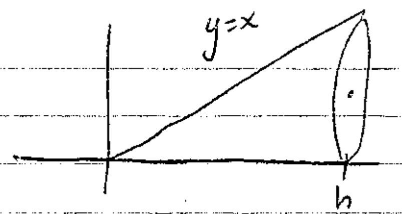
Volume by Slicing · page 264

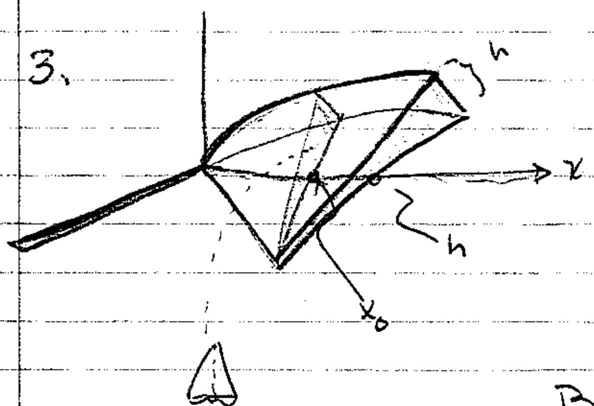


1. $A = \pi R^2 =$ $D = x$ $r = \frac{x}{2}$

$A = \left[\pi\left(\frac{x}{2}\right)^2\right]$ $V = \int A(x)\,dx$

$V = \frac{\pi}{4}\int_0^h x^2\,dx = \left[\frac{\pi}{12}x^3\right]_0^h = \boxed{\frac{\pi}{12}h^3}$

3.



$y = \sqrt{x}$

Base $= x_0$
height $= \sqrt{x}$

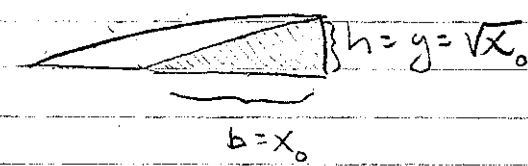
$A = \frac{bh}{2} = \frac{x_0 h}{2}$

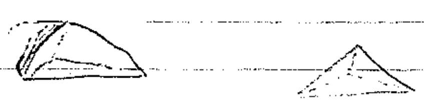
$A = \frac{x_0\sqrt{x}}{2}$ $V = \frac{1}{2}\int x_0\sqrt{x_0}\,dx = \frac{1}{2}\int x^{3/2}dx$

$= \frac{1}{2}\frac{2}{5}x^{5/2} + C = \boxed{\frac{1}{5}x^{5/2} \cancel{+C}}\Big|_0^h =$ Volume

Book Answer $= \boxed{\frac{1}{5}h^{5/2}} =$ volume

If base $= x_0$,
AND $h = y = \sqrt{x_0}$





2. A solid of Circular base or Radius R has vertical xsect. that are squares

Circle = $x^2+y^2=R^2$ $\quad r=\sqrt{x^2+y^2}$ $\quad D=2r=2(x^2+y^2)$

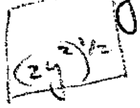
 

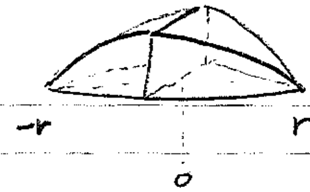
a. one square (in center) Has demension: $2\sqrt{x^2+y^2}$

$(2\sqrt{x^2+y^2})$

B. $A(x)=\left[2\sqrt{x^2+y^2}\right]^2 = 4(x^2+y^2)$

Volume By Slicing.





2 Circle = $x^2+y^2=R^2$

Since Square $y=x$ $\quad r=(x^2+y^2)^{1/2}=(2y^2)^{1/2}$

Diameter $=2r=2(2y^2)^{1/2}$

Area (x) = $\left[2(2y^2)^{1/2}\right]^2$

Vol = $\displaystyle\int_{-r}^{r}\left[2(2y^2)^{1/2}\right]^2 dy$ $\qquad \displaystyle\frac{V}{2}=\int_0^r 4(2y^2)\,dy$

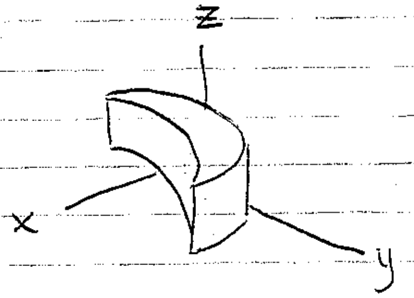
$\displaystyle\frac{V}{2}=8\int_0^r y^2\,dy=\frac{8y^3}{3}$ $\qquad \displaystyle V=\left[\frac{16y^3}{3}\right]_0^r=\boxed{\frac{16r^3}{3}}$

Problem # 7, Page 265.

7. Solid of height h is contained between two Surfaces

$x_1 = 2y^2$

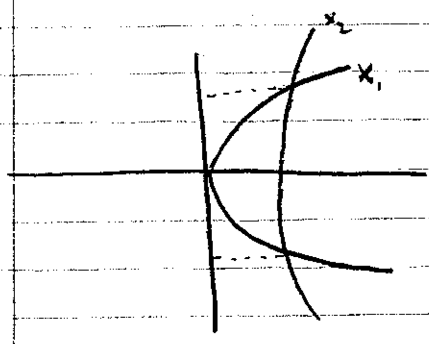
$x_2 = y^4 + 1$



A. $A = \int f(x)\,dx$

$Vol = \int A(x)\,dx$

$\hookrightarrow$ A(x) = AREA of Cross Section Perpendicular to X-Axis.



$A = \int y^4 + 1 - 2y^2 \, dx$

B. The Bounds are At points where $x_1 = x_2$

$2y^2 = y^4 + 1$

$y^4 - 2y^2 + 1 = 0$ $\quad (y^2 - 1)(y^2 - 1) = 0 \quad (y^2 - 1) = 0$

$y^2 = 1 \quad y = \pm 1$

$A = \int_{-1}^{1} y^4 + 1 - 2y^2 \, dx \qquad A = 2\int_{0}^{1} y^4 + 1 - 2y^2 \, dx$

Method of Cylindrical Shells

1. Find vol of Solid generated when the Area bounded by

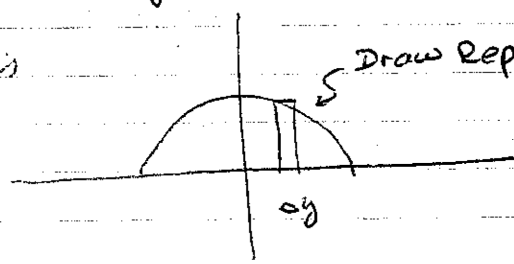
$$y = 1-x^2$$

$y=0$ is revolved About the x-Axis

A. Sketch

1. Parabola $\cap$ symmetric about x-Axis.

2. $y=0$ is x axis



B. Draw Representative Strip; vol is the sum of All such strips

$y = 1-x^2$ $\quad V = \sum \pi y^2 \Delta x = \sum \pi (1-x^2)^2 \Delta x$

$$= \sum \pi (1-2x^2+x^4)\,\Delta x$$

$$\pi \int_{-1}^{1} (1-2x^2+x^4)\,\Delta x$$

Volume By Slicing

Books answer $16(\ln 2)^2 - 16\ln 2 + 6 = 2.596$

11. Solid $h = 3$ $\quad A = h^2 \quad A =$

$y = h = \ln x \Rightarrow A = (\ln x)^2 = \ln^2 x$

$\text{Vol} = \int A(x)\,dx$

$\text{Vol} = \int_1^4 \ln^2 x\,dx$

$dv = dx \qquad v = x$

$u = (\ln x)^2 \qquad du = \frac{2\ln x}{x}\,dx \qquad \left[du = 2\ln x \left[\frac{d}{dx}\ln x\right]\right]$

$x\ln^2 x - \int_1^4 2\ln x\,dx = x\ln^2 x - 2\int \ln x\,dx$

$dv = dx \qquad v = x$

$u = \ln x \qquad du = \frac{1}{x}dx$

$x\ln^2 x - 2\left[x\ln x - \int \frac{x}{x}dx\right] = \left(x\ln^2 x - 2\left[x\ln x - x\right]\right)_0^3$

$3\ln^2 3 - 2(3\ln 3 - 3) = 3\ln^2 3 - 6\ln 3 - 3$

IMP. Interval must begin at $x = 1$ since $\ln 1 = 0$ and $h_0 = \emptyset$

$\left[x\ln^2 x - 2x\ln x + 2x\right]_1^4$

$\left[4\ln^2 4 - 8\ln 4 + 8\right] - \left[\ln^2 1 - 2\ln 1 + 2\right] = 4\ln^2 4 - 8\ln 4 + 8 - 2$

$4\ln^2 4 - 8\ln 4 + 6 \approx 2.59689 \approx 2.60$

(i) $= \lim_{b\to\infty} \int_1^b \frac{1}{x^3}\,dx$

$= \int_1^b x^{-3}\,dx = \lim_{b\to\infty}\left[-\frac{1}{2b^2} - \left(-\frac{1}{2\cdot 1^2}\right)\right]$

$= \lim_{b\to\infty}\left[-\frac{1}{2b^2} + \frac{1}{2}\right] = \frac{1}{2}$

$\int u \, du$ $\qquad u = x^2 + x$

$du = (2x+1)$

4. $\int (2x+1)(x^2+x)^{-1/2} \, dx$

$\int u^{-1/2} \, du = 2u^{1/2} =$ $\qquad 4 \qquad 2x^2 + 6x$

2. $\int \frac{(x^2+1)}{(x^3+3x^2)^2} \, dx \qquad (x^2+1)(x^3+3x^2)^{-2}$

$\frac{(x^2+1)}{(x^3+3x^2)(x^3+3x^2)} \qquad x^? + 6x^5 + 6x^?$

$x(x^2+3) \qquad x^4(x^5+6x+6)$

① Find the volume generated by revolving about the Y-axis the region bounded by $y=x^2$, $y=0$, and $x=2$.

② A colony of bacteria increases in such a way that at each instant, the rate of increase per hour is equal to twice the size of the colony at that instant. How big will the colony be at the end of 1 hour? Assume that when $t=0$ the number present is N_0.

③ Find the volume of the solid generated when the area bounded by $y=x^2-\sqrt{2x}$ and $y=0$ is revolved about the x-axis.

④ A stone dropped from a balloon which was rising at the rate of 15 feet per second reached the ground in 8 seconds. How high was the balloon when the stone was dropped? It is assumed that the acceleration acting on the stone through gravity is $a=-32$ feet/sec^2.

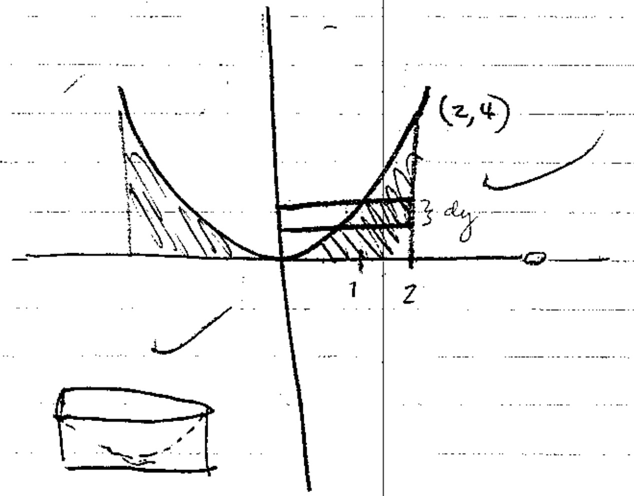
⑤ Integrate the following:

(a) $\int x e^{3x}\,dx$

(b) $\int \frac{2x+1}{\sqrt{x^2+x}}\,dx$

CLINE

QUIZ IV



1. $y_1 = x^2 \quad y_2 = 0 \quad x_2 = 2$

$y = x^2 = x = 2 \quad y = 2^2 = 4$

$x^2 = y_1$

$x_1 = \sqrt{y_1} \qquad x_2 = 2$

$$\text{Vol} = \pi \int_0^4 (2)^2\, dy - \pi \int_0^4 (\sqrt{y})^2\, dy$$

$$= 4\pi \int_0^4 dy - \pi \int_0^4 y\, dy = \Big[4\pi y\Big]_0^4 - \frac{\pi}{2}\Big[y^2\Big]_0^4$$

$$= (4 \cdot 4\pi) - \left[\frac{16}{2}\pi\right] = (16-8)\pi = \boxed{8\pi}$$

Quiz IV

2

(2)

