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Planck, units, and modern metrology

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In honour of Max Planck (1858–1947) on the occasion of his 150th birthday.

Max Planck was an early proponent of the use of units based on fundamental constants, his particular system included one involving the Planck constant which emerged from his work on black-body radiation. In this review we discuss a number of such systems and how they have a strong interplay with modern metrology and the SI system of units.

1 Introduction

One can only express the magnitude of a dimension of an object by expressing it in terms of a property of the same kind: a length in terms of another length, a mass in terms of another mass and so on. Where the same reference quantities are used with sufficient frequency they acquire the status of 'units'. Depending on their convenience of use and their popularity, these reference quantities may be supported by international agreement and be incorporated into a national or international unit system.

Over the centuries people have looked for a set of natural quantities to form the basic units of their measurement systems. Initially these started with standards of a familiar human human dimensions, such as the length of the forearm and the masses of certain seeds and properties of water. Development of these systems led to the centimetre, gram, second, system of units for scientific use based on properties of the Earth. This has subsequently evolved into the SI of today which has increasingly made use of atomic properties to define its base units.

Suggestions for a universal basis for a unit system emerged as certain invariant properties of nature were discovered, such as the speed of light and gravitational constant. Early examples of such systems were that of Johnstone Stoney and of Planck which we return to later.

Scientific theories are expressed in terms of quantity equations. These have the important property that the dimensions of the expressions on either side of the equations have to balance: if one side represented an energy then the other side had to be equivalent. The exploitations of the method of dimensions in a predictive sense received considerable attention, notably in the early part of the twentieth century by such proponents as Bridgeman [1].

2 Origin of the Planck constant

Max Karl Ernst Ludwig Planck was born in Kiel, Germany, on April 23, 1858, so 2008 marks the 150th anniversary of his birth. His theory describing the black-body radiation [2,3] led on to quantum physics, a topic which still fascinates and puzzles scientists, while continuing to explain large areas of physics. Our

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ability to investigate the ramifications of his theory by experimenting with simple quantum systems has made huge advances following, for example, the advent of laser cooled macroscopic atomic systems and we are now able to directly investigate quantum phenomena such as entanglement.

The Planck constant h, sometimes termed the constant of action, along with what is now known as the Boltzmann constant, k emerged from the above theory (see Petley [4]) as an important fundamental physical constant which is also involved in a wider range of physics. It permeates the physics of quantisation, for example, of the electromagnetic field, QED, whereby the energy E, associated with a photon of frequency ν, is given by:

$$E = h\nu. \tag{1}$$

It is also associated with the spins of fundamental particles for their spin angular momenta are found to be exact integer multiples of $h/2\pi$. This combination appears sufficiently frequently to be given a symbol of its own $\hbar$, read 'aitch bar', or sometimes 'aitch cross' and is also sometimes termed the reduced Planck constant or the Dirac constant. For many purposes this constant is more convenient to use as a fundamental constant than h. This duality helps underline the fact that nature has not provided us with a unique set of fundamental units — and probably never will. The deep role that h plays in science has been discussed by Bordé [5].

The latest recommended value for h and other fundamental constants have been evaluated by Mohr, Taylor and Newell [6, 7] under the auspices of the CODATA Task group on fundamental constants. At the time of writing the review of the 2006 values are in press and the values are posted on the National Institute of Standards and Technology, USA (NIST) Website (http://physics.nist.gov/cuu/Constants/).

2.1 Are the constants really constant?

The question of whether the fundamental constants are really constant has been investigated since their existence was formulated. One cannot investigate variations of dimensioned constants for one has no assurance that one has invariant units. The best that one could do hitherto was to compare physics at one epoch with the physics at another, by looking for variation of dimensionless constants or ratios, notably astronomically as evidenced by the radiation received from distant stars, or geologically by looking at the relative atomic abundances of the atoms produced in radioactive decay products of recent and much earlier sources. Experimentally the former involved looking for differences in various aspects of the fine structure of the radiation after the effects of the red-shift have been removed.

2.2 The Planck constant and the Dirac big numbers

Dirac excited interest in the subject after his suggestion involving the 'big numbers' provided a possible time-scale for the variations. This meant that the postulated variations might be of the order of 100% over the age of the universe, currently estimated to be $\sim 4.3 \times 10^{17}$ s. This soon proved not to be the case but the search for smaller variations continued. Although work on this problem has hitherto relied on astronomical evidence, today the accuracy of laboratory atomic frequency standards has reached the point where one can investigate near instantaneous changes in the dimensionless fine structure constant, $\alpha = e^2/4\pi\varepsilon_0\hbar c$, which involves the Planck constant. Current measurements set limits to changes of h at less than a fractional part in 10^{-15} per year [8].

