An introduction to Causality and Growth Rates

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May 22, 2024

Today's plan:

- An introduction to causality

- Dealing with growth rates

- Review of slopes

Causality

Economists often are interested in answering the following question: Does an event A cause an event B?

Some examples:

- Does smoking (Event A) increase the chance of developing lung cancer (Event B)?
- Does democracy (Event A) cause growth (Event B)?

In most cases, it is **really** hard to answer these questions. Seeing that both events happen (correlation) is not sufficient! Why?

- Omitted variable
- Reverse causality

Causality

Omitted variable

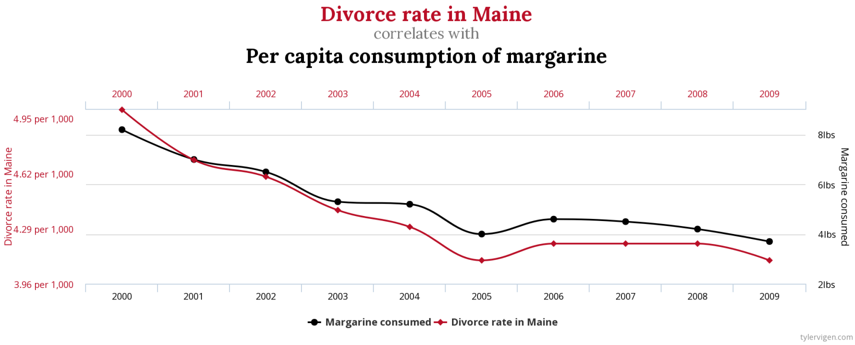
- A study group observes a strong relationship between two variables: the number of cigarette lighters that a person owns and the probability that they will develop cancer in the future.
- The study group advises the government to discourage people from owning cigarette lighters.
- **Problem:** They did not hold smoking amount constant!

Reverse causality

- Study plots the number of violent crimes per thousand people in major cities against the number of police officers per thousand people. The study notes the curve being upward sloping and argues that because police increase rather than decrease the amount of urban violence, law enforcement should be abolished.
- **Problem**: More dangerous cities have more police officers. So rather than police causing crime, crime causes police.

Causality

Correlation does not imply causation!!!



Source: `https://www.tylervigen.com/spurious-correlations`

Causality

Correlation does not imply causation!!!

Funny video

Growth rates

If some variable Y grows at a **constant rate** g, then there is an easy way to find the value of Y at some instant t given its value at time 0:

$$Y_t = Y_0 \cdot (1+g)^t$$

Let's see some examples:

- Suppose the US GDP grows at a constant rate of 2%. How long will it take US to double its GDP?

- US GDP grows at a constant rate of 2% while China's GDP grows at a constant rate of 5%. China's GDP today is 12.6% of US GDP. How long it will take China to surpass the US?

A useful approximation

We can generalize the result we found in the first example we solved in the previous slide.

Suppose now US GDP grows at a constant rate of g and you want to find how long it will take for it to be x times the value it is today. Then:

$$t = \frac{\ln(x)}{\ln(1+g)}$$

Now, for small values of g, we can use the following beautiful approximation:

$$\ln(1+g) \approx g$$

Which gives us:

$$\boxed{t = \frac{\ln(x)}{g}}$$

⚠ Make sure you are using natural logarithm when applying this approximation. Otherwise you are going to be very sad with the result.

Slopes

The slope of a line is the "Rise" over the "Run".

$$\text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta Y}{\Delta X} = \frac{Y_2 - Y_1}{X_2 - X_1}$$

Important: We will use the symbol Δ a lot in this class!

Consider a line represented by the equation $Y = a + b \cdot X$, then the slope of this line is b.

Why should I care about this?

Slopes

Suppose a variable Y grows at a constant rate of 10% each year and at $t = 0$ $Y_0 = 100$. Using the formula we just learned, Y_t is given by:

$$Y_t = 100 \cdot 1.1^t$$

Taking ln on both sides:

$$\ln(Y_t) = \ln(100) + t \ln(1.1)$$

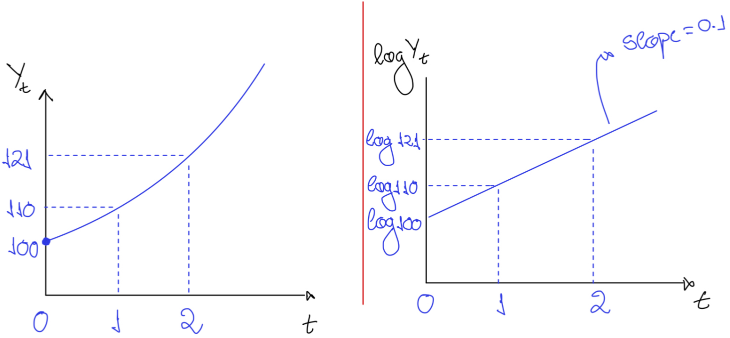
which is a line with slope $\ln(1.1)$.

Using the beautiful approximation we just learned: $\ln(1 + 0.1) \approx 0.1$. Then:

$$\ln(Y_t) = \ln(100) + 0.1 \cdot t$$

The slope of the line in the log-scale plot is approximately the growth rate!

Slopes



Average Growth Rates

Suppose we have a time series on a variable x, represented by: x_1, x_2, x_3, where the subscript refers to time.

The rate of growth of x between $t = 1$ and $t = 2$, g_1, is:

$$1 + g_1 = \frac{x_2}{x_1}$$

and the rate of growth of x between $t = 2$ and $t = 3$, g_2, is:

$$1 + g_2 = \frac{x_3}{x_2}$$

Note that:

$$x_1 \cdot (1 + g_1) \cdot (1 + g_2) = x_1 \cdot \frac{x_2}{x_1} \cdot \frac{x_3}{x_2} = x_3$$

Average Growth Rates

This gives us a motivation to find the geometric average rate of growth of x.

Let's call it $\bar{g}$. $\bar{g}$ is a constant growth rate that satisfies:

$$x_1 \cdot \underbrace{(1+\bar{g}) \cdot (1+\bar{g})}_{(1+\bar{g})^2} = x_3$$

Isolating $\bar{g}$:

$$\bar{g} = \left(\frac{x_3}{x_1}\right)^{1/2} - 1$$

We can also express $\bar{g}$ in terms of the rate of growth of x for each period.

Note that $x_3 = x_1 \cdot (1+g_1) \cdot (1+g_2)$:

$$\begin{aligned}
\cancel{x_1} \cdot (1+\bar{g})^2 = x_3 = \cancel{x_1} \cdot (1+g_1) \cdot (1+g_2) \Rightarrow \\
(1+\bar{g})^2 = (1+g_1) \cdot (1+g_2) \Rightarrow \\
\bar{g} = [(1+g_1) \cdot (1+g_2)]^{1/2} - 1
\end{aligned}$$

Average Growth Rates

Let's now generalize this result. Suppose we have a time series on the variable x, represented by: $x_1, x_2, x_3 \dots x_N$, where the subscript refers to time.

The rate of growth of x between $t = i + 1$ and $t = i$, g_i, is:

$$1 + g_i = \frac{x_{i+1}}{x_i}$$

The geometric average rate of growth of x, $\bar{g}$, is:

$$\bar{g} = \left(\frac{x_N}{x_1}\right)^{\frac{1}{N-1}} - 1$$

and expressed in terms of the rate of growth of x for each period:

$$\bar{g} = \left[\prod_{i=1}^{N-1} (1 + g_i)\right]^{\frac{1}{N-1}} - 1$$