MASSACHUSETTS INSTITUTE OF TECHNOLOGY Department of Physics

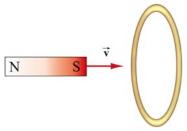
8.02 Fall 2007

Turn in at your table during class labeled with your name and group (e.g. L01 6B)

Problem Set 9 Solutions

Problem 1: Magnet Moving Through a Coil of Wire

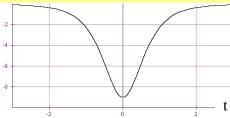
Suppose a bar magnet is pulled through a stationary conducting loop of wire at constant speed, as shown in the figure below. Assume that the positive direction for flux is to the right.



Assume that the north pole of the magnet enters the loop of wire first, and that the center of the magnet is at the center of the loop at time t = 0.

(a) Sketch qualitatively the magnetic flux Φ_B through the loop as a function of time.

Here the field points to the left through the loop at all times, so the flux is negative and the plot of flux vs. time looks like:



(b) Sketch qualitatively a graph of the current I in the loop as a function of time. Take the direction of positive current to be clockwise in the loop as viewed from the left.

The current will first oppose a leftward flux by going clockwise. It will then oppose the loss of leftward flux by going counterclockwise



(c) What is the direction of the force on the permanent magnet due to the current in the coil of wire just before the magnet enters the loop?

As the magnet approaches the loop, the force on the magnet must be resistive so the force on the magnet points to the left. Note that the induced current flows clockwise (as seen from the left hence $I_{ind} < 0$) as the magnetic approaches the loop, so the current acts as a magnetic dipole with the dipole moment pointing to the right. We can model the current loop as a magnet, with the north pole facing to the right, hence the permanent magnet and the dipole have south poles facing each other indicating that the magnetic force is repulsive.

(d) What is the direction of the force on the magnet just after it has exited the loop?

The force is still resistive so it points to the left on the permanent magnet. The induced current has changed direction and now flows counterclockwise (as seen from the left hence $I_{ind} > 0$). Thus dipole moment of the loop points to the left. Again, we model the current loop as a magnet, with the north pole facing to the left, hence the permanent magnet and the dipole have opposite poles facing each other indicating that the magnetic force is now attractive.

(e) Do your answers in (c) and (d) agree with Lenz's law?

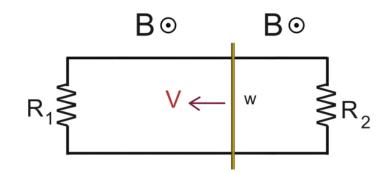
Yes. See solutions to part (c) and (d).

(f) Where does the energy come from that is dissipated in ohmic heating in the wire?

The magnet is pulled a constant speed so the energy is coming from the source that is pulling the magnet.

Problem 2: Faraday's Law

A conducting rod with zero resistance and length w slides without friction on two parallel perfectly conducting wires. Resistors R_I and R_2 are connected across the ends of the wires to form a circuit, as shown. A constant magnetic field \mathbf{B} is directed out of the page. In computing magnetic flux through any surface, take the surface normal to be out of the page, parallel to \mathbf{B} .



- (a) The magnetic flux in the right loop of the circuit shown is (circle one)
 - 1) decreasing
 - 2) increasing.

Problem 2: Faraday's Law continued...

What is the magnitude of the rate of change of the magnetic flux through the right loop?

$$\frac{d\Phi(t)}{dt} = \frac{d}{dt}BA = B\frac{d}{dt}A = BwV$$

(b) What is the current flowing through the resistor R_2 in the right hand loop of the circuit shown? Gives its magnitude and indicate its direction on the figure.

The flux out of the page is increasing so the current is clockwise to make a flux into the page. The magnitude we can get from Faraday:

$$I = \frac{\left|\mathcal{E}\right|}{R_2} = \frac{1}{R_2} \frac{d\Phi(t)}{dt} = \frac{BwV}{R_2}$$

- (c) The magnetic flux in the left loop of the circuit shown is (circle one)
 - 1) decreasing
 - 2) increasing.

What is the magnitude of the rate of change of the magnetic flux through the right loop?

$$\frac{d\Phi(t)}{dt} = \frac{d}{dt}BA = B\frac{d}{dt}A = -BwV$$

"Magnitude" is ambiguous – either a positive or negative number will do here. I use the negative sign to indicate that the flux is decreasing.

