In[1]:= **T0 = 1**

Out[1]= 1

In[2]:= **T1 = (1 / 2) * c1**

Out[2]= $\dfrac{c1}{2}$

In[3]:= **T2 = (1 / 12) * (c1 ^ 2 + c2)**

Out[3]= $\dfrac{1}{12}\left(c1^2 + c2\right)$

In[4]:= **T3 = (1 / 24) * (c1 * c2)**

Out[4]= $\dfrac{c1\ c2}{24}$

In[5]:= **T4 = – (1 / 720) * (c1 ^ 4 – 4 * c1 ^ 2 * c2 – 3 * c2 ^ 2 – c1 * c3 + c4)**

Out[5]= $\dfrac{1}{720}\left(-c1^4 + 4\ c1^2\ c2 + 3\ c2^2 + c1\ c3 - c4\right)$

In[6]:= **T5 = – (1 / 1440) * (c1 ^ 3 * c2 – 3 * c1 * c2 ^ 2 – c1 ^ 2 * c3 + c1 * c4)**

Out[6]= $\dfrac{-c1^3\ c2 + 3\ c1\ c2^2 + c1^2\ c3 - c1\ c4}{1440}$

In[7]:=

T6 = (1 / 60 480) * (2 * c1 ^ 6 – 12 * c1 ^ 4 * c2 + 11 * c1 ^ 2 * c2 ^ 2 + 10 * c2 ^ 3 + 5 * c1 ^ 3 * c3 +
 11 * c1 * c2 * c3 – c3 ^ 2 – 5 * c1 ^ 2 * c4 – 9 * c2 * c4 – 2 * c1 * c5 + 2 * c6)

Out[7]= $\dfrac{1}{60\,480}\left(2\ c1^6 - 12\ c1^4\ c2 + 11\ c1^2\ c2^2 + 10\ c2^3 + \right.$
$\left. 5\ c1^3\ c3 + 11\ c1\ c2\ c3 - c3^2 - 5\ c1^2\ c4 - 9\ c2\ c4 - 2\ c1\ c5 + 2\ c6\right)$

In[8]:= **C0 = r**

Out[8]= r

In[9]:= **C1 = d1**

Out[9]= d1

In[10]:= **C2 = (1 / 2) * (d1 ^ 2 – 2 * d2)**

Out[10]=

$\dfrac{1}{2}\left(d1^2 - 2\ d2\right)$

In[11]:= **C3 = (1 / 6) * (d1 ^ 3 – 3 * d1 * d2 + 3 * d3)**

Out[11]=

$\dfrac{1}{6}\left(d1^3 - 3\ d1\ d2 + 3\ d3\right)$

In[12]:= `C4 = (1 / 24) * (d1^4 - 4 * d1^2 * d2 + 4 * d1 * d3 + 2 * d2^2 - 4 * d4)`

Out[12]=

$$\frac{1}{24}\left(d1^4 - 4\,d1^2\,d2 + 2\,d2^2 + 4\,d1\,d3 - 4\,d4\right)$$

In[13]:= `C5 = (1 / 120) *`
`(d1^5 - 5 * d1^3 * d2 + 5 * d1 * d2^2 + 5 * d1^2 * d3 - 5 * d2 * d3 - 5 * d1 * d4 + 5 * d5)`

Out[13]=

$$\frac{1}{120}\left(d1^5 - 5\,d1^3\,d2 + 5\,d1\,d2^2 + 5\,d1^2\,d3 - 5\,d2\,d3 - 5\,d1\,d4 + 5\,d5\right)$$

In[14]:= `C6 = (1 / 720) * (d1^6 - 6 * d1^4 * d2 + 9 * d1^2 * d2^2 - 2 * d2^3 + 6 * d1^3 * d3 -`
`12 * d1 * d2 * d3 + 3 * d3^2 - 6 * d1^2 * d4 + 6 * d2 * d4 + 6 * d1 * d5 - 6 * d6)`

Out[14]=

$$\frac{1}{720}\left(d1^6 - 6\,d1^4\,d2 + 9\,d1^2\,d2^2 - 2\,d2^3 + \right.$$
$$\left. 6\,d1^3\,d3 - 12\,d1\,d2\,d3 + 3\,d3^2 - 6\,d1^2\,d4 + 6\,d2\,d4 + 6\,d1\,d5 - 6\,d6\right)$$

