④ $\frac{d}{dy} a^v = a^v \ln a \frac{dv}{dy}$ $\qquad\qquad$ $\int a^v\,dv = \frac{1}{\ln a} a^v + C$

5. $\frac{d}{dy} u^v = v u^{v-1} \frac{du}{dy} + u^v \ln u \frac{dv}{dy}$

as x^n $\qquad$ as a^v

Properties of Natural Logarithms

1. $\frac{d}{dy}(\ln x) = \frac{1}{x}$ $\qquad$ since $\frac{d}{dy}\int_1^x \frac{1}{t}\,dt = \frac{1}{x}$

2. $\ln 1 = \emptyset$ $\qquad$ $\ln 1 = \int_1^1 \frac{1}{t}\,dt = 0$

3. $\ln(ab) = \ln a + \ln b$

4. $\ln \frac{1}{a} = -\ln a$

5. $\ln \frac{a}{b} = \ln a - \ln b$

6. $\ln a^r = r \ln a$

7. $\ln a < \ln b$ $\quad$ when $a < b$

Definition one p. 219

$C^x = \exp(x \ln C) = e^{x \ln C}$

Proof

Theorem ONE p. 215 Properties of Exponentials

1. $\ln(\exp a) = a$ $\exp(\ln b) = b$
2. $\exp a = e^a$
3. $\exp 0 = e^0 = 1$
4. $\exp(-a) = \dfrac{1}{\exp a}$
5. $\exp(a+b) = (\exp a)(\exp b) = (e^a)(e^b)$
6. $\exp a > 0$ $\exp a < \exp b$ when $a < b$

Proof $C^r = \exp \ln C^r = \exp[r \ln C]$

$C^r = C^r$ $\ln C^r = r \ln C$ Take ln both sides: Reduce one side

$\exp(\ln C^r) = \exp(r \ln C)$ Take exp both sides

$\exp(\ln C^r) = C^r = \exp(r \ln C)$ Solve

$$\log_b A = \frac{\ln A}{\ln B}$$

memory aid $\dfrac{\log}{b} A = \dfrac{\ln A}{\ln B}$

Page 225

OVER

Derivitives of exponents

1. $\frac{d}{dx} e^x = e^x$

2. $\frac{d}{dx} e^v = e^v \frac{dv}{dx}$

3) $\frac{d}{dx} a^x = a^x \ln a$ $a>0$

4) $\frac{d}{dx} a^v = a^v \ln a \frac{dv}{dx}$ $a>0$

Derivitive of $\ln x$

Memorize:

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

when $v = f(x)$ such that $v > 0$, by Chain Rule

$\frac{d}{dx}(\ln v) = \frac{1}{v}\frac{dv}{dx}$

$\int \frac{1}{x}dx = \ln x + C$ for $x>0$

Thoerem Two p. 221

$\log_c x = \frac{1}{\ln c} \ln x$

Solve for x

Proof

5. $\frac{d}{dy} u^v = v \cdot u^{v-1} \cdot \frac{du}{dy} + u^v \ln u \cdot \frac{dv}{dy}$

Properties of exponents

1. $\ln(\exp a) = a$

2. $\exp 0 = 1$

3 $\exp a = e^a$

4. $\exp(a+b) = (\exp a)(\exp b)$

5. $\exp(-a) = \frac{1}{\exp a}$

6. $\exp a > 0$

7. $\exp a < b$ when $a < b$

Logarithms

$\log_b A^k = k \log_b A$ $\log_b 1 = 0$

$\log_b A = x$ $b^x = A$

$= x$ $b^x = N$ $\text{Log } A = \underline{1}$

$$\log_{10} AB = \log_{10} A + \log_{10} B$$

$$\log_{10} \frac{A}{B} = \log_{10} A - \log_{10} B$$

$$\log_{10} A^k = k \log_{10} A \qquad \log_{10} 1 = \emptyset$$

The Derivitive of a Natural Log

$$\ln x = \int_1^x \frac{1}{t}\,dt \quad \text{where } x>0$$

$$\frac{dt}{dy}(\ln x) = \frac{dt}{dy}\int_1^x \frac{1}{t}\,dy = \frac{1}{x}$$

The exponential function

Definition ONE

exp a read "exponential a" is the unique number whose natural log is a

$$\exp a = e^a$$

The soln of $\ln b = 2$ is called exponential 2

$\ln b = \exp 2$ if $\ln b = 2$

OR $\exp a = b \equiv \ln b = a$

$\exp 2 = 7.3891$

$e = \exp 1 \simeq 2.7183$ $\ln e = 1$

e is Transcendental = irrational No:

(a) Not a Root
b) Not a fraction where $\frac{m}{N}$ are integers
c) π is a transcendental.

IMPROPER INTEGRALS

$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$$

$$\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty} \int_a^b f(x)\,dx$$

If Limits Exist. — They ARE said to Converge

If Limits Don't Exist — — Diverge

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx \quad \text{for } a<b$$

$$\int_a^a f(x)\,dx = 0$$

Method for Evaluating Definite Integrals

(box)

$$\int x^n dx = \frac{x^{n+1}}{n+1} \qquad n \neq -1$$

$$\int x^{-1} dx = \int \frac{dx}{x} = \ln x + C$$

$$\int_a^b f(x) dx = F(b) - F(a)$$

Since from Mean Value $\frac{f(b) - f(a)}{b-a} = f'(c)$

Properties of Indefinite Integrals

1. $\int F'(x)\, dx = F(x) + C$
2. $\int A f(x) dx = A \int f(x) dx$ for any constant A
3. $\int [f(x) + g(x)]\, dx = \int f(x) dx + \int g(x) dx$
4. $\int [f(x) - g(x)] dx = \int f(x) dx - \int g(x) dx$
5. $\int x^a dx = \frac{1}{a+1} x^{a+1} + C$ for $A \neq -1$
6. $\int \frac{1}{x} dx = \ln x + C$

Indefinite Integrals

$$\int f(x)\, dx = F(x) + C$$

$$F'(x) = f(x)$$

Properties of Integrals

1. $\int_a^b c\,dx = c(b-a)$ for any constant C (Rectangle)

2. $\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$ for any constant C

3. $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$

Definitions

Limits of Integration

dummy variable

$$\int_0^x f(t)\,dt$$

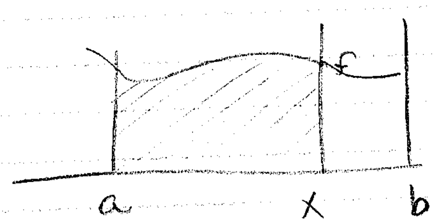
variables of Integration

Fundamental Theorem of Calculus (P.17

If $f(x)$ is continuous on $[a,b]$, then

$\phi(x) = \int_a^x f(t)\,dt$ has a derivative at each point x_0 in (a,b)

$$\phi'(x_0) = f(x_0)$$



Example 1. $\boxed{\text{Page 179.}}$

$f(x) = x^2$ $[a,b] = [0,1]$, then for each x, $[0,1]$:

$$\phi(x) = \int_0^x f(t)\,dt = \int_0^x t^2\,dt = \tfrac{1}{3}x^3$$

NOTE $\phi'(x) = f(x)$

?VVV $\phi(x) = F(b) - F(a)$

AND According to Fundamental theorem of calculus.