~~$P'(t) = 2N$ what is $P(t)$ after 1 hr.~~

~~$P(t) = \int 2N_0\,dt = N_0 t = 2N_0 t + C$~~

~~$P(1) = N_0(1) = \boxed{N_0^2}$ population after one hr.~~

~~$P(0) = N_0 \quad t=0 \qquad N_0 = N_0(0) + C \Rightarrow C = N_0$~~

~~$P(t) = N_0 t + N_0 \qquad \boxed{P(1) = 2N_0(1) + N_0}$~~

$\dfrac{dN}{dt} = 2N \qquad N = \int 2N\,dt = 2Nt + C$

$t=0 \quad N = N \Rightarrow C = N$

$2Nt + N = N(t) \qquad$ at $t=1$ $\boxed{N(1) = 2N + N}$

/10

$\dfrac{dN}{dt} = 2N$

$\dfrac{dN}{N} = 2dt$

$\ln N = 2t + C_1$

$N = e^{2t + C_1} = e^{C_1} e^{2t} = N_0 e^{2t}$

when $t = 0$, $N = N_0$

Quiz IV

3

~~2. Rate of increase per unit of time = $P'(t) = 2N_0$~~

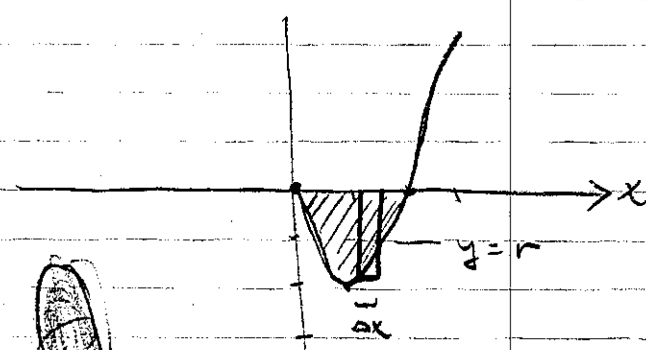
~~when $t=0$ Pop. size $= N_0$~~

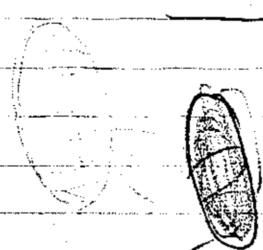
~~$P(t) = \int 2N_0\,dt = N_0^2 t + C$~~

3. Vol of $y=0$ and $y=x^2-2x$ about x axis

x	y
0	0
1	-2
2	0
3	3

$V = \pi r^2 h$





$r = x^2-2x$

$h = \Delta x$

limits $0 \rightarrow 2$

$(x^2-2x)(x^2-2x)$

$$V = \pi\int_0^2 (x^2-2x)^2\,dx = \pi\int_0^2 x^4-4x^3+4x^2\,dx$$

$$V = \pi\left[\frac{x^5}{5} - x^4 + \frac{4}{3}x^3\right]_0^2 = \pi x^3\left[\frac{x^2}{5} - x + \frac{4}{3}\right]_0^2$$

$$8\pi\left[\frac{4}{5} - 2 + \frac{4}{3}\right] = 8\pi\left[\frac{12+20}{15} - 2\right] = 8\pi\frac{(32-30)}{15} = \boxed{\frac{16\pi}{15}}$$

Quiz IV

4

4. $V = 15$ ft/sec
$a = -32$ ft/sec^2
$t = 8$ sec

$a = -32$ ft/sec^2 $\quad a = \frac{dv}{dt} \quad V = -32\int dt = -32t + C$

$t=0 \quad V=15 \quad \Rightarrow C = 15$

$V(t) = -32t + 15 \quad V(t) = \frac{ds}{dt} \quad S = \int -32t + 15\, dt$

$S(t) = -\frac{32t^2}{2} + 15t + C \quad t=0 \quad S=0 \Rightarrow C=0$

$S(8) = -16t^2 + 15t \quad = -16(8)^2 + 15(8) + C = 0$

$-1024 + 120 = -904$ feet

The Balloon was $\boxed{904 \text{ feet}}$ high when the Stone was dropped.

Quiz IV

5

5. a $\int x e^{3x}\, dy =$

$dv = e^{3x}$ $\quad V = \frac{1}{3} e^{3x}$ $\quad v = 3x$ $\quad dv = 3\,dx$ $\quad dy = \frac{du}{3}$

$u = x$ $\quad \cancel{du = dy}$

$$\frac{x e^{3x}}{3} - \frac{1}{3}\int e^{3x}\, dy = \frac{x e^{3x}}{3} - \frac{e^{3x}}{9} = \boxed{\frac{e^{3x}}{3}\left[x - \frac{1}{3}\right]}$$

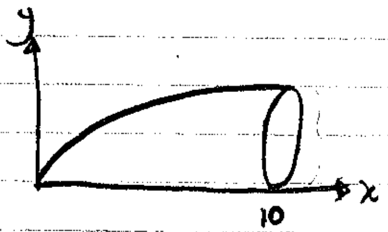
b) $\int \frac{2x+1}{\sqrt{x^2+x}}\, dx = \int (2x+1)(x^2+x)^{-1/2}\, dx$

$u = (x^2+x)$ $\quad du = (2x+1)\, dx$

$$\int u^{-1/2}\, dx = \boxed{\cancel{(x^2+x)^{-1/2}}}$$

$$= 2u^{1/2} = \boxed{2(x^2+x)^{1/2} +}\, C$$

9.



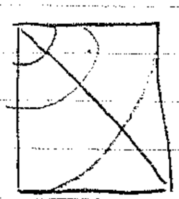
$y = \sqrt{x}$ $\quad A = \pi r^2$ $\quad D = 2r = \sqrt{x}$ $\quad r = \dfrac{\sqrt{x}}{2}$

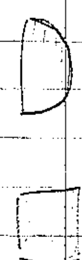
$A = \pi \left(\dfrac{\sqrt{x}}{2}\right)^2 = \dfrac{\pi x}{4}$

$V = \int_0^{10} A(x)\,dx = \int_0^{10} \dfrac{\pi x}{4}\,dx = \dfrac{\pi}{4}\int_0^{10} x\,dx = \dfrac{\pi}{2\cdot 4}\Big[x^2\Big]_0^{10}$

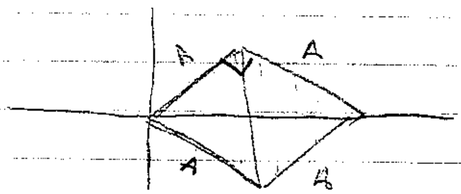
$\dfrac{\pi}{8}(100) = \dfrac{100\pi}{8} = \dfrac{25\pi}{2}$

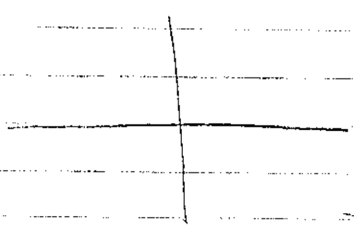
10.

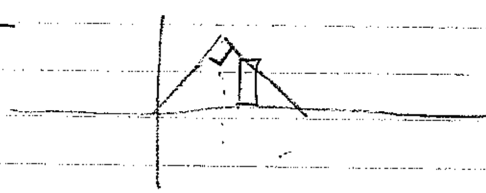




$V = \int A(x)\,dx$





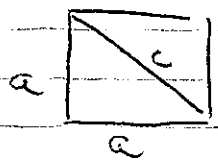


AIR Pallet Control
228-8311

10°30

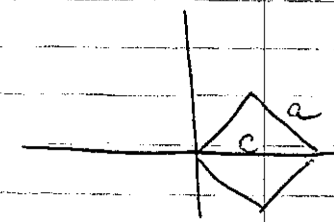
Applications of Integration - Volumes by Slicing

10.



$c^2 = a^2 + a^2 = 2a^2$

$c = a\sqrt{2}$



$D = a\sqrt{2}$ $\quad r = \dfrac{a\sqrt{2}}{2}$ $\quad \text{area} = \dfrac{\pi r^2}{2}$

$$\text{area} = \frac{\pi}{2}\left[\frac{a\sqrt{2}}{2}\right]^2 = \frac{\pi}{2}\,\frac{a^2\,2}{4} = \frac{\pi}{4}a^2$$

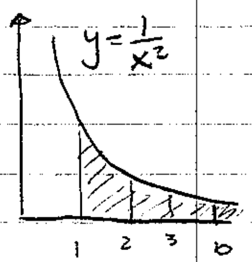
$$\text{Vol} = \int_a^b A(x)\,dx = \int_0^c \frac{\pi a^2}{4}\,da = \frac{\pi}{4}\int_0^c a^2\,da$$

$$= \left[\frac{\pi}{4}\,\frac{a^3}{3}\right]_0^c = \frac{\pi}{12}\left[a^3\right]_0^c = \frac{\pi}{12}\left(a\sqrt{2}\right)^3$$

$$= \frac{\pi}{12}\,a^3\,2\sqrt{2} = \boxed{\frac{\pi\sqrt{2}}{6}\,a^3} = \text{Volume}$$

IMPROPER INTEGRALS over INfinite INTERVALS P.267.

Example ONE:



$$\int_1^b \frac{1}{x^2}\,dx =$$

$$\int_1^b x^{-2}\,dx = \left[-\frac{1}{x}\right]_1^b = -\frac{1}{b} - (-1) = 1-\frac{1}{b} = 1-\frac{1}{\infty} = 1$$

Exampl 2;
Show that the improper ~~band~~ integral $\int_2^\infty \frac{1}{x}\,dx$ diverge.

$$\int_2^\infty \frac{dx}{x} = \lim_{b\to\infty}\int_2^b \frac{dx}{x} = \left[\ln x\right]_2^b = \left[\ln\infty - \ln 2\right] = \infty$$

ln ∞ does not exist ∴ Limits Diverge.

Example 3 – Both Limits of INTEGration are infinite

$$\int_{-\infty}^0 e^x\,dx = \lim_{a\to-\infty}\int_a^0 e^x\,dx = \left[e^x\right]_a^0 = \left[e^0 - e^{\infty}\right] = \left[1-0\right]$$

$= 1$ ∴ limits converge

$$\int_{-\infty}^{\infty} xe^{-x^2}\,dx =$$

Over

$u = x \qquad du = dx$

$dv = e^{-x^2} \qquad V = -\frac{du}{2x}e^{-x^2} \qquad du = -2x\,dx \qquad dx = \frac{du}{-2x}$

$$\int_{-\infty}^{\infty} x e^{-x^2} dx \qquad e^{v} dv \quad dv = -2x \, dx$$

$$dx = \frac{dv}{-2x}$$

$$\int_{-\infty}^{\infty} x e^{v} \frac{dv}{-2x} = -\frac{1}{2}\int_{-\infty}^{\infty} e^{v} dv = \left[-\frac{1}{2} e^{-x^2}\right]_a^b$$

$$= -\frac{1}{2} e^{-b^2} + \frac{1}{2} e^{-a^2}$$

a. $\lim_{a \to -\infty} \left(-\frac{1}{2} e^{-b^2} + \frac{1}{2} e^{-a^2}\right) = -\frac{1}{2} e^{-b^2} + \phi$

b. $\lim_{b \to \infty} \left(-\frac{1}{2} e^{-b^2}\right) = \phi$

Therefore: $\int_{-\infty}^{\infty} x e^{-x^2} dx = 0$

Example 5:

$$\int_{-\infty}^{\infty} e^{-|x|}\,dx \qquad e^{-|x|} = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ e^{x} & \text{if } x \leq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} e^{-|x|}\,dx = \int_{-\infty}^{0} e^{x}\,dx + \int_{0}^{\infty} e^{-x}\,dx$$

$$= \lim_{a\to-\infty} \int_{a}^{0} e^{x}\,dx + \lim_{b\to\infty} \int_{0}^{b} e^{-x}\,dx = \lim_{a\to-\infty} \Big[e^{x}\Big]_{a}^{0} + \lim_{b\to\infty} \Big[-e^{-x}\Big]_{0}^{b}$$

$$= [e^{0} - 0] + [0 + e^{-0}] = 2$$

Improper Integrals with unbounded Integrands
Exercise 8.5 page 270

1. $\displaystyle\int_1^{\infty} \frac{1}{x^3}\,dx = \lim_{b\to\infty}\left[\int_1^b x^{-3}\,dx\right] = \lim_{b\to\infty}\left[\frac{1}{2x^2}\right]_1^b$

$= \displaystyle\lim_{b\to\infty}\left[\frac{1}{2b^2} - \frac{1}{2}\right] = -1/2$ ∴ converges

$\boxed{\text{Book Answer } = \ln 2}$

3. $\displaystyle\int_0^{\infty} \frac{e^{-x}}{e^{-x}+1}\,dx = \int_0^{\infty} e^{-x}(e^{-x}+1)^{-1}\,dx \qquad u=(e^{-x}+1)$

$= \displaystyle -\int_0^{\infty} u^{-1}\,du = -\int_0^{\infty}\frac{du}{u} = \left[-\ln u\right]_0^{\infty}$

$= \displaystyle\lim_{b\to\infty}\left[-\ln(e^{-x}+1)\right]_0^b = \lim_{b\to\infty}\left(\left[-\ln(e^{-b}+1)\right] - \left[-\ln(e^{0}+1)\right]\right)$

$(-\ln(1) + \ln(2))$

$\cancel{\displaystyle\lim_{b\to\infty}\left[-\ln e^{-b}+1\right] - [0] = 1}$

$\displaystyle\lim_{b\to\infty}\left(\left[-\ln(e^{-b}+1)\right] + \ln(e^{0}+1)\right) = \left[-\ln 1\right] + \ln(1+1)$

