MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Fall 2007**

Turn in at your table during class labeled with your name and group (e.g. L02 3C)

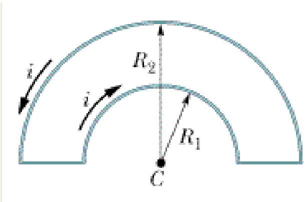
Problem Set 7

Due: Wednesday, October 24th at beginning of class (before 10:15/1:15)

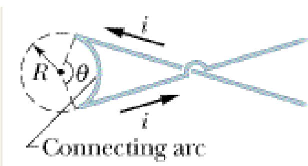
This problem set focuses on material from classes 15-17 (week 7). Textbook references can be found at the top of the summaries from these three days.

<u>Warm Up</u>

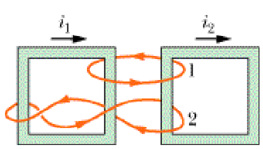
Problem 1: Quickies…



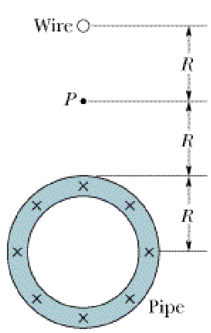
a) Two semicircular arcs have radii R_2 and R_1, carry current i, and share the same center of curvature C. What is the magnitude of the net magnetic field at C?



b) A wire with current i is shown at left. Two semi-infinite straight sections, both tangent to the same circle with radius R, are connected by a circular arc that has a central angle θ and runs along the circumference of the circle. The connecting arc and the two straight sections all lie in the same plane. If $B = 0$ at the center of the circle, what is θ?



c) The figure at left shows two closed paths wrapped around two conducting loops carrying currents i_1 and i_2. What is the value of the integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for **(a)** path 1 and **(b)** path 2?

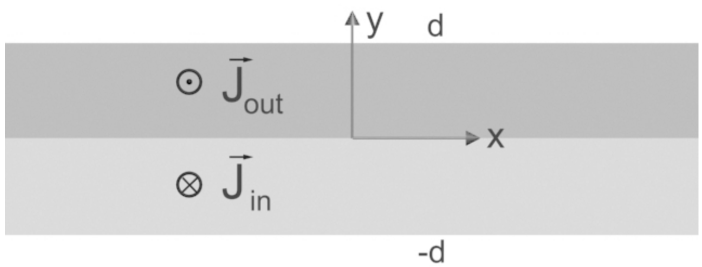


Problem 2: Two Currents
In the figure at left a long circular pipe with outside radius R carries a (uniformly distributed) current i into the page. A wire runs parallel to the pipe at a distance of 3.00R from center to center. Find the current in the wire such that the ratio of the magnitude of the net magnetic field at point P to the magnitude of the net magnetic field at the center of the pipe is x, but it has the opposite direction.

Sample Exam Problems (two this week)

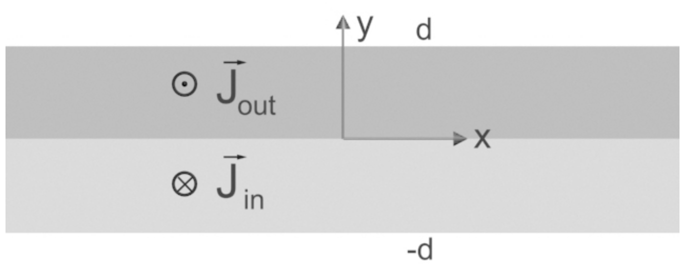
Problem 3: Current Slabs

The figure below shows two slabs of current. Both slabs of current are infinite in the x and z directions, and have thickness d in the y-direction. The top slab of current is located in the region $0 < y < d$ and has a constant current density $\vec{\mathbf{J}}_{out} = J\,\hat{\mathbf{z}}$ out of the page. The bottom slab of current is located in the region $-d < y < 0$ and has a constant current density $\vec{\mathbf{J}}_{in} = -J\,\hat{\mathbf{z}}$ into the page.