$$\frac{d}{dx}\int_1^x (t^3 - \tfrac{1}{4}t^2)\,dt = x^3 - \tfrac{1}{4}x^2$$

Absolute Values

$$|x-1| = \begin{cases} (x-1) & \text{if } x > 0 \\ -(x-1) = (-x+1) & \text{if } x < 0 \end{cases}$$

NOTE: From page 16 –
Evaluate

$|x-2| < 1$ $\qquad -1 < (x-2) < 1$

$\qquad\qquad 1 < x < 2$

Page 195

Piecewise Continuous functions and Absolute Values

(Integrating functions for which No $F(x)$ exists over entire Range of Integration).

$$\int_{-1}^{3} |x-2|\,dx \qquad |x-2| = \begin{cases} x-2 & \text{for } x \geq 2 \\ -(x-2) & \text{for } x \leq 2 \end{cases}$$

Then

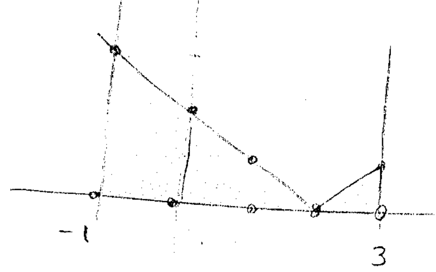
$$\int_{-1}^{3} |x-2|\,dx = \int_{-1}^{2} |x-2|\,dx + \int_{2}^{3} |x-2|\,dx$$

$$= \int_{-1}^{2} -(x-2)\,dx + \int_{2}^{3} (x-2)\,dx$$

Graph OVER

$$|x-2| = \begin{cases} x-2 & \text{if } x > 2 \quad \left(\frac{du}{dx} = 1 \text{ thus } f(x) \text{ is increasing}\right) \\ -(x-2) & \text{if } x < 2 = 2-x \quad \left(\frac{du}{dx} = -1 \text{ thus } f(x) \text{ is decreasing}\right) \end{cases}$$

x	f(x)
-1	3
0	2
1	1
2	0
3	1

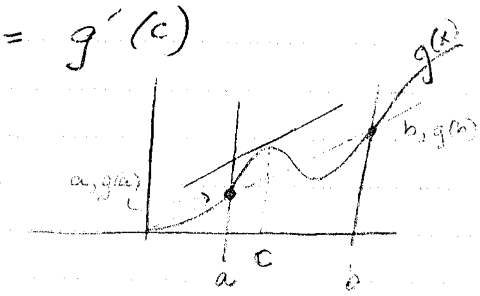


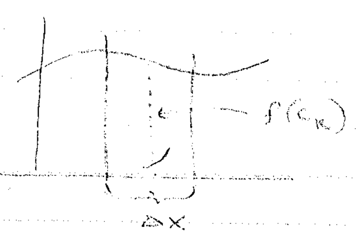
$|x-2| =$

$$\boxed{\int_{-1}^{3} f(x)\,dx = \int_{-1}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx}$$

Fundamental Theorem of Integral Calculus
Proof

Mean Value $\quad \dfrac{g(b) - g(a)}{b-a} = g'(c)$

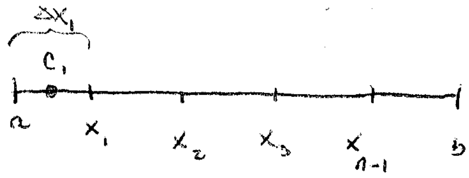




$m = \frac{dy}{dx}$

Thus $\underbrace{g(b) - g(a)}_{dy} = \underbrace{g'(c)}_{m}\ \underbrace{(b-a)}_{dx}$

$$F(b) - F(a) = \int_a^b f(x)\,dx$$



$F(x_1) - F(a) = F'(c_1)(x-a) = f(c)\,\Delta x_i$

From mean Value

See notes for Cancellation

$F(x_2) - F(x_1) = F'(c_2)(x_2 - x_1) = f(c_2)\,\Delta x$

$F(b) - F(x_{n-1}) = F'(c_n)(b - x_{n-1}) = f(c_n)\,\Delta x_n$

$F(b) - F(a) = f(c_n)\,\Delta x$

$= f(c_1)\Delta x + f(c_2)\Delta x \ldots + f(c_n)\Delta x_n.$

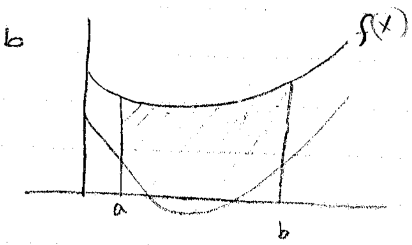
$\lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\,\Delta x_k = \int_a^b f(x)\,dx$

$$\boxed{F(b) - F(a) = \int_a^b f(x)\,dx}$$

AREA between Two Curves

$A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

if $f(x) \geq g(x)$ for $a \leq x \leq b$



Substitution IN Definite Integrals

(Proof - over)

Theorem 2

$u = g(x)$

$$\int_a^b f[g(x)]\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du = F(u)\Big|_{g(a)}^{g(b)} =$$

Proof

$$\int f(u)\,du = F(u) + C$$

$$\int_{g(a)}^{g(b)} f(u)\,du = F(u)\Big|_{g(a)}^{g(b)} = F[g(b)] - F[g(a)]$$

$$\int_a^b f[g(x)]\,g'(x)\,dx = F[g(x)]\Big|_a^b = F[g(b)] - F[g(a)]$$

INVERSE Functions

$y = \ln x$ is the inverse of $x = \exp y$

A function $f(x)$ has an inverse when
$y = f(x)$ has a unique solution x for each y

The Symbol $f^{-1}(y)$ is used for inverse of f

$$[y = f(x)] = [x = f^{-1}(y)]$$

$$\lim_{\theta \to 0} \frac{\theta}{\sin\theta} = 1$$

Thuy Ngo
Thuy Ngo

INTEGRATION By PARTES p. 231

$\int u\,dv = uv - \int v\,du$

INDEFINITE INTEGRAL

page 185

FIND A function $f(x)$ that Satisfies

$f'(x) = 5 + 3x - 6x^2$

$f(1) = -2$

$f(x) = \int (5 + 3x - 6x^2)\,dx = 5x + \frac{3}{2}x^3 + \cancel{2}x^3 + C$

Solve for C

$f(1) = -2 \qquad 5 + \frac{3}{2} + (-2x^3) - \frac{13}{2} = f(1)$

$C = -\frac{13}{2} \qquad f(x) = 5x + \frac{3}{2}x^2 - 2x^3 - \frac{13}{2}$

Theorem 2

P. 177

If $F(x)$ and $G(x)$ are both antiderivatives of $f(x)$ on interval (a, b), then there is a constant C, such that

$G(x) = F(x) + C$

Double Angle formulas p.71 Flanders

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\cos 2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

