Growth Theory

Rodrigo Pacheco

Columbia University

August 1, 2024

Introduction

So far we've been analyzing a static picture of the economy.

Our closed economy model, for example, analyzes the economy in a specific point in time.

The same is true for the monetary system we've been analyzing.

We are shifting gears now. We will analyze the economy in a dynamic setting.

We will go from analyzing a picture to analyzing a movie.

Introduction

Average income (real GDP per person) varies hugely between different countries.

Country	Real GDP per capita 2022 (PPP)
United States	58,487
Germany	46,648
South Korea	41,321
United Kingdom	38,407
Japan	38,269
Mexico	16,235
Brazil	14,640
India	7,766
Ethiopia	2,289
Mozambique	1,088

Source: Bolt and van Zanden - Maddison Project Database 2023

Average income in the US is more than 20 times higher than in Ethiopia!

These large differences in income are reflected in large differences in "quality of life".

Introduction

Growth rates of GDP per capita also vary substantially among countries.

Average Growth Rate of GDP per Capita: 1990 - 2022

Country	**Growth Rate**	**Years to Double GDP per Capita**
United States	1.44%	48.6
Germany	1.91%	37.2
South Korea	3.47%	20.9
United Kingdom	1.20%	60.0
Japan	0.77%	90.9
Mexico	1.62%	43.2
Brazil	1.96%	36.7
India	4.19%	17.2
Ethiopia	2.91%	24.0
Mozambique	−1.52%	-

Source: Bolt and van Zanden - Maddison Project Database 2023

Introduction

The last table shows the impact of small differences in growth rates over time.

Take US and Germany as an example: the difference in their growth rates is just 0.47%.

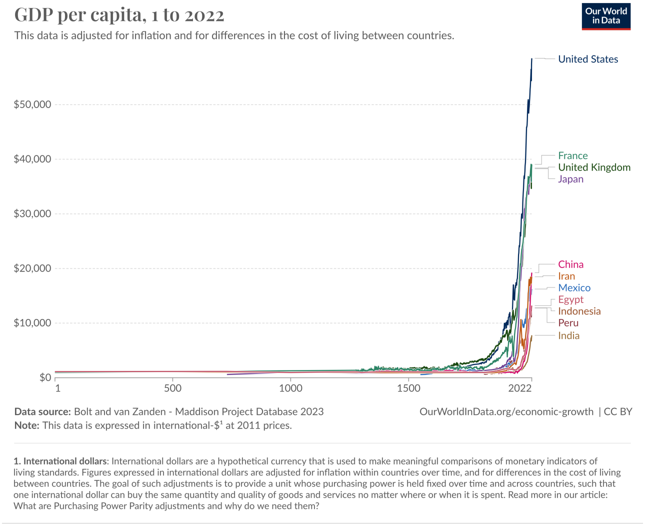
- Germany doubles its GDP per capita every 37.2 years.
- The US doubles its GDP per capita every 48.6 years.
- At this rate, Germany will surpass the US in GDP per capita in less than 50 years.

Inspired by these facts, in this lecture we will try to answer the following questions:

- What explains these differences in income and growth rates?
- How can the rich countries be sure to maintain their high standard of living?
- What policies should the lower income countries pursue to promote more rapid growth?

Before that, let's see a bit of the history of economic growth.

Introduction



Introduction

Last picture shows the evolution of GDP per capita over time.

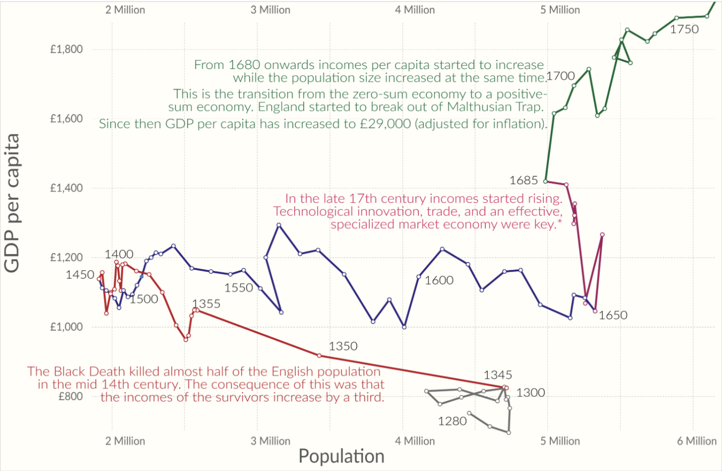
For most of the history, the standards of living were extremely low, not much different from that of Ethiopia today.

