Finding Submarines

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Abstract

Acoustic pressure data sampled over 24 hours in the Puget Sound was analyzed with the goal of locating a submarine known to be generating an unknown frequency signature within the signal. The frequency signature was identified by applying the Fourier transform method to the signal samples and then examining the time average to find the dominating frequency. Once known, a Gaussian filter was applied to remove non dominating frequencies from the transform to remove noise. The signal was then recovered with the inverse Fourier transform method allowing the submarine's location to be tracked precisely throughout the 24 hour period.

1 Introduction

A submarine with unknown characteristics was known to have traveled through the Puget Sound over a 24 hour time period. Due to the new technologies implemented on the submarine little was known about it and it was crucial that the submarine's location could be tracked so it could be monitored in the future. A broad spectrum recording of acoustic pressure taken in half-hour increments was available. While the data likely contained the frequency signal from the submarine many other signals were also captured causing the data to be noisy and the submarine's location was difficult to identify. The main goal with this study was to identify the signature frequency emitted by the submarine as well as effectively remove noise from the signal to be able to track the submarine's path. To accomplish these tasks the discrete Fourier transform technique was implemented to analyze the signal and filter out the unwanted frequencies. The Fourier transform is a powerful technique in signal processing that is used on data to examine the signal within the frequency domain. By examining the frequency content of the signal one can obtain useful information about important features in the signal and guide future analysis [3]. The signal can also be modified in this domain causing which will manifest themselves when the original signal is recovered through the inverse Fourier Transform. This makes the Fourier transform a useful tool for removing noise as you can filter a signal by its most dominant frequencies which will keep major features of the signal while removing unwanted or unneeded features.

2 Theoretical Background

The discrete Fourier transform, an approximation of the continuous Fourier transform used for discrete signals, is defined as such

$$\hat{f}(k_n) \approx \frac{1}{N} \sum_{n=0}^{N-1} f(x_n) e^{-i2\pi k_n n/N}$$

where $f(x_n)$ is each sample of the original signal and N is the number of samples in the signal. The transform can be extended to multiple dimensions:

$$\hat{f}(\overline{K}_n) \approx \frac{1}{N_x N_y \dots} \sum_{n_x=0}^{N_x-1} \sum_{n_y=0}^{N_y-1} \dots f(x_n, y_n, \dots) e^{-i2\pi k_{nx} n_x/N_x - i2\pi k_{ny} n_y/N_y \dots}$$

where $\overline{K}_n$ is a tensor of the desired dimensions. One of the most important properties of the Fourier transform is that it can be reversed via the inverse Fourier transform to recover the original signal:

$$f(x_n) \approx \sum_{n=0}^{N-1} \hat{f}(k_n) e^{i2\pi k_n n/N}$$

This means adjustments can be made to the signal within the frequency domain which will then manifest themselves once transformed back into the original signal. There are a multitude of useful techniques one can apply within the transformed domain with different applications [3]. One such technique is to filter the frequencies of the signal to remove noise and single out the important features of the signal. This is possible because random noise, when averaged over many samples, will have a low overall frequency compared to a consistent feature within the signal. By filtering out these lower frequencies in the frequency domain you will remove irrelevant noise when you take the inverse transform while maintaining the important high frequency features of the signal. There are many types of filters one can apply. In this study a 3D Gaussian filter was chosen because it includes the largest frequencies which will be clustered together at its peak and smoothly removes the lower frequencies towards the tails of the filter which makes it ideal for de-noising. The three dimensional Gaussian function is defined as such:

$$f(x, y, z) = e^{-((x-\mu_x)^2+(y-\mu_y)^2+(z-\mu_z)^2)/2\sigma^2}$$

Where μ is the mean of the distribution to be located at the center and σ is the standard deviation chosen to best filter the data.

3 Algorithm Implementation and Development

The data was initially flattened into 49 samples of a one dimensional array so it was first reshaped back into its original shape of 49 time samples of a 3D spatial signal. To perform the needed transforms the Numpy fourier transform toolset `numpy.fft`, named after the Fast Fourier Transform algorithm it employs, was used [1]. It contains methods for doing Fourier transforms and inverse Fourier transforms in multiple dimensions. Using these functions the Fourier transform was calculated for each time sample in the signal. After performing the transform the indices were shifted using the `fftshift` method to center the zero frequency component at the center of the array for visualization and interpretation purposes. The

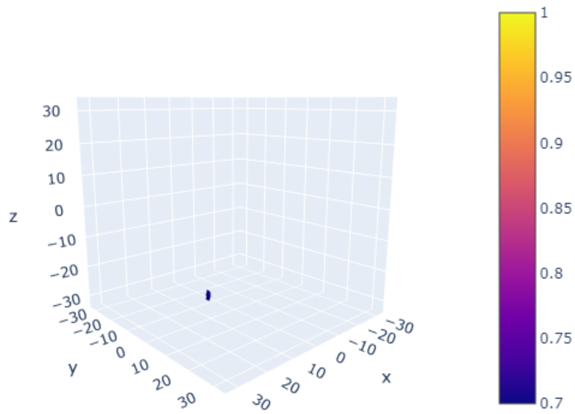


Figure 1: Top 30% of the frequencies. It is clear there is a distinct maximum frequency.

transforms were then averaged over time into a single array and visualized using the Plotly library [2]. By examining different isosurfaces (Figure 1) it was clear there was a distinct signature frequency that stood out as the largest frequency.