Definition ANTIDerivitive
1. $f(x)$ is defined on Interval I

Then $F(x)$ is the ANTIDERIVITIVE if

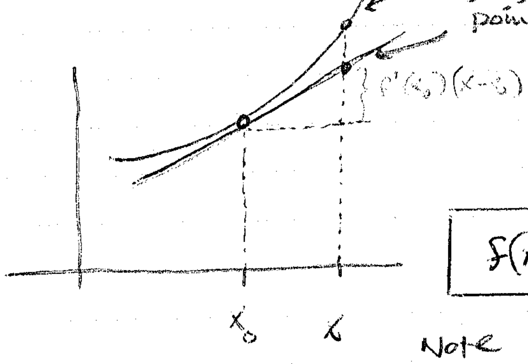
$F'(x) = f(x)$ for all points I

If $f(x) = \dfrac{1}{x^2}$ the $F(x) = \dfrac{-1}{x}$

Page 147

LINEAR Approximations

$f(x)$ is close to point on the tangent Line



$y - y_0 = m(x - x_0)$
$y - f(x_0) = f'(x_0)(x - x_0)$
$y = f(x_0) + f'(x_0)(x - x_0)$

$$\boxed{f(x) \approx f(x_0) + f'(x_0)(x - x_0)}$$

Note if $x_0 = 0$ $f(x) = f(0) + f'(0)x$

see pr 3

$\sqrt{0.24}$

$f(x) = x^{1/2}$ $\quad f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

Let $x_0 = 0.25$ $\quad x \approx x_0 = 0.25$

$f(x) = f(x_0) + f'(x_0)(x - x_0)$

$f(.24) = 0.5 + (1)(.24 - .25)$

$f(.24) = (.5) + (-0.1) = 0.49$

Theorem 1 $\quad$ Substitution of Integrals (page 189)

$\int f(u)\,du = F(u) + C$ $\quad$ then

$\int f[g(x)]g'(x)\,dx = F[g(x)] + C$ $\quad$ <u>IMPORTANT</u>

proof

$\frac{d}{dx} F[g(x)] = F'[g(x)]\,g'(x) = f(g(x))\,g'(x)$

Corollary 1

If $\int f(u)\,du = F(u) + C$

then

$$\int f(x-a)\,dx = F(x-a) + C$$

and

$$\int f(bx)\,dx = \frac{1}{b} F(bx) + C$$

because if $u = bx$

$\frac{du}{dx} = b \qquad du = b\,dx$

$\frac{du}{b} = dx$

where $u=(bx)$, $\int f(bx)\,dx = \int f(u)\,\frac{du}{b}$

The Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x_k = F(b) - F(a)$$

where $F'(x) = f(x)$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Identities P. 65 Flanders + Price Trig.

$\sin = O/h$	$\csc = h/O$	$\sin^2\theta + \cos^2\theta = 1$
$\cos = A/h$	$\sec = h/a$	$1 + \tan^2\theta = \sec^2\theta$
$\tan = O/A$	$\cot \; a/O$	$1 + \cot^2\theta = \csc^2\theta$

NUMERICAL INTEGRATION

1. Approx. by Rectangles

$$A = \Delta x\,(y_1 + y_2 + y_3 \cdots + y_n)$$

2. Trapezoidal Rule

$$A = \Delta x\left(\frac{y_0}{2} + y_1 + y_2 + \cdots + y_{n-1} + \frac{y_n}{2}\right)$$

3. Simpson's Rule (Fitting To A parabola) $h = \Delta x$

$$A = \frac{h}{3}\,(y_0 + 4y_1 + y_2)$$

(Note n must be even, $\Delta x = \frac{b-a}{n}$)

Numerical INTEGRATION
APPROXIMATION WITH RECTANGLES

$\Delta x = \dfrac{b-a}{n}$ $\qquad x_k = a + k\Delta x \qquad f(x_k) = y_k$

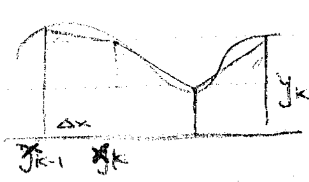
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x$$

$$\approx \sum_{k=1}^{n} f(x_k)\,\Delta x$$

$$\approx \Delta x\,(y_1 + y_2 + \cdots y_n)$$

NUMERICAL INTEGRATION – TRAPEZOIDAL RULE

Area of Trapezoid = (base)(ave. height) $= b\left[\dfrac{h_1 + h_2}{2}\right]$



$y_k = f(x)_k$ Area = (Σ trapezoids)

$\Delta x = \dfrac{b-a}{n} \qquad f(x_k) = y_k$

$$\text{Area} \approx \cancel{\sum_{k=1}^{n} f(x_k)\,\Delta x} \approx \sum_{k=1}^{n} y_k\,\Delta x \approx \Delta x\,($$

$$\text{AREA} \approx \sum \frac{f x_{k-1} + f(x_k)}{2} \Delta x = \sum \left(\frac{y_{k-1} + y_k}{2} \right) \Delta x$$

$$\approx \Delta x \left(\frac{y_0 + y_1}{2} + \frac{y_1 + y_2}{2} + \frac{y_2 + y_3}{2} \cdots \frac{y_{n-1} + y_n}{2} \right)$$

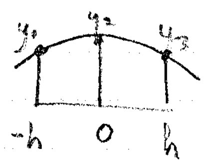
$$\approx \Delta x \left(\frac{y_0}{2} + y_1 + y_2 + y_3 \cdots \frac{y_n}{2} \right)$$

Numerical Integration

n must be even

Simpson's Rule

$\Delta x = \frac{b-a}{n}$



$y = Ax^2 + bx + c$

$$A = \int_{-h}^{h} ax^2 + bx + c \, dx = \left[\frac{ax^3}{3} + \frac{bx^2}{2} + Cx \right]_{-h}^{h}$$

$$A = \left(\frac{ah^3}{3} + \cancel{\frac{bx^2}{2}} + Ch \right) - \left(-\frac{ah^3}{3} + \cancel{\frac{bh^2}{2}} - Ch \right)$$

$$= \frac{2ah^3}{3} + 2Ch = \frac{h}{3}\left(2ah^2 + 6C \right)$$

$y_0 = ah^2 - bh + c$

$y_1 = c$

$y_2 = ah^2 + bh + C$

$y_0 + y_2 = 2Ah^2 + 2C$

$+ 4y_1 = 4C$

$y_0 + 4y_1 + y_2 = 2Ah^2 + 6C$

$$\therefore \Rightarrow A = \frac{h}{3}\left[y_0 + 4y_1 + y_2 \right] \equiv \frac{h}{3}\left(2ah^2 + 6C \right)$$

RATIONAL functions GRAPHS & Continuity

Find VERTICAL + HORIZONTAL Asymptotes $\frac{P(x)}{Q(x)}$

1. Find Critical Values of the denominator these are the VERTICAL Asymptotes as Long As they aren't the Critical Values of the Numerator

$\frac{x^2+1}{(x+1)(x-3)}$ $\quad q(-1)=0 \;+\; q(3)=0 \;=$ VERTICAL Asy.