In[15]:= `C7 = (1 / 5040) * (d1^7 - 7 * d1^5 * d2 + 14 * d1^3 * d2^2 - 7 * d1 * d2^3 +`
`7 * d1^4 * d3 - 21 * d1^2 * d2 * d3 + 7 * d2^2 * d3 + 7 * d1 * d3^2 - 7 * d1^3 * d4 +`
`14 * d1 * d2 * d4 - 7 * d3 * d4 + 7 * d1^2 * d5 - 7 * d2 * d5 - 7 * d1 * d6 + 7 * d7)`

Out[15]=

$$\frac{1}{5040}\left(d1^7 - 7\,d1^5\,d2 + 14\,d1^3\,d2^2 - 7\,d1\,d2^3 + 7\,d1^4\,d3 - 21\,d1^2\,d2\,d3 + 7\,d2^2\,d3 + \right.$$
$$\left. 7\,d1\,d3^2 - 7\,d1^3\,d4 + 14\,d1\,d2\,d4 - 7\,d3\,d4 + 7\,d1^2\,d5 - 7\,d2\,d5 - 7\,d1\,d6 + 7\,d7\right)$$

In[16]:= `Expand[r * Chi + C1 * T3 + C2 * T2 + C3 * T1 + C4 * T0]`

Out[16]=

$$\frac{c1\,c2\,d1}{24} + \frac{c1^2\,d1^2}{24} + \frac{c2\,d1^2}{24} + \frac{c1\,d1^3}{12} + \frac{d1^4}{24} - \frac{c1^2\,d2}{12} -$$
$$\frac{c2\,d2}{12} - \frac{c1\,d1\,d2}{4} - \frac{d1^2\,d2}{6} + \frac{d2^2}{12} + \frac{c1\,d3}{4} + \frac{d1\,d3}{6} - \frac{d4}{6} + \text{Chi }r$$

In[17]:= `Expand[6 * (-r * d + %16 + (d4 / 6))]`

Out[17]=

$$\frac{c1\,c2\,d1}{4} + \frac{c1^2\,d1^2}{4} + \frac{c2\,d1^2}{4} + \frac{c1\,d1^3}{2} + \frac{d1^4}{4} - \frac{c1^2\,d2}{2} -$$
$$\frac{c2\,d2}{2} - \frac{3\,c1\,d1\,d2}{2} - d1^2\,d2 + \frac{d2^2}{2} + \frac{3\,c1\,d3}{2} + d1\,d3 + 6\,\text{Chi }r - 6\,d\,r$$

In[18]:= `Expand[r * Chi + C1 * T4 + C2 * T3 + C3 * T2 + C4 * T1 + C5 * T0]`

Out[18]=

$$-\frac{c1^4\,d1}{720} + \frac{1}{180}c1^2\,c2\,d1 + \frac{c2^2\,d1}{240} + \frac{c1\,c3\,d1}{720} - \frac{c4\,d1}{720} + \frac{1}{48}c1\,c2\,d1^2 + \frac{c1^2\,d1^3}{72} + \frac{c2\,d1^3}{72} +$$
$$\frac{c1\,d1^4}{48} + \frac{d1^5}{120} - \frac{c1\,c2\,d2}{24} - \frac{1}{24}c1^2\,d1\,d2 - \frac{c2\,d1\,d2}{24} - \frac{1}{12}c1\,d1^2\,d2 - \frac{d1^3\,d2}{24} + \frac{c1\,d2^2}{24} +$$
$$\frac{d1\,d2^2}{24} + \frac{c1^2\,d3}{24} + \frac{c2\,d3}{24} + \frac{c1\,d1\,d3}{12} + \frac{d1^2\,d3}{24} - \frac{d2\,d3}{24} - \frac{c1\,d4}{12} - \frac{d1\,d4}{24} + \frac{d5}{24} + \text{Chi }r$$

In[19]:= **Expand[24 * (r * d - %18 + (d5 / 24))]**

Out[19]=

$$\frac{c1^4\,d1}{30} - \frac{2}{15}\,c1^2\,c2\,d1 - \frac{c2^2\,d1}{10} - \frac{c1\,c3\,d1}{30} + \frac{c4\,d1}{30} - \frac{1}{2}\,c1\,c2\,d1^2 - \frac{c1^2\,d1^3}{3} - \frac{c2\,d1^3}{3} -$$

$$\frac{c1\,d1^4}{2} - \frac{d1^5}{5} + c1\,c2\,d2 + c1^2\,d1\,d2 + c2\,d1\,d2 + 2\,c1\,d1^2\,d2 + d1^3\,d2 - c1\,d2^2 -$$

$$d1\,d2^2 - c1^2\,d3 - c2\,d3 - 2\,c1\,d1\,d3 - d1^2\,d3 + d2\,d3 + 2\,c1\,d4 + d1\,d4 - 24\,Chi\,r + 24\,d\,r$$

