MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Fall 2007**

Turn in at your table labeled with your name and group (e.g. L02 3A)

Problem Set 4

Due: Wednesday, October 3rd at beginning of class (before 10:15/12:15)

Warm Up
Problem 1: Parallel Plate Capacitor

A parallel-plate capacitor is charged to a potential V_0, charge Q_0 and then disconnected from the battery. The separation of the plates is then halved. What happens to
(a) the charge on the plates?
(b) the electric field?
(c) the energy stored in the electric field?
(d) the potential?
(e) How much work did you do in halving the distance between the plates?

Exam Problem
Problem 2: Capacitor

A parallel plate capacitor has capacitance C. It is connected to a battery of EMF ε until fully charged, and then disconnected. The plates are then pulled apart an extra distance d, during which the measured potential difference between them changed by a factor of 4. Below are a series of questions about how other quantities changed. Although they are related you do not need to rely on the answers to early questions in order to correctly answer the later ones.

a) Did the potential difference increase or decrease by a factor of 4?

b) By what factor did the electric field change due to this increase in distance? Make sure that you indicate whether the field increased or decreased.

c) By what factor did the energy stored in the electric field change? Make sure that you indicate whether the energy increased or decreased.

d) A dielectric of dielectric constant κ is now inserted to completely fill the volume between the plates. Now by what factor does the energy stored in the electric field change? Does it increase or decrease?

e) What is the volume of the dielectric necessary to fill the region between the plates? (Make sure that you give your answer only in terms of variables defined in the statement of this problem, fundamental constants and numbers)

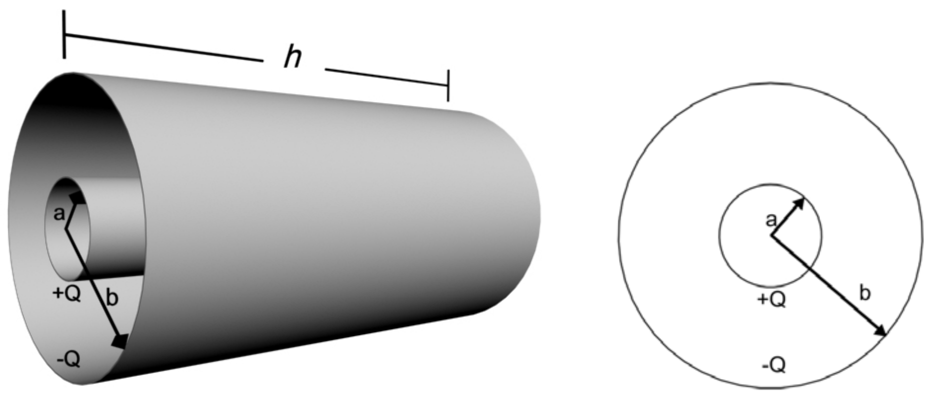
Experiment 2: Faraday Ice Pail Pre-Lab Questions

Don't forget to read the pre-lab before attempting these questions so you'll have some idea what we are talking about!

Problem 3: Capacitance of our Experimental Set-Up

In this experiment we will measure the potential difference between the pail and the shield, and make statements about the charge on the pail based on this. Here you will calculate the relationship between charge and potential.

Consider two nested cylindrical conductors of height h and radii a & b respectively. A charge $+Q$ is evenly distributed on the outer surface of the pail (the inner cylinder), $-Q$ on the inner surface of the shield (the outer cylinder).



(a) Calculate the electric field between the two cylinders ($a < r < b$).
NOTE: Yes, this is the 3rd time you've done this calculation. If we ask you this on the exam you'll thank us for the practice…

(b) Calculate the potential difference between the two cylinders:

$$\Delta V = V(a) - V(b)$$

(c) Calculate the capacitance of this system, $C = Q/\Delta V$

(d) Numerically evaluate the capacitance for your experimental setup, given:
$h \cong 15$ cm, $a \cong 4.75$ cm and $b \cong 7.25$ cm

The capacitance should be given in Farads (1 F = 1 Coulomb/Volt) or some fraction thereof (mF, μF, …) ***<u>Write this number in your notes</u>***, as you will use it to convert from the measured voltage difference ΔV to a charge on the outer surface of the pail (inner cylinder) Q, using $Q = C\,\Delta V$

Problem 4: What about charge on the other surface?

In the previous problem we assumed that charge was located on the outer surface of the inner cylinder and the inner surface of the outer one, in other words, if both cylinders were charged. What if instead the cylinders were both neutral but exhibited charge separation due to the presence of a positive charge Q sitting in the interior of the inner cylinder? What now is the potential difference between the two cylinders?