3 The role of the Planck constant in the units of science

3.1 The nature of units

A fundamental aspect of any measurement is that of comparison: one can only compare the value of a quantity Q_1, with that of a like quantity Q_2. We usually make our measurements in internationally agreed standard quantities known as units. Thus the value of a quantity is given by

$$\text{Quantity} = \{\text{numerical value}\} \cdot [\text{unit}] \tag{2}$$

that is

$$Q_1 = \{Q_1\} \cdot [Q_1] \text{ and } Q_2 = \{Q_2\} \cdot [Q_2]. \tag{3}$$

A frequent problem with using the results of a measurement is that of transferring the value of a quantity from one point in space-time $Q_i(r_1, t_1)$ to another point in space-time $Q_i(r_2, t_2)$. Since one can only measure relative to a unit one must ensure that the unit is constant, that is $[Q](r_1, t_1) = [Q](r_2, t_2)$ or any variation is accounted for. That is, one needs to know whether, how, and to what extent the reference quantity may change with changes of space-time. Further, in order to transfer measurements from one person to another, each person needs to know how to convert measurements between their respective systems of reference quantities. This leads to the convenience of having a minimum set of agreed reference quantities.

The requirements for such a system have gradually become ever more stringent as science and technology have evolved. Today requirements vary from a universal system appropriate to all sciences and technologies to more specialised systems that are better suited for particular purposes. The globally adopted system is termed the International System of Units (SI — from the French Système international d'unités). The reference quantities used by science involve the quantities which are termed fundamental physical constants, and their values in SI units are essential for the continued coherence of science and technology.

National laboratories in most countries of the world sustain the best possible representations of the SI units. These are co-ordinated at the International Bureau of Weights and Measures (BIPM) which is operated under the auspices of the General Conference on Weights and Measures (CGPM). We consider the unit systems involving the fundamental constants in the next section and return to the SI later in the paper.

3.2 The Planck constant and the units of science

Planck is commemorated in many of the fundamental constant based systems of units used by science via the Planck constant and in particular in the system of units he introduced which are known as the Plank units. A number of such system have been introduced over the years, as well as Planck units, commonly used systems are known as the atomic units and the natural units, but note that these names are not always used for the same systems by different authors. The principal systems are summarised in Table 1. Of these the most common systems, the Planck, atomic and natural units are expanded on below. See Tables 2, 3, and 4.

Ever since the standard group of fundamental constants was established at the beginning of the twentieth century scientists have been used to forming 'units' in order to simplify expressions when developing a theory. Statements such as 'let $\hbar = m_\mathrm{e} = c = 1$' or 'let $\hbar = m_e = e = 1$' indicate that the person is working in terms of either the natural units or atomic units respectively (these unit systems usually also set

Table 1 The constants used as 'units' in the leading systems.

Unit Scheme	c	e	$\hbar$	G	m_e	m_p	ε_0	k
Stoney units	1	1		1			1/4π	1
Planck units	1		1	1			1/4π	1
Schrödinger units		1	1	1			1/4π	1
atomic units, Hartree units		1	1		1		1/4π	1
electronic system of units, Dirac units	1	1			1		1/4π	1
natural units, quantum relativistic system (Ruark)	1		1		1		1/4π	1
QED system of units (Stille)	1	1	1			1		1

Quantity	Symbol	Expression	Value in SI units
Planck mass	m_P	$(\hbar c/G)^{1/2}$	$2.176\,44(11)\times10^{-8}$ kg
energy equivalent		$m_\mathrm{P}c^2$	$1.220\,892(61)\times10^{19}$ GeV
Planck length	l_P	$(\hbar G/c^3)^{1/2}$	$1.616\,252(81)\times10^{-35}$ m
Planck time	t_P	$(\hbar G/c^5)^{1/2}$	$5.391\,24(27)\times10^{-44}$ s
Planck temperature	T_P	$(\hbar c^5/G)^{1/2}/k$	$1.416\,785(71)\times10^{32}$ K

Table 2 The Planck units.

Quantity	Atomic Unit	Value in SI units
Time	$a_0/\alpha^2 c = \hbar/(\alpha m_\mathrm{e} c^2)$	$2.418\,884\,326\,505(16)\times10^{-17}$ s
Length	$a_0 = 4\pi\varepsilon_0\hbar^2/m_\mathrm{e}c^2$	$5.291\,772\,0859(36)\times10^{-11}$ m
Mass	m_e	$9.109\,382\,15(45)\times10^{-31}$ kg

Table 3 The atomic units.

Quantity	Natural Unit	Value in SI units
Length	$\lambda_\mathrm{c} = \hbar/m_\mathrm{e}c$	$3.861\,592\,6459(53)\times10^{-13}$ m
Mass	m_e	$9.109\,382\,15(45)\times10^{-31}$ kg
Time	$\hbar/m_\mathrm{e}c^2$	$1.288\,088\,6570(18)\times10^{-21}$ s

Table 4 The natural units.

Table 5 The units of length, mass, time, electric charge, and temperature in unit systems based on various of the fundamental constants: $\hbar, c, e, G, \varepsilon_0$.