TAKE Limit of $f(x)$ as $x \to \infty$ Horizontal Asymptote

$f(x) = \frac{x^2+1}{x^2-2x-3} = \frac{1+1/x^2}{1-\frac{2}{x}-\frac{3}{x^2}}$ $\lim_{x \to \infty} f(x) = 1$

$y = 1$ is Horizontal asymptote

The Chain Rule

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$\frac{d(\ln u)}{dx}$ where $u = f x$

$f(x) = u$ $f(u) = y$ $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Find $\frac{dy}{dx}$ when $y = \ln(x^2+2x+3)$

$= \frac{dy}{dx} \int_1^x \frac{1}{u}\, du = \frac{1}{u}$

$u = x^2+2x+3$ $\frac{du}{dx} = 2x+2$ $\frac{dy}{dx} \ln u = \frac{1}{u}$

Parallel Lines

$y = mx + b_1$ $\qquad$ $y = mx + b_2$

with same slope m

$$mx_0 + b_2 = mx_0 + b_1 + (b_2 - b_1)$$

Thus, for any given point x_0; line $mx_0 + b_1$ is $b_2 - b_1$ units from $mx_0 + b_2$

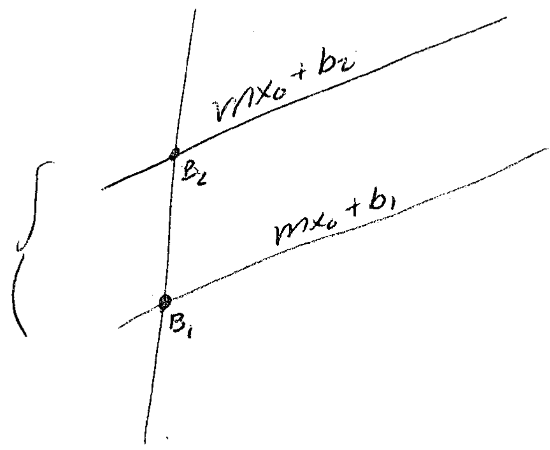
(over)

$mx_0 + b_2 = mx_0 + b_1$

$mx_0 + b_1 - b_2 = mx_0 + b_1 - b_1 \longrightarrow mx_0 + b_2 = mx_0 + b_1 + (b_2 - b_1)$

Distance

$B_2 - B_1$



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Question. See problems on bounds,
Statement: when degree of Numerator is less than the numerator divide and will find the horizontal asymptote.

1

The function:

a collection of ordered pairs such that -
no two distinct ordered pairs have the same first element

See Text 121
Class Notes

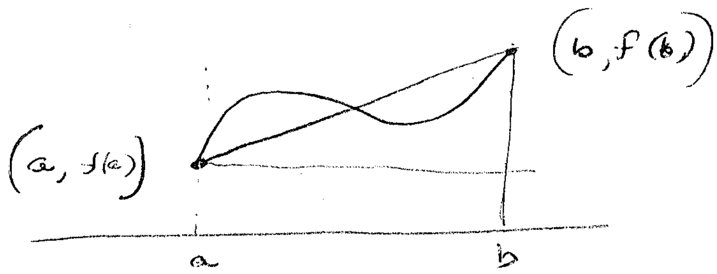
Mean Value Theorem

(i) $f(x)$ is continuous on closed interval $[a, b]$.

(ii) $f(x)$ is differentiable on (a, b)

then there is a point c, $a < c < b$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



$L(x)$ = the line $(f(a), f(b))$ $\quad L'(x) = \frac{f(b)-f(a)}{b-a}$

$g(x) = f(x) - L(x)$

Derivatives of Absolute Values

$|x^3|-1$ $\qquad f(x) = \begin{cases} x^3-1 & \text{if } x>0 \\ -x^3-1 & \text{if } x<0 \end{cases}$

Max. + minima.

If $f''(x_0) > 0$, curve $y = f(x_0)$ is Above Tan
at $[x_0, f(x_0)]$ for x in interval $(x-\delta, x+\delta)$.

If $f''(x_0) < 0$, curve $y = f(x_0)$ is Below Tange

OVER

$g'(x) = 0$ INDICATES CRITICAL VALUES

$g''(x)$ INDICATES IF TANGENT IS ABOVE OR below curve $y = g(x)$

$g''(x) > 0$, CURVE ABOVE TANGENT and CONCAVE UP.

$g''(x) < 0$ Curve Below Tangent and CONCAVE DOWN.

INFLECTION POINT – Curve changes direction of Concavity.

TESTS for Inflection points: $f'(x)$ has inflection point at x_0 for $\delta > \phi$, if

$f''(x) < 0$ Curve below tan
$f''(x) = 0$ Curve crosses TAN
$f''(x) > 0$ Curve Above tang.

see p. 101

$f(x)$ has inflection point AT x_0 if $f''(x)$ changes sign as x changes from below x_0 to Above x_0.

Test for LOCAL MAX. & min.

1. If $f'(x_0) = 0$ and $f''(x_0) > 0$ then $f(x)$ has local <u>min.</u> at x_0. (AND Tangent $[x_0, f(x_0)]$ is horizontal.

2. <u>Max</u> if $f'(x_0) = 0$ + $f''(x_0) < 0$ then $f(x)$ has local max.

<u>MAX</u> —	$f'(x_0) = 0$	$f''(x_0) < 0$
<u>MIN</u> —	$f'(x_0) = 0$	$f''(x_0) > 0$

Curve Lies Above The tangent.

Tests for Local MAX and min (p 99)

If $f''(x) > 0$ in an interval, then curve $y = f(x)$ Lies above its tangents.

If $f'(x_0) = 0$ then tangent $(x_0, f(x_0))$ is the horizontal line $y = f(x_0)$

TEST FOR LOCAL MINIMUM

If $f'(x_0) = 0$ i.e. the critical values $= x_0$

$f''(x_0) > 0$, then $f(x)$ has Local min. at x_0

TEST for Local MAX

$f''(x_0) < 0$ then

Graphs + Continuity Imp.

Solution of A parabola - a Second method.

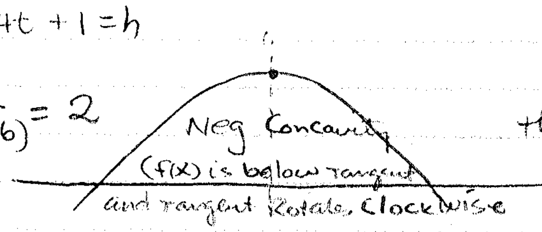
See problem 14, page 98.

USE the Solution for the Axis of Symetry about the Quadratic $ax^2 + bx + c$

Axis of Symmetry $= \dfrac{-b}{2A} = x$

$-16t^2 + 64t + 1 = h$

$h = \dfrac{-64}{(2)(-16)} = 2$



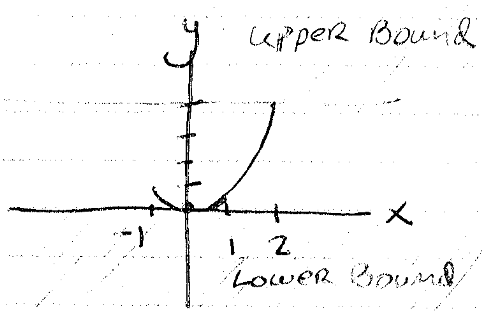
If the Coefficient of the FIRST TERM is:
+ then Concavity is up
— the Concavity is DOWN

Bounds P. 116

$f(x) = x^2$ on Interval $[-1, 2]$ is bounded

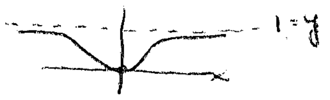
As x varies from -1 to 2, f(x) decreases from 1 to 0 then increases to 4.

$0 \le f(x) \le 4$



Find bounds for $f(x) = \frac{x^2}{x^2+1}$

1. graph



2. $0 \le x^2 < x^2+1$ $\qquad 0 \le \frac{x^2}{x^2+1} < 1$

Graphs and Continuity Imp.

