SYDE Capstone Optimization

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1 Introduction

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\begin{aligned}
\textbf{Objective:} \quad & \min_{x^{(t)}} \quad \sum_{a\in A}\sum_{i\in R_t} \frac{1}{w_i}\Big(d\big(\mathrm{pos}_a^{(t)}, i\big) + d\big(i, h^*(i)\big)\Big) x_{ai}^{(t)} \\
\textbf{subject to:} \quad & \sum_{i\in R_t} x_{ai}^{(t)} \le 1, && \forall a \in A \\
& \sum_{a\in A} x_{ai}^{(t)} \le 1, && \forall i \in R_t \\
& \sum_{a\in A}\sum_{i\in R_t} x_{ai}^{(t)} = k_t \\
& x_{ai}^{(t)} \in \{0,1\}, && \forall a \in A,\ i \in R_t.
\end{aligned}
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The first constraint ensures each ambulance serves at most one incident, the second ensures each incident is served at most once, and the third ensures exactly k_t assignments per round.

where:

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\begin{aligned}
A &: \text{set of available ambulances, indexed by } a, \\
R_t &: \text{set of unserved incidents at round } t, \\
H &: \text{set of hospitals (fixed locations), indexed by } h, \\
\mathrm{pos}_a^{(t)} &: \text{current position of ambulance } a \text{ at round } t, \\
d(p,q) &: \text{Google Distance Matrix API distance between coordinates } p \text{ and } q, \\
h^*(i) &: \text{nearest hospital to incident } i, \\
w_i \in \{1,2,4,8,16\} &: \text{priority weight for incident } i \text{ (larger } w_i \text{ implies higher priority)}, \\
k_t = \min(|A|, |R_t|) &: \text{number of assignments in round } t, \\
x_{ai}^{(t)} &= \begin{cases} 1, & \text{if ambulance } a \text{ is assigned to incident } i \text{ at round } t, \\ 0, & \text{otherwise.} \end{cases}
\end{aligned}
$$

Interpretation:
The optimization seeks to minimize the total weighted travel cost of all ambulance–incident–hospital assignments during round t. Each term in the objective function combines:

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d\big(\mathrm{pos}_a^{(t)}, i\big) + d\big(i, h^*(i)\big),
$$

which represents the total distance an ambulance travels to reach incident i and then deliver the patient to the nearest hospital. The weighting factor $1/w_i$ ensures that higher-priority incidents (larger w_i) are given more influence in the optimization, effectively reducing their associated cost relative to lower-priority incidents.