Until 1700 (pre-growth economy), the world was in a Malthusian trap.

- There was a tight link between population growth and living standards.
- More births, lower incomes.
- More deaths, higher incomes.
- A zero-sum game:
  - The size of the pie (total output) was not changing much.
  - The only way to get a bigger slice was to take it from someone else.

The best way to understand the Malthusian trap is to see the effects of the Black Death in England in the 14th century.

Introduction



Data: Broadberry, S., Campbell, B. M., Klein, A., Overton, M., & Van Leeuwen, B. (2015). British Economic Growth, 1270–1870 and Bank of England for the period thereafter. Data prior to 1700 refers to England; data thereafter refers to the UK. Averages over 5 year intervals are shown here.
*An additional factor was another pandemic, the Great Plague of London hit England in 1665 and reduced population size.

OurWorldinData.org – Research and data to make progress against the world's largest problems.

Introduction

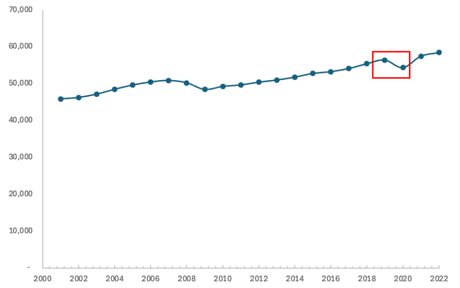
The Black Death killed about 50% of the population of England.

However, GDP per capita increased by 1/3 in the period.

This is an extremely different world from the one we live in today.

Remember the COVID-19 pandemic? Look what happened to GDP per capita in the US:

US Real GDP per capita



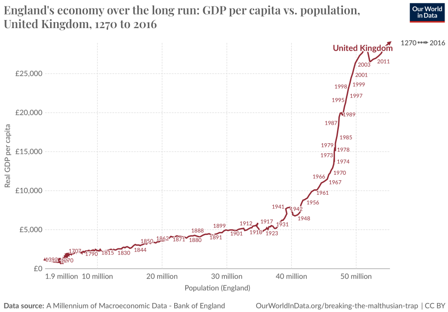
Introduction

The first country to break free from the Malthusian trap was England.

Starting in 1685, England experienced a sustained increase in living standards.

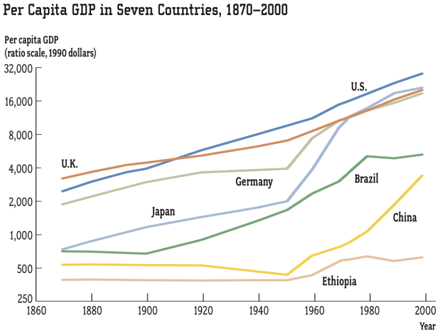
For the first time, the speed of innovation became so fast that the size of the population and income per capita started to grow at the same time.

England left the zero-sum game economy and entered a positive-sum game economy.



Introduction

After England, sustained growth emerged in many other countries, at different times.



Source: Maddison, *The World Economy* (see Figure 3.1). Observations are presented every decade after 1950 and less frequently before that as a way of smoothing the series.

Arguably, the Industrial Revolution was the most important event in the history of humanity since the domestication of animals and plants.

Introduction

Unfortunately, a large part of the world is still living on very low incomes.

Understanding the sources of economic growth is crucial to help these countries.

We will start our journey by analyzing a model of economic growth called the Solow model.

The Solow model

Our task in this section is to develop a theory of economic growth called the Solow Growth Model.

The model is named after Robert Solow, who won the Nobel Prize in Economics in 1987 for his work on this model.

The model shows how savings, population growth and technological progress affect the economy's output and growth rate.

Robert M. Solow in 1987, when he won the Nobel Memorial Prize in Economic Sciences. Mark Lennihan/Associated Press

The Solow model

We will start by analyzing the role of saving!

For now, there's no population growth and no technological progress.

Capital accumulation will be the engine of growth in this model!

Let's see the assumptions of the model.

Solow Model: Supply of Goods

The economy produces a single good.

The output is described by a production function F:

$$Y = F(K, L)$$

where Y is output, K is capital and L is labor.

