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For: IGN's Code Foo

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I will first start by defining the dimensions of the various objects that were used in calculating this problem. According to the USA Table Tennis Board(<http://www.usatt.org/news/40mm_5-08-00.html>) as of the year 2000, the diameter of a standard ping pong ball is 40 mm. Since the radius of a sphere is radius = diameter/2, the radius is 20 mm. In order to keep all the units in this problem the same, I can convert this radius of 20 mm to 0.02 m. According to the National Transportation Library (<http://ntl.bts.gov/lib/10000/10600/10694/MBTC1054-1.pdf>) a "conventional" school bus is a "Type C" bus. The "recommended design" of this type of bus has a width of 2.4 m(8.0 ft), a length of 11.1 m(36.4 ft), and a height of 3.3 m(10.9 ft). Finding official information for the dimensions of the seats proved to be quite more difficult than I had expected. Luckily my mom is a teacher at an elementary school and was able to find this information for me by taking the measurements of the seats inside one of the actual school buses that is used at the school(a picture of the actual seat is placed in the appendix at the end of this document). Measurements for the seats were taken in centimeters, but were then converted into meters for this problem. Starting with the back part of the seat(the cushion where your back rests): the length is 0.98 m, the width is 0.74 m, and the height(thickness) is 0.09 m. The bottom part of the seat(the cushion where you sit) has a length of 0.98 m, a width of 0.40 m, and a height(thickness) of 0.10 m. The two front legs of the seat have a height of 0.29 m, while the two back legs are slightly taller having a height of 0.32 m. The measurements around the leg(the circumference) is 0.09 m. This bus contained a total of 24 seats(or could seat 67 passengers which is typical of a "Type C" bus).

The thought process to calculate the number of balls that could be contained inside a typical school bus is actually quite simple. All one would need to do is divide the volume of the bus by the volume of one ping pong ball. This could be shown as the following formula:  
  
NumberofBalls = Vbus / Vpingpongball

Using the information provided in the first paragraph, one can calculate the volume of the bus and the ping pong ball. Since the main part of the school bus is generally considered to be a rectangular shape, one could use the formula Vrectangle = length \* width \* height.  
  
Vbus = 2.4 m \* 11.1 m \* 3.3 m

Calculating this gives:  
  
 Vbus = 87.92 m3

Since the shape of a ping pong ball is a sphere, one could use the formula Vsphere = (4/3)(pi)r3 to calculate the volume of a ping pong ball.  
  
Vpingpongball = (4/3)(3.14)(0.02 m)3  
  
Calculating this gives:

Vpingpongball = 0.000034 m3

Substituting these numbers back into the original NumberofBalls formula:  
  
NumberofBalls = Vbus / Vpingpongball  
  
NumberofBalls = 87.92 m3 / 0.000034 m3

Calculating this gives approximately:

NumberofBalls = 2,623,430  
  
2,623,430, however, is not the final answer. Although the calculations are correct, this answer assumes that the bus is completely empty. In the real world, school buses are filled with all sorts of stuff that occupies volume inside the bus such as air conditioning units, various lights and their fixtures, and the steering wheel. Most importantly, the bulk of the volume on the inside is occupied by the seats. So in order to get a more accurate "real world" answer to this problem, the volume of the bus is going to have to be recalculated to take into account all the extra volume inside the bus. Since Vbus is no longer accurate we will reassign the volume 87.92 m3 to be Vemptybus or:  
  
Vemptybus = 87.92 m3  
  
The main idea in recalculating the volume of the bus is to find the volume of everything that is inside the bus and subtract it from the volume of the empty bus. That would give the usable volume inside the bus. The new formula for the volume of the bus becomes:

Vbus = Vemptybus - V1 - V2 - V3 - V4.......Vn

where Vn could be anything with calculable volume. For example V1 could be the volume of the steering wheel and V2 could be the volume of the air condition unit. Obviously I realize the accuracy of the answer will depend on how many elements are inserted into the formula. For the sake of simplicity and time(finding official measurements can be very time consuming), I am only going to subtract the volume of the total number of seats from the volume of the empty bus. The volume for all the seats will largely dwarf the volume of all the other miscellaneous stuff in the bus rendering the volume of the miscellaneous stuff almost negligible. The formula then becomes:  
  
Vbus = Vemptybus - V24seats

In order to calculate the volume of the 24 seats, one must first calculate the volume of one seat. In order to calculate the volume of one seat, one must break up the seats into smaller recognizable shapes. The back part of the seat is rectangular, the bottom part of the seat is rectangular, and the legs are shaped approximately like a cylinder(for simplicity assuming only four legs holding up the seat). Adding the volume of these parts will give use the total volume for one seat:  
  
Voneseat = Vseatback + Vseatbottom + Vlegs

The volume of the seat back can be calculated using the formula for the volume of a rectangle and the information from the first paragraph:  
  
Vseatback = 0.98 m \* 0.74 m \* 0.09 m

Calculating this gives:

Vseatback = 0.065268 m3

The volume of the seat bottom can be calculated using the formula for the volume of a rectangle and the information from the first paragraph:  
  
Vseatbottom = 0.98 m \* 0.40 m \* 0.10 m

Calculating this gives:

Vseatbottom = 0.0392 m3

Since the legs are approximately shaped like a cylinder, one can use the formula for the volume of a cylinder and the information from the first paragraph to calculate the volume of the legs. Since the back and front legs have slightly different heights, the formula from calculating the volume of the legs must be broken up further:  
  