1. When $f(x)$ involves an Absolute Value.

a. Remove Abolute — express ~~as~~ terms of x

eg. $|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$

b. Take the derivative for each case

2. then Solve problem

Rolle's Theorem Test

1. Is $f(x)$ Continuous
   a) $f(A)$ exists
   b) $\lim_{x \to a} f(x)$ exist
   c) $\lim_{x \to a} f(x) = f(a)$

2. Does $f'(x)$ exist i.e. is there a no. for x such that $f'(x)$ fails to exist?

3. $f(a) = f(b) = 0$
4. Then $f'(c) = 0$ (slope at c on $f(x) = 0$ & is ∥ to a, b)

then there exists at Least ONE NO. C $a < c < b$ such that $f'(c) = 0$

i.e. there is at least one tangent line ∥ TO the x axis (with slope $= \phi$)

Note if $f(x)$ is continuous on closed interval $[a, b]$ and is differentiable on open interval (a, b) but $f(a) \neq f(b) \neq 0$ then there exists a point c such that $a < c < b$ and $f'(c) = \frac{f(b) - f(a)}{b - a}$

AND $f'(x)$ exists! i.e. is differentiable on (a, b)

Graphs and Continuity Sec 4.4

TO Find MAX + Min Values

1. FIND $\frac{Dy}{dx}$

2. Set $\frac{dy}{dx} = 0$ and solve for x to find Critical Values.

3. make Chart to show sign changes in $f'(x)$ This reveals max + min, and where $f(x)$ is increasing and decreasing.

(OVER)

Possible Candidates for MAX + min. are Critical values which occur:

1. $f'(x) = 0$
2. $f'(x)$ does not exist $= \left(f'(x) = \frac{u}{0} \right)$ see Rational funct. VERT [illegible]
3. At the END Points of A Closed Domain

<u>Note</u>: the Cusp

tangent becomes vertical at cusp. Becoming a max value where x is undefined. (see #2)

Continuity – See Text p. 115 + notes

(posses a max. + min. on a closed interval)

Def. $f(x)$ is continuous at x_0 if it is defined on an interval $(x_0 - \delta,\ x_0 + \delta)$

AND $\lim_{x \to x_0} f(x) = f(x_0)$

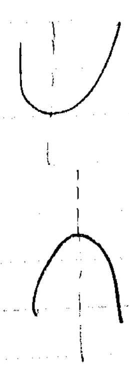
But a function may be continuous at a point but Not Defined there, – i.e. At A Cusp.

Quadratic Soln

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2A}$$

$f(x) = ax^2 + bx + c$

$f(x) = -ax^2 + bx + c$



Axis of Symmetry $= \frac{-b}{2A}$

Quiz One P. 125

Give AN EXAmple of A function that is defined on the closed interval $[-1, 1]$ and discontinuous exactly At the points -1 and 1

$$f(x) = \begin{cases} 1 & \text{if } -1 < x < 1 \\ 0 & \text{if } x = -1,\ x = 1 \end{cases}$$

Maxima + Minima

Absolute Maxima or Minima = The greatest max or minima.

Local Maxima or Min.

DISTANCE FORMULA

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Addition formulas for Sine + Cosine

1) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

2) $\cos(A+B)\ \cos A \cos B - \sin A \sin B$

3) $\sin(A+B) = \sin A \cos B + \sin B \cos A$

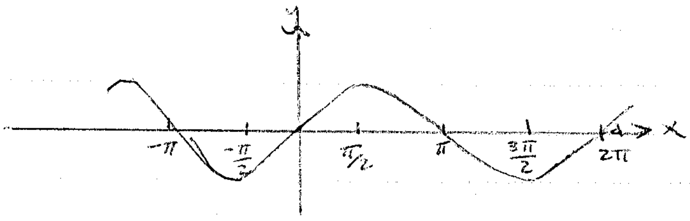
4) $\sin(A-B) = \sin A \cos B - \sin B \cos A$

OVER

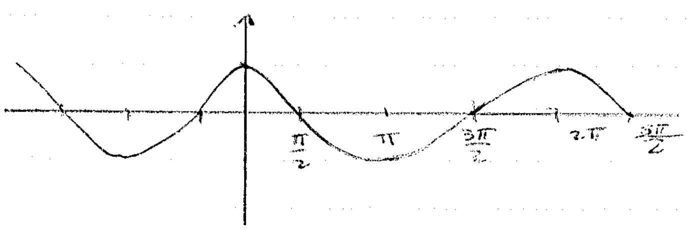
$$\tan(a+B) = \frac{\tan a + \tan B}{1 - \tan a \tan B}$$

<u>Memorize</u> Graphs p. 289-305.

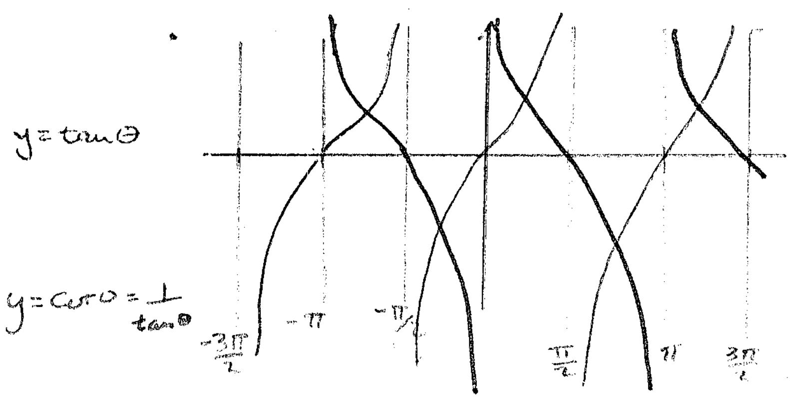
Sinθ

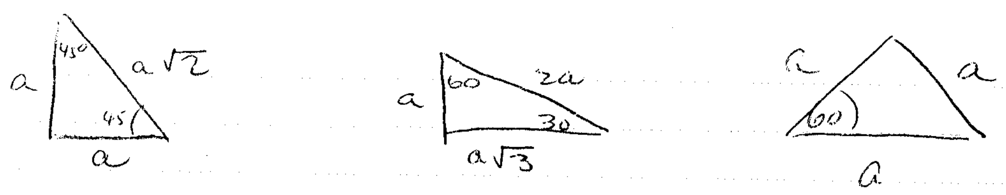


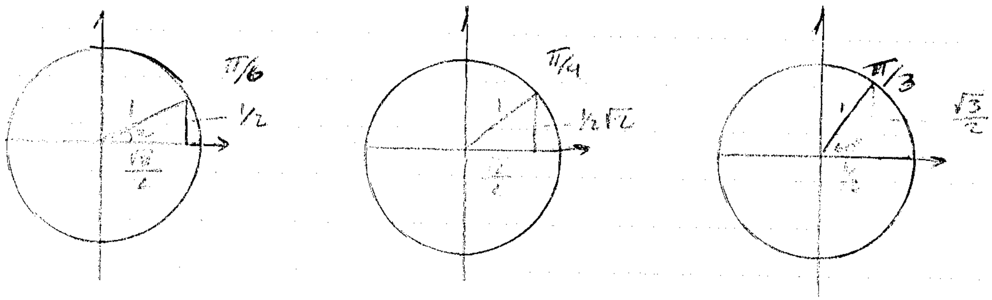
$y = \cos\theta$



(a la vuelta)


y = tan θ
y = cot θ = 1/tan θ
-3π/2
-π
-π/2
π/2
π
3π/2





$$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

Memorize

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	-1	0
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0	-1

INDEFINITE INTEGRALS of TRIG functions
page 326.