F has constant returns to scale (CRS): $F(zK, zL) = zF(K, L)$

CRS allows us to analyze all quantities in the economy relative to the size of the labor force. In the definition of CRS, let $z = 1/L$:

$$Y = F(K, L) = L \cdot F(K/L, 1)$$

This means that we can analyze the economy in terms of per worker (or per capita) variables.

Solow Model: Supply of Goods

We will refer to per capita (or per worker) quantities with lowercase letters.

Then, output per worker is:

$$y = Y/L = F(k, 1) = f(k)$$

where:

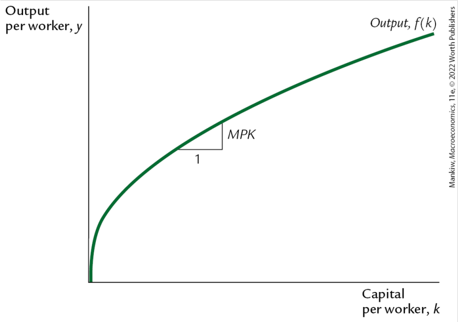
- $f(k) = F(k, 1)$ is the per worker production function
- y is output per worker
- k is capital per worker

CRS implies that L does not affect the relationship between y and k!

Example: Suppose the production function is $F(K, L) = K^{\alpha} L^{1-\alpha}$. What is the per worker production function?

Solow Model: Supply of Goods

The beautiful production function $f(k)$:



Remember your old friend MPK:

- It is the slope of the production function.
- It tells us how much output increases when we increase capital by a little bit.
- We will assume diminishing marginal product of capital, i.e, MPK is decreasing!

Solow Model: Demand for Goods

Demand for goods comes from consumption and investment:

$$y = c + i$$

Output per worker is divided between:

- c consumption per worker.
- i investment per worker.

This equation is the per worker version of the national income identity:

- It omits government spending, which we ignore for simplicity.
- It omits net exports, since we assume a closed economy.

We will assume that people save a constant fraction s of their income.

Solow Model: Demand for Goods

Then, they must consume a fraction $1 - s$ of their income:

$$c = (1 - s)y$$

We call s the savings rate.

- It is a fraction, so it varies between 0 and 1!
- Many things can influence s such as interest rates, income, etc.
- For now, we will assume that s is constant and is given!

How much is investment per worker then?

$$i = y - c = y - (1 - s)y = s \cdot y$$

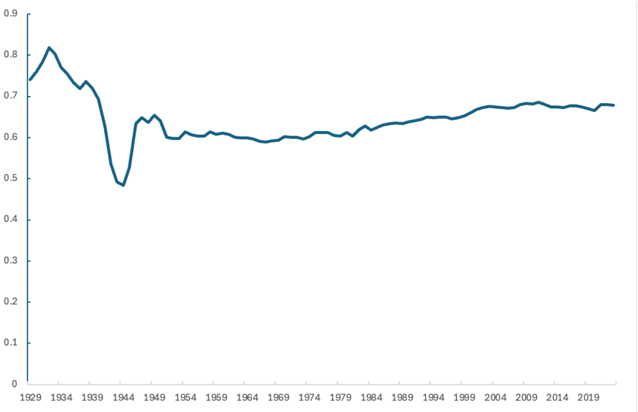
Nothing new here! Savings equals investment, always!

We can also refer to s as the investment rate!

Savings

Assuming people save a constant fraction of their income is a simplification. How good is this assumption?

Consumption as a fraction of income in the US



At least in the aggregate, the assumption seems to hold!

Capital accumulation

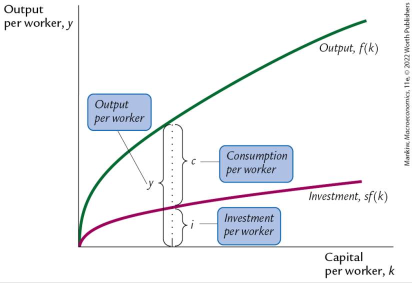
The capital stock determines the level of output in the Solow model.

Then, changes in capital stock can lead to economic growth.

Two forces determine the change in the capital stock: investment and depreciation.

Investment: $i = s \cdot y = s \cdot f(k)$

- Expenditure on new plant and equipment.
- Increases the capital stock.