$= 0 + \ln 2 = \boxed{\ln 2}$

Converges 7. $\int_{-\infty}^{-1} \frac{1}{x^2}\, dx = \int_{-\infty}^{-1} x^{-2}\, dx = \left[\frac{x^{-1}}{-1}\right]_{-\infty}^{-1} = \left[-\frac{1}{x}\right]_{-\infty}^{-1}$

$\lim_{a \to -\infty} \left(1 - \left[\frac{1}{a}\right]\right) = \boxed{1}$

converges 9. $\int_{-\infty}^{0} x^3 e^x\, dx = x^3 e^x - \int e^x 3x^2\, dx$

$dv = e^x \quad v = e^x$

$u = x^3 \quad du = 3x^2 dx$

$dv = e^x \quad v = e^x$

$u = 3x^2 \quad du = 6x\, dx$

$x^3 e^x - 3x^2 e^x - \int 6x e^x\, dx$

$x^3 e^x - 3x^2 e^x - \int 6x e^x\, dx = x^3 e^x - 3x^2 e^x - 6x e^x - \int 6 e^x\, dx$

$dv = e^x \quad v = e^x$

$u = 6x \quad du = 6\, dx$

$x^3 e^x - 3x^2 e^x - 6x e^x - 6e^x$

$\int_{-\infty}^{0} x^3 e^x\, dx = \lim_{a \to -\infty} \left[e^x (x^3 - 3x^2 - 6x - 6)\right]_a^0$

$= \left[e^0 (-6)\right] - \left[0\right] = \boxed{-6}$

11. ~~7~~ over

Diverges 11. $\int_{-\infty}^{\infty} x e^{-x}\, dx = -xe^{-x} - \int -e^{-x}\, dx$

$u = x \quad du = dx \qquad = \left[-xe^{-x} - e^{-x}\right]_{-\infty}^{\infty}$

$dv = e^{-x} \quad v = -e^{-x}$

$\lim_{a \to -\infty} \left[-xe^{-x} - e^{-x}\right]_a^{\infty} = \left[-e^{-x}(x+1)\right]$

$\lim_{a \to -\infty} = (0) - \left[-e^{-a}(a+1)\right] = e^{\infty}(\infty+1) = \boxed{\infty \text{ therefore the INTEGRAL Diverges}}$

Converges 13. $\int_{-\infty}^{\infty} e^{-|x-3|}\, dx$

$$e^{-|x-3|} = \begin{cases} -(x-3) = (3-x) \text{ if } x \geq 3 \\ -|3-x| = (x-3) \text{ if } x \leq 3 \end{cases}$$

$\int_{-\infty}^{3} e^{x-3}\, dx + \int_{3}^{\infty} e^{3-x}\, dx$

$\left[e^{x-3}\right]_{-\infty}^{3} + \left[-e^{3-x}\right]_{3}^{\infty} = \cancel{\lim_{a \to \infty}} \left[e\right.$

$\lim_{a \to -\infty} \left[e^{0} - e^{a-3}\right] + \lim_{b \to \infty} \left[-e^{3-b} + e^{0}\right]$

$[1] + [1] = \boxed{2}$

Improper Integrals w unbounded Integrands p. 271.

8. $\int_{-\infty}^{-2} \frac{1}{(x+1)^4}\,dx = \int_{-\infty}^{-2} (x+1)^{-4}\,dx = \left[\frac{-(x+1)^{-3}}{3}\right]_{-\infty}^{-2}$

$\lim_{a\to-\infty} \left[\left(\frac{-(-2+1)^{-3}}{3}\right) + \left(\frac{(a+1)^{-3}}{3}\right)\right] \quad \frac{-1}{\sqrt[3]{-1}(3)} + \frac{1}{3(a+1)}$

$\frac{1}{3} + \frac{1}{-\infty} = \frac{1}{3} + \infty = \boxed{\infty}$ Therefore $\int f(x)\,dx$ diverges

10. $\int_{-\infty}^{4} x^{2/3}\,dx = \left[\frac{3\,x^{5/3}}{5}\right]_{-\infty}^{4} = \lim_{a\to-\infty} \frac{3}{5}\left(4^{5/3} - a^{5/3}\right]$

$\left(\frac{3}{5}\right)4^{5/3} = \boxed{6.05}$

Page 274 text

1. $\int_0^1 x^{-1/3}\,dx =$ $\quad \epsilon > 0 \quad$ $\int_\epsilon^1 x^{-1/3}\,dx = \left[\frac{3x^{2/3}}{2}\right]_\epsilon^1$

$= \left[\frac{3}{2} - \frac{\epsilon^{2/3}\,3}{2}\right] = \frac{3}{2}$

2. $\cancel{\int_0^1 \frac{1}{x^2-1}\,dx}$ $\quad \epsilon > 0 \quad$ $\cancel{\int_\epsilon^1 \frac{1}{(x^2-1)}\,dx}$

$\lim\limits_{\epsilon \to 0+} \left[\ln(x^2-1)\,2x\right]_\epsilon = \left[2(1)\ln(0)\right] - \left[\ln(-1)\cdot(0)\right]$

$2\ln(0) = \infty$ DIVERGES

? 5. $\int_{-1}^1 |x|^{-3/4}\,dx$ $\qquad f(x) = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$

$\int_{-1}^\epsilon -x^{-3/4}\,dx + \int_\epsilon^1 x^{-3/4}\,dx$

$\lim\limits_{\epsilon \to 0(-)} \left[-4x^{1/4}\right]_{-1}^\epsilon + \lim\limits_{\epsilon \to 0+} \left[4x^{1/4}\right]_\epsilon^1$

$\lim\limits_{\epsilon \to 0-} \left[(-4\epsilon^{1/4}) - (-4(-1)^{1/4})\right] + \lim\limits_{\epsilon \to 0+} \left[4 - 4(\epsilon)^{1/4}\right]$

Improper Integrals — # 3 page 294

PARTIAL FACTIONS

3. $\int_0^1 \frac{1}{x^2-1}\,dx$ $\qquad \frac{1}{x^2-1} = \frac{A}{(x-1)} + \frac{B}{(x+1)}$

$= \frac{A(x+1) + B(x-1)}{x^2-1} = \frac{Ax + A + Bx - B}{(x^2-1)}$

$= \frac{x(A+B) + A - B}{x^2-1}$

$\frac{x(A+B) + A - B}{x^2-1} = \frac{1}{x^2-1}$ $\qquad (x^2-1)(1) = \cancel{(x^2-1)}\left[x(A+B) + A - B\right]$

$x(A+B) + (A-B) = 1$

$A + B = 1$
$A - B = 0$
$2A = 1$ $\qquad A = 1/2$
$B = 1/2$

OR $A(x+1) + B(x-1) = (1)$

If $x = 1$ $\quad A(x+1) = 1$ $\qquad A = 1/2$

If $x = -1$ $\quad B(x-1) = 1$ $\qquad B = -1/2$

$\int_0^1 \frac{1/2}{(x-1)}\,dx + \int_0^1 \frac{1/2}{(x+1)}\,dx = 1/2\left(\int \frac{1}{(x-1)} + \frac{1}{(x+1)}\,dx\right)$

$\left[\frac{(\ln(x-1) + \ln x+1)}{2}\right]_0^1 = \frac{(\ln 0 + \ln 2)}{2} - \frac{\ln(-1) + \ln 1}{2}$

$= \ln 2$

$$\left(\cancel{-4(0)^{1/4}} - \left[-4(-1)^{1/4}\right]\right) + \left(4(1)^{1/4} - 4(0)\right)$$

$$4(-1)^{1/4} + 4(1)^{1/4} =$$

$$-4(1)^{1/4} + 4(1)^{1/4} = 0$$

Page 274 text

~~3.~~ $\int_0^1 \frac{1}{x^2-1}\,dx = \lim_{\epsilon \to 0} \int_0^{1-\epsilon} \frac{1}{x^2-1}\,dx = \cancel{\lim_{\epsilon\to 0}\left[\ln(x^2-1)\right]}$

$= \lim_{\epsilon \to 0} \Big[\ln(x^2-1)\,2x\Big]_0^{1-\epsilon}$

$= \lim_{\epsilon \to 0} \Big[\ln\big[(1-\epsilon)^2-1\big]\,2(1-\epsilon)\Big] - \Big[\ln(-1)(0)\Big]$

$2\big(\ln(1-1)\big) = 2\ln(0) = \boxed{\infty \therefore \text{diverges}}$

5. $\int_{-1}^{1} |x|^{-3/4}\,dx \qquad f(x) = \begin{cases} x^{-3/4} & \text{if } x \geq 0 \\ -x^{-3/4} & \text{if } x \leq 0 \end{cases}$

$\int_{-1}^{0} -x^{-3/4}\,dx + \int_0^1 x^{-3/4}\,dx$

$\leftarrow \lim_{\epsilon \to 0}\Big[-4x^{1/4}\Big]_{-1}^{0-\epsilon} + \lim_{\epsilon \to 0}\Big[4x^{1/4}\Big]_{0+\epsilon}^{1} = \cancel{\lim_{\epsilon\to 0}}$

$\lim_{\epsilon \to 0}\Big[\big(-4\epsilon^{1/4}\big) - \big(-4(-1)^{1/4}\big)\Big] + \lim_{\epsilon \to 0}\Big[4 - 4\epsilon^{1/4}\Big]$

$= \Big[4(-1)^{1/4} + 4\Big]$

?

3, cont.

$$\int_0^1 \frac{1}{x^2-1}\,dx = \int_0^1 \left[-\tfrac{1}{2}\cdot\frac{1}{x+1} + \frac{1}{2}\left[\frac{1}{x-1}\right]\right] dx$$

$$= -\tfrac{1}{2}\int_0^1 \frac{dx}{x+1} + \frac{1}{2}\int_0^1 \frac{dx}{x-1}$$

$$-\tfrac{1}{2}\Big[\ln(x+1)\Big]_0^1 + \tfrac{1}{2}\lim_{\epsilon\to 0}\int_0^{1-\epsilon} \frac{dx}{x-1}$$

$$-\tfrac{1}{2}\Big[\ln(x+1)\Big]_0^1 + \tfrac{1}{2}\lim_{\epsilon\to 0}\Big[\ln(x-1)\Big]_0^{1-\epsilon}$$

$$-\tfrac{1}{2}\Big[\ln 2 - \ln 1\Big] + \frac{1}{2}\Big[\lim_{\epsilon\to 0} \ln 0 - \ln(-1)\Big]$$

$$= \boxed{-\tfrac{1}{2}\ln 2}$$

Improper Integrals - unbounded Integrands

p. 274

Cannot let the integrand have $\emptyset$ in denominator

$$7 \quad \int_{-1/2}^{4} \frac{1}{\sqrt{2x+1}}\,dx \qquad \int_{-1/2+\epsilon}^{4} (2x+1)^{-1/2}\,dx$$

$$\lim_{\epsilon \to 0} \left[\frac{2(2x+1)^{1/2}}{2} \right]_{-1/2+\epsilon}^{4} = \lim_{\epsilon \to 0} \left[(2x+1)^{1/2} \right]_{-1/2+\epsilon}^{4}$$

$$\lim_{\epsilon \to 0} \left[9\right]^{1/2} - \left[2(-1/2+\epsilon)+1\right]^{1/2} = \boxed{3}$$

ok Book answer = 3

Exercise 8.6 P. 274

Improper integrals
Unbounded integrands

9. $\int_{-10}^{10} \frac{x}{|x|}\,dx$ $\quad f(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$

$$\int_{-10}^{\epsilon} -\,dx + \int_{\epsilon}^{10} dx = \lim_{\epsilon \to 0(-)} \left[-x\right]_{-10}^{\epsilon} + \lim_{\epsilon \to 0+} \left[+x\right]_{\epsilon}^{10}$$

$$\lim_{\epsilon \to 0-} \left[-\epsilon - (10)\right] + \lim_{\epsilon \to 0+} \left[10 - \epsilon\right] = \phi$$

11. $\int_{-1}^{1} \frac{|x|}{\sqrt{1-x^2}}\,dx$ $\quad f(x) = \begin{cases} \frac{x}{\sqrt{1-x^2}} & \text{for } x > \phi \\ \frac{-x}{\sqrt{1-x^2}} & \text{for } x < \phi \end{cases}$

$$-\int_{-1+\epsilon}^{0} x\,(1-x^2)^{1/2}\,dx + \int_{0}^{1+\epsilon} x\,(1-x^2)^{1/2}\,dx$$

$u = (1-x^2)$ $\quad du = -2x\,dx$ $\quad dx = \frac{du}{-2x}$

$$\frac{1}{2}\int_{-1+\epsilon}^{0} u^{1/2}\,du + -\frac{1}{2}\int_{0}^{1+\epsilon} u^{1/2}\,du =$$