Unit system	Length	Mass	Time	Electric charge	Temperature
Stoney units	$\left(\frac{Ge^2}{4\pi\varepsilon_0 c^4}\right)^{1/2}$	$\left(\frac{e^2}{4\pi\varepsilon_0 G}\right)^{1/2}$	$\left(\frac{Ge^2}{4\pi\varepsilon_0 c^5}\right)^{1/2}$	e	$\left(\frac{e^2c^4}{4\pi\varepsilon_0 Gk^2}\right)^{1/2}$
Planck units	$(\hbar G/c^3)^{1/2}$	$(\hbar c/G)^{1/2}$	$(\hbar G/c^5)^{1/2}$	$(4\pi\varepsilon_0\hbar c)^{1/2}$	$((\hbar c^5/G)/k^2)^{1/2}$
Natural units	$\hbar/m_\mathrm{e}c$	m_e	$\hbar/m_\mathrm{e}c^2$	$e/\alpha^{1/2}$	$m_\mathrm{e}c^2/k$
Atomic units	$4\pi\varepsilon_0\hbar^2/m_\mathrm{e}e^2$	m_e	$\hbar^3 4\pi\varepsilon_0/m_\mathrm{e}e^4$	e	$m_\mathrm{e}c^2/k$

$4\pi\varepsilon_0$ = 1 or $\mu_0 = 1$). It is the value of the constant in the unit system that is being set equal to one, that is the charge on the electron is one e unit. In the notation of Eq. (2) above $\{\hbar\} = \{m_\mathrm{e}\} = \{c\} = 1$ indicates that only the numerical values and not the units are equal.

In addition to these constants [9], there is also the very important fine structure constant, α. This dimensionless quantity may be used to form other natural units in combination with h, m_e, e, G and c.

The corresponding expressions for the units of length, mass, time, charge, and temperature in the leading systems are shown in Table 5. When equations are written in terms of these unit systems the constants set at unity are omitted throughout. It is a straightforward process to recover the full expressions at the end of the calculation.

The quantities $4\pi\varepsilon_0$ and k were implicitly assumed to be unity by physicists of that era. It was Stille [10] who realised that by relaxing the fixing of $4\pi\varepsilon_0$ it was possible to fix c, h and e without constraining the value of α.

By inspection of the dimensions quantity involved in the results of the theory the constants required can easily be recovered by using the appropriate combination of the expressions given in the table. Thus, if the output quantity of a calculation in terms of Planck units is a time one simply inserts the quantities in the time unit in Table 1.

It is apparent that, although eminently suitable for the particular purposes for which they are used, the sizes of the units generated by these systems are mostly far too small for use in technology and commerce.

Other systems of units may be developed for use elsewhere, such as high energy nuclear physics. The fact that there are so many systems demonstrates that nature has not provided a unique set at this stage of our knowledge. This may change if it ever becomes possible to calculate the values of any of the constants in terms of other constants.

3.3 The Planck units

These units were introduced by Planck [3] as an alternative to those suggested by Johnstone Stoney [11]. Planck introduced derived units of mass, length, and time, which were based on h, G, and c as his standard fixed quantities and these had a numerical value of unity in his system. The use of three units mirrored the practical units of the centimetre, gram, second system that was in widespread use in the science of the nineteenth century and was based on properties of the Earth. The modern system of Planck units differs from Planck's original by a factor of 2π as $\hbar$ is customarily used as the unit of action instead of h. The modern version of the Planck units are shown in Table 2 with their values in SI units.

More recently it has been realised that the Planck units are significant for cosmology and fundamental physics, in that for a quantity, having a numerical value $\{1\}$ in Planck units shows the limit in our understanding of that quantity. For example the Planck length is the quantum short length limit of applicability of General Relativity. The Planck scale is the scale of quantum fluctuations in space-time. For a recent discussion of many quantities in terms of the Planck units and aspects of dimensionless quantities of cosmological significance see Tegmark et al [12], for an interesting discussion on Planck and the question of whether nature has provided absolute units see Wilczek [13–15].

3.4 The atomic units

These units are of principal usefulness in atomic physics and were proposed by D Hartree to simplify the Schrödinger equation. Hence they are sometimes known as Hartree units. The values in terms of their SI equivalent are shown in Table 3.

3.5 The natural units

The natural units are similarly a frequently used tool to simplify theory and their values are known relative to their SI units and are given in Table 4. They were one of a pair of similar systems introduced by Ruark [16] and are sometimes known as the quantum relativistic system. Again, there is no universal nomenclature, some authors use 'natural units' as a generic name for all fundamental constant based systems and some use it as an alternative name for the Planck units.

The expression for five physical quantities in the four main system are compared with one another in Table 5. Other quantities such as force, velocity, electric current etc may be derived by forming the appropriate combinations of the former.

4 Historic measurements of the Planck constant

From the viewpoint of measurement, the recommended value has not been a constant, for the recommended numbers and their estimated accuracy have changed dramatically since Birge evaluated the best values of the fundamental constants in 1929. Earlier work on the value of h came via measurements of h/e. Interestingly, Birge first evaluated the best value of the Planck constant in 1919 [17] from the available direct measurements but not as part of a full evaluation of the fundamental constants.

