

# How to calculate Z scores in R

**Cholesterol levels in men 18-24 years of age are normally distributed with a mean of 178 and a SD of 40.** <sup>1</sup>

## 1. In what percentile is a man with cholesterol level of 190?

To convert a Z score to a percentile, you need to use the `pnorm()` function in R to calculate the cumulative probability up to the Z score. This cumulative probability represents the percentile below the Z score, not above it.

- $Z = \text{Obs} - \text{mean} / \text{SD}$
- $Z = (190 - 178) / 40$

```
[1] 0.3
```

- The Z score of 0.3 is the SD above the mean.  
To convert to a percentile, use `pnorm() * 100`
- Convert Z score to Probability
- `percentile <- pnorm(Zscore)`

```
percentile <- pnorm(0.3)
percentile
```

```
[1] 0.6179114
```

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<sup>1</sup>These problems are found in the tutorial from David Longstreet's *statisticsisfun* channel, "How to calculate Z scores" (2011), <https://www.youtube.com/watch?v=Ypf6eAP9aFg>

**2. What is the probability that a man in this age group has a cholesterol level greater than 145?**

*i.e. All of the are to the right of 145.*

```
z145 <- (145-178)/40  
1- pnorm(z145)
```

```
[1] 0.7953142
```

**3. What is the probability that a man in this age group has a cholesterol level greater than 178?**

```
z178 <- (178-178)/40  
1-pnorm(z178)
```

```
[1] 0.5
```

**4. What Cholesterol level corresponds to the 75th percentile?**

$$Z = (X - \text{mean})/SD$$

You would apply the inverse cdf to the percentile ranks to convert them to quantiles, so if you want standard normals,  $z=\Phi^{-1}(p)$  should do what you seem to be asking for.

- To convert a value, let's call it X, to the 75th percentile in R when the mean is 178 and the standard deviation is 40, you need to calculate the corresponding Z score and then find the value that corresponds to that Z score in a standard normal distribution.

- The formula to calculate the Z score is:

$$- Z = (X - \text{mean}) / \text{standard deviation}$$

- **To convert the Z score to the corresponding percentile, you can use the `qnorm()` function in R.**

- The `qnorm()` function calculates the quantile (value) of a standard normal distribution given a probability.

- $Z \leftarrow \text{qnorm}(\text{percentile}/100)$  = Z score corresponding to the 75th percentile, then,  $X \leftarrow Z \times \text{SD} + \text{mean}$  calculates the X corresponding to that Z score.

```
mean <- 178
std_dev <- 40
percentile <- 75

Z <- qnorm(percentile/100)
X <- Z * std_dev + mean
X
```

```
[1] 204.9796
```

**5. What is the probability that a man in this age group has a cholesterol level between 155 and 185?**

```
probability1 <- pnorm((185-178)/40) - pnorm((155-178)/40)
probability1
```

```
[1] 0.2868145
```

**6. If a sample of 100 men is selected, what is the probability that the mean cholesterol level is greater than 175?**

$$Z = \frac{X - \text{mean}}{\frac{\text{SD}}{\sqrt{\text{sample size}}}}$$

```
Z100 <- (175-178)/(40/10)

p <- 1-pnorm(Z100)
p
```

```
[1] 0.7733726
```