P. 274

11. Continued $\quad u = (1-x^2)$

$$\frac{1}{2}\int_{-1+\epsilon}^{0} u^{1/2}\,du - \frac{1}{2}\int_{0}^{1+\epsilon} u^{1/2}\,du = \left[\frac{1}{2}\,\frac{2}{3}\,u^{3/2}\right]_{-1+\epsilon}^{0} - \frac{1}{2}\left[\frac{2}{3}\,u^{3/2}\right]_{0}^{1+\epsilon}$$

$$\left[\frac{1}{3}\left(u^{3/2}\right)\right]_{-1+\epsilon}^{0} - \frac{1}{3}\left(u^{3/2}\right)_{0}^{1+\epsilon} = \left[\frac{(1-x^2)^{3/2}}{3}\right]_{-1+\epsilon}^{0} - \frac{1}{3}\left[(1-x^2)^{3/2}\right]_{0}^{1+\epsilon}$$

$$\left[\frac{1}{3}\right] - [0] - \left[0 - \frac{1}{3}\right] = \frac{1}{3} + \frac{1}{3} = \boxed{2/3}$$

Book answer = 2

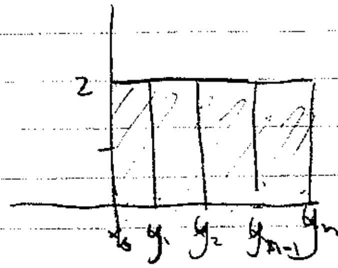
P. 282

Excercise 8.7

Numerical Integration

Method I – Approximation with Rectangles

1. $\int_0^4 2\,dx \qquad n=4$



$\int_0^4 2\,dx = 2\int_0^4 dx$

$\approx \Delta x\,(y_1 + y_2 + y_n) \qquad \Delta x = \frac{b-a}{n} = \frac{4-0}{4} = \frac{4}{4} = 1$

$\approx 1\,(y_1 + y_2 + \dots + y_n)$

$\approx 1\,(2+2+2+2) = 8$

3. $\int_1^5 e^x\,dx \qquad \Delta x = \frac{b-a}{n} = \frac{5-1}{4} = \frac{4}{4} = 1$

$x_k = a + k\Delta x = 1 + k\cdot 1 = 1+k$

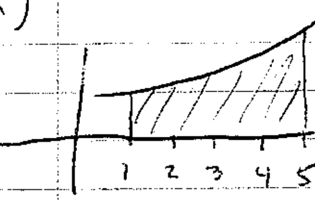
$f(x_k) = y_k$

$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x$

$\approx \sum_{K=1}^{n} f(x_k)\,\Delta x \approx \sum_{K=1}^{n} y_k\,\Delta x \approx \Delta x\left(\sum_{k=1}^{n} y_k\right)$

$\int_1^5 e^x\,dx = \Delta x\left(\Sigma y_k\right) = (+y_1 + y_2 + \dots y_{n-1} + y_n)$

$= (1)(+e^2 + e^3 + e^4 + e^5) = 230.48$



$e^5 - e^1$

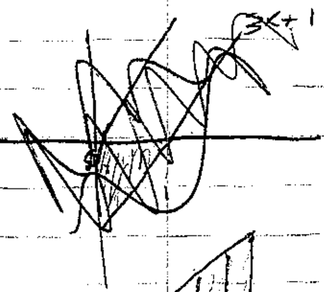
$\boxed{\text{Book Answer } 230.48}$

Approximation with Rectangles

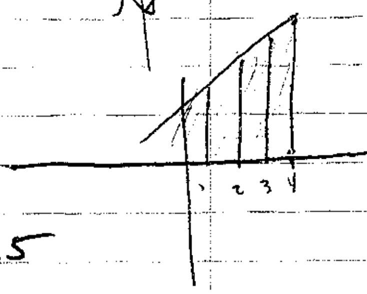
2. $\int_0^1 (3x+1)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} (3x+1)\Delta x$

$\approx \Delta x(\Sigma y_k)$ $\qquad \Delta x = \frac{b-a}{n} = \frac{1-0}{4} = .25$

$\approx .25(\Sigma y_k)$



x	y
1	4
2	7
3	10
4	13



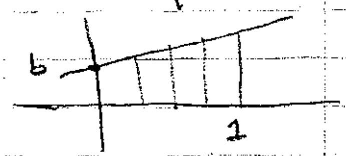
$\approx 0.25(\Sigma y_k) = \frac{4+7+10+13}{4} = \frac{34}{4} = 8.5$

Method II Trapezoidal Rule

Area of trapezoid $=$ (base)(ave height) $=$ base $\left(\frac{y_1+y_2}{2}\right) = (x)\left[\frac{y_1+y_2}{2}\right]$

Area $= \Delta x \left(\frac{y_0}{2} + y_1 + y_2 + y_{n-1} + \frac{y_n}{2}\right)$

4. $\int_0^1 (ax+b)\,dx$ $\qquad n=4$ $\qquad \Delta x = \frac{1-0}{4} = 0.25$



x	y
1	a+b
2	2A+b
3	3a+b
4	4a+b

Area $= \Delta x\left(\frac{a+b-1}{2} + 2a+b + 3a+b + \frac{4a+b}{2}\right)$

$= \Delta x\left(\cancel{\frac{a}{2}} + \cancel{\frac{b}{2}} - \frac{1}{2} + \cancel{5a} + \cancel{2b} + \cancel{2a} + \cancel{\frac{b}{2}}\right)$

$= \Delta x\left(7a + \frac{a}{2} + 3b\right) = \Delta x\left(\frac{15a}{2} + 3b\right)$

$\boxed{= \frac{15a}{8} + \frac{b}{3}}$

$1/4 \quad 2/4 \quad 3/4 \quad 4/4$

x	y
0	$1 = y_0$
$1/4$	$16/17 = y_1$
$2/4$	$16/20 = y_2$
$3/4$	$16/25$
1	$1/2$

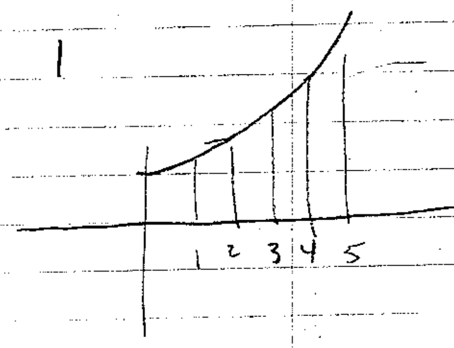
$A \approx 1/4\left(\frac{1}{2} + \frac{16}{17} + \frac{16}{20} + \frac{16}{25} + [illegible]\right)$

Method II Trapezoidal Rule

Area of trapezoid $= x\dfrac{(y_1+y_2)}{2}$

$$\int f(x)\,dx = \left(\frac{y_0}{2} + y_1 + y_2 + y_3 + y_{n-1} + \frac{y_n}{2}\right)\Delta x$$

5. $\displaystyle\int_1^5 e^x\,dx \qquad \Delta x = \frac{b-a}{n} = \frac{5-1}{4} = 1$



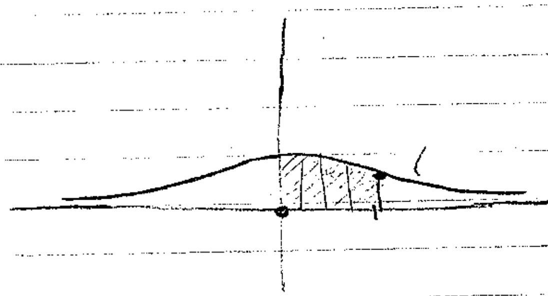
$$\int_1^5 e^x\,dx = \Delta x\left(\frac{y_0'}{2} + y_1 + y_2 + y_{n-1} + \frac{y_n}{2}\right)$$

$$= \Delta x\left(\frac{e^1}{2} + e^2 + e^3 + e^4 + \frac{e^5}{2}\right)$$

$\boxed{\text{Book answer} = 157.64}$

$$= 0.25(157.64) = 39.41$$

7. $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx =$



x	y
1	1/2
2	1/5
3	1/10
4	1/17

$$\text{Area} = \int_0^1 \frac{1}{1+x^2}\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\,dx$$

$$= \Delta x\left(\frac{y_0}{2} + y_1 + y_2 + y_3 + \frac{y_4}{2}\right)$$

$$= \Delta x\left(\frac{0}{2} + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \left(\frac{1}{17}\cdot\frac{1}{2}\right)\right]$$

$$= 0.25(0.829) = .20735$$

Method IV. Simpson's Rule – numerical integration

Page 282

$n = 4$

$$A = \frac{\Delta x}{3}\left[y_0 + 4y_1 + y_2\right] + \frac{\Delta x}{3}\left[y_2 + 4y_3 + y_4\right] \qquad \Delta x = \frac{b-a}{n} = \frac{4}{4} = 1$$

11. $\int_0^4 \sqrt{1+x^2}\,dx =$

x	y	
0	1	y_0
1	$\sqrt{2}$	y_1
2	$\sqrt{5}$	$= y_2$
3	$\sqrt{10}$	$= y_3$
4	$\sqrt{17}$	$= y_4$

$$A = \frac{1}{3}\left[1 + 4(\sqrt{2}) + \sqrt{5} + \sqrt{5} + 4\sqrt{10} + \sqrt{17}\right] = 9.3004$$

$\boxed{\text{Book answer} = 9.3004}$

Derivitives of Sine + Cosine

$$\boxed{\begin{aligned}\frac{d}{dx}(\sin x) &= \cos x \\ \frac{d}{dx}(\cos x) &= -\sin x\end{aligned}}$$

$$\frac{d}{dx}\sin u = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}\cos u = -\sin u \frac{du}{dx}$$

Circle $= x^2 + y^2 = 1$

See Properties of Sine + Cosine functions page 287

ADDITION Formulas for Sine + Cosine.

1. $y = \sin(e^x)$

$$\frac{dy}{dx} = (\cos e^x)\frac{de^x}{dx} = \quad e^x \cos e^x$$

$\frac{u}{v}$

3. $y = \dfrac{\cos x}{\sin x} = \dfrac{\sin x \frac{d}{dx}\cos x - \cos x \frac{d}{dx}\sin x}{\sin^2 x}$

$$= \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-1(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$$

5. $y = \frac{1}{\cos x}$ $\frac{dy}{dx} = -\frac{1}{\cos^2 x}\frac{d}{dx}\cos x = -\frac{(-\sin x)}{\cos^2 x}$

$= \frac{\sin x}{\cos^2 x}$

6. $y = \sin(\ln x)$

$\frac{dy}{dx} = \cos(\ln x)\frac{d}{dx}\ln x = \frac{\cos \ln x}{x}$

7. $y = \sin x \cdot \cos x$ $\frac{dy}{dx} = \sin x \frac{d}{dx}\cos x + \cos x \frac{d}{dx}\sin x$

$(\sin x)(-\sin x) + \cos x(\cos x)$ $\boxed{\text{Book answer } (\cos 2x)}$

$-\sin^2 x + \cos^2 x =$

$\cos^2 x - \sin^2 x = (\cos x - \sin x)(\cos x + \sin x) = 0$

Double Angle formula p. 71 Flanders + Price

$\cos^2 - \sin^2 = \cos 2\theta$

$\cos^2 x - \sin^2 x = \cos 2\theta$

9. $y = \sin x^4 \qquad \frac{dy}{dx} = \cos x^4 \cdot 4x^3$

10. $y = \cos 5x \qquad y' = -5\sin 5x$

12. $y = x \sin \frac{1}{x} \qquad y' = x \frac{d}{dx} \sin \frac{1}{x} + \sin \frac{1}{x} \frac{dx}{dx}$

$x \cos \frac{1}{x} \frac{d}{dx} \frac{1}{x} + \sin \frac{1}{x} \qquad x^{-1}$

$x \cos \frac{1}{x}\left(-x^{-2}\right) + \sin \frac{1}{x} = -\frac{x}{x^2} \cos \frac{1}{x} + \sin 1/x$

$= -1/x \cos 1/x + \sin 1/x$

13. $y = (\sin x)^x \qquad y'(u^v) = vu^{v-1}\frac{du}{dx} + u^v \frac{1}{\ln u} \frac{dv}{dx}$

$x \sin x^{x-1} \cos x + \frac{(\sin x)^x}{\ln(\sin x)}$

$x \sin x^{x-1} + \frac{(\sin x)^x}{\ln(\sin x)} \cos x$

IMPLICIT Differentation

$x^2+1=3y-1$ $\quad 2x=3$ $\quad$ implicit

$x^2+1=y$ $\quad y'=2x$ $\quad$ Explicit

17. $\sin y+\cos x=0$ $\quad \cos y - \sin x=0$

Book answer $= \boxed{\frac{\sin x}{\cos x}}$

$\cos y\, dy + (-\sin x)dx=0$ $\quad \cos y\, dy = \sin x\, dx$

$\frac{dy}{dx}=\frac{\sin x}{\cos y}$

19. $x=\sin y$ $\quad 1=\cos y$

$dx=\cos y\, dy$ $\quad \frac{dy}{dx}=\frac{1}{\cos y}$

21. $\sin^2 x+\sin^2 y=C$

$2\sin x\cos x\, dx + 2\sin y\cos y\, dy=0$

$2\sin x\cos x\, dy = -2\sin y\cos y\, dy$

$\frac{dy}{dx}=-\frac{2\sin x\cos x}{2\sin y\cos y}=-\frac{\sin x\cos x}{\sin y\cos y}$

Double angle formula $2\sin\theta\cos\theta=\sin 2\theta$

$\frac{dy}{dx}=-\frac{\sin 2x}{\sin 2y}$

Limits of Sin and Cos. page 301. IMPORTANT!