Problem 5: Experimental Predictions

In each of the three parts of this question you are asked to sketch potential difference versus time as you move charges into or around the system. You should mark the given actions on the time axis (assume they are each done very quickly but with some time in between) and indicate what happens to the potential difference when each action is made. In each case the potential difference should start at zero volts. NOTE: Of course you can't give any numbers, we just want you to indicate if the potential increases or decreases or goes to zero or back to some previous value.

A. Prediction: Charging by Contact

Sketch your prediction for the graph of potential difference vs. time for part 2 of this experiment. Indicate the following events on the time axis:

(a) Insert positive charge producer into pail
(b) Rub charge producer against inner surface of pail
(c) Stop rubbing (move away from surface, but leave inside pail)
(d) Remove charge producer

B. Prediction: Charging by Induction

Sketch your prediction for the graph of potential difference vs. time for part 3 of this experiment. Indicate the following events on the time axis:

(a) Insert positive charge producer into pail
(b) Ground pail to shield
(c) Remove ground contact between pail and shield
(d) Remove charge producer

C. Prediction: Electrostatic Shielding

Sketch your prediction for the graph of potential difference vs. time for part 4 of this experiment. Indicate the following events on the time axis:

(a) Bring positive charge producer near (but outside of) shield
(b) Move charge producer away

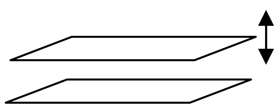
Approximations, Fermi Problems, Back of the Envelope Calculations…

Problem 6: UROP Interview (AKA Shameless Plugging)

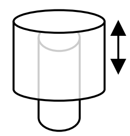
A graduate student in my lab has developed a motion control system (we call it a "walker") that can move objects over about 5 millimeters with 0.1 to 1 nm resolution ("step size"). We want to publish the details of this device but first of all have to characterize its motion. The problem is that there aren't many conventional measurement systems that can measure distances of several millimeters with sub-nanometer resolution.

Your job on this assignment is to design such a system. We have a couple of ideas we'd like you to flesh out, both involving capacitive measuring devices. The first involves designing a parallel plate capacitor, with one plate fixed and the other moving towards or away from the first as the walker moves forward or backward. The second involves a cylindrical capacitor, where the inner plate of the capacitor is fixed and the outer plate of the capacitor moves forward or backward with the walker.

Parallel Plate Method



Cylindrical Capacitor Method



You have at your disposal one of the finest commercial capacitance bridges, an Andeen-Hagerling 2550A, pictured below. It can read up to 1.5 μF with 0.8 aF (1 aF = 10^{-18} F) resolution (although only 8 digits of accuracy).

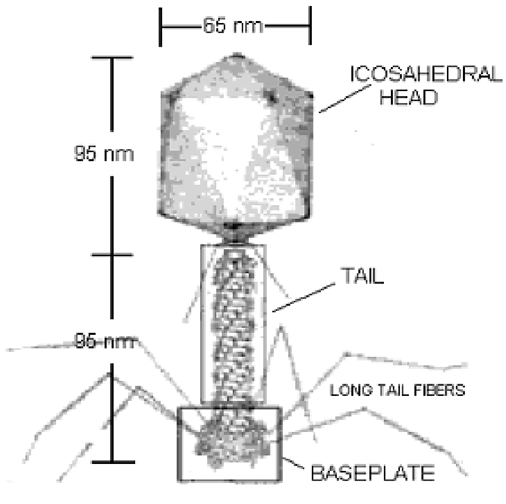
Which method should we use (or is there another you can suggest)? What will our measurement (distance) resolution be? Will it change with position? We have room for about 3 cm outer diameter (I've drawn the parallel plates as squares, but feel free to make them circular). Machining precision limits spacing to about 0.1 mm at the smallest (in both methods – you can do better if it really matters but we want this to be robust).

Problem 7: Human Capacitor

What, approximately, is the capacitance of a typical MIT student? Check out the exhibit in Strobe Alley (4^{th} floor of building 4) for a hint or just to check your answer.

Problem 8: DNA in T4 Phage

The T4 phage (pictured at right) has a strand of DNA about 50 μm long wound up inside of its head. DNA has a net charge of one electron every 1.7 Å.



(a) About how much energy does it take to put that charge into the head?

(b) About how much pressure is exerted on the head? (HINT: $P = -dU/dV$ and one atmosphere ~ 10^5 Pascal, the SI unit)

(c) Is this reasonable? Is there something that we are forgetting about?

Analytic Problems

Problem 9: More Reflections on a Conducting Plane

In the last problem set you considered a charge q a distance d above an infinitely large, uncharged, perfectly conducting, shiny piece of metal. Thinking about the answers to that problem,

(a) What is the charge density on the surface of the conductor as a function of position r?

(b) What is the total charge on the surface of the conductor (the part of the surface that is facing the charge, *not* the bottom side facing away)? Does this make sense?