Usually there are more determinations available than there are constants and a careful statistical analysis of the results enables one to show that the understanding of particular measurements is less than was thought. Such analyses contributed to advances in science. A particular example of this is to be found in the Birge evaluation of 1929 [18].

Table 6 The 1929 estimates of the Planck constant (with probable errors: 0.675 σ).

Method	Value/10^{-34} Js	Power of e
Rydberg constant	6.547(11)	5/3
Ionisation potential	6.560(15)	3/3
x-ray limit	6.550(9)	4/3
Photo-electron	6.543(10)	3/3
Second radiation constant, c_2	6.548(15)	3/3
Stefan-Boltzmann constant, σ	6.539(10)	4/3
CODATA 2006 value	6.626 068 96(33)	

Although some of the changes in the recommended values are much larger than the estimated uncertainties, these observed changes are not due to any secular changes, but they arise from an improper understanding of the physics and metrology of the measurements from which h was deduced. (See Sect. 2.1.)

The sources of the 1929 experimentally measured values came from measurements of h in combination with other constants including various powers of e, These are summarised in Table 6, and include measurements of ionisation potentials, electron x-ray limit, photo-electrons, the radiation constant c_2 and the Stefan-Boltzmann constant.

The 1929 values all depended on the value of the elementary charge [19] raised to various powers, as summarised in the third column. This value obtained by Millikan in his famous oil-drop experiment was $1.59108(16)\times10^{-19}$ C. His value was subsequently shown to be underestimated by about 0.7% due to an error in the measurement of the viscosity of air which he had used. The present CODATA 2006 recommended value is $1.602\,176\,487(40)\times10^{-19}$ C. After he had revised his values following the uncovering of the error in Harrison's value for the viscosity of air, Millikan [20,21] recommended the value of h to be 6.61×10^{-34} Js, a value that is in much better agreement with the present value. (Note that the value of h obtained from the Rydberg constant also involved the e/m_e measurements, further increasing the reliance on the value of e.)

5 Modern measurements of the Planck constant

As science has progressed so too have the methods of measuring different combinations of constants which include the Planck constant and these have led to greatly increased accuracy for the value of h. This progress is summarised in Table 7.

Year	Value: $h/10^{-34}$ Js	Estimated fractional uncertainty $\sigma_T/10^{-6}$
1919	6.5543(90)	1400
1929	6.547(8)	1200
1955	6.625 17(23)	35
1965	6.625 59(16)	24
1973	6.626 176(36)	5.4
1986	6.626 0755(40)	0.60
1998	6.626 068 76(52)	0.078
2002	6.626 0693(11)	0.17
2006	6.626 068 96(33)	0.050

Table 7 Some recommended values for the Planck constant.

This progress illustrates both the increasing accuracy of measurement science as well as the steady convergence towards today's value. The uncertainties in the experimental values comprise two components, one including quantities that have fluctuated during the measurement process and the other of quantities on which no information can be gleaned from a study of the results. Traditionally the component groups were termed 'random' and 'systematic' errors or uncertainties respectively. More recently an internationally agreed method of assessing uncertainties, the Guide to Measurement Uncertainties(GUM) [22] has adopted the terms Type A and Type B uncertainties to denote and enlarge upon the former terms 'random' and 'systematic' respectively. These designations take into account that many experimental fluctuations are far from random in origin but have a source that can often be identified, for example, a thermostat controlling the temperature of the apparatus may well be switching the heating on and off at unknown times. Similarly the Type B component may contain 'random' fluctuations over a much slower time-scale than the duration of the measurement. With hindsight, in many cases, we can see that it would have been safer to double or triple the estimated standard deviation uncertainties.

5.1 The Josephson effect and the quantised Hall resistance

Two major advances in measurement have transformed the situation. These involved superconducting and normal electron effects at cryogenic temperatures. The first was the discovery of the Josephson effects by Brian Josephson in 1962 [23] whereby the potential difference, V, between two closely separated superconductors ($\sim 1\,\mathrm{nm}$ apart) in the presence of (microwave) radiation of frequency ν, satisfies the equation

$$2eV = n_{\mathrm{J}} h\nu, \tag{4}$$

where n_{J} is an integer. The Josephson effects appearing across such a *Josephson junction* were soon verified experimentally and the work at the University of Pennsylvania led to a more accurate measurement of $2e/h$ and an improved set of values by them for the fundamental constants [24]. Today, by cascading 20 000 or so junctions in series, steps up to potential differences of some 10 V are routinely observed. The accuracy achieved was such that from about 1972 the various national standards laboratories throughout the world were able to maintain a more stable representation of the SI volt than hitherto by utilising an internationally agreed value for $2e/h$.