In[20]:= **Expand[r * Chi + C1 * T5 + C2 * T4 + C3 * T3 + C4 * T2 + C5 * T1 + C6 * T0]**

Out[20]=

$$-\frac{c1^3\,c2\,d1}{1440} + \frac{1}{480}\,c1\,c2^2\,d1 + \frac{c1^2\,c3\,d1}{1440} - \frac{c1\,c4\,d1}{1440} - \frac{c1^4\,d1^2}{1440} + \frac{1}{360}\,c1^2\,c2\,d1^2 + \frac{c2^2\,d1^2}{480} +$$

$$\frac{c1\,c3\,d1^2}{1440} - \frac{c4\,d1^2}{1440} + \frac{1}{144}\,c1\,c2\,d1^3 + \frac{c1^2\,d1^4}{288} + \frac{c2\,d1^4}{288} + \frac{c1\,d1^5}{240} + \frac{d1^6}{720} + \frac{c1^4\,d2}{720} -$$

$$\frac{1}{180}\,c1^2\,c2\,d2 - \frac{c2^2\,d2}{240} - \frac{c1\,c3\,d2}{720} + \frac{c4\,d2}{720} - \frac{1}{48}\,c1\,c2\,d1\,d2 - \frac{1}{72}\,c1^2\,d1^2\,d2 -$$

$$\frac{1}{72}\,c2\,d1^2\,d2 - \frac{1}{48}\,c1\,d1^3\,d2 - \frac{d1^4\,d2}{120} + \frac{c1^2\,d2^2}{144} + \frac{c2\,d2^2}{144} + \frac{1}{48}\,c1\,d1\,d2^2 + \frac{d1^2\,d2^2}{80} - \frac{d2^3}{360} +$$

$$\frac{c1\,c2\,d3}{48} + \frac{1}{72}\,c1^2\,d1\,d3 + \frac{c2\,d1\,d3}{72} + \frac{1}{48}\,c1\,d1^2\,d3 + \frac{d1^3\,d3}{120} - \frac{c1\,d2\,d3}{48} - \frac{d1\,d2\,d3}{60} +$$

$$\frac{d3^2}{240} - \frac{c1^2\,d4}{72} - \frac{c2\,d4}{72} - \frac{c1\,d1\,d4}{48} - \frac{d1^2\,d4}{120} + \frac{d2\,d4}{120} + \frac{c1\,d5}{48} + \frac{d1\,d5}{120} - \frac{d6}{120} + Chi\,r$$

In[21]:= **Expand[120 * (%20 - r * d + (d6 / 120))]**

Out[21]=

$$-\frac{1}{12}\,c1^3\,c2\,d1 + \frac{1}{4}\,c1\,c2^2\,d1 + \frac{1}{12}\,c1^2\,c3\,d1 - \frac{c1\,c4\,d1}{12} - \frac{c1^4\,d1^2}{12} + \frac{1}{3}\,c1^2\,c2\,d1^2 + \frac{c2^2\,d1^2}{4} +$$

$$\frac{1}{12}\,c1\,c3\,d1^2 - \frac{c4\,d1^2}{12} + \frac{5}{6}\,c1\,c2\,d1^3 + \frac{5\,c1^2\,d1^4}{12} + \frac{5\,c2\,d1^4}{12} + \frac{c1\,d1^5}{2} + \frac{d1^6}{6} + \frac{c1^4\,d2}{6} -$$

$$\frac{2}{3}\,c1^2\,c2\,d2 - \frac{c2^2\,d2}{2} - \frac{c1\,c3\,d2}{6} + \frac{c4\,d2}{6} - \frac{5}{2}\,c1\,c2\,d1\,d2 - \frac{5}{3}\,c1^2\,d1^2\,d2 - \frac{5}{3}\,c2\,d1^2\,d2 -$$

$$\frac{5}{2}\,c1\,d1^3\,d2 - d1^4\,d2 + \frac{5\,c1^2\,d2^2}{6} + \frac{5\,c2\,d2^2}{6} + \frac{5}{2}\,c1\,d1\,d2^2 + \frac{3\,d1^2\,d2^2}{2} - \frac{d2^3}{3} + \frac{5\,c1\,c2\,d3}{2} +$$

$$\frac{5}{3}\,c1^2\,d1\,d3 + \frac{5\,c2\,d1\,d3}{3} + \frac{5}{2}\,c1\,d1^2\,d3 + d1^3\,d3 - \frac{5\,c1\,d2\,d3}{2} - 2\,d1\,d2\,d3 + \frac{d3^2}{2} -$$

$$\frac{5\,c1^2\,d4}{3} - \frac{5\,c2\,d4}{3} - \frac{5\,c1\,d1\,d4}{2} - d1^2\,d4 + d2\,d4 + \frac{5\,c1\,d5}{2} + d1\,d5 + 120\,Chi\,r - 120\,d\,r$$