23. $\lim_{\theta \to \emptyset} \frac{1-\cos^2\theta}{\theta} = \lim_{\theta \to \emptyset} \frac{\sin^2\theta}{\theta} = 1$

25. $\lim_{x \to 0} \frac{\sin 3x}{x} = \lim_{x \to \emptyset} \frac{3\sin 3x}{3x} = 3(1) = 3$

24. $\lim_{x \to 0} \frac{\sin x \cos x}{x} = \lim_{x \to 0} \frac{2\sin x \cos x}{2x} = \frac{\sin 2x}{2x} = 1$

26. $\lim_{x \to \pi/2} \frac{\cos\theta}{\theta} = \frac{\cos \pi/2}{\pi/2} = \frac{0}{\pi/2} = \emptyset$

27. $\lim_{x \to 0} \frac{\sin x}{\sin 2x} = \frac{\sin x}{2\cos x \sin x} = \frac{1}{2\cos x} = 1/2$

28. $\lim_{\pi/2} \frac{\cos x}{\pi/2 - x}$ Let $\theta = \pi/2 - x$
Then $x = \pi/2 - \theta$

$\lim_{x \to \pi/2} \frac{\cos(\pi/2 - \theta)}{\theta} = \lim_{x \to \pi/2} \frac{\cos \pi/2 \cos\theta + \sin \pi/2 \sin\theta}{\theta}$

$= \frac{(0)(\cos\theta) + (1)\sin\theta}{\theta} = \boxed{\frac{\sin\theta}{\theta} = 1}$ which is an identity.

TRIGONOMETRIC FUNCTIONS
P. 306

1. Show that $\cot x$ is periodic with π

$$\cot(x+\pi) = \cot x$$

$$\frac{\cos(x+\pi)}{\sin(x+\pi} = \cot x = \frac{\cos x \cos\pi - \sin x \sin\pi}{\sin x \cos\pi + \sin\pi \cos\pi} = \cot x$$

$$\frac{\cos x(1) - (0)}{\sin x(1) + (0)} = \frac{\cos x}{\sin x} = \cot x$$

2. Show that $\sec x$ and $\csc x$ are periodic with period 2π

A. $\sec(2\pi + x) = \sec x$

$$\frac{1}{\cos(2\pi + x)} = \frac{1}{\cos 2\pi \cos x - \sin 2\pi \sin x} = \frac{1}{\cos x} = \sec x$$

B. $\csc x = \csc(2\pi + x)$

$$\csc(2\pi + x) = \frac{1}{\sin(2\pi + x)} = \frac{1}{\sin 2\pi \cos x + \sin x \cos 2\pi}$$

$$= \frac{1}{\sin x} = \csc x$$

3. Verify the differentiation formulas

$$\frac{d}{dy}\cot x = -\csc^2 x$$

$$\frac{\cos x}{\sin x} = \frac{(\sin x)\frac{d}{dy}\cos x - \cos x \frac{d}{dy}\sin x}{\sin^2 x}$$

$$\frac{\sin x(-\sin x) - \cos x \cos x}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$$

b. $\frac{d}{dy} \sec x = \sec x \tan x$.

$y = \sec x = \frac{1}{\cos x}$ $\quad y' = \frac{-1}{\cos^2 x} \frac{d}{dx} \cos x$

$= \frac{+\sin x}{\cos^2 x} = +\tan x \frac{1}{\cos x} = \sec x \tan x$

c. $\frac{d}{dy} \csc x =$ $\quad y = \csc x = \frac{1}{\sin x}$

$y' = \frac{-1}{\sin^2 x} \frac{d}{dy} \sin x = \frac{-\cos x}{\sin^2 x} = \frac{-\cot x}{\sin x} = -\csc x \cot x$

9. $y = \ln(\tan x)$

$y' = \frac{1}{\tan x} \frac{d}{dy} \tan x = \cot x \sec^2 x$

10. $y = \cos x \tan x$ $\quad y' = \cos x \frac{d}{dy} \tan x + \tan x \frac{d}{dx} \cos x$

$= \cos x \sec^2 x + \tan x(-\sin x) =$

$= \frac{\cos x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2} = \frac{\cos x - \sin^2 x}{\cos^2 x}$?

$$\boxed{y = \cos x \frac{\sin x}{\cos x} = \sin x \quad y' = \cos x}$$

Trigonometric Functions
P. 306

1. Show that $\tan x$ is periodic with period π

Therefore $\tan(\pi + x) = \tan x$

then $\frac{\sin(\pi + x)}{\cos(\pi + x)} = \frac{\sin x}{\cos x}$

$\frac{\sin \pi \cos x + \sin x \cos \pi =}{\cos \pi \cos x - \sin \pi \cos \pi =}$ $\frac{\tan x}{\tan x}$

$\frac{0 + \sin x(1)}{\cos x - (0)} = \tan x$ $\frac{\sin x}{\cos x} = \tan x = \frac{\sin x}{\cos x}$

2. Find the Derivative of $\tan x$

$y = \tan x = \frac{\sin x}{\cos x}$ $y' = \frac{\cos x \frac{d}{dx} \sin x - \sin x \frac{d}{dx} \cos x}{\cos x^2}$

$= \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$= \sec^2 x$

Therefore $\boxed{\frac{d}{dx} \tan x = \sec^2 x}$

2. Find $\frac{d}{dy}(\cot x)$

$y = \cot x = \frac{\cos x}{\sin x}$ $\qquad y' = \frac{\sin x \frac{d}{dx} \cos x - \cos x \frac{d}{dy} \sin x}{\sin^2 x}$

$= \frac{\sin x(-\sin x) - \cos x \cos x}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$= \frac{-1}{\sin^2 x} = -\csc^2 x$

$\frac{d}{dy} \cot x = -\csc^2 x$

3. $\frac{d}{dy}(\sec x)$ $\qquad y = \sec x = \frac{1}{\cos x}$

$y' = \frac{-1}{\cos^2 x} \cdot \frac{d}{dy} \cos x = \frac{-1}{\cos^2 x} - \sin x = \frac{\sin x}{\cos^2 x}$

$= \frac{\sin x}{\cos x \ \cos x} = \frac{\tan x}{\cos x} = \tan x \sec x$

$\frac{d}{dy} \sec x = \tan x \sec x$

4. $\frac{d}{dy} \csc x$ $\qquad y = \csc x = \frac{1}{\sin x}$

$y' = \frac{-1}{\sin^2 x} \frac{d}{dy} \sin x = \frac{-1}{\sin^2 x} \cos x = \frac{-\cos x}{\sin x \ \sin x}$

$= \frac{\cot x}{\sin x} = \cot x \csc x$

$\frac{d}{dy} \csc x = \cot x \csc x$

P. 306

11. $y = \cot\sqrt{x^2-1} = \cot(x^2-1)^{1/2}$

$\frac{d}{dy}\cot u = -\csc^2 u \frac{du}{dy}$

$-\csc^2(x^2-1)^{1/2} \frac{d}{dy}(x^2-1)^{1/2} = -\csc^2(x^2-1)^{1/2} \cdot \cancel{\tfrac{1}{2}} \cdot (x^2-1)^{-1/2} \cdot \cancel{2}x$

$= \dfrac{-x \cdot \csc^2(x^2-1)^{1/2}}{(x^2-1)^{1/2}}$

12. $y = \dfrac{\tan x}{\cot x} = \tan^2 x \qquad y' = 2\tan x \cdot \frac{d}{dy}\tan x$

$= 2\tan x \sec^2 x$

13. $y = x^{\sec x}$

$\boxed{u^v = v u^{v-1}\frac{du}{dx} + u^v \ln u \frac{dv}{dy}}$

$y' = (\sec x)x^{\sec x - 1}\frac{dx}{dx} + x^{\sec}\ln x \frac{d}{dy}\sec x$

* $(\sec x)\, x^{\sec x - 1} + x^{\sec x}\ln x \cdot \sec x \cdot \tan x \quad dx$

$(\sec x)\, x^{\sec - 1}\left[x + \tan x \ln x + 1\right]$

P. 306

15. $x = \cot(x+y)$

$1 = -\csc^2(x+y) \cdot \frac{d}{dy}(x+y)$ $\quad 1 = -\csc^2(x+y)\left(1 + \frac{dy}{dx}\right)$

$1 = -\csc^2(x+y) + \frac{dy}{dx}\left(-\csc^2[x+y]\right)$

$\frac{dy}{dx}\left[-\csc^2(x+y)\right] = 1 + \csc^2(x+y)$

$\frac{dy}{dx}\csc^2(x+y) = -\left[\csc^2(x+y) + 1\right]$

$\frac{dy}{dx} = \frac{-\csc^2(x+y) + 1]}{\csc^2(x+y)}$

$= \frac{-\csc^2(x+y)}{\csc^2(x+y} - \frac{1}{\csc^2(x+y)} = -1 - \sin^2(x+y)$

?

<u>Example 3</u>: Find $\frac{dy}{dx}$ if $y + \cot xy = 1$

$$\frac{dy}{dx} + \left(-\csc^2 xy\right)\frac{d(xy)}{dx} = 0$$

$$\frac{dy}{dx} = \csc^2 xy\left(\frac{dy}{dx} + y\right) = \frac{dy}{dx}\csc^2 xy + y\csc^2 xy$$

$$= 2\csc xy\left(-\csc xy\right)\left(\frac{dy}{dx} + y\right)$$

$$= 2y\csc xy\left(-\csc xy\right) + \left[\frac{dy}{dx}\, 2\csc xy\left(-\csc xy\right)\right]$$

$$-2y\csc^2 xy + \frac{dy}{dx}\left(-2\csc^2 xy\right)$$

$$= -2y\csc^2 xy + \left[-4\csc xy \cdot \frac{d}{dx}\csc xy\right]$$

$$= -2y\csc^2 xy - 4\csc xy\left(-\cot xy \csc xy\right)\frac{dy}{dx}$$

TRigonometric defferentiation p. 306

17. $\tan x + \tan y = 1$

$\sec^2 x + \sec^2 y \frac{dy}{dx} = 0$

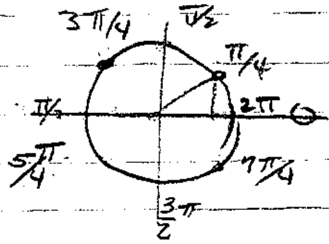
$\frac{dy}{dx} = -\frac{\sec^2 x}{\sec^2 y}$

? 16. $y = \sec y$ $\frac{dy}{dx} = \sec y \cdot \tan y \frac{dy}{dx}$

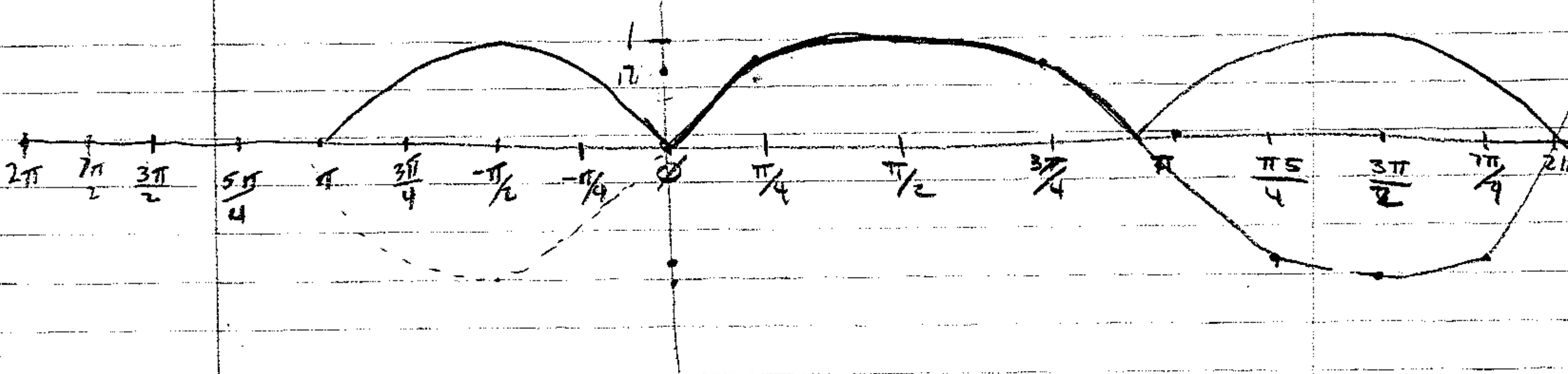
31. $y = |\sin x|$ $y = \begin{cases} = (\sin x) & \text{if } 0 < x < \pi \\ = -(\sin x) = y & \pi < x < 2\pi \end{cases}$

therefore $y = \sin x$

π



x	y	$(\cos)$
0	0	1
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/2$	1	0
$3\pi/4$	$\sqrt{2}/2$	$-\sqrt{2}/2$
π	0	-1
$5/4\,\pi$	$-\sqrt{2}/2$	$-\sqrt{2}/2$
$\pi\,3/4$	-1	0
$\pi\,7/4$	$-\sqrt{2}/2$	$\sqrt{2}/2$
2π	0	1