The measurements between different apparatus for measurement of $2e/h$ suggest that the method is good to a few parts in 10^{-9}. Indirect tests of the material independence of the voltages induced on Josephson junctions made from different superconductors when irradiated with the same frequency and biased on the equivalent Shapiro step suggests that the ratio $2e/h$ is material independent at the level of around one part in 10^{16} [25]. The Josephson constant K_{J} is defined by

$$K_{\mathrm{J}} = \frac{2e}{h}. \tag{5}$$

To maintain worldwide consistency between laboratories an exact value of $2e/h$ for maintaining the volt, $K_{\mathrm{J}-90} = 483\,597.9 \times 10^{9}\,\mathrm{Hz\,V^{-1}}$ was internationally adopted in 1990. The value of this 'as maintained' volt is more stable than our knowledge of the SI volt and being based on the 1990 estimate of $2e/h$ differs slightly from the current best value which is $K_{\mathrm{J}} = 483\,597.891(12) \times 10^{9}\,\mathrm{Hz\,V^{-1}}$.

The Josephson effects were followed in 1980 by the discovery by Klaus von Klitzing [26] that at cryogenic temperatures and strong flux densities the Hall resistance of a two dimensional electron gas layer existing in a high purity semiconductor was such that its conductance was quantised in steps of h/e^2, that is

$$R_{\mathrm{H}} = \frac{h}{e^2}\frac{1}{n_{\mathrm{H}}} \tag{6}$$

where n_{H} is an integer. The fundamental constants involved are known as the von Klitzing constant, R_{K}, given by

$$R_{\mathrm{K}} = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}. \tag{7}$$

Similarly, in 1990 this led to an internationally agreed value for the von Klitzing constant to maintain resistance standards of $R_{\mathrm{K}-90} = 25812.807\,\Omega$. The present value for R_{K} recommended by CODATA is 25 812.807 557(88) Ω.

The metal film is usually of gallium arsenide on a silicon substrate. There is a slight temperature effect that can be readily corrected for. Tests of the material independence of the von Klitzing constant [27, 28] show that under appropriate operating conditions the impedance of a silicon quantum Hall device agrees with a gallium arsenide one at the level of one in 10^{10}, similarly devices have been shown to be independent of operating conditions to 8 in 10^{11} [29].

The above measurements may be checked by a further measurement involving the charging of a very small evaporated film capacitor having a gate electrode. These provide possibilities to measure the elementary charge e and hence allow a fuller investigation of whether the three types of measurement are consistent with unique values for e and h. The possibilities for checking the quantum triangle, that is the consistency between independent measurements of h/e^2, $2e/h$ and e, have been discussed in the review by Gallop [30]. Essentially the gate electrode may be used to charge the capacitance with an integral number of electrons and by suitable design provide a possible quantum capacitance standard. Keller et al [31–33] made a seven junction electron pump which provided an electron counter operating with an error of 15 parts in 10^9 per electron pumped.

More recently a coulomb blockade device, namely a radio-frequency single electron transistor (RF-SET), has been used as a quantum current standard operating at the picoampere level. The error rate increases with frequency and so far this has limited the radio frequency to about 10 MHz. Quantised acoustic current techniques at the Physikalisch-Technischen Bundesanstalt, Germany (PTB) and the National Physical Laboratory, UK (NPL) have been discussed by Ebbecke et al [34], and recently Blumenthal et al [35] have reported on gigahertz quantised charge pumping.

Hopefully the error rates of the various possible configurations can be reduced sufficiently to make a through check of the consistency of the three methods to the fractional accuracies of < 1 in 10^8.

5.2 The watt balance

By combining measurement of the Josephson voltage and quantised Hall resistance one may obtain a measurement of h, see, for example Petley et al [36]. However, if both the voltage and resistance measurements are made in terms of the as maintained electrical units, the accuracy is ultimately limited by the accuracy with which the SI electrical units may be realised. While measuring the high field value of the gyromagnetic ratio of the proton, Bryan Kibble realised [37] that one could combine the electrical current measurement involved in the high field and low field methods by using the same coil and flux density in both cases and hence obtain the current absolutely without the necessity to measure either the dimensions of the coil involved, or the magnetic flux density. Since the current involved could also be measured in terms of the laboratory standards of voltage and resistance the method essentially provided a way to realise the watt absolutely. Implicitly the method realised a watt via the Planck constant in terms of the metre, kilogram and second. Thus, if a rectangular coil is suspended from the arm of a balance in a uniform magnetic flux density B, and a current I, is passed through it, the mass required on the other arm to balance the electrical force is given by

$$mg = Bk_cI \tag{8}$$

where k_c is the coil constant.

If now the coil is moved vertically with a velocity u, the induced voltage V is given by

$$V = Bk_cu \tag{9}$$

Combining the two equations to eliminate Bk_c yields

$$VI = mgu \tag{10}$$

although great care is needed to ensure that the product Bk_c remains constant in both cases.