In[22]:= `Expand[r * Chi + C1 * T6 + C2 * T5 + C3 * T4 + C4 * T3 + C5 * T2 + C6 * T1 + C7 * T0]`

Out[22]=

$$\frac{c1^6\, d1}{30240} - \frac{c1^4\, c2\, d1}{5040} + \frac{11\, c1^2\, c2^2\, d1}{60480} + \frac{c2^3\, d1}{6048} + \frac{c1^3\, c3\, d1}{12096} + \frac{11\, c1\, c2\, c3\, d1}{60480} - \frac{c3^2\, d1}{60480} -$$

$$\frac{c1^2\, c4\, d1}{12096} - \frac{c2\, c4\, d1}{6720} - \frac{c1\, c5\, d1}{30240} + \frac{c6\, d1}{30240} - \frac{c1^3\, c2\, d1^2}{2880} + \frac{1}{960}\, c1\, c2^2\, d1^2 + \frac{c1^2\, c3\, d1^2}{2880} -$$

$$\frac{c1\, c4\, d1^2}{2880} - \frac{c1^4\, d1^3}{4320} + \frac{c1^2\, c2\, d1^3}{1080} + \frac{c2^2\, d1^3}{1440} + \frac{c1\, c3\, d1^3}{4320} - \frac{c4\, d1^3}{4320} + \frac{1}{576}\, c1\, c2\, d1^4 +$$

$$\frac{c1^2\, d1^5}{1440} + \frac{c2\, d1^5}{1440} + \frac{c1\, d1^6}{1440} + \frac{d1^7}{5040} + \frac{c1^3\, c2\, d2}{1440} - \frac{1}{480}\, c1\, c2^2\, d2 - \frac{c1^2\, c3\, d2}{1440} +$$

$$\frac{c1\, c4\, d2}{1440} + \frac{c1^4\, d1\, d2}{1440} - \frac{1}{360}\, c1^2\, c2\, d1\, d2 - \frac{1}{480}\, c2^2\, d1\, d2 - \frac{c1\, c3\, d1\, d2}{1440} + \frac{c4\, d1\, d2}{1440} -$$

$$\frac{1}{144}\, c1\, c2\, d1^2\, d2 - \frac{1}{288}\, c1^2\, d1^3\, d2 - \frac{1}{288}\, c2\, d1^3\, d2 - \frac{1}{240}\, c1\, d1^4\, d2 - \frac{d1^5\, d2}{720} +$$

$$\frac{1}{288}\, c1\, c2\, d2^2 + \frac{1}{288}\, c1^2\, d1\, d2^2 + \frac{1}{288}\, c2\, d1\, d2^2 + \frac{1}{160}\, c1\, d1^2\, d2^2 + \frac{d1^3\, d2^2}{360} - \frac{c1\, d2^3}{720} -$$

$$\frac{d1\, d2^3}{720} - \frac{c1^4\, d3}{1440} + \frac{1}{360}\, c1^2\, c2\, d3 + \frac{c2^2\, d3}{480} + \frac{c1\, c3\, d3}{1440} - \frac{c4\, d3}{1440} + \frac{1}{144}\, c1\, c2\, d1\, d3 +$$

$$\frac{1}{288}\, c1^2\, d1^2\, d3 + \frac{1}{288}\, c2\, d1^2\, d3 + \frac{1}{240}\, c1\, d1^3\, d3 + \frac{d1^4\, d3}{720} - \frac{1}{288}\, c1^2\, d2\, d3 -$$

$$\frac{c2\, d2\, d3}{288} - \frac{1}{120}\, c1\, d1\, d2\, d3 - \frac{1}{240}\, d1^2\, d2\, d3 + \frac{d2^2\, d3}{720} + \frac{c1\, d3^2}{480} + \frac{d1\, d3^2}{720} - \frac{c1\, c2\, d4}{144} -$$

$$\frac{1}{288}\, c1^2\, d1\, d4 - \frac{c2\, d1\, d4}{288} - \frac{1}{240}\, c1\, d1^2\, d4 - \frac{d1^3\, d4}{720} + \frac{c1\, d2\, d4}{240} + \frac{d1\, d2\, d4}{360} -$$

$$\frac{d3\, d4}{720} + \frac{c1^2\, d5}{288} + \frac{c2\, d5}{288} + \frac{c1\, d1\, d5}{240} + \frac{d1^2\, d5}{720} - \frac{d2\, d5}{720} - \frac{c1\, d6}{240} - \frac{d1\, d6}{720} + \frac{d7}{720} + Chi\, r$$