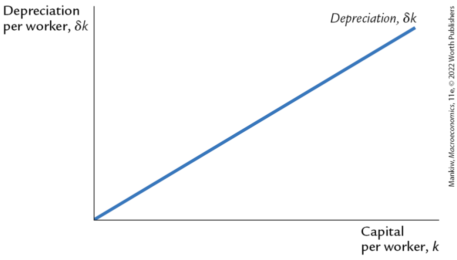


Capital accumulation

Depreciation:

- The wearing out of old capital.
- Decreases the capital stock.
- We assume that a fraction δ of the capital stock wears out each year.
- δ is the depreciation rate.

$$\boxed{\text{Depreciation} = \delta k}$$



Capital accumulation

The change in the capital stock is:

$$\text{Change in Capital Stock} = \text{Investment} - \text{Depreciation}$$
$$\Delta k = i - \delta k$$
$$\Delta k = sf(k) - \delta k$$

Now suppose you start with a capital stock k_0 at time $t = 0$.

The capital stock at time $t = 1$ is:

$$k_1 = k_0 + \Delta k_0 = k_0 + sf(k_0) - \delta k_0$$

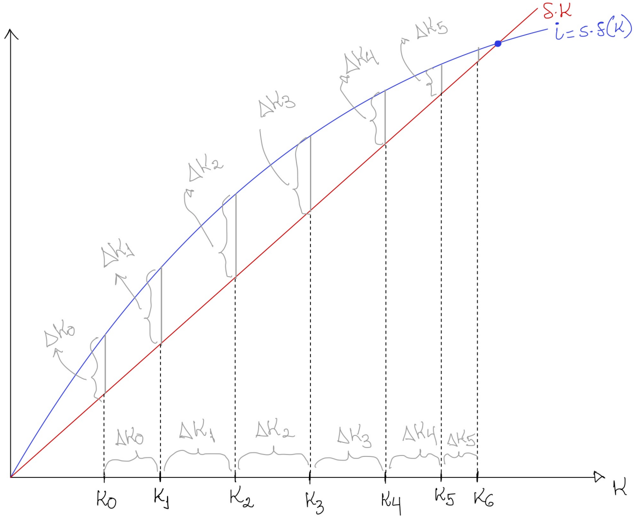
The capital stock at time $t = 2$ is:

$$k_2 = k_1 + \Delta k_1 = k_1 + sf(k_1) - \delta k_1$$

And so on...

Given k_0, we can determine the capital stock at any point in time, by applying the equation above successively.

Let's see how the capital stock evolves over time:



It seems that the capital stock is slowly converging to a point. What is that point? That point is the steady state of the economy.

Steady State

Steady state:

- It is the point where the capital stock does not change over time.
- The investment in the economy is just enough to replace the capital that wears out.
- We will refer to it as k^*.
- At the steady state, $\Delta k^* = 0$.

This gives us an expression to find the steady state:

$$\Delta k^* = sf(k^*) - \delta k^* = 0 \implies$$
$$\boxed{sf(k^*) = \delta k^*}$$

What determines the steady state?

- The savings rate s.
- The depreciation rate δ.
- The production function $f(k)$.

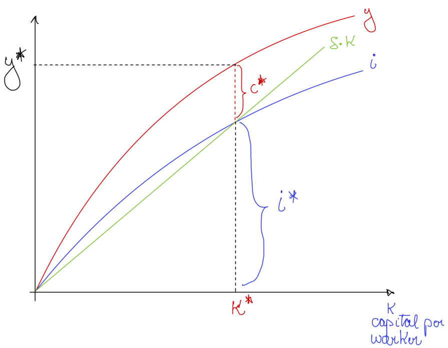
Steady State

We will refer to the variables at the steady state with a star!

At the steady state, the capital stock doesn't change, then:

- Output per worker is constant: $y^* = f(k^*)$
- Investment per worker is constant: $i^* = s \cdot y^*$
- Consumption per worker is constant: $c^* = (1 - s)y^*$

You can find the steady state graphically:



Steady State

Let's see how changes in the savings rate and the depreciation rate affect the steady state: Steady State Solow Model App

A decrease in the depreciation rate:

- Increases the steady state capital stock.
- Increases the steady state output per worker.
- Increases the steady state consumption per worker.

What about an increase in the savings rate?

- Increases the steady state capital stock.
- Increases the steady state output per worker.
- The effect on consumption per worker is ambiguous. Why?

Steady State

The effect on consumption is ambiguous because:

- An increase in the savings rate increases output per worker.
- But it also decreases the share of output that people consume $1 - s$.