Thus, if the power is measured in terms of a Josephson voltage standard as k_1/K_J , and a resistance standard $k_2 R_\mathrm{K}$ then we have

$$VI = \frac{V_2}{R} = \frac{k_1^2}{(K_\mathrm{J})^2\, k_2 R_\mathrm{K}} = \left(\frac{k_1^2}{k_2}\right) \cdot \left(\frac{1}{(2e/h)^2\,(h/e^2)}\right) = \left(\frac{k_1^2}{k_2}\right) \cdot \frac{h}{4} \tag{11}$$

and finally

$$h = \frac{4mguk_2}{k_1^2} \tag{12}$$

Thus measurement by this method leads to an absolute value of h that is independent of the electrical units. To date measurements have been made at NIST by Steiner *el al* [38,39] and at NPL by Robinson and Kibble [40,41]. Their values are $h = 6.626\,068\,91(24) \times 10^{-34}$ Js and $h = 6.626\,070\,95(44) \times 10^{-34}$ Js respectively, which are somewhat discrepant. Work on watt balances continues in a number of laboratories around the world.

5.3 The silicon Avogadro experiment

A non-electrical route to the Planck constant, the present day method of measuring the Avogadro constant, was pioneered by Richard Deslattes et al at NIST in 1974 [42,43]. This method involves measuring the lattice parameter, a, the relative atomic mass $A_\mathrm{r}(\mathrm{Si})$ and the density, $\rho(\mathrm{Si})$, of a crystal of pure silicon. The Avogadro constant was then obtained from the Bragg equation:

$$N_\mathrm{A} = \frac{M(\mathrm{Si})}{\rho(\mathrm{Si})a^3/n} \tag{13}$$

where $M(\mathrm{Si})$ is the mean molar mass of silicon. Because of the accuracy of relative atomic masses, for the purpose of redefinition this result is transferable to any microscopic mass such as ^{12}C, m_p or m_e.

The lattice parameter was measured by an x-ray method involving Moiré fringes, whereby the crystal was displaced by a known amount along the required crystal direction and each time it was moved by one lattice spacing the fringe count increased or decreased by one. The major advantage of the method was that by displacing the crystal by a large number of x-ray fringes it enabled the lattice parameter to be measured in a helium-neon optical interferometer. Thus the spacing could be measured directly in terms of a visible wavelength instead of the x-unit used by earlier methods. This improved the accuracy to the part per million level. The density was obtained by weighing a precise 1 kg sphere of silicon obtained from the same sample of silicon as that used in the interferometer. The atomic mass of the silicon was obtained from an isotopic-abundance-weighted combination of the nuclidic masses involved.

Since that time the accuracy of the method has been steadily improved by increasing the crystal displacement and lately by the availability of kilogram spheres of isotopically pure silicon. The earlier work on N_A has been reviewed by Becker [44]. The complexity of the project has led to collaboration between metrologists from several countries. At the time of writing the value of h derived from the N_A measurements is still slightly discrepant by up to a part in 10^6, Becker [45,46], but further work is in progress.

5.4 Potential use for measuring mass: the inconstant kilogram

The general principles required for the properties and definitions of the base SI units have been discussed, for example, by Kose et al [47] (see also the comment by Moldover [48]).

All of the estimated values of the values of the quantities in Table 8 depend on the uncertainty in the laboratory measurements of the Planck constant (or equivalently of the Avogadro constant). If all other components of these latter uncertainties are negligible compared with the mass on the scale-pan, then

Table 8 Fundamental Constants and units improved by the moving coil realization of the watt.

1. Fundamental Constants 2006 value		
Quantity	Symbol	Value(uncertainty)
Planck constant	h	$6.626\,068\,96(33)\times10^{-34}$ Js
Josephson constant, K_{J}	$2e/h$	483 597.891(12) GHz/V
Electron mass	m_e	$9.109\,382\,15(45)\times10^{-31}$ kg
Proton mass	m_{p}	$1.672\,621\,637(83)\times10^{-27}$ kg
Avogadro constant	N_{A}	$6.022\,141\,79(30)\times10^{23}$ mol^{-1}
Elementary charge	e	$1.602\,176\,487(40)\times10^{-19}$ C
magnetic flux quantum, $h/2e$	ϕ_0, $h/2e$	$2.067\,833\,667(52)\times10^{-15}$ Wb
von Klitzing constant, R_{K}	h/e^2, $\mu_0 c/2\alpha$	25 812.807 557(18) Ω
2. Units approved for use with the SI		
Unit	Symbol	Value(uncertainty)
Electron volt	eV	$1.602\,176\,487(40)\times10^{-19}$ J
Unified mass unit	u	$1.660\,538\,782(83)\times10^{-27}$ kg

the method may be used to monitor the stability of the Prototype kilogram and, if internationally agreed, may lead to a change in the definition of the kilogram. A consequence of this would be a considerable improvement of the accuracies of the quantities in Table 8.

From the discussion in Sect. 3 it is apparent that there are considerable differences between the basic quantities of the SI units and the 'natural' contenders. In particular it is interesting that time does not have a natural unit and emerges as a derived quantity. The quantity cR_∞ (where c is the speed of light and R_∞ the Rydberg constant for infinite mass) can be measured to parts in 10^{12}, theory and measurement would have to be improved a thousandfold to surpass the performance of modern atomic clocks but this may be possible. A further possibility might be to involve the Planck time t_{P}, but such possibilities are for the future to decide.