(d) What is the current flowing through the resistor R_I in the left hand loop of the circuit shown? Gives its magnitude and indicate its direction on the figure.

The flux out of the page is decreasing so the current is counterclockwise to make a flux out of the page to make up for the loss. The magnitude we can get from Faraday:

$$I = \frac{\left|\mathcal{E}\right|}{R_1} = \frac{1}{R_1} \frac{d\Phi(t)}{dt} = \frac{BwV}{R_1}$$

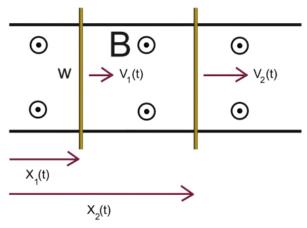
(e) What is the magnitude **and direction** of the magnetic force exerted on this rod?

The total current through the rod is the sum of the two currents (they both go up through the rod). Using the right hand rule on $\vec{\mathbf{F}} = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$ we see the force is to the **right**. You could also get this directly from Lenz. The magnitude of the force is:

$$F = \left| \vec{IL} \times \vec{B} \right| = ILB = \left(BwV \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right) wB = \left[B^2 w^2 V \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right]$$

Problem 3: Coupled Rods

Two rods slide without friction on two conducting rails separated by a distance w, in the presence of a constant background magnetic field **B**, as shown in the diagram. The mass of each rod is m and each rod has a resistance R. The rails have no resistance. At time t, the rod on the left is a distance $x_1(t)$ from the origin, and the rod on the right is a distance $x_2(t)$ from the origin. The respective



speeds of the rods are given by $V_1(t) = \frac{dx_1(t)}{dt}$ and $V_2(t) = \frac{dx_2(t)}{dt}$.

At t = 0 the rod on the right is moving to the right with speed V_o , and the rod on the left is at rest. We want to find the subsequent speed of the rods for t > 0.

(a) At time *t*, what is the magnetic flux through the circuit consisting of the two rods and the rails connecting them?

$$\Phi(t) = wB(x_2(t) - x_1(t))$$

(b) At time t, what is the current through the circuit consisting of the two rods and the rails connecting them. Give its magnitude and also give its sense (clockwise or counterclockwise) using a Lenz's Law argument. You can make this argument at t = 0 if you wish, when $V_1(t)|_{t=0} = 0$ and $V_2(t)|_{t=0} = V_0$.

The EMF is given by:
$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{d}{dt}wB(x_2(t) - x_1(t)) = -wB(V_2(t) - V_1(t))$$

The minus sign means that the EMF (and the direction of driven current) is clockwise. We could also get this from Lenz's Law, as a clockwise current will produce a self-field into the page inside the rails, trying to offset the increase in flux due to the increasing area.

The current is thus:
$$I = \frac{|\varepsilon|}{2R} = \frac{wB(V_2(t) - V_1(t))}{2R}$$
 clockwise

(c) At time t, what is the magnetic force $\vec{\mathbf{F}}_{right}$ on the rod on the right, and what is the magnetic force $\vec{\mathbf{F}}_{left}$ on the rod on the left?

The magnitude of the magnetic force is wIB for both rods, to the left on the right rod and to the right on the left rod (trying to bring them together). So:

$$\vec{\mathbf{F}}_{left} = -\vec{\mathbf{F}}_{right} = \frac{\left(wB\right)^2 \left(V_2(t) - V_1(t)\right)}{2R}\hat{\mathbf{i}}$$

Problem 3: Coupled Rods continued...

(d) Using $F_{right} = m \frac{d}{dt} V_2(t)$, $F_{left} = m \frac{d}{dt} V_1(t)$, and the results of (c), derive equations for $\frac{d}{dt} V_1(t)$ and $\frac{d}{dt} V_2(t)$. Using these equations, derive differential equations for the two new functions defined by $P(t) = V_1(t) + V_2(t)$ and $M(t) = V_2(t) - V_1(t)$.

$$\frac{d}{dt}V_{1}(t) = +\frac{\left(wB\right)^{2}\left(V_{2}(t) - V_{1}(t)\right)}{2mR} \text{ and } \frac{d}{dt}V_{2}(t) = -\frac{\left(wB\right)^{2}\left(V_{2}(t) - V_{1}(t)\right)}{2mR}$$