37. FIND $y^{(8)}$ when $y = \cos x$

$y' = (-\sin x)$
$y^2 = (-\cos x)$
$y^3 = (\sin x)$
$y^4 = (\cos x)$
$y^5 = (-\sin x)$
$y^6 = (-\cos x)$
$y^7 = (\sin x)$
$y^8 = (\cos x)$

38 y^7 when $y = \sin 2x$

$y' = 2\cos 2x$

$y'' = 2[-\sin 2x]2$

$y''' = 4[-\cos 2x]2$
$y^{IV} = 8[\sin 2x]2$
$y^{V} = 16[\cos 2x]2$

$y^{(6)} = 32[-\sin 2x]2 = -64 \sin 2x$

39. $y^{(2)} + y$ when $y = \sin x$

$y' = \cos x$
$y^2 = -\sin x$

$y^2 + y = -\sin x + \sin x = 0$

40. $y^{(2)} + y$ when $y = x \sin x$

y' $x \frac{d \sin x}{dx} + \sin x \frac{d(x)}{dx} = x \cos x + \sin x$

$y^2 = -x \sin x + \cos x + \cos x = 2\cos x - x \sin x$

$y^2 + y = 2\cos x - x \sin x + x \sin x = \boxed{2\cos x}$

Quiz IV

1. $\displaystyle\int_{2+\epsilon}^{3} \frac{dx}{\sqrt{x-2}}\, dx = \int_{2+\epsilon}^{3} (x-2)^{-1/2}\, dx$

$\displaystyle\lim_{\epsilon \to 0} = \left[2(x-2)^{1/2}\right]_{2+\epsilon}^{3} = 2(3-2)^{1/2} - 2(2-2)^{1/2} = 2(1)^{1/2} = \boxed{2}$

2. $\displaystyle\int_{0+\epsilon}^{1} \frac{dx}{x^2} = \lim_{\epsilon \to 0} \int_{0+\epsilon}^{1} x^{-2}\, dx = \left[-x^{-1}\right]_{0+\epsilon}^{1} = \lim_{\epsilon \to 0}\left[-\frac{1}{x}\right]_{0+\epsilon}^{1}$

$\displaystyle = 2\left[-1 + \frac{1}{e}\right] \lim_{\epsilon \to 0}$ is undefined

3. $\displaystyle\int_{1}^{b} x^{-2}\, dx = \left[-x^{-1}\right]_{1}^{b} = \lim_{b \to \infty}\left[-\frac{1}{x}\right]_{1}^{b} = \frac{-1}{\infty} + 1$

4. $\displaystyle\int_{-\infty}^{\infty} x^{-2/3}\, dx = \int_{a}^{0-\epsilon} x^{-2/3}\, dx + \int_{0+\epsilon}^{b} x^{-2/3}\, dx$

$\displaystyle = \left[3x^{1/3}\right]_{a}^{0-\epsilon} + \left[3x^{1/3}\right]_{0+\epsilon}^{b}$

$\displaystyle = \lim_{\substack{a \to \infty \\ \epsilon \to 0}} \left[3(-\infty)^{1/3}\right] + \lim_{\substack{b \to \infty \\ \epsilon \to 0}} \left[\right.$

Divergent

Quiz IV

6. $y = \sin(x^2 - 2x)$,

$y' = \cos(x^2-2x)\dfrac{d}{dx}(x^2-2x) = \cos(x^2-2x)(2x-x)$

7. $y = \cos x^3 \quad y' = -\sin x^3 \cdot 3x^2$

8. $y = \tan 6x \quad 6\sec^2 6x$

9. $y = \sec 8x \quad y' = 8\sec 8x \cdot \tan 8x$

10. $y = \sec^2 x \quad y' = 2\sec x \cdot \sec x \tan x = 2\sec^2 \tan x$

11. $y = \ln \sec x \quad y' = \dfrac{1}{\sec x} \cdot \sec x \tan x = \tan x$

$= \cancel{\cos x \sec x \tan x}$

12. $y = \ln \sin x \quad y' = \dfrac{1}{\sin x}\cos x$

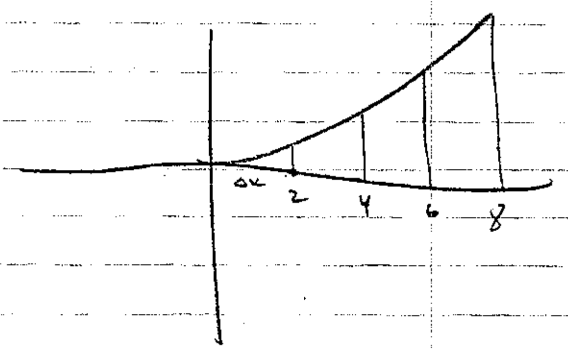
→ 13. $\lim_{x \to 0} \dfrac{1 - \cos^2 x}{x} = \dfrac{\sin^2 x}{x}$

$$\frac{\sin^2 x}{x} = \left[\frac{\sin^{\cancel{2}} x}{x}\right]\left[\sin x\right] \quad 1 \; ($$

14 $$\lim_{x \to 0} \frac{\sin 2x}{\sin x} = \lim_{x \to 0} \frac{2 \sin x \cos x}{\sin x} = \lim_{x \to 0} 2 \cos x$$

15. Trapezoidal Rule $\Delta x \left(\frac{y_0}{2} + y_1 + y_2 + \frac{y_n}{2} \right)$

$$\int_0^8 x^2 \, dx \qquad \Delta x = \frac{8-0}{4} = 2$$

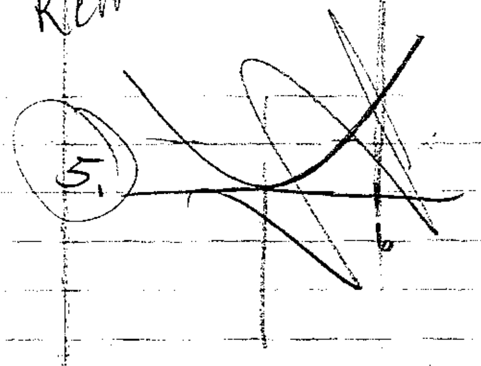


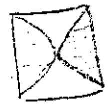
x	y	
0	0	$= y_0$
1	1	$= y_1$
2	4	$= y_2$
3	9	$= y_3$
4	16	$= y_4$

$$A = 2\left(\frac{0}{2} + 1 + 4 + 9 + \frac{16}{2}\right) = 2(1+4+9+8)$$
$$= 44$$

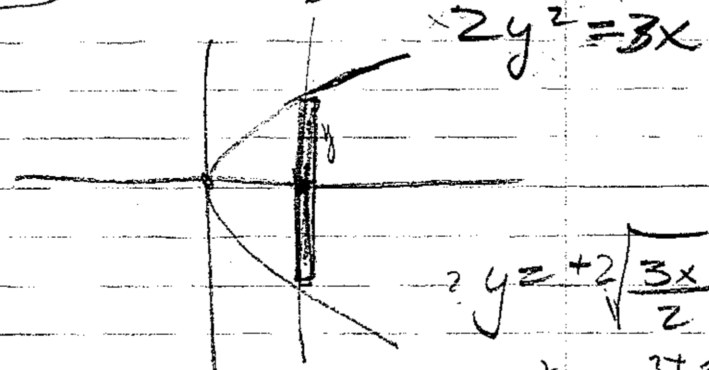
Rework

Quiz IV





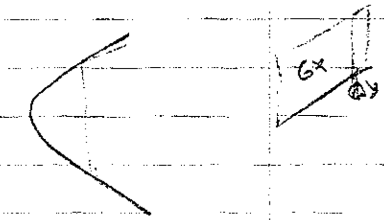
$2y^2 = 3x$



$y = \pm 2\sqrt{\frac{3x}{2}}$

$(2y)^2 = 4 \cdot \frac{3x}{2} = 6x$

$\underline{x=} \quad y$



Area of Square $= 2b$

$b = 2y = 2\sqrt{\frac{3x}{2}}$

$(2y)^2 =$ Area $= 4\sqrt{\frac{3x}{2}} =$

$\text{VoL} = \int (a(x)dx) = \frac{\sqrt{3}}{2} 4 \int_0^6 \sqrt{x}\, dx = 2\sqrt{3}\int_0^6 x^{1/2}\, dx$

$2\sqrt{3}\left[\frac{2x^{3/2}}{3}\right]_0^6 = \frac{4\sqrt{3}}{3}\left[x^{3/2}\right]_0^6 = \frac{4\sqrt{3}}{3}\left(6\sqrt{6}\right)$

$= 8\sqrt{3}\sqrt{6} = 8 \cdot 3\sqrt{2} = \boxed{24\sqrt{2}}$

36
6
36
216
3 | 72
3 | 24
3 | 8
4 | 2

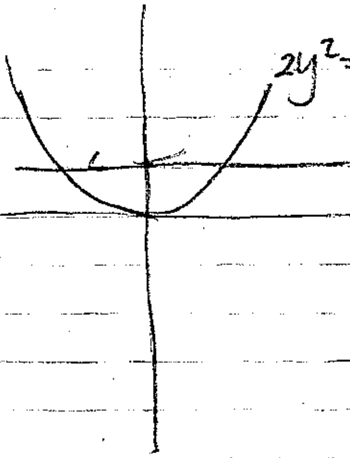
$3 \cdot 2\sqrt{6}$

36
$3 \cdot 3$
108

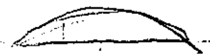
Quiz 4-22-76

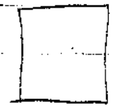
Quiz IV

5)



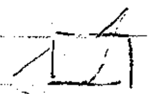
$y = \sqrt{\frac{3x}{2}}$ $x = \frac{2y^2}{3}$

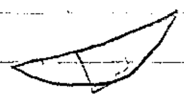




$y = x^2$

x y





9. $8 \sec 8x \tan 8x$ $\quad 8\left[\dfrac{1}{\cos 8x} \dfrac{\sin 8x}{\cos 8x}\right]$ $\quad \dfrac{\sin 8x}{1-\sin 8x}$

imp

Integrate $\int_0^{\pi} \sin^2 x \, dx$

use trig Identities $\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

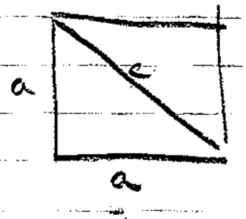
$\cos 2A = 1 - 2\sin^2 A$

$\int$

Supplemental Problem #10, page 266.

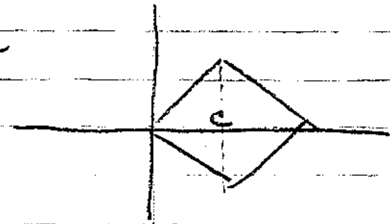
Due 4-20-76

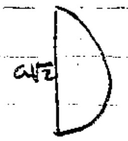
10



$$c^2 = a^2 + a^2 = 2a^2$$

$$c = a\sqrt{2}$$





$D = a\sqrt{2}$ $\qquad r = \dfrac{a\sqrt{2}}{2}$ $\qquad \text{Area} = \dfrac{\pi r^2}{2}$

$$\text{Area} = \frac{\pi}{2}\left[\frac{a\sqrt{2}}{2}\right]^2 = \frac{\pi}{2}\left[\frac{a^2\, 2}{4}\right] = \frac{\pi a^2}{4}$$

$$V = \int_a^b A(x)\,dx = \int_0^c \frac{\pi a^2}{4}\,da = \frac{\pi}{4}\int_0^c a^2\,da$$

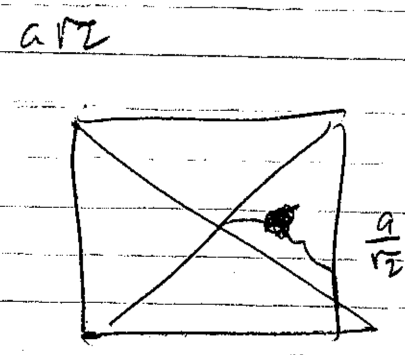
$$= \left[\frac{\pi}{4}\,\frac{a^3}{3}\right]_0^c = \frac{\pi}{12}\left[a^3\right]_0^c = \frac{\pi}{12}(a\sqrt{2})^3$$

$$= \frac{\pi}{12}\, a^3\, 2\sqrt{2} = \boxed{\frac{\sqrt{2}\,\pi a^3}{6}} = \text{Vol.}$$

$$\frac{\pi a^3\sqrt{2}}{12}$$

$$V = 2 \cdot \pi/2 \int_0^{a/\sqrt{2}} x^2\,dx = \frac{\pi}{3}\left[x^3\right]_0^{a/\sqrt{2}} =$$