As far as the SI is concerned, time and frequency are by far the most accurately measurable quantities, and the definitions of the other SI base units are steadily being replaced in terms of fundamental constants, either in combination with the second or with the hertz. It would be highly inconvenient if the definition of the second had amended whenever a more accurate atomic clock as developed.

The accuracy of measurements of the Planck constant, the Josephson constant and the von Klitzing constant has reached the stage where a considerable simplification of the best values of the constants might be achieved by revising the definitions of the SI units along the lines suggested by Taylor and Mohr [49] and Mills et al [50–52].

It is unlikely that the prototype kilogram is really constant in some absolute reference frame such as the masses of fundamental particles would provide. Davis [53, 54] compared the measurements of the Faraday constant with the high-field gyromagnetic ratio of the proton which were some ten years apart to set limits on the possible drift rate of the prototype kilogram of ± 20 μg/yr. The realizations of the watt by the moving coil method by Williams et al (1998) and Kibble et al (1990) were nine years apart, and may be used to infer a value for the drift rate of the watt of:

$$\frac{W_{90}}{W_{1988}} - \frac{W_{90} - W_{1988}}{1998 - 1988} = (1 \pm 2) \times 10^{-8}/\mathrm{yr} \tag{14}$$

If we ascribe any drift in the value as being due to drift in the mass of the prototype kilogram, this puts a tighter limit of on its possible drift than the estimate obtained by Davis. We could do slightly better if we took the value of Steiner et al (2005) [38] instead:

$$\frac{W_{2005}}{W_{1988}} - \frac{W_{2005} - W_{1988}}{2005 - 1988} = (0.7 \pm 1.4) \times 10^{-8}/\mathrm{yr} \tag{15}$$

but the limit is still limited by the accuracy of the earlier measurement. Comparison of the latest NIST value with the earlier value gives $(W_{90}/\mathrm{W}) - 1 = 8(36)$ nW/W. This suggests that the value of the 'electrical' kilogram disseminated by W_{90} is very close to the kilogram, and also that the method might be used to monitor the stability of the kilogram with the required accuracy.

There is also an alternative approach via the Faraday constant route in progress at the PTB [55]. This is based on an ion current method for tracing the atomic mass unit to the kilogram by ion accumulation. However this method will require significant development to reach the uncertainty offered by the methods discussed above.

6 Proposed revision to the SI

The prototype kilogram has only been accessed on about three occasions since it was introduced in 1889 and comparisons between the six BIPM maintained copies of the prototype have shown changes within one another of up to seventy microgram over their hundred year existence [56]. It has been clear for some time that a fundamental constant based definition is desirable but until recently the experimental realization of the proposed methods did not approach the requisite level of accuracy. The alternatives presently being debated are a redefinition of mass in terms of the Planck constant (similar to the QED units of Stille in Table 1) or one based on atomic mass, that is defining the atomic mass unit or Avogadro constant (similar to the Dirac units).

6.1 Redefinition based on the Planck constant

In the past changes to definitions have often been made following a demonstrated instability in the prototype unit. In the case of the kilogram we have only recently had the method involving the Planck constant that promises to show up any absolute drifts and the target of fractional part in 10^8 per year accuracy suggested by Quinn [57] is well in hand.

With further refinement the moving coil method of 'measuring' h might enable metrologists to rapidly check for any absolute drift of the mass of a secondary mass standard. Such rapidity is unlikely to be achievable via the lattice spacing of silicon measurements of the Avogadro constant. Mills et al [51], proposed changing the definitions of some of the SI base units in order to provide a more stable system. Their proposed changes which are shown in Table 9, involve four of the SI Base units: the kilogram, ampere, kelvin and mole. For these units, they propose both a style of definition similar to the present one and also an alternative version more explicitly based on fundamental constants which sidesteps some of the difficulties in framing the traditional style of definition.

They rely on the constancy of h for the kilogram, of e, for the ampere (coulomb), and of k, for the kelvin respectively. The first two also implicitly rely on the constancy of $2e/h$, and h/e^2, and hence on correctness of the expressions (5) and (7) that would be demonstrated by closure of the quantum triangle discussed in Sect. 5.1 above. If definitions similar to those in the table are adopted, it would also remove a common (correlated), uncertainty from the estimates of the values of many fundamental constants, and would also generate a more explicit role for h in the SI.

The historical distinction between base and derived units was made in order to develop a logical hierarchy. It does not imply that velocity, for example, is a less fundamental physical quantity than length or time. However, it is clear from the proposed revisions that for the future SI the definitions of frequency and time must be introduced ahead of the other quantities. Interestingly, if one is using the systems of units discussed in Sect. 3 that are commonly used in theoretical science, time becomes more of a derived quantity.

