18.02 Problem Set 7

Due Thursday 10/25/07, 12:45 pm in 2-106.

Part A (15 points)

Hand in the $\underline{\text{underlined}}$ problems only; the others are for more practice.

Lecture 18. Thu Oct. 18 Double integrals.
Read: 14.1–14.3; Notes I.1. Skip the theory. Concentrate on:
Iterated integrals, pp. 942–944, Examples 2–4;
Evaluation of double integrals, pp. 950–952 and Notes I.1;
Calculation of area and volume 14.3.
Work: 3A/ 1, 2ab$\underline{\text{c}}$, 3a$\underline{\text{b}}$, 4b$\underline{\text{c}}$, 5$\underline{\text{a}}$bc, 6.

Lecture 19. Fri Oct. 19 Double integrals in polar coordinates.
Read: 14.4 (concentrating on the examples), Notes I.2;
14.5 centroid, pp. 968–970; moment of inertia pp. 973–975.
Work: 3B/ 1$\underline{\text{a}}$cd, 2b$\underline{\text{d}}$, 3$\underline{\text{ac}}$; 3C/ $\underline{1}$, $\underline{\text{2a}}$, $\underline{4}$; 14.4/ $\underline{4}$, $\underline{17}$, 25; 14.5/ 11.

Lecture 20. Tue Oct. 23 Change of variables.
Read: Notes CV; 14.9 Examples 1–4.
Work: 3D/ $\underline{1}$, $\underline{2}$, 3, $\underline{4}$ (integrate in the order $du\,dv$).

Part B (23 points)

Directions: Attempt to solve *each part* of each problem yourself. If you collaborate, solutions must be written up independently. It is illegal to consult materials from previous semesters. With each problem is the day it can be done.

Write the names of all the people you consulted or with whom you collaborated and the resources you used.

Problem 1. (Thursday, 3 points: 1+2)

Evaluate $\displaystyle\int_0^\infty \frac{e^{-x} - e^{-ax}}{x}\,dx$ as follows:

a) Compute $\displaystyle\int_1^a e^{-xy}\,dy$.

b) Use part (a) to rewrite the integral you wish to evaluate as a double integral. Evaluate the double integral by exchanging the order of integration.

Problem 2. (Friday, 2 points)

Show that the average distance of the points of a disk of radius a to its center is $2a/3$.

Problem 3. (Friday, 4 points)

In general, the moment of inertia around an axis (a line) L is

$$I_L = \iint_R \operatorname{dist}(\cdot, L)^2\,\delta\,dA.$$

The collection of lines parallel to the y-axis have the form $x = a$. Let $I = I_y$ be the usual moment of inertia around the y-axis

$$I = \iint_R x^2\,\delta\,dA.$$

Let $\bar{I}$ be the moment of inertia around the axis $x = \bar{x}$, where $(\bar{x}, \bar{y})$ is the center of mass. Show that

$$I = \bar{I} + M\bar{x}^2.$$

This is known as the parallel axis theorem.

Problem 4. (Friday, 5 points)

Find the average area of an inscribed triangle in the unit circle. Assume that each vertex of the triangle is equally likely to be at any point of the unit circle and that the location of one vertex does not affect the likelihood the location of another in any way. (Note that, as seen in Problem set 4, the maximum area is achieved by the equilateral triangle, which has side length $\sqrt{3}$ and area $3\sqrt{3}/4$. How does the maximum compare to the average?)

<u>Hint</u>: in order to reduce the problem to the calculation of a double integral, place one of the vertices of the triangle at $(1, 0)$, and use the polar angles θ_1 and θ_2 of the two other vertices as variables. What is the region of integration?

Problem 5. (Tuesday, 5 points) Notes 3D/7.

Problem 6. (Tuesday, 4 points)

Find the area of the ellipse $(2x + 5y - 3)^2 + (3x - 7y + 8)^2 < 1$.