In[23]:= **Expand[720 * (r * d - %22 + (d7 / 720))]**

Out[23]=

$$-\frac{c1^6 \, d1}{42} + \frac{1}{7} \, c1^4 \, c2 \, d1 - \frac{11}{84} \, c1^2 \, c2^2 \, d1 - \frac{5 \, c2^3 \, d1}{42} - \frac{5}{84} \, c1^3 \, c3 \, d1 - \frac{11}{84} \, c1 \, c2 \, c3 \, d1 + \frac{c3^2 \, d1}{84} +$$
$$\frac{5}{84} \, c1^2 \, c4 \, d1 + \frac{3 \, c2 \, c4 \, d1}{28} + \frac{c1 \, c5 \, d1}{42} - \frac{c6 \, d1}{42} + \frac{1}{4} \, c1^3 \, c2 \, d1^2 - \frac{3}{4} \, c1 \, c2^2 \, d1^2 - \frac{1}{4} \, c1^2 \, c3 \, d1^2 +$$
$$\frac{1}{4} \, c1 \, c4 \, d1^2 + \frac{c1^4 \, d1^3}{6} - \frac{2}{3} \, c1^2 \, c2 \, d1^3 - \frac{c2^2 \, d1^3}{2} - \frac{1}{6} \, c1 \, c3 \, d1^3 + \frac{c4 \, d1^3}{6} - \frac{5}{4} \, c1 \, c2 \, d1^4 -$$
$$\frac{c1^2 \, d1^5}{2} - \frac{c2 \, d1^5}{2} - \frac{c1 \, d1^6}{2} - \frac{d1^7}{7} - \frac{1}{2} \, c1^3 \, c2 \, d2 + \frac{3}{2} \, c1 \, c2^2 \, d2 + \frac{1}{2} \, c1^2 \, c3 \, d2 - \frac{c1 \, c4 \, d2}{2} -$$
$$\frac{1}{2} \, c1^4 \, d1 \, d2 + 2 \, c1^2 \, c2 \, d1 \, d2 + \frac{3}{2} \, c2^2 \, d1 \, d2 + \frac{1}{2} \, c1 \, c3 \, d1 \, d2 - \frac{c4 \, d1 \, d2}{2} + 5 \, c1 \, c2 \, d1^2 \, d2 +$$
$$\frac{5}{2} \, c1^2 \, d1^3 \, d2 + \frac{5}{2} \, c2 \, d1^3 \, d2 + 3 \, c1 \, d1^4 \, d2 + d1^5 \, d2 - \frac{5}{2} \, c1 \, c2 \, d2^2 - \frac{5}{2} \, c1^2 \, d1 \, d2^2 - \frac{5}{2} \, c2 \, d1 \, d2^2 -$$
$$\frac{9}{2} \, c1 \, d1^2 \, d2^2 - 2 \, d1^3 \, d2^2 + c1 \, d2^3 + d1 \, d2^3 + \frac{c1^4 \, d3}{2} - 2 \, c1^2 \, c2 \, d3 - \frac{3 \, c2^2 \, d3}{2} - \frac{c1 \, c3 \, d3}{2} +$$
$$\frac{c4 \, d3}{2} - 5 \, c1 \, c2 \, d1 \, d3 - \frac{5}{2} \, c1^2 \, d1^2 \, d3 - \frac{5}{2} \, c2 \, d1^2 \, d3 - 3 \, c1 \, d1^3 \, d3 - d1^4 \, d3 + \frac{5}{2} \, c1^2 \, d2 \, d3 +$$
$$\frac{5 \, c2 \, d2 \, d3}{2} + 6 \, c1 \, d1 \, d2 \, d3 + 3 \, d1^2 \, d2 \, d3 - d2^2 \, d3 - \frac{3 \, c1 \, d3^2}{2} - d1 \, d3^2 + 5 \, c1 \, c2 \, d4 +$$
$$\frac{5}{2} \, c1^2 \, d1 \, d4 + \frac{5 \, c2 \, d1 \, d4}{2} + 3 \, c1 \, d1^2 \, d4 + d1^3 \, d4 - 3 \, c1 \, d2 \, d4 - 2 \, d1 \, d2 \, d4 + d3 \, d4 -$$
$$\frac{5 \, c1^2 \, d5}{2} - \frac{5 \, c2 \, d5}{2} - 3 \, c1 \, d1 \, d5 - d1^2 \, d5 + d2 \, d5 + 3 \, c1 \, d6 + d1 \, d6 - 720 \, \text{Chi} \, r + 720 \, d \, r$$