$\int \sin x \, dx = -\cos x + C$

$\int \cos x \, dx = \sin x + C$

$\int \sec^2 x \, dy = \tan x + C$

$\int \csc^2 x \, dx = \cot x + C$

$\cancel{\int \csc x \tan x} =$

over

$\int \sec x \tan x \, dx = \sec x + C$

$\int \csc x \cot x = -\csc x + C$

$\int \tan x \, dx = -\ln|\cos x| + C$

$\int \cot x \, dx = \ln|\sin x| + C$

INDEFINITE INTEGRALS of Trig. functions

1. $\int \sin x \, dx = -\cos x + C$

$\int \cos x \, dx = \sin x + C$

$\int \sec^2 x \, dx = \tan x + C$

$\int \csc^2 x \, dx = -\cot x + C$

$\int \sec x \tan x \, dx = \sec x + C$

$\int \csc x \cot x \, dx = -\csc x + C$

over

$\int \tan x \, dx = -\ln|\cos x| + C$

$\int \cot \, dx = \ln|\sin x| + C$

Properties of Sin & Cos function

1. $-1 \le \sin\theta \le 1$
2. $-1 \le \cos\theta \le 1$
3. $\sin^2\theta + \cos^2\theta = 1$
4. $\sin(-\theta) = -\sin(\theta)$
5. $\cos(-\theta) = \cos\theta$
6. $\sin(\theta \pm 2\pi) = \sin\theta$
7. $\cos(\theta \pm 2\pi) = \cos\theta$

over

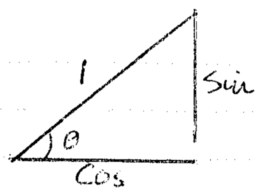
IDentities

$\sin^2\theta + \cos^2\theta = 1$

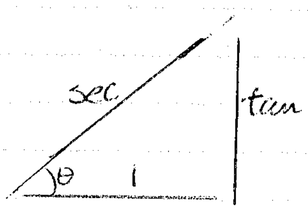
$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

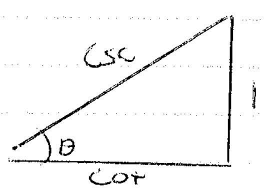
IDENTITIES Memorize



θ measured in Radians
Radius of Unit Circle = 1



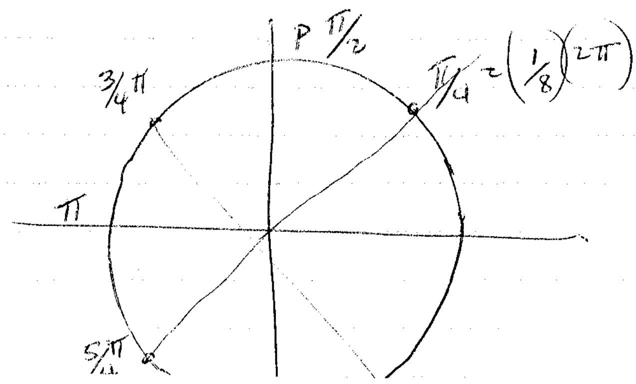
Not defined for $\theta = (k + \frac{1}{2})\pi$



Not defined for $\theta = k\pi$

Memorize Page 288

θ	$\sin\theta$	$\cos\theta$
0	0	1
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/2$	1	0
$3/4\,\pi$	$\sqrt{2}/2$	$-\sqrt{2}/2$
π	0	-1
$5\pi/4$	$-\sqrt{2}/2$	$-\sqrt{2}/2$

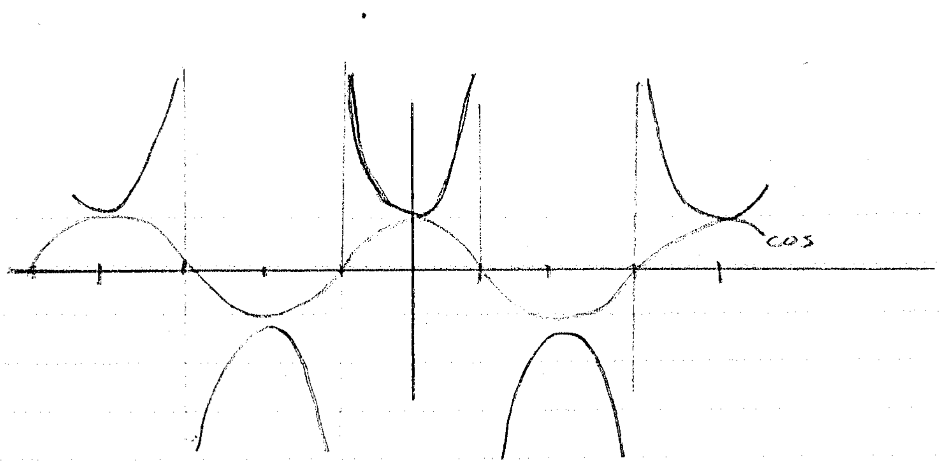


Derivitives of INVERSE TRIG. functio

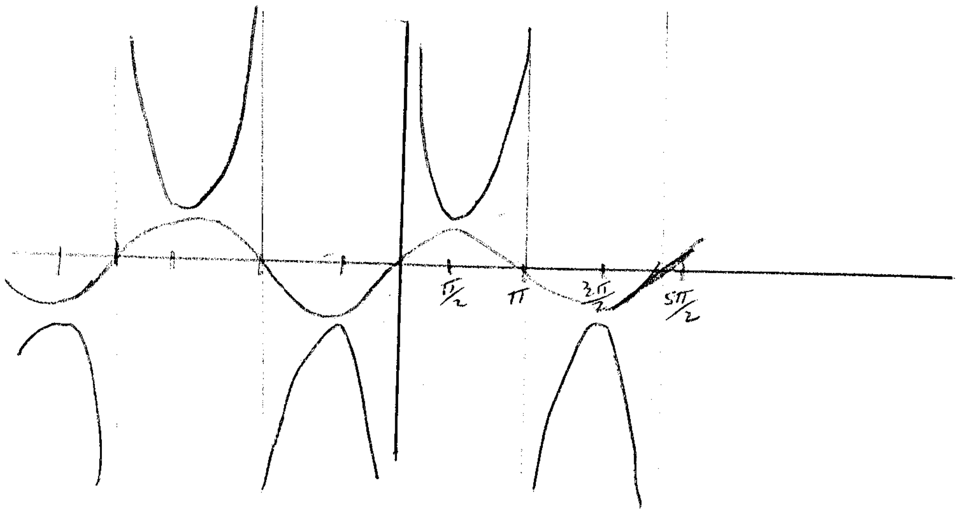
$$\frac{d}{dx}\left(\sin^{-1} u\right) = \frac{1}{\sqrt{1-u^2}}\,\frac{du}{dx}$$

$$\frac{d}{dx}\left(\cos^{-1} u\right) = \frac{-1}{\sqrt{1-u^2}}\,\frac{du}{dx}$$

$$\frac{d}{dx}\left(\tan^{-1} u\right) = \frac{1}{1+u^2}\,\frac{du}{dx}$$



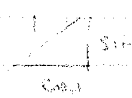
$$y = \sec x = \frac{1}{\cos x}$$



$y = \csc x = \frac{1}{\sin x}$

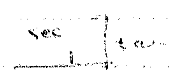
TRIGONOMETRIC Differentiation

$\frac{d}{dy}(\tan u) = \sec^2 u \frac{du}{dy}$

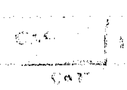


$\frac{d}{dy}(\cot u) = -\csc^2 u \frac{du}{dy}$

$\frac{d}{dy}(\sec u) = \sec u \tan u \frac{du}{dy}$



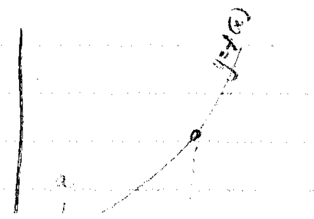
$\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dy}$