Adding these: $\frac{d}{dt}[V_1(t) + V_2(t)] = \boxed{\frac{dP}{dt} = 0}$

Subtracting:
$$\frac{d}{dt} \left[V_2(t) - V_1(t) \right] = \boxed{\frac{dM}{dt} = -\frac{\left(wB \right)^2 \left(V_2(t) - V_1(t) \right)}{mR} = -\frac{M(t)}{\tau}} \quad \text{where } \tau = \frac{mR}{w^2 B^2}$$

(e) What are the solutions for $P(t) = V_1(t) + V_2(t)$ and $M(t) = V_2(t) - V_1(t)$ given the initial conditions? Remember that $V_1(t)\big|_{t=0} = 0$ and $V_2(t)\big|_{t=0} = V_0$. (Note: if the function D(t) satisfies the equation $\frac{d}{dt}D(t) = -\frac{D(t)}{\tau}$, then $D(t) = D_o e^{-t/\tau}$; if the function E(t) satisfies the equation $\frac{d}{dt}E(t) = 0$, then $E(t) = E_o$).

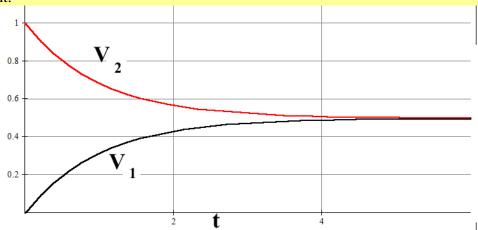
$$P(t) = P_o$$
 and $M(t) = M_o e^{-t/\tau}$. Given our initial conditions, $P(t) = V_o$; $M(t) = V_o e^{-t/\tau}$

(f) Solve for the speeds of rods 1 and 2 at time *t* using the time dependence of *P* and *M*. After a very long time, what are the speeds of rods 1 and 2?

We can now solve for $V_1 = \frac{1}{2}(P-M)$ and $V_2 = \frac{1}{2}(P+M)$:

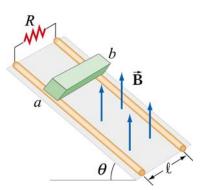
$$V_1(t) = \frac{1}{2}V_o \left[1 - e^{-t/\tau}\right]; \quad V_2(t) = \frac{1}{2}V_o \left[1 + e^{-t/\tau}\right]$$

After a long time the exponentials decay to zero and both rods move at speed V0/2 to the right.



Problem 4: Sliding Bar on Wedges

A conducting bar of mass m and negligible resistance slides down two frictionless conducting rails which make an angle θ with the horizontal, separated by a distance ℓ and connected at the top by a resistor R, as shown in the figure. In addition, a uniform magnetic field $\vec{\mathbf{B}}$ is applied vertically upward. The bar is released from rest and slides down. At time t the bar is moving along the rails at speed v(t).



(a) Find the induced current in the bar at time t. Which way does the current flow, from a to b or b to a?

$$\Phi_{\scriptscriptstyle B} = B \, \ell \, x(t) \cos \theta$$

where x(t) is the distance of the bar from the top of the rails.

Then,

$$\varepsilon = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}B\ell x(t)\cos\theta = -B\ell v(t)\cos\theta$$

Because the resistance of the circuit is R, the magnitude of the induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{B \ell v(t) \cos \theta}{R}$$

By Lenz's law, the induced current produces magnetic fields which tend to oppose the change in magnetic flux. Therefore, the current flows clockwise, from b to a across the bar

(b) Find the terminal speed v_T of the bar.

At terminal velocity, the net force along the rail is zero, that is gravity is balanced by the magnetic force:

$$mg\sin\theta = IB\ell\cos\theta = \left(\frac{B\ell v_t(t)\cos\theta}{R}\right)B\ell\cos\theta$$

or

$$v_t(t) = \frac{Rmg\sin\theta}{\left(B\ell\cos\theta\right)^2}$$

Problem 4: Sliding Bar on Wedges continued...