Table 9 The proposed definitions of the kilogram, ampere, kelvin and mole that link these units to exact values of the Planck constant h, elementary charge e, Boltzmann constant k and Avogadro constant N_A, respectively (after Mills et al [51]).

Unit	traditional format definition	explicit fundamental constant definition
kilogram	The kilogram is the mass of a body whose equivalent energy is equal to that of a number of photons whose frequencies sum to exactly $[(299\,792\,458)^2/662\,606\,93]\times10^{41}$ hertz. OR The kilogram is the mass of a body whose de Broglie–Compton frequency is equal to exactly $[(299\,792\,458)^2/(6.626\,069\,3\times10^{-34})]$ hertz.	The kilogram, unit of mass, is such that the Planck constant is exactly $6.626\,069\,3\times10^{-34}$ joule second.
ampere	The ampere is the electric current in the direction of the flow of exactly $1/(1.602\,176\,53\times10^{-19})$ elementary charges per second	The ampere, unit of electric current, is such that the elementary charge is exactly $1.602\,176\,53\times10^{-19}$ coulomb.
kelvin	The kelvin is the change of thermodynamic temperature that results in a change of thermal energy kT by exactly $1.380\,650\,5\times10^{-23}$ joule.	The kelvin, unit of thermodynamic temperature, is such that the Boltzmann constant is exactly $1.380\,650\,5\times10^{-23}$ joule per kelvin
mol	The mole is the amount of substance of a system that contains exactly $6.022\,141\,5\times10^{23}$ specified elementary entities, which may be atoms, molecules, ions, electrons, other particles or specified groups of such particles.	The mole, unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles, is such that the Avogadro constant is exactly $6.022\,141\,5\times10^{23}$ per mole.

The proposed changes to the basic units of the SI have excited considerable interest and are presently being discussed by most national and international organisations representing scientists and technologists throughout the world. The proposals are provisionally scheduled for discussion by the General Conference on Weights and Measures in 2011.

6.2 Redefinition based on the unified atomic mass unit

The atomic mass unit, m_u, is largely an anthropic unit which had a compromise origin [58] which was contrived in order to rationalise the Chemical and Physical Atomic Weight scales. One could argue that a kilogram defined in terms of the atomic mass unit or the mass of ^{12}C is functionally equivalent to using the electron or proton mass and provides just as 'natural' a basis for an SI base unit as h does for a SI mass unit as any of the fundamental constants.

Mills et al [50] have earlier suggested the unified atomic mass unit as an alternative basis for defining the kilogram. It has the scientific merit that the definition of the kilogram would not be affected by any failure to close the 'quantum triangle' (as too would their proposed definition of the coulomb: comparisons of the watt realised by the moving coil method against W_{90} could still be used to sustain the kilogram constant). Although this would leave a small correlated uncertainty between the SI values of many fundamental physical constants, a definition in terms of a fixed m_u might be more readily acceptable to the less scientifically educated members of the public. However, in the proposed new definitions [52] of Table 9, the mass of one mole of ^{12}C would be $12(1\pm1.4\times10^{-9})$ g. Although the difference from unity is negligible in terms of present-day uncertainties, atomic mass measurements are already being made relative to $m_{\rm u}$ with fractional part in 10^{10} accuracy and have usually been made with greater accuracy than measurements at the laboratory mass scale.

Aside from aesthetic aspects, the overriding consideration must be the suitability of the SI for global use and whether the assumptions are fully justified. Thus, in part, the acceptability of the proposals also depends on to what degree one accepts the underlying scientific aspects such as $E = mc^2$ (presently experimentally verified to $1 - mc^2/E = -1.44(4) \times 10^{-7}$), $E = h\nu$ and, perhaps more realistically, the Josephson and von Klitzing relationships (5) and (7). If in future modifications are found to the expressions used or the quantum triangle fails to show closure to parts in 10^9 this would affect the way the kilogram is defined.

There would also be an additional implication for science in that the assumption that μ_0 or ε_0 is numerically equal to one that is made by many theoreticians. It would affect the relation between the units that they use (which are related to the former CGS units) and the SI. The unit-system based dependence of the impedance of free space on α might affect the α-dependence of the frequencies of different types of atomic clocks and hence on the interpretation of laboratory investigations of the constancy of the fine structure constant.

Debate continues, and, at the time of writing, the choice of h for the definition seems favoured. The size of the redefined SI units must in any case be chosen to lie within the tolerance of those produced by the earlier definitions and this requires consistent data on the value of the constants chosen for the replacement definitions in terms of the previous definitions, so much still depends on on-going measurements of the quantities concerned.

7 Conclusion

The on-going role of the Planck constant in the physical systems of units discussed in Sect. 3 has already extended to the maintained SI units and may lead to its incorporation, implicitly or explicitly in the definition of the kilogram. It will almost certainly in any case lead to a more stable mass unit.

It would, of course, be pleasing to many if h the centenarian were to be incorporated either explicitly or implicitly in the SI unit of mass chosen, rather than the relative newcomer: the twenty-seven year old $m_{\rm u}$!

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