Newton's Method (page 143)

$m = \frac{(y - \Delta y)}{(x - \Delta x)} = \frac{[y - f(x_0)]}{[x - x_0]}$ $\qquad y - f(x_0) = f'(x_0)[x - x_0]$

$m = f'(x_0)$



AND (c.f. p. 144) FINDING Square Roots

Find Square Root of $b>0$

$x^2-b=0$ $\qquad x_1 = x_0 - \dfrac{x_0^2-b}{2x_0}$ The No. you need the Root of

x_0 = the estimate

simplifies to

$$x = \tfrac{1}{2}\left(x_0 + \frac{b}{x_0}\right)$$

INVERSE TRIGONOMETRIC functions p. 313

1. $x = \sin\theta$ same as $\theta = \sin^{-1} x$

2. $\dfrac{d\theta}{dy} = \dfrac{1}{\frac{dy}{d\theta}}$ $\qquad \dfrac{dx}{dy} = \dfrac{1}{\frac{dy}{dx}}$

3. $\dfrac{d}{dx}\left(\sin^{-1} u\right) = \dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$

MAX OR MINIMA (Problem 17, page 105)

Example: Find local max and min for

$f'(x) = (x-1)^2 (x+1)^2$

Since $f'(x)$ is never < 0, there is no max or min.

Slope-Intercept

6.

$y = mx + b$

(when $x = 0$, $y = b$)

Limits Definition 1

If the function f approaches a single number L as x approaches x_0, we write

$\lim f(x) = L$

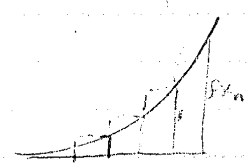
Definition

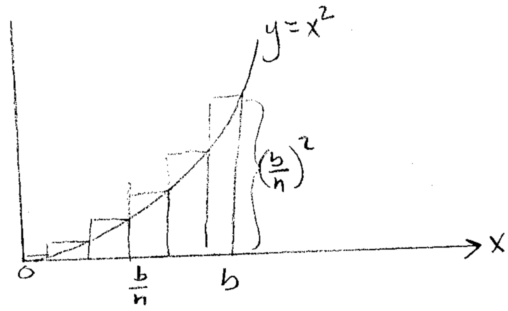
The integral $\int_a^b f(x)\,dx$ **imp.**

is the limit of the sum

$$S_n = f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$$

$$\Delta x = \frac{b-a}{n}$$





Approximating area under $y=x^2$

$$S_n = \frac{b}{n}\left(\frac{b}{n}\right)^2 + \frac{b}{n}\left(2\frac{b}{n}\right)^2 \cdots + \frac{b}{n}\left(n\frac{b}{n}\right)^2.$$

= Sum of the areas of rectangles with base $\frac{b}{n}$ and height $\left[\frac{b}{n}\right]^2$, $\left[2\frac{b}{n}\right]^2$ etc.

$$\Delta x = \frac{b-a}{n}$$

The Quadratic Equation

$$f(x) = ax^2 + bx + c$$

Always a parabola

 positive a

 negative a

Slope of Line

$y = mx + b$ $\qquad$ $m = \dfrac{\Delta y}{\Delta x} = \dfrac{\text{increment } y}{\text{increment } x}$

Implicit Function (page 84)

Eg.

① $xy + 2 = y$ $\quad xy - y = -2 \quad y(x-1) = -2$

and $y = \dfrac{-2}{x-1}$ and $= \dfrac{2}{1-x}$

② $x^2 + y^2 = 1 \qquad y = \sqrt{1-x^2}$ or $-\sqrt{1-x^2}$

(See page 84 for implicit differentiation)

Growth and Decay page 246

Rate of growth ($P'(t)$) is proportional to Population Size $[P(t)]$

$P'(t) = \alpha P(t)$ $\qquad \alpha$ = Constant of Proportionality

Size of Population at $t=0$ $\quad P(0) = Ae^0 = A$ pop. size

$P(t) = Ae^{\alpha t}$

$= P(0)e^{\alpha t}$

$$P(t) = P(0)\, e^{\alpha t}$$

Proof: $\frac{f'(t)}{f(t)} = \frac{\text{Rate of growth}}{\text{Population Size}} = \alpha$ = Constant of Proportionality

1) $\frac{d}{dt}[\ln f(t)] = \frac{1}{f(t)} f'(t) \Rightarrow = \alpha$ (FACT)

2) $\ln f(t) = \int \alpha \, dt = \alpha t + C$ (Then follows)

by exponential differentiation

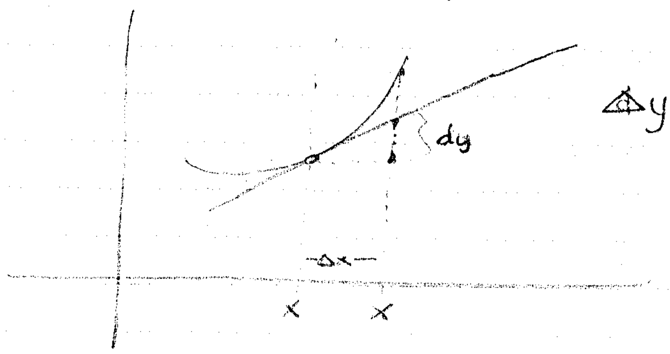
3) $e^{\ln f(t)} = f(t) = e^{\alpha t + C} = e^{\alpha t} e^{C}$)

Since $f(0) = e^{C}$

4) $f(t) = f(0) e^{\alpha t}$

Differentials

P. 150.



$dx = \Delta x$

Def $dy = f'(x)\,dx$

$\Delta y = f(x_0 + \Delta x) - f(x_0) \approx f'(x_0)\Delta x = dy$

$f'(x) = \frac{\Delta y}{\Delta x}$ $f'(x)\Delta x \approx \Delta y$ $dy = f'(x)\Delta x$

$\Delta y \approx dy$

Definition Two

the nth derivitive of $f(x)$ is the Result of N Successive differentations of f

$\frac{d^n y}{d x^n}$, $f^n(x) = y^n$

Sec. 3.5

Definition:

the derivitive of $\frac{dy}{dx} = f'(x)$ is the second derivitive of $y = f(x)$

designated — $\frac{d^2 y}{d x^2} = f''(x)$

d-two-y of d x-square = f double prime of x

Problem 8 page 69 Derivative,

use Product Rule For derivative, to find

$\frac{d}{dx}(uvw)$

3

FINITE DISCONTINUITY

$$g(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{for } x \neq 1 \\ 2 & \text{for } x = 1 \end{cases}$$

OVER

Let $A = vw$

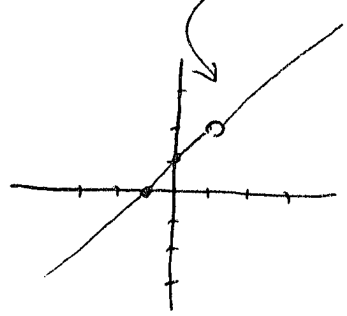
then $\frac{d}{dx} uA = u\frac{dA}{dx} + A\frac{du}{dx}$

$= u\frac{d\,vw}{dx} + vw\frac{du}{dx}$

$= uv\frac{dw}{dx} + uw\frac{dv}{dx} + vw\frac{du}{dx}$

$\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$ the Limit

To fill in the Hole



(when $x=1$, in above formula, there is no def.)