In[24]:= **ReplaceAll[(1 / 2) * (d1^2 - d1 * KX) * H^{n - 2} +**
(r / 12) * (KX^2 + c2 - ((3 * n^2 + 5 * n + 2) / 2) * H^2) * H^{n - 2},
{d1 → (r / 2) * (d - 1) * H, KX → (d - n - 2) * H,
c2 → (Binomial[n + 2, 2] + d * (d - n - 2)) * H^2}]

Out[24]=

$$\left\{ \frac{1}{12} \, H^{-2+n} \left(H^2 \, (-2 + d - n)^2 - \frac{1}{2} \, H^2 \, (2 + 5 \, n + 3 \, n^2) + H^2 \left(d \, (-2 + d - n) + \frac{1}{2} \, (1 + n) \, (2 + n) \right) \right) r + \right.$$
$$\left. \frac{1}{2} \, H^{-2+n} \left(-\frac{1}{2} \, (-1 + d) \, H^2 \, (-2 + d - n) \, r + \frac{1}{4} \, (-1 + d)^2 \, H^2 \, r^2 \right) \right\}$$

In[25]:= **Expand[%]**

Out[25]=

$$\left\{ -\frac{H^n \, r}{6} + \frac{1}{4} \, d \, H^n \, r - \frac{1}{12} \, d^2 \, H^n \, r + \frac{H^n \, r^2}{8} - \frac{1}{4} \, d \, H^n \, r^2 + \frac{1}{8} \, d^2 \, H^n \, r^2 \right\}$$

In[26]:= **ReplaceAll[Out[25], {H^n → d}]**

Out[26]=

$$\left\{ -\frac{d \, r}{6} + \frac{d^2 \, r}{4} - \frac{d^3 \, r}{12} + \frac{d \, r^2}{8} - \frac{d^2 \, r^2}{4} + \frac{d^3 \, r^2}{8} \right\}$$

In[27]:= **Factor[%]**

Out[27]=

$$\left\{ \frac{1}{24} \, (-1 + d) \, d \, r \, (4 - 2 \, d - 3 \, r + 3 \, d \, r) \right\}$$

In[28]:= `Expand[% / d]`

Out[28]=

$$\left\{-\frac{r}{6}+\frac{d\,r}{4}-\frac{d^2\,r}{12}+\frac{r^2}{8}-\frac{d\,r^2}{4}+\frac{d^2\,r^2}{8}\right\}$$

In[29]:= `Factor[%]`

Out[29]=

$$\left\{\frac{1}{24}\,(-1+d)\,r\,(4-2\,d-3\,r+3\,d\,r)\right\}$$

In[30]:= `Expand[1 - Binomial[4 - d, 4]]`

Out[30]=

$$\frac{25\,d}{12}-\frac{35\,d^2}{24}+\frac{5\,d^3}{12}-\frac{d^4}{24}$$

In[31]:= `ReplaceAll[2 * r * (d - %30) + d1 * d2 - (1 / 3) * d1 ^ 3 +`

    `(1 / 2) * KX * (d1 ^ 2 - 2 d2) - (1 / 6) * (KX ^ 2 + c2) * d1, {d1 → (r / 2) * (d - 1) * H,`

    `KX → (d - 5) * H, d2 → ` $\frac{1}{24}$ ` * (-1 + d) * r * (4 - 2 * d - 3 * r + 3 * d * r) * H ^ 2,`

    `c2 → (Binomial[5, 2] + d * (d - 5)) * H ^ 2}]`

Out[31]=

$$2\left(-\frac{13\,d}{12}+\frac{35\,d^2}{24}-\frac{5\,d^3}{12}+\frac{d^4}{24}\right)r-\frac{1}{12}\,(-1+d)\,H\left((-5+d)^2\,H^2+(10+(-5+d)\,d)\,H^2\right)r-$$

$$\frac{1}{24}\,(-1+d)^3\,H^3\,r^3+\frac{1}{48}\,(-1+d)^2\,H^3\,r^2\,(4-2\,d-3\,r+3\,d\,r)+$$

$$\frac{1}{2}\,(-5+d)\,H\left(\frac{1}{4}\,(-1+d)^2\,H^2\,r^2-\frac{1}{12}\,(-1+d)\,H^2\,r\,(4-2\,d-3\,r+3\,d\,r)\right)$$