This frequency was found and its coordinates were used to center a three dimensional Gaussian filter around the maximum frequency. To determine the standard deviation for the Gaussian function multiple sigma values were tested on a single sample and visualized as a 2D slice through the submarine's location (Figure 2).

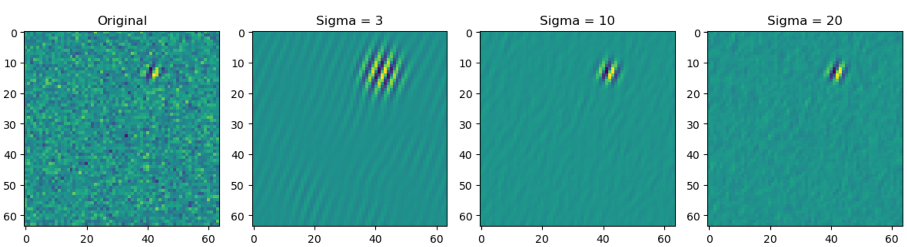
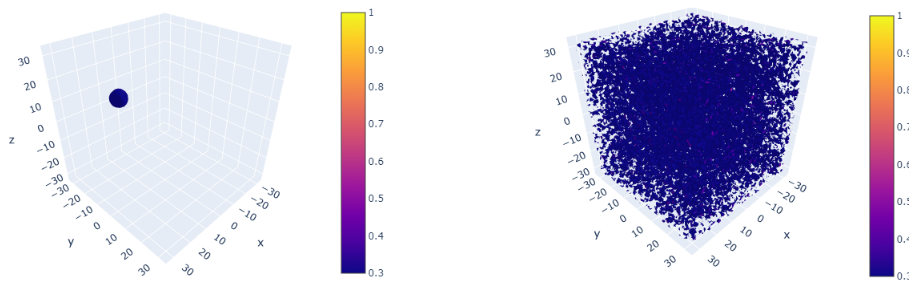


Figure 2: Result of noise filtering with various standard deviations.

A standard deviation of 10 was ultimately chosen and the filter was applied to each sample. The inverse Fourier transform was then performed on each sample returning the original signal and successfully removing much of the noise. After the filtering process the location of the submarine could be easily determined as the maximum value within the signal.

4 Computational Results

Through the averaging of the Fourier Transform over the samples the signature of the frequency was identified to be $161.72 + 70.90i$. A Gaussian filter optimized for the data was successfully applied around this center frequency and the noise present in the original signal was greatly reduced. A visual representation is the clearest way to see the effects of the de-noising treatment. A view of the top 70% of the cleaned data (Figure 3a) had no discernible noise allowing the submarine's location to be easily seen in the 3D space. In contrast taking the same view of the original signal (Figure 3b) makes the submarine's location indiscernible.

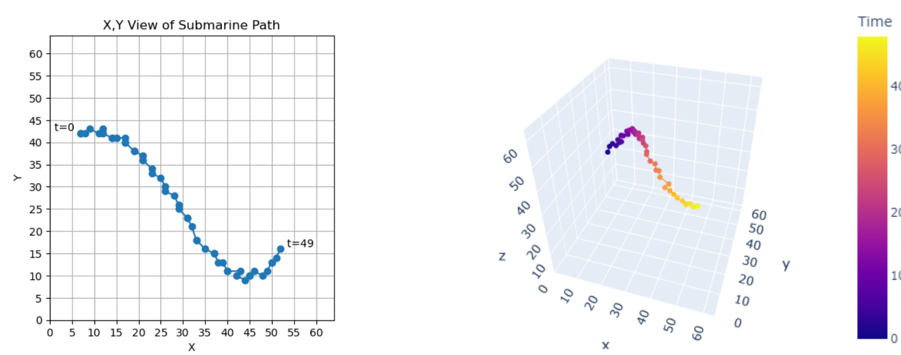


(a) The clean signal

(b) The original signal

Figure 3: A comparison of the top 70% of the data of a single sample between the original signal and the result of the Gaussian filtering.

From the now cleaned samples the location of the submarine could be easily identified as the maximum value in the signal and tracked over the 24 hour period in 30 minute increments. (Figure 4).



(a) An overhead view of the path

(b) A snapshot of the 3D path

Figure 4: The path of the submarine over the 24 hour period seen from an overhead view (a) and a snapshot from a 3D rendering of the path (b).

5 Summary and Conclusions

The Fourier Transform allows for powerful analysis of signals in any dimension. Major features of the signal are seen as high frequencies in the signal domain which allowed us to identify the signature frequency of the submarine amidst a noisy signal. Filtering techniques within the frequency domain allow important features to be extracted from a signal by removing the lower frequencies and using the inverse Fourier Transform to return to the signal domain. This technique allowed us to remove most of the noise from the signal and clearly track the submarine over a 24 hour period. The analysis performed in this paper has useful applications in underwater surveillance allowing any object with a signature frequency to be tracked precisely using recordings of acoustic data. The methods could be automated to search for objects and alert interested persons of the appearance of objects of interest. which has applications in search and rescue operations, ecological studies, and national security.

6 Acknowledgements

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