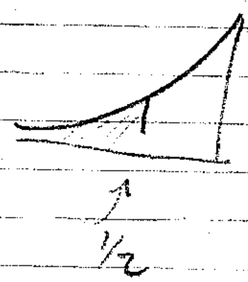
Robert C. Cline
Math 202, 1
4-7-76
Supplemental Problem



$$\frac{\pi a^3 \sqrt{2}}{12}$$

$$V = 2\frac{\pi}{2}\int_0^{\frac{a}{\sqrt{2}}} x^2\,dx$$

Must INTEGrate Then Double because



INVERSE functions. - Changing Roles of the Dependent + Independent Variables

Exercise 9.4, page 307

FIND the Explicit formulas for the inverse functions

#1

1. To Determine if inverse function $\left[f^{-1}(y)\right]$ of $f(x)$ Exists

1. If $\frac{dy}{dx}$ of $f(x)$ is Always increasing or Decreasing
2. $f^{-1}(y)$ of $f(x) = x$

$y = \frac{x-1}{x+1}$ $y' = \frac{2}{(x+1)^2}$ $x \neq -1$

$y = \frac{(x-1)}{x+1}$ $y(x+1) = x-1$ $x = y(x+1)+1 = yx+y+1$

$y+1 = x - yx = x(1-y) \Rightarrow x = \frac{1+y}{1-y} =$

therefore $f^{-1}(y) = \frac{1+y}{1-y}$

2. $y = \sqrt{\frac{x-1}{x+1}}$ $y^2 = \frac{x-1}{x+1}$ $y^2(x+1) = x-1$

$x = y^2x + y^2 + 1$ $x - y^2x = y^2+1$ $x(1-y^2) = y^2+1$

$x = f^{-1}(y) = \frac{y^2+1}{1-y^2}$

5. $x = \dfrac{\sqrt{y}+1}{\sqrt{y}-1}$ $(y>1)$ $\qquad x(\sqrt{y}-1) = \sqrt{y}+1$

$x\sqrt{y} - x = \sqrt{y} + 1 \qquad \sqrt{y} = x\sqrt{y} - x - 1$

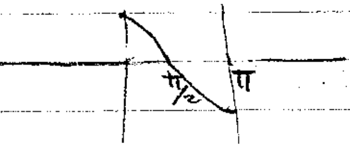
$\sqrt{y} - x\sqrt{y} = -x-1 \qquad \sqrt{y}\,(1-x) = -x-1 \qquad \sqrt{y} = \dfrac{-x-1}{1-x}$

$y = \dfrac{(-x-1)^2}{(1-x)^2} \qquad f^{-1}(x) = \dfrac{(-x-1)^2}{(1-x)^2} = \dfrac{x^2+2x+1}{x^2-2x+1} \quad (x>1)$

Decide which functions $y = f(x)$ has inverse $x = f^{-1}(y)$
Find $\dfrac{dy}{dx}$ and $\dfrac{dx}{dy}$ when it exists. (Deriv of inverse)

7. $y = \cos x \quad (0 \le x \le \pi)$

cos x



$y' = -\sin x$

− sin x

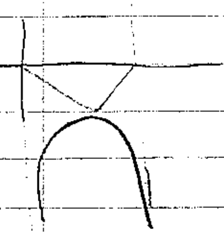
As INVERSE exists

$\dfrac{dx}{dy} = \dfrac{1}{dy/dx}$

$\dfrac{dx}{dy} = \dfrac{1}{-\sin x} = -\csc x \quad (0 \le x \le \pi)$

(−sin x)

(−csc x)



9. $y = \sqrt{x^2+1}$

$$y' = \tfrac{1}{2}(x^2+1)^{-1/2}\, 2x = \frac{x}{(x^2+1)^{1/2}}$$

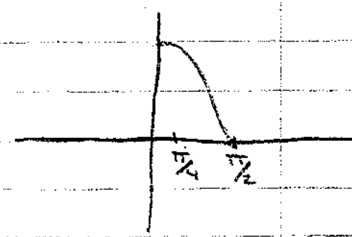
* ~~No derivative exists at $x=1$~~

$f'(x)$ is decreasing if $x<0$

increasing if $x>0$

Therefore no inverse exists.

11. $y = x\cos x \qquad 0 \le x \le \pi/4$



$$\frac{dy}{dx} = x(-\sin x) + \cos x$$

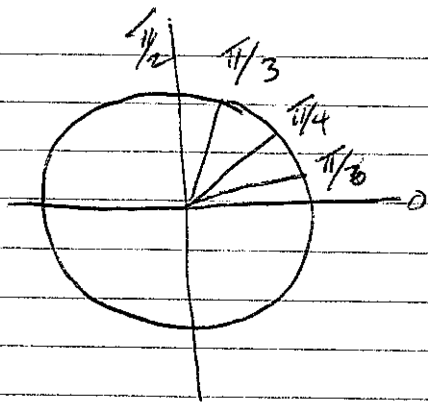
$$\frac{dx}{dy} = \frac{1}{dy/dx} = \frac{1}{-x(\sin x) + \cos x} = \frac{1}{\cos x - x\sin x}$$

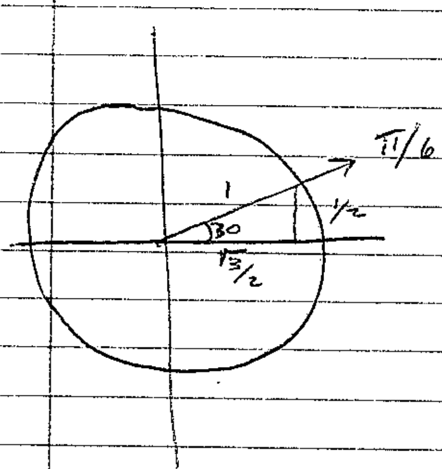
13. $y = x + \ln x$

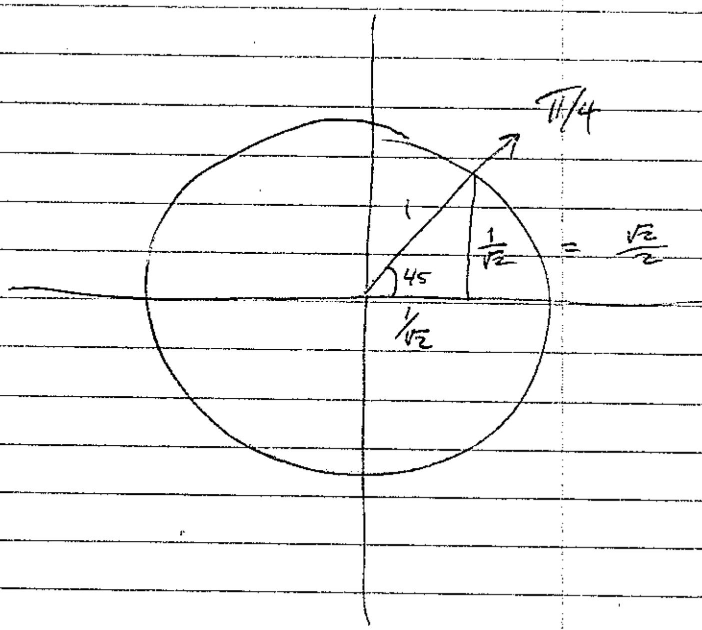
$$y' = 1 + \frac{1}{x} = \frac{x+1}{x}$$

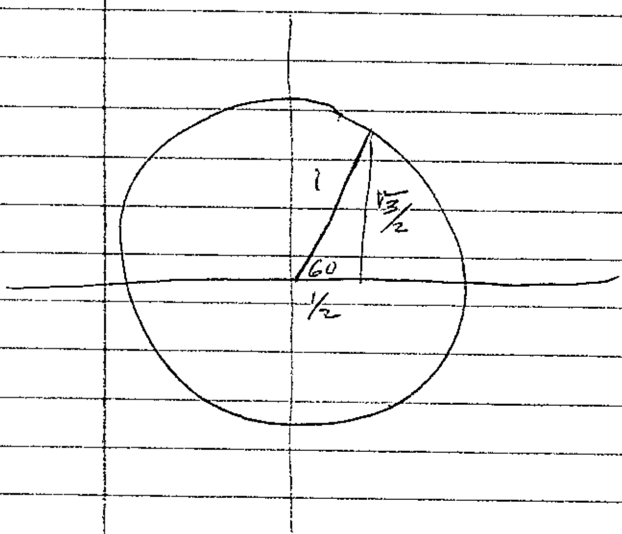
$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{\frac{x+1}{x}} = \frac{x}{x+1}$$

Memorize





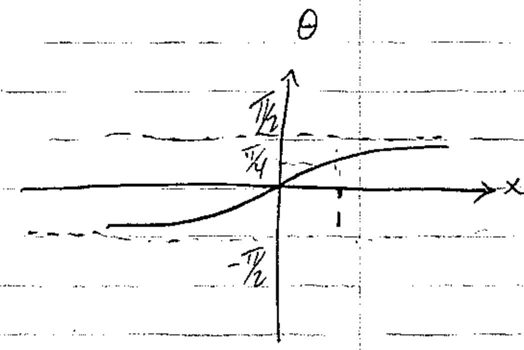




Class notes

4 $x = \tan\theta$ $\theta = \tan^{-1} x$

say same thing — theta is an angle whose tangent is x



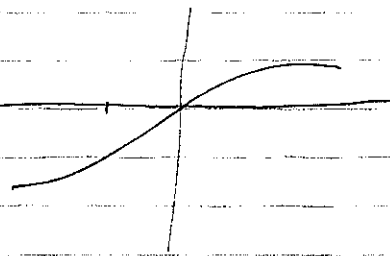
$-\infty < x < \infty$

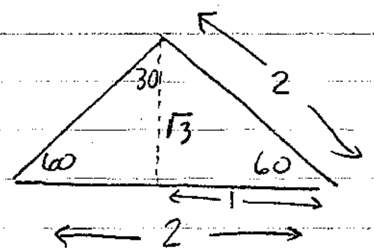
$-\pi/2 < \tan^{-1} x < \pi/2$

Evaluate

$\tan^{-1} = 1$ $\theta = 45$

$\tan^{-1}(-\sqrt{3}) =$ $\tan^{-1}(71.7)$



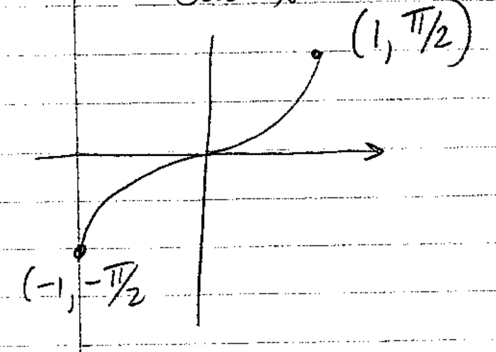


$h^2 + 1^2 = 2^2$

$h^2 =$

Class notes

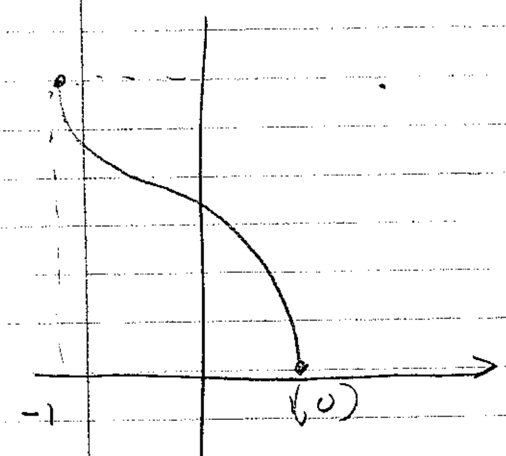
$\sin^{-1} x$



$x = \sin\theta$

domain $- \le x \le 1$
Range $-\pi/2 \le \sin^{-1} x \le \pi/2$

$\theta = \cos^{-1} x$



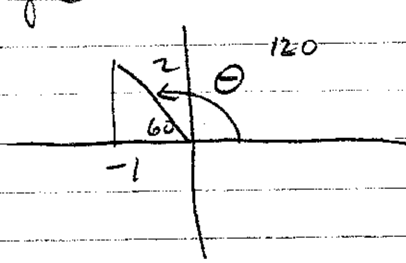
EVALuate

$\cos^{-1} \frac{\sqrt{2}}{2} = \pi/4 = 45^{0}$

$\cos^{-1}(-1/2) = 2\pi/3$

$-1 \le x \le 1$
$0 \le \cos^{-1} x \le \pi$

Reference angle



7. $\theta = \sin^{-1} \dfrac{1}{1+x^2}$

$$\frac{d\theta}{dx} = \frac{1}{\sqrt{1 - \frac{1}{(1+x^2)^2}}} \frac{du}{dy}(1+x^2)$$

$$= \frac{1}{\sqrt{\frac{(1+x^2)^2 - 1^2}{(1+x)^2}}} \quad \frac{-2x}{(1+x^2)^2}$$

$$\theta = \cos^{-1} x$$
$$\cos\theta = x$$

11. $\theta = \sin(\cos^{-1} x)$

$$\frac{d\theta}{dx} = \underbrace{\cos(\cos^{-1} x)}_{x} \frac{d}{dx} \cos^{-1} x$$

$$= x \frac{d}{dx} \cos^{-1} x$$

$$= -\underline{x}$$

---

proof

$$\int \tan x \, dx = \int \frac{\sin}{\cos}$$

Let $u = \cos x$
$du = -\sin x \, dx$

$$\int \cot x \, dx = \int \frac{\cos x \, dx}{\sin x} = \ln|\sin x| + C$$

$u = \sin x \quad du = \cos x \, dx$

$x^{3+1} = \frac{x^{11}}{11}$

12 $\int (\sin^3 x - \cos^3 x)\,dx$

$\int \sin^3 x \,\cancel{dx} \int \cos^3 x\, dx$

$\frac{\sin^4}{4} - \frac{\cos^4}{4} =$

12. $\int (\sin^3 x - \cos^3 x)\,dx$

$x^3 - y^3 = (x-y)(x^2 + xy + y^3)$

$= \int (\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)\,dx$

$\int (\sin x - \cos x)(1 + \sin x \cos x)\,dx$

$\int (\sin x + \sin^2 x \cos x - \cos x - \sin x \cos^2 x)\,dx$

$\int \sin x\, dx + \int \sin^2 x \cos x\, dx - \int \cos x\, dx - \int \cos^2 x \sin x\, dx$

$-\cos x + \frac{1}{3}\sin^3 x - \sin x + \frac{1}{3}\cos^3 x + C$

$-\ln|\cos x| + C$

10. $\int e^x \tan e^x \, dx$

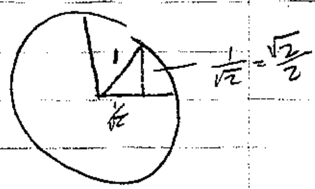
tan

INVERSE TRIGONOMETRIC functions
Page 316.