Vlegs = Vfrontlegs + Vbacklegs

The formula to calculate the volume of a cylinder is Vcylinder = (pi)r2h. Since the radius of the leg is not known, it must be calculated from the measurements taken. The circumference of the legs were measured to be 0.09 m. Since the measurement taken around the leg is essentially a circle created by cross section of the cylinder, one could use the formula of the circumference of a circle and a little bit of algebra to figure out the value of the radius. Ccircle = 2(pi)r. Using this formula and substituting Ccircle with 0.09 m:  
  
Ccircle = 2(pi)r

0.09 m = 2(pi)r

Using algebra to solve for r:  
  
0.09 m / (2(pi)) = r

Calculating this gives:  
  
0.014324 m = r

Since r has been calculated to be 0.014324 m, it can now be substituted back into the formula of a cylinder. Using the measurements for the height of the legs, one can calculate the volume of the legs. Starting with the front legs:  
  
Vfrontleg = (pi)(0.014324 m)2(0.29 m)  
  
Calculating this gives:  
  
 Vfrontleg = 0.000187 m3

Since there are two front legs, one must multiply by two to get the volume of both front legs:  
  
Vfrontlegs = Vfrontleg \* 2

Calculating this gives:  
  
Vfrontlegs = 0.000374 m3  
  
Similarly for the backlegs:  
  
Vbackleg = (pi)(0.014324 m)2(0.32 m)  
  
Calculating this gives:  
  
 Vbackleg = 0.000206 m3

Since there are two back legs, one must multiply by two to get the volume of both back legs:

Vbacklegs = Vbackleg \* 2

Calculating this gives:  
  
Vbacklegs = 0.000431 m3  
  
Now substitute these numbers back into the Vlegs formula:  
  
Vlegs = Vfrontlegs + Vbacklegs

Vlegs = 0.000374 m3 + 0.000431 m3

Vlegs = 0.000786 m3

All the information that was needed to calculate the volume for one seat is now ready to be inserted into the formula:  
  
Voneseat = Vseatback + Vseatbottom + Vlegs

Voneseat = 0.065268 m3 + 0.0392 m3 + 0.000786 m3

Calculating this gives:  
  
Voneseat = 0.105254 m3

Since the volume for one seat is now known, it is possible to calculate the volume of the 24 seats by simply multiplying the volume of one seat by 24:  
  
V24seats = Voneseat \* 24  
  
Calculating this gives:  
  
V24seats = 2.52611 m3

Now that the volume of the 24 seats that occupies the inside of the bus is known, it can be subtracted from the empty volume of the bus so that a more accurate representation of amount of balls that could fit inside can be calculated. Substituting V24seats back into the formula for the bus:

Vbus = 87.92 m3 - 2.52611 m3  
  
Calculating this gives:  
  
Vbus = 85.3939 m3  
  
Now that there is a better representation of the volume that the ping pong balls can occupy, one can recalculate the number of ping pong balls that can fit in a school bus:

NumberofBalls = 85.3939 m3 / 0.000034 m3

Calculating this gives approximately:  
  
NumberofBalls = 2,511,590

Unfortunately, although this number looks a little more accurate, it still isn't accurate enough. Since the goal is to find the most "realistic" number possible, it is imperative that the physical properties of the sphere are also taken into account; more specifically, the physicall properties of spheres when stacked onto and next to each other. Spheres only touch at one point on thier surface, as seen in Figure 1.

  
  
Figure 1

This creates empty space between each ball as they are piled onto each other. Therefore, the numbers of ping pong balls ***must be*** ***less*** than the latest calculated value of 2,511,590 because this calculation does not take into account the volume occupied by the empty space that remains as a result of the physical properties of stacking spheres onto each other. One can, however, account for this empty space by pretending that the spheres were cubes with the same diameter of the sphere, 0.04 m. This concept can be seen in Figure 2.

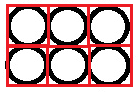


Figure 2

The formula to calculate the volume of a cube is Vcube = length \* width \* height, but since all the sides are the same, it can be rewritten as Vcube = s3, where s is one side of the cube. From the information contained in the first paragraph, the diameter of the sphere is 40 mm or 0.04 m. This can be used as s since the diameter is the same size as any side of the cube:  
  
Vcube = (0.04 m)3

Calculating this gives:  
  
Vcube = 0.000064 m3

Now this number can be substituted for Vpingpongball in our original formula:

NumberofBalls = Vbus / Vpingpongball

Substituting:

NumberofBalls = Vbus / Vcube

Replacing:  
  
NumberofBalls = 85.3939 m3 / 0.000064 m3  
  
Calculating this gives approximately:  
  
NumberofBalls = 1,334,280

As one can note, taking into account this small change in mentality can change the result drastically. An important lesson that can be learned from this problem is although the answer may seem simple, one must break down the problem completely in order to achieve maximum accuracy.   
  
As mentioned earlier, this answer can be improved even further by recalculating the volume of the bus as you add more elements that are contained on the inside of the bus. However, I believe that doing this would make very little difference, especially after the calculation for the twenty-four seats only subtracted around 2.5 m3 from the overall volume. I think it is safe to say that almost all other things contained within the bus take up volume is negligible in the scope of this problem. ***The final answer then would be that approximately 1,334,280 ping pong balls fit inside a typical "Type C" school bus with 24 seats.***

**Appendix**

**Websites used to find official information:**  
  
Ping Pong Ball Information:

<http://www.usatt.org/news/40mm_5-08-00.html>

School Bus Information:

<http://ntl.bts.gov/lib/10000/10600/10694/MBTC1054-1.pdf>  
  
**Picture of school bus seat that measurements were taken from:**  
  