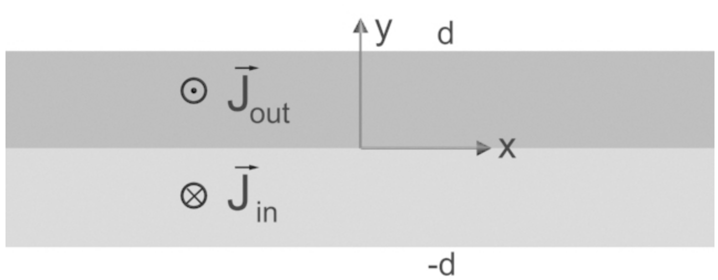


(a) What is the magnetic field for $|y| > d$? Justify your answer.

(b) Use Ampere's Law to find the magnetic field at $y = 0$. On the figure below show the Amperian Loop that you use and give the magnitude and direction of the magnetic field.

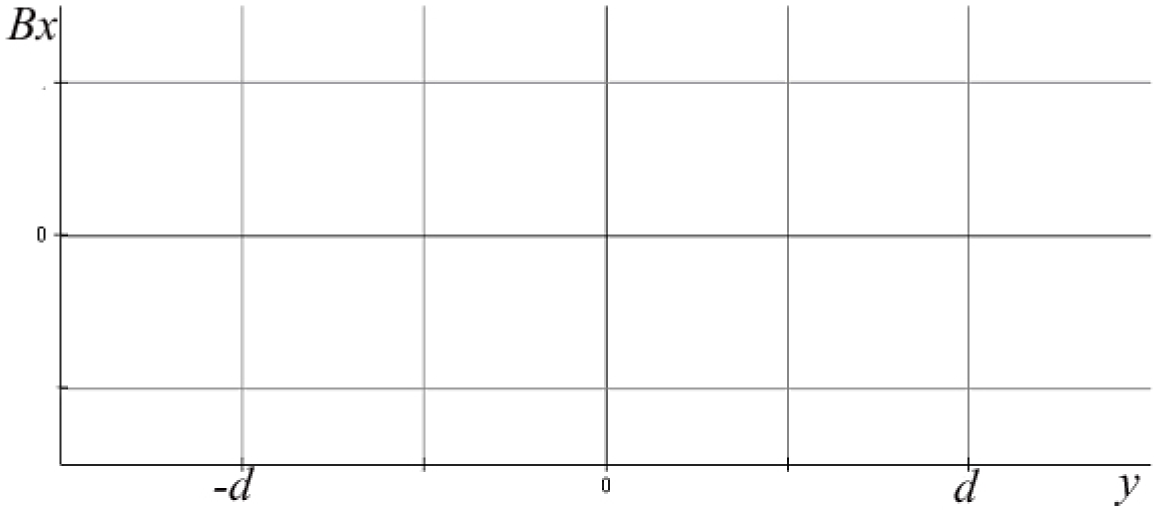


(c) Use Ampere's Law to find the magnetic field for $0 < y < d$. On the figure below show the Amperian Loop that you use and give the magnitude and direction of the magnetic field.



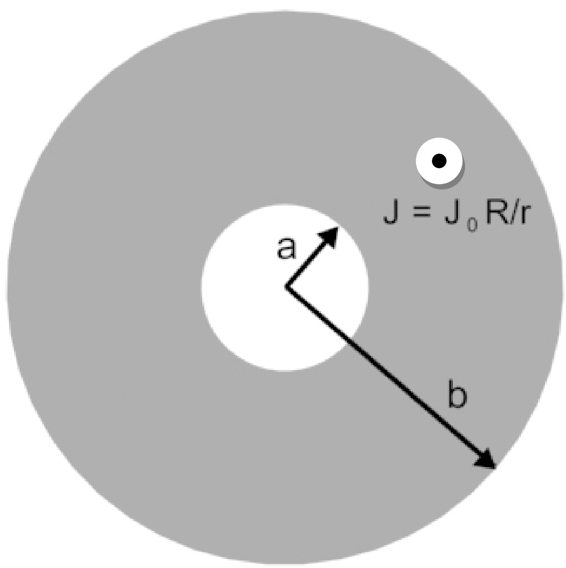
Problem 3: Current Slabs *continued*…

(d) Plot the x-component of the magnetic field as a function of the distance y on the graph below. Label your vertical axis.