(Decomposition of Fractions)

PARTIAL FRACTIONS: a proper fraction

(degree of Numerator less than degree of denominator)

may be WRITTEN as the Sum of Partial fractions

① If Linear factor $(ax+b)$ occurs once in denominator then;

$$\frac{x+4}{(x+7)(2x-1)} = \frac{A}{x+7} + \frac{B}{2x-1}$$

(2) Linear factors which are Repeated

$$\frac{3x-1}{(x+4)^3} = \frac{A}{(x+4)} + \frac{B}{(x+4)^2} + \frac{C}{(x+4)^3}$$

Solution

1. $$\frac{2x^2+10x-3}{(x+1)(x^2-9)} = \frac{A}{x+1} + \frac{B}{x+3} + \frac{C}{x-3}$$

2. $2x^2+10x-3 = A(x^2-9) + B(x+1)(x-3) + C(x+1)(x+3)$

TO FIND A, Let $x=-1$ $\quad 2-10-3 = A(1-9) + (0) + (0) = 11/8$

TO FIND B, Let $x=-3$ $\quad 18-30-3 = B(-3+1)(-3-3) = -5/4$

To Find C, Let $x=3$; $18+30-3 =$

$(0)+(0)+C(3+1)(3+3) \qquad C = 15/8$

Hence $$\frac{2x^2+10x-3}{(x+1)(x^2-9)} = \frac{11}{8(x+1)} - \frac{5}{4(x+3)} + \frac{15}{8(x-3)}$$

Quadratic Factors – None Repeated

If ax^2+bx+C occurs ONCE in Denominator

Then $A \neq 0$, $B \neq 0$ $\quad \dfrac{Ax+B}{ax^2+bx+C}$

$$\frac{x^2-3}{(x-2)(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+4}$$

$$\frac{2x^3-6}{x(2x^2+3x+8)(x^2+x+1)} = \frac{A}{x} + \frac{Bx+C}{2x^2+3x+8} + \frac{Dx+E}{x^2+x+1}$$

Domain 2

Set of x values — all permissable values of the independant variable.

Range - Set of y values
The dependant Variable

$f(x) = x^3$ independant variable

4

EVEN Function (Symmetrical about the y axes)

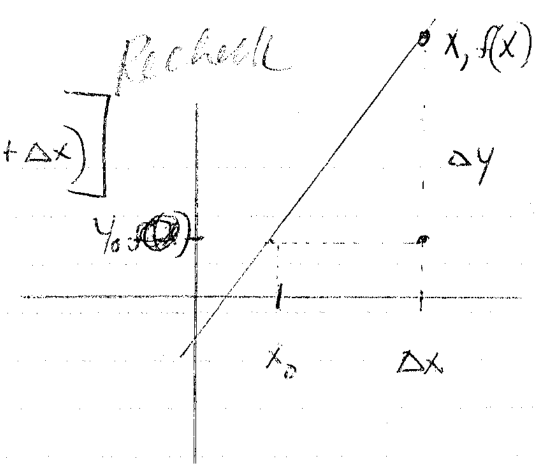
$f(x) = f(-x)$

ODD Function

$f(-x) = -f(x)$ Symmetrical about the origin

Rechell

$(x, f(x))$ $[(x_0 + \Delta x), f(x_0 + \Delta x)]$



$y = f(x) - y_0$

$= f(x_0) -$

The Chain Rule - Composition of functions

If

$y = f(u) \quad u = g(x) \quad y = f[g(x)]$

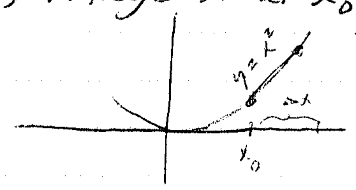
$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ whenever $\frac{dy}{du}$ and $\frac{du}{dx}$ exist

TANGENTS

DEFINITION ONE (page 41)

(m of TANGENT at x_0)

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 2x_0$



The Limit (Limiting value) of $\frac{\Delta y}{\Delta x}$ as Δx approaches 0 is $2x_0$

The derivitive of x^2 at x_0 is $2x_0$.

The No. $2x_0$ is m of Tangent to $y=x^2$ at (x_0, x_0^2)

(OVER)

TANGENTS

Definition 2 (p. 42; See card for Def #ONE)

$y - x_0^2 = 2x_0(x - x_0)$

is TANGENT to the Curve $y = x^2$ at $P(x_0, x_0^2)$

The Slope of the tangent $y = x^2$ at (x_0, x_0^2)

$= (3, 9)$ is $2 \cdot 3 = 6$

The tangent at $(3,9)$ is given by $y - 9 = 6(x-3)$

$y - y_0 = m(x - x_0)$ Point Slope

NOTE may find this by $\Delta y = f(x_0 + \Delta x) - f(x_0)$

$\Delta y = (x_0 + \Delta x)^2 - x_0^2$ [illegible]

Definition Tangents $y = f(x)$

$m = f'(x_0)$

The Tangent to $y = f(x)$ at $x_0, (f(x_0))$

is the line $y - f(x_0) = m(x - x_0)$

where slope m is given by

$$m = \lim_{\Delta x \to} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

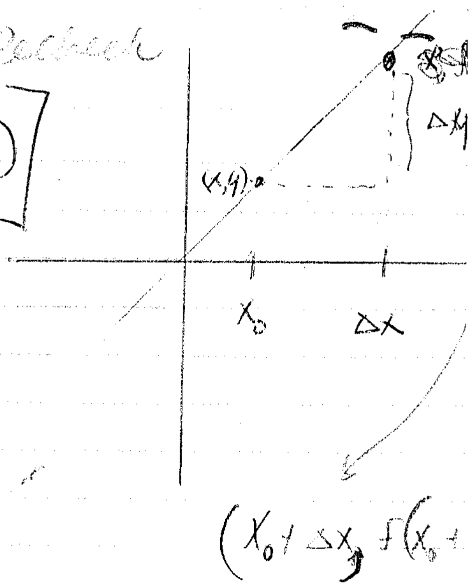
(over)

The number m is the derivitive of f at x_0

$$m = f'(x_0)$$

Definition Recheck

$$(x, f(x)) = [(x_0 + \Delta x), f(x_0 + \Delta x)]$$



$$\Delta y = f(x) - y$$

$$= -f(x) + f(x_0 + \Delta x)$$

Therefore

$$\frac{\Delta y}{\Delta x} = \frac{-f(x) + f(x_0 + \Delta x)}{\Delta x} = m$$