Conclusion: higher savings rates lead to higher output per worker, but not necessarily to higher consumption per worker!

Why do we care so much about the steady state? Two reasons:

- An economy at the steady state will stay there forever, unless something changes.
- An economy not at the steady state will move towards it!
    - No matter the level of capital with which the economy begins, it ends up with the steady-state level of capital!
    - That's why the steady state represents the long-run equilibrium of the economy.

Saving rate and growth in the Solow Model

This is really important: an economy not at the steady state will move towards it!

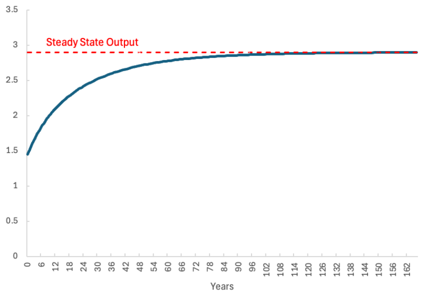
Let's see how the economy moves towards the steady state.

The following plot shows the evolution of output per capita in the Solow model, with the following parameters:

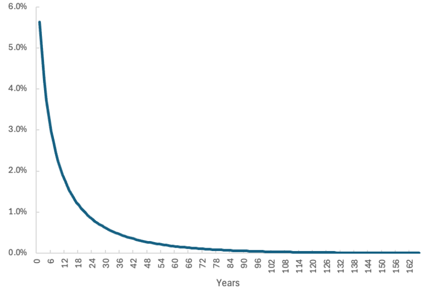
- $s = 0.6$
- $\delta = 0.05$
- $f(k) = k^{0.3}$
- $k_0 = 0.1 \cdot k^*$=3.48

Savings rate and growth in the Solow Model

Output per capita



Growth rate of output per capita



The economy is growing, but the growth rate is decreasing!

The growth rate approaches zero as the economy approaches the steady state.

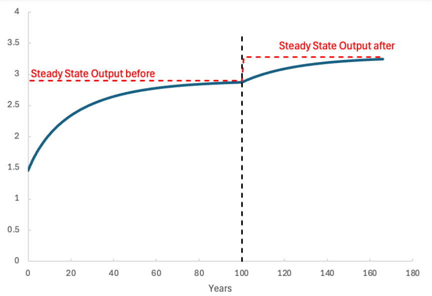
Can we have sustained growth in this Solow model?

Savings rate and growth in the Solow Model

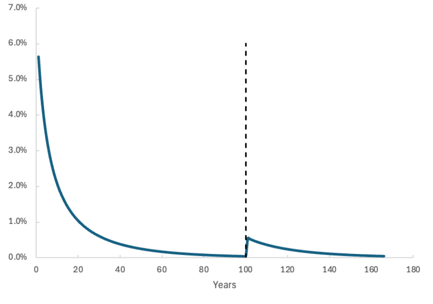
We could try increasing the savings rate.

Let's increase the saving rate to 0.8 at year 100 and see what happens.

Output per capita



Growth rate of output per capita



The growth rate jumps up but then decreases and approaches zero as the economy approaches the new steady state.

Savings rate and growth in the Solow Model

The increase in the growth rate is temporary!

We could try increasing the savings rate again and again.

But we can't do this forever. Why?

We can't have a savings rate greater than 1!

- People can't save more than their income!

Conclusion: We can't have sustained growth in the Solow model.

We will have to introduce population growth and technological progress to have sustained growth. Next Lecture!

Golden Rule

We saw that the effect of the savings rate on consumption per worker is ambiguous.

Let's think about a policymaker that wants to maximize economic well-being by choosing the savings rate.

What people really care about is consumption, not output.

So the policymaker should choose the savings rate that maximizes consumption per worker.

This is the Golden Rule savings rate.

Let's play a bit with the savings rate here: Golden Rule Solow App

Golden Rule

We will now see how to find the Golden Rule savings rate.

We will follow a 2-step process:

- First, find the capital per worker that maximizes consumption per worker, at the steady state. This is called the Golden rule level of capital.
- Second, find the savings rate that leads to that capital per worker.

Let's start with the first step:

Consumption at the steady state is given by:

$$c^* = \underbrace{f(k^*)}_{y^*} - i^*$$

Now, at the steady state investment is equal to depreciation, then:

$$c^* = f(k^*) - \delta k^*$$

We want k^* that maximizes c^*. We will call it k_g.