Problem 4: Cable

A long cylindrical cable consists of a conducting cylindrical shell of inner radius a and outer radius b. The current density $\vec{\mathbf{J}}$ in the shell is out of the page (see sketch) and varies with radius as $J(r) = J_o \dfrac{R}{r}$ for $a < r < b$ and is zero outside of that range.



Problem 4: Cable *continued…*

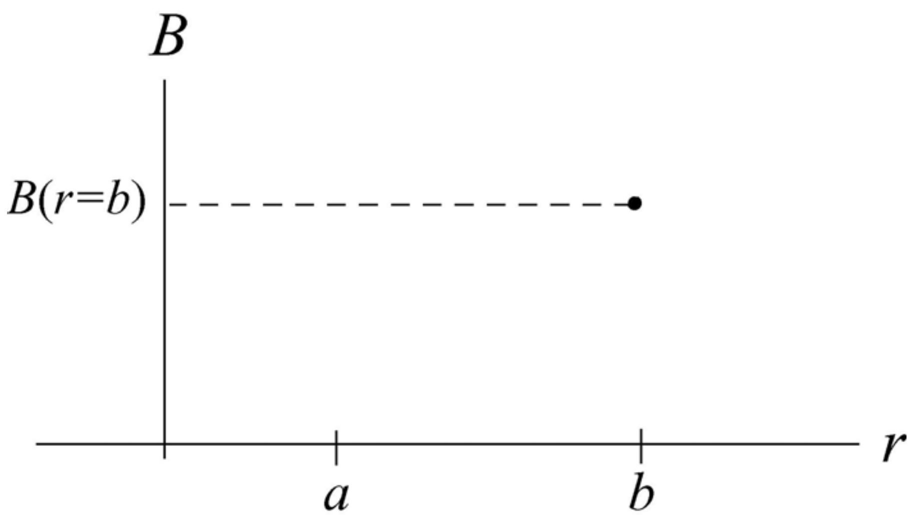
Find the magnetic field in each of the following regions, indicating both magnitude and direction. Show your work and your Amperian loops.

(a) $r < a$

(b) $a < r < b$

(c) $r > b$. Give also $B(r = b)$, the value of the magnetic field at $r = b$.

(d) Plot your previous answers for the magnitude of B on the graph below.



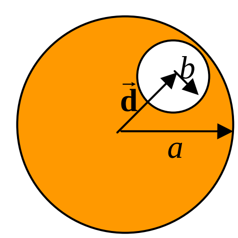
Analytic Problems…

Problem 5: Rotating Cylinder

A very long uniformly charged solid cylinder of charge $+Q$ and radius R and length L lies coaxial with the z-axis and is rotating counter clockwise about it with angular velocity ω. Ignoring end effects due to the finite length of the cylinder, what is the direction and magnitude of the magnetic field $\vec{\mathbf{B}}_{cylinder}$ inside and outside the cylinder? HINT: What does this look like? Is there any place that the field is zero?

Problem 6: Cut up wire

A long solid copper wire of radius *a* has a cylindrical hole (radius *b*) bored out of it that is offset from the center by a vector $\vec{\mathbf{d}}$. A current *I* is flowing through the copper wire. Find the magnetic field everywhere inside the hole.



Problem 7: Magnetic Field Strengths

A friend asks you to help with his problems with magnetic fields in his new house. Here are his comments:

> I had my inspector inspect the house and he concluded that the electromagnetic field levels inside the house were unacceptable. They ranged from a low of about 3 mG to a high of about 5.8 mG in the master bedroom, which is closest to the power lines running parallel to the back of the house at a distance of about 35 feet away. The inspector indicated to me that he considered levels above 2 mG as unacceptable and recommended that levels under 1 mG would be preferred. I could bury the lines (with dubious chances of reducing the EMF levels: a worker at the power company told me that it could make things worse because the lines would be closer to the house and the earth doesn't stop or mitigate EMFs but the inspector and some literature I've read postulated that with the wires in closer physical proximity the EMFs would cancel each other out to a greater extent) at a cost exceeding $20,000 just for the 50 feet or so in the back of my house, or live in the house and risking the health and possibly lives of myself and my family including any unborn children which may be especially susceptible to adverse health effects when they are in the womb.