In[32]:= `Expand[%]`

Out[32]=

$$-\frac{13\,d\,r}{6}+\frac{35\,d^2\,r}{12}-\frac{5\,d^3\,r}{6}+\frac{d^4\,r}{12}+\frac{25\,H^3\,r}{12}-\frac{11}{4}\,d\,H^3\,r+\frac{3}{4}\,d^2\,H^3\,r-\frac{1}{12}\,d^3\,H^3\,r+\frac{H^3\,r^2}{12}-$$

$$\frac{5}{24}\,d\,H^3\,r^2+\frac{1}{6}\,d^2\,H^3\,r^2-\frac{1}{24}\,d^3\,H^3\,r^2-\frac{H^3\,r^3}{48}+\frac{1}{16}\,d\,H^3\,r^3-\frac{1}{16}\,d^2\,H^3\,r^3+\frac{1}{48}\,d^3\,H^3\,r^3$$

In[33]:= `ReplaceAll[Out[32], {H ^ 3 → d}]`

Out[33]=

$$-\frac{d\,r}{12}+\frac{d^2\,r}{6}-\frac{d^3\,r}{12}+\frac{d\,r^2}{12}-\frac{5\,d^2\,r^2}{24}+\frac{d^3\,r^2}{6}-\frac{d^4\,r^2}{24}-\frac{d\,r^3}{48}+\frac{d^2\,r^3}{16}-\frac{d^3\,r^3}{16}+\frac{d^4\,r^3}{48}$$

In[34]:= **Expand[% / d]**

Out[34]=

$$-\frac{r}{12} + \frac{d\,r}{6} - \frac{d^2\,r}{12} + \frac{r^2}{12} - \frac{5\,d\,r^2}{24} + \frac{d^2\,r^2}{6} - \frac{d^3\,r^2}{24} - \frac{r^3}{48} + \frac{d\,r^3}{16} - \frac{d^2\,r^3}{16} + \frac{d^3\,r^3}{48}$$

In[35]:= **Factor[%]**

Out[35]=

$$\frac{1}{48}\,(-1+d)^2\,(-2+r)\,r\,(2-r+d\,r)$$

In[36]:= **Expand[1 - Binomial[5 - d, 5]]**

Out[36]=

$$\frac{137\,d}{60} - \frac{15\,d^2}{8} + \frac{17\,d^3}{24} - \frac{d^4}{8} + \frac{d^5}{120}$$

In[37]:= **ReplaceAll$\Big[$Out[17], $\Big\{$d1 → (r / 2) * (d - 1) * H,**

d2 → $\frac{1}{24}$ * (-1 + d) * r * (4 - 2 * d - 3 * r + 3 * d * r) * H^2, d3 → Out[35] * H^3,

c1 → - (d - 6) * H, c2 → (Binomial[6, 2] + d * (d - 6)) * H^2, Chi → %36$\Big\}\Big]$

Out[37]=

$$-6\,d\,r + 6\,\left(\frac{137\,d}{60} - \frac{15\,d^2}{8} + \frac{17\,d^3}{24} - \frac{d^4}{8} + \frac{d^5}{120}\right)r + \frac{1}{8}\,(6-d)\,(-1+d)\,(15 + (-6+d)\,d)\,H^4\,r +$$

$$\frac{1}{16}\,(6-d)^2\,(-1+d)^2\,H^4\,r^2 + \frac{1}{16}\,(-1+d)^2\,(15 + (-6+d)\,d)\,H^4\,r^2 + \frac{1}{16}\,(6-d)\,(-1+d)^3\,H^4\,r^3 +$$

$$\frac{1}{64}\,(-1+d)^4\,H^4\,r^4 + \frac{1}{32}\,(6-d)\,(-1+d)^2\,H^4\,(-2+r)\,r\,(2-r+d\,r) +$$

$$\frac{1}{96}\,(-1+d)^3\,H^4\,(-2+r)\,r^2\,(2-r+d\,r) - \frac{1}{48}\,(6-d)^2\,(-1+d)\,H^4\,r\,(4 - 2\,d - 3\,r + 3\,d\,r) -$$