Golden Rule

How do we find k_g? First order condition!

Take the derivative of c^* with respect to k^* and set it equal to zero:

$$f'(k_g) - \delta = 0 \implies$$
$$\boxed{MPK = f'(k_g) = \delta}$$

The capital per worker that maximizes consumption per worker is the one that makes the MPK equal to the depreciation rate.

Second step: find the savings rate that leads to k_g.

At the steady state, investment is equal to depreciation. Then:

$$sf(k_g) = \delta k_g \implies$$
$$\boxed{s_g = \frac{\delta k_g}{f(k_g)}}$$

Golden Rule

Example: The production function is given by a Cobb-Douglas $F(K, L) = K^\alpha L^{1-\alpha}$. The depreciation rate is δ. Find the Golden Rule savings rate. *Answer*: $s_g = \alpha$

⚠ The economy does not automatically gravitate toward the Golden Rule steady state.

- If we want a particular k^*, such as the golden one, we need a saving rate that leads to that k^*.

Now, is it easy for this policymaker to implement the Golden Rule savings rate?

Will people support this policy?

As it is always the case in economics, the answer is: it depends!

The transition to the Golden Rule steady state

Suppose the economy has reached a steady state other than the Golden Rule.

A policymaker sets a new savings rate that leads to the Golden Rule steady state.

What happens to consumption, investment and capital when the economy transition from the old steady state to the new one?

We will have to consider two cases:

- The current savings rate is higher than the Golden Rule savings rate.
- The current savings rate is lower than the Golden Rule savings rate.

The transition to the Golden Rule steady state

Let's start with the first case: the current savings rate is higher than the Golden Rule savings rate.

We will call t_0 the time when the policymaker implements the golden rule savings rate.

The economy is in the old steady state and, at time t_0, the savings rate is reduced to the Golden Rule savings rate.

Let's analyze what happens to all variables following this change.

The transition to the Golden Rule steady state

At t_0, there's an immediate decrease in investment and, consequently, an immediate increase in consumption.

Now, investment is lower than depreciation $\implies$ We are no longer in the steady state.

Capital slowly decreases, leading to reductions in output, consumption, and investment.

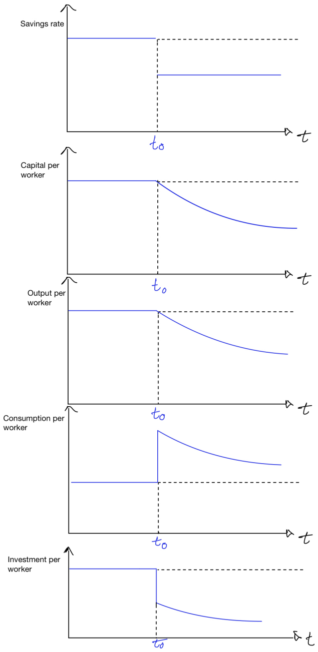
These variables decrease until a new steady state is reached.

Since we are assuming the new steady state is the golden rule, consumption in the new steady state must be higher than it was in the old steady state.

Conclusion:

- Consumption is higher at any point in time after the change in the savings rate.
- Easy to convince people to support this policy.

Transition to the Golden Rule steady state



Transition to the Golden Rule steady state

Now, let's analyze the second case: the current savings rate is lower than the Golden Rule savings rate.

Again, we will call t_0 the time when the policymaker implements the golden rule savings rate.

The economy is in the old steady state and, at time t_0, the savings rate is increased to the Golden Rule savings rate.

Let's analyze what happens to all variables following this change.

Transition to the Golden Rule steady state

At t_0, there's an immediate increase in investment and, consequently, an immediate decrease in consumption. Not good =(

Now, investment is higher than depreciation $\implies$ We are no longer in the steady state.

Capital slowly increases, leading to increases in output, consumption, and investment.

These variables increase until a new steady state is reached.

Again, the new steady state is the golden rule, then consumption in the new steady state must be higher than it was in the old steady state!

Conclusion:

- Consumption is higher in the very long-run, but lower immediately after the change in the savings rate.
    - Trade-off between reduced consumption now and higher consumption later.
- It might not be easy to convince people to support this policy.
    - Policymaker might have to sacrifice consumption of current generations for future generations.

Transition to the Golden Rule steady state