Let's see what you should tell your friend.

a) Based on your real life experiences, how far apart do you think the wires are now? For the purposes of this problem assume that there are just two parallel wires carrying equal current in opposite directions – this is basically correct.

b) Based on (a) and the numbers he gave you how much current are they carrying?

c) If you bury them how much closer will they be to the house (how deep is a trench that power lines are buried in?) and how much closer to each other will the wires be?

d) How much will this change the field in the master bedroom?

e) Most importantly, we need to know what to think about these field sizes. I'm sitting next to a desk lamp with an old fashioned power cord (the kind with the two wires running parallel in a molded plastic cover) less than a foot away from me. How does the field from this compare to the fields mentioned above?

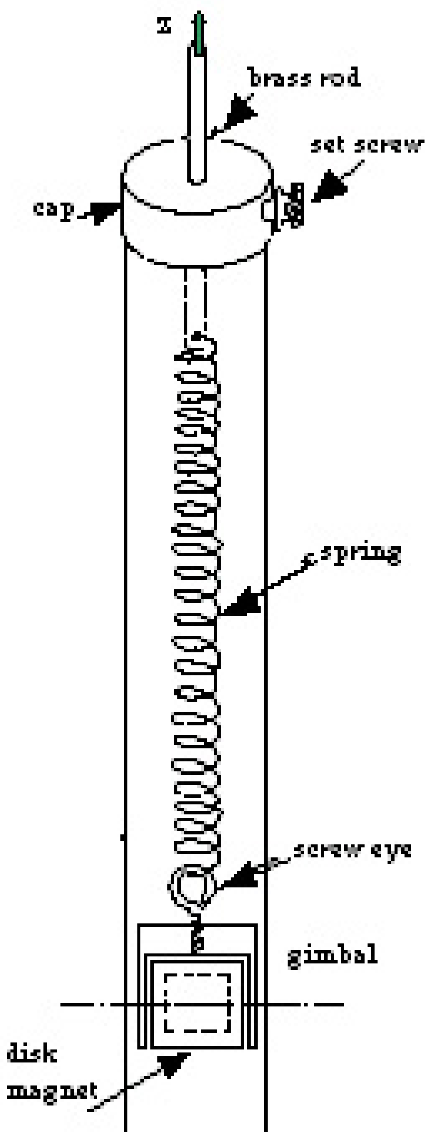
Experiment 4: Forces and Torques on Magnetic Dipoles

Problem 8: Force on a Dipole in the Helmholtz Apparatus

In class you calculated the magnetic field along the axis of a coil to be given by:

$$B_{axial} = \frac{N\,\mu_0\,I\,R^2}{2}\frac{1}{(z^2+R^2)^{3/2}}$$

where z is measured from the center of the coil.



In this lab we will have a disk magnet (a dipole) suspended on a spring, which we will use to observe forces on dipoles due to different magnetic field configurations.

(a) Assuming we energize only the top coil (current running counter-clockwise in the coil, creating the field quoted above), and assuming that the dipole is always well aligned with the field and on axis, what is the force on the dipole as a function of position? (HINT: In this situation $F = \mu \; dB/dz$)

(b) The disk magnet (together with its support) has mass *m*, the spring has spring constant *k* and the magnet has magnetic moment μ. With the current on, we lift the brass rod until the disk magnet is sitting a distance z_0 above the top of the coil. Now the current is turned off. How does the magnet move once the field is off (give both direction and distance)?

(c) At what height(s) is the force on the dipole the largest?

(d) What is the force where the field is the largest?

(e) Our coils have a radius R = 7 cm and N = 168 turns, and the experiment is done with I = 1 A in the coil. The spring constant k ~ 1 N/m, and μ ~ 0.5 A m^2. The mass m ~ 5 g is in the shape of a cylinder ~ 0.5 cm in diameter and ~ 1 cm long. If we place the magnet at the location where the spring is stretched the furthest when the field is on, at about what height will the magnet sit after the field is turned off?