$$\frac{1}{48}\,(-1+d)\,(15 + (-6+d)\,d)\,H^4\,r\,(4 - 2\,d - 3\,r + 3\,d\,r) -$$

$$\frac{1}{32}\,(6-d)\,(-1+d)^2\,H^4\,r^2\,(4 - 2\,d - 3\,r + 3\,d\,r) -$$

$$\frac{1}{96}\,(-1+d)^3\,H^4\,r^3\,(4 - 2\,d - 3\,r + 3\,d\,r) + \frac{(-1+d)^2\,H^4\,r^2\,(4 - 2\,d - 3\,r + 3\,d\,r)^2}{1152}$$

In[38]:= **Expand[%]**

Out[38]=

$$\frac{77\,d\,r}{10} - \frac{45\,d^2\,r}{4} + \frac{17\,d^3\,r}{4} - \frac{3\,d^4\,r}{4} + \frac{d^5\,r}{20} - \frac{31\,H^4\,r}{4} + \frac{91}{8}\,d\,H^4\,r -$$

$$\frac{13}{3}\,d^2\,H^4\,r + \frac{3}{4}\,d^3\,H^4\,r - \frac{1}{24}\,d^4\,H^4\,r + \frac{H^4\,r^2}{18} - \frac{1}{6}\,d\,H^4\,r^2 + \frac{49}{288}\,d^2\,H^4\,r^2 -$$

$$\frac{1}{16}\,d^3\,H^4\,r^2 + \frac{1}{288}\,d^4\,H^4\,r^2 - \frac{H^4\,r^3}{48} + \frac{7}{96}\,d\,H^4\,r^3 - \frac{3}{32}\,d^2\,H^4\,r^3 + \frac{5}{96}\,d^3\,H^4\,r^3 -$$

$$\frac{1}{96}\,d^4\,H^4\,r^3 + \frac{H^4\,r^4}{384} - \frac{1}{96}\,d\,H^4\,r^4 + \frac{1}{64}\,d^2\,H^4\,r^4 - \frac{1}{96}\,d^3\,H^4\,r^4 + \frac{1}{384}\,d^4\,H^4\,r^4$$

In[39]:= **ReplaceAll[Out[38], {H^4 → d}]**

Out[39]=

$$-\frac{d\,r}{20} + \frac{d^2\,r}{8} - \frac{d^3\,r}{12} + \frac{d^5\,r}{120} + \frac{d\,r^2}{18} - \frac{d^2\,r^2}{6} + \frac{49\,d^3\,r^2}{288} - \frac{d^4\,r^2}{16} + \frac{d^5\,r^2}{288} -$$

$$\frac{d\,r^3}{48} + \frac{7\,d^2\,r^3}{96} - \frac{3\,d^3\,r^3}{32} + \frac{5\,d^4\,r^3}{96} - \frac{d^5\,r^3}{96} + \frac{d\,r^4}{384} - \frac{d^2\,r^4}{96} + \frac{d^3\,r^4}{64} - \frac{d^4\,r^4}{96} + \frac{d^5\,r^4}{384}$$

In[40]:= **Expand[% / d]**

Out[40]=

$$-\frac{r}{20} + \frac{d\,r}{8} - \frac{d^2\,r}{12} + \frac{d^4\,r}{120} + \frac{r^2}{18} - \frac{d\,r^2}{6} + \frac{49\,d^2\,r^2}{288} - \frac{d^3\,r^2}{16} + \frac{d^4\,r^2}{288} -$$

$$\frac{r^3}{48} + \frac{7\,d\,r^3}{96} - \frac{3\,d^2\,r^3}{32} + \frac{5\,d^3\,r^3}{96} - \frac{d^4\,r^3}{96} + \frac{r^4}{384} - \frac{d\,r^4}{96} + \frac{d^2\,r^4}{64} - \frac{d^3\,r^4}{96} + \frac{d^4\,r^4}{384}$$

In[41]:= **Factor[%]**

Out[41]=

$$\frac{1}{5760}$$

$$(-1 + d)\,r\,\big(288 - 432\,d + 48\,d^2 + 48\,d^3 - 320\,r + 640\,d\,r - 340\,d^2\,r + 20\,d^3\,r + 120\,r^2 - 300\,d\,r^2 +$$

$$240\,d^2\,r^2 - 60\,d^3\,r^2 - 15\,r^3 + 45\,d\,r^3 - 45\,d^2\,r^3 + 15\,d^3\,r^3\big)$$