1. EVALUATE

a) $\sin^{-1}\frac{1}{\sqrt{2}} = \sin^{-1}\frac{\sqrt{2}}{2}$ $\quad \sin \pi/4 = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

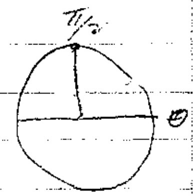
$\theta = 45° = \pi/4$



B.) $\cos^{-1}\frac{1}{\sqrt{2}} = 45°$

C.) $\sin^{-1}(1) - \sin^{-1}(-1)$

$\pi/2 - \pi/2$



d) $\tan^{-1}(\tan \pi/4) = \pi/4$

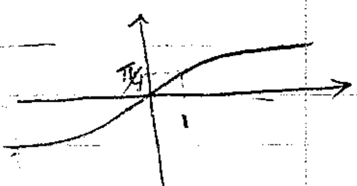
$\theta = \tan^{-1} x$

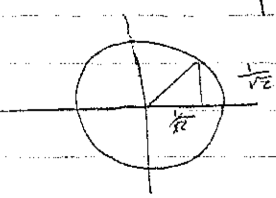
$x = \tan\theta$

$\tan^{-1}(x) = \theta$

2. $\tan^{-1}(2) + \tan^{-1}(-2)$

answer $= \emptyset$





$\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

$\tan 45° = \tan \pi/4 = 1$

2. Evaluate (con't)

b) $\sin^{-1}\left(\sin \pi/6\right) = \pi/6$

c) $\sin^{-1}\left[\sin\left(2\pi + \pi/6\right)\right] = 2\pi + \pi/6$

INVERSE TRIGONOMETRIC functions.

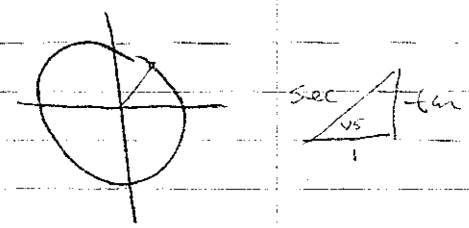
$\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ $\quad \frac{d}{dx}(\cos^{-1} u) = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$ $\quad \frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2}\frac{du}{dx}$

1. (a) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) =$

b) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

c) $\sin^{-1}(1) - \sin^{-1}(-1)$

d) $\tan^{-1}\left(\tan \pi/4\right) = \pi/4$



3. $\theta = \tan^{-1}\frac{a}{x}$

$$\frac{d\theta}{dx} = \frac{d}{dx}\left(\tan^{-1}\frac{a}{x}\right) = \frac{1}{1+\left(\frac{a}{x}\right)^2}\frac{d}{dx}\frac{a}{x} = \frac{1}{1+\frac{a^2}{x^2}}\cdot\frac{-a}{x^2}$$

$$= \frac{1}{\frac{x^2+a^2}{x^2}}\cdot\frac{-a}{x^2} = \frac{x^2}{x^2+a^2}\cdot\frac{-a}{x^2} = \boxed{\frac{-a}{x^2+a^2}}$$

5. $\theta = \sin^{-1}\frac{x}{2} + \cos^{-1}\frac{x}{2}$

$$\frac{d\theta}{dx} = \frac{1}{\left(1-\left(\frac{x}{2}\right)^2\right)^{1/2}}\cdot\frac{d}{dx}\left(\frac{x}{2}\right) + \frac{-1}{\sqrt{1-\frac{x^2}{4}}}\frac{d}{dx}\frac{x}{2}$$

$$= \frac{1}{2\left(1-\frac{x^2}{4}\right)^{1/2}} - \frac{1}{2\left(1-\frac{x^2}{4}\right)^{1/2}} = \emptyset$$

$$(1+x^2)(1+x^2) = 1+2x^2+x^4$$

7. $$\theta = \sin^{-1}\frac{1}{1+x^2} \Rightarrow$$

$$\frac{d\theta}{dx} = \frac{1}{\left(1-\left(\frac{1}{1+x^2}\right)^2\right)^{1/2}} \cdot \frac{d}{dx}\frac{1}{1+x^2}$$

$$\frac{1}{1-\frac{1}{1+2x^2+x^4}} \cdot \frac{-2\cancel{x}}{(1+x^2)^2} = \left[\frac{1}{1-\frac{1}{(1+x^2)^2}}\right]\frac{-2x}{(1+x^2)^2}$$

$$= \frac{(1+x^2)^2-1}{(1+x^2)^2} \cdot \frac{-2x}{(1+x^2)^2} = \frac{-2x(1+x^2)^2+2x}{(1+x^2)^4}$$

$$= \frac{-2x(1+2x^2+4x^4-1)}{(1+x^2)^4} = \frac{-2x(2x^2+4x^4)}{(1+x^2)^4} = \frac{-4x^3(1+2x^2)}{(1+x^2)^4}$$

INTEGRATION of TRIGONOMETRIC functions

P.330

1. $\int \sin 7x\,dx = -\frac{1}{7}\cdot \cos 7x + C$

$u = 7x$

$du = 7dx \qquad dx = \frac{du}{7}$

$-\frac{1}{7}\int \sin u \underbrace{7dx}_{du} = -\frac{1}{7} \sin 7x\,dx$

2. $\int x^2 \sin x^3\,dx \qquad u = x^3 \qquad du = 3x^2\,dx$

$\frac{du}{3} = x^2 dx$

$\frac{1}{3}\int x^2 \sin x^3 dx = \boxed{-\frac{1}{3} \sin x^3 + C}$

3. $\int \cos(2x+1)\,dx = \frac{1}{2} \sin(2x+1) + C$

4. $\int \frac{1}{x} \cos(\ln x)\,dx$

$u = \ln x \qquad du = \frac{1}{x}\,dx$

$= \cos(\ln x)$

5. $\int (\sin x + \cos x)^2 \, dx$

~~use~~ $\int \left(\sin^2 x + 2 \sin x \cos x + \cos^2 x \right) dx$

$\cos 2A = \cos^2\theta - \sin^2\theta$ $\qquad$ $\cos^2\theta = 1 - \sin^2\theta$

$= 1 - \sin^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$\cos 2a - 1 = -2\sin^2\theta$

$\sin^2\theta = (\cos 2A - 1)/\!-\tfrac{1}{2} = \tfrac{1}{2}(1 - \cos 2a)$

$\cos 2A = \cos^2\theta - \sin^2\theta$ $\qquad$ $\sin^2\theta = 1 - \cos^2$

$= \cos^2\theta - (1 - \cos^2\theta)$

$= 2\cos^2\theta - 1$

$\cos 2A + 1 = 2\cos^2\theta$

$\cos^2\theta = \tfrac{1}{2}(\cos 2A + 1)$

$\int (\sin^2 x + 2\sin x \cos x + \cos^2 x \, dx$

$= \int \tfrac{1}{2}(\cos 2A + 1)\, dx + \int 2 \sin x \cos x \, dx + \int \tfrac{1}{2}(1 - \cos 2A)$

Integ of Trig Funct.

5. $\int (\sin x + \cos x)^2 \, dx$

$\int \sin^2 x + 2 \sin x \cos x + \cos^2 x) \, dx$

$\int (2 \sin x + \cos x + 1) \, dx$

$\int (\sin 2\cancel{x} + 1) \, dx = \int \sin 2x \, dx + \int dx$

$= -\frac{1}{2} \cos 2x + x = \boxed{x - \frac{\cos 2x}{2}} + C$

6. $\int (\sin^2 x + \cos^2 x) \, dx = x$

7. $x \sec^2 (2x^2 + 1) \, dx$

$u = 2x^2 + 1 \quad du = 4x \, dx$

$\frac{du}{4} = x \, dx$

$= \frac{1}{4} \tan (2x^2 + 1) + C$

$\int \tan x \, dv = - \ln |\cos x| + C$

8. $\int \tan \frac{1}{5} x \, dx$

$u = \frac{1}{5} x \quad du = \frac{1}{5} \, dx$

$5 \ln |\cos x| + C$

9. $\int \cot \sqrt{x+1} \, \frac{dx}{\sqrt{x+1}}$

P. 330 Integ.

$\int \cot x = \ln|\sin x| + C$

9. $\int \cot\sqrt{x+1}\,dx$

$u = (x+1)^{1/2}$ $du = 1/2\,(x+1)^{-1/2}\,dx$

10. $\int e^x \tan e^x\,dx$ $= -\ln|\cos e^x| + C$

$u = e^x$ $du = e^x\,dx$

$dx = \frac{du}{e^x}$

$1-\cos^2$ $1-\cos^2$

$1-2\cos^2+\cos^4$

11. $\int \sin^4 x - \cos^4 x\,dx$ $\sin^2 = 1-\cos^2$

$\sin^4 = (1-\cos^2)^2 = 1-2\cos^2+\cos^4$

$\int 1-2\cos^2+\cos^4-\cos^4\,dx$

$\sin^2 x = 1-\cos^2 x$

$\int 1-2\cos^2 x\,dx$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = \cos^2 x - (1-\cos^2 x)$

$\int 1-(\cos 2x - 1)\,dx$

$= 2\cos^2 x + 1$

$\cos 2x - 1 = 2\cos^2 x$

$-\int \cos 2x$

$-1/2 \sin 2x + C$

$\boxed{-\frac{\sin 2x}{2} + C}$

$$4\int \overbrace{\sin^2 x}^{u^2}\ \overbrace{\cos x\,dx}^{du}$$

$$= \frac{4}{3}\sin^3 x + C$$

13. $= \int (\cos^3 x - 3\sin^2 \cos x)\,dx$

$[\cos^2 \cos x - 3\sin^2 x \cos x)$

$[(1-\sin^2 x)\cos x - 3\sin^2 x \cos x)$

$= (\cos x - \sin^2 x \cos - 3\sin 2 \cos x$

Important Problem

INTEGRATION OF TRIGONOMETRIC functions.

12. $\int (\sin^3 x - \cos^3 x)\, dx$

$= \int (\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)\, dx$

$\int (\sin x - \cos x)(1 + \sin x \cos x)\, dx$

$\int (\sin x + \sin^2 x \cos x - \cos x - \sin x \cos^2 x)\, dx$

$\int \sin x\, dx + \int \sin^2 x \cos x\, dx - \int \cos x\, dx - \int \sin x \cos^2 x\, dx$

$-\cos x - \sin x + \int \sin^2 x \cos x\, dx - \int \sin x \cos^2 x\, dx$

$-\cos x - \sin x + \frac{1}{3}\sin^3 x + \frac{1}{3}\cos^3 + C$

$\int (\sin x)^2 \cos x\, dx$

$u = \sin x \quad du = \cos x$

$\int u^2\, du = \frac{1}{3}u^3 = \frac{1}{3}\sin^3 x$

P. 330 Integ.

12. $\int (\sin^3 x - \cos^3 x)\,dx$

$\int \sin^3 x - \int \cos^3 x\,dx$ $\sin^2 = 1-\cos^2$

$\sin^3 = \sin(1-\cos^2)$

$\cos^2 = 1-\sin^2$

$\cos^3 = \cos(1-\sin^2)$

$\sin x - \sin\cos^2 x - [\cos x - \sin^2\cos x$

$\sin - \cos - \sin\cos^2 + \sin^2\cos x$

$\sin - \cos + \sin\cos(\sin - \cos)$

$(\sin - \cos)(1 + \sin\cos)$ $2\sin\cos = \sin 2x$

$\sin\cos = \frac{\sin 2x}{2}$

$(\sin - \cos)\left(1 - \frac{\sin 2x}{2}\right)$

$\sin x - \cos x \frac{\sin 2x}{2} - \cos x + \sin x \frac{\sin 2x}{2}$

$\sin x - \cos x - \cos x \frac{\sin 2x}{2} + \sin x \frac{\sin 2x}{2}$