After the terminal speed has been reached,

(c) What is the induced current in the bar?

$$I = \frac{B \ell v_t(t) \cos \theta}{R} = \frac{B \ell \cos \theta}{R} \left(\frac{Rmg \sin \theta}{(B \ell \cos \theta)^2} \right) = \frac{mg \sin \theta}{B \ell \cos \theta} = \frac{mg}{B \ell} \tan \theta$$

(d) What is the rate at which electrical energy is being dissipated through the resistor?

The power dissipated in the resistor is

$$P = I^2 R = \left(\frac{mg}{B\ell} \tan \theta\right)^2 R$$

(e) What is the rate of work done by gravity on the bar? The rate at which work is done is $\vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$. How does this compare to your answer in (d)? Why?

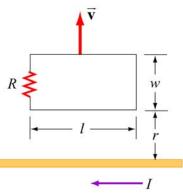
$$\vec{F} \cdot \vec{V} = (mg \sin \theta) v_t(t) = mg \sin \theta \left(\frac{Rmg \sin \theta}{(B\ell \cos \theta)^2} \right) = \left(\frac{mg}{B\ell} \tan \theta \right)^2 R = P$$

That is, they are equal. All of the work done by gravity is dissipated in the resistor, which is why the rod isn't accelerating past its terminal velocity.

Problem 5: Moving Loop

A rectangular loop of dimensions l and w moves with a constant velocity $\vec{\mathbf{v}}$ away from an infinitely long straight wire carrying a current l in the plane of the loop, as shown in the figure. The total resistance of the loop is R.

(a) Using Ampere's law, find the magnetic field at a distance *s* away from the straight current-carrying wire.



Consider a circle of radius s centered on the current-carrying wire. Then around this Amperian loop, $\oint \vec{\bf B} \cdot d \vec{\bf s} = B(2\pi s) = \mu_0 I$ which gives

$$B = \frac{\mu_0 I}{2\pi s}$$
 (into the page)

Problem 5: Moving Loop *continued...*

(b) What is the magnetic flux through the rectangular loop at the instant when the lower side with length l is at a distance r away from the straight current-carrying wire, as shown in the figure?

$$\Phi_B = \iint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int_r^{r+w} \left(\frac{\mu_0 I}{2\pi s} \right) l ds = \boxed{\Phi_B = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{r+w}{r}\right) \text{ (into the page)}}$$

(c) At the instant the lower side is a distance r from the wire, find the induced EMF and the corresponding induced current in the rectangular loop. Which direction does the induced current flow?

The induced EMF is

$$\varepsilon = -\frac{d}{dt}\Phi_B = -\frac{\mu_0 Il}{2\pi} \frac{r}{(r+w)} \left(\frac{-w}{r^2}\right) \frac{dr}{dt} = \left[\varepsilon = \frac{\mu_0 Il}{2\pi} \frac{vw}{r(r+w)}\right]$$

The flux into the page is decreasing as the loop moves away because the field is growing weaker. By Lenz's law, the induced current produces magnetic fields which tend to oppose the change in magnetic flux. Therefore, the current flows clockwise, which produces a self-flux that is into the page.

The induced current is:
$$I = \frac{|\mathcal{E}|}{R} = \frac{\mu_0 Il}{2\pi R} \frac{vw}{r(r+w)}$$
 clockwise

Problem 6: Measuring Voltage and Current

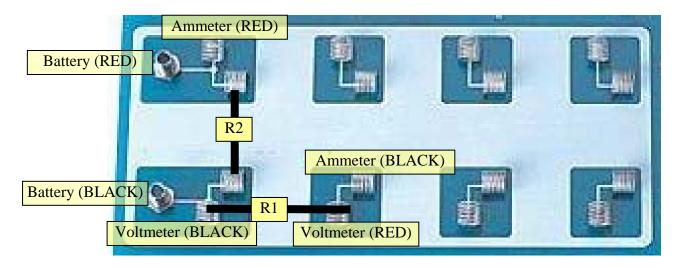
In Part 1 of this experiment you will measure the potential drop across and current through a single resistor attached to the "variable battery." Below is a picture of the circuit board you will use for wiring up the experiment. Each pad (dark green area) has two spring clips (the little slinky shaped objects) connected by a wire (indicated by the white lines), aside from the left most pads, which also have a third connection post. The pads themselves are isolated. If you want to connect two of them you need to run a wire between them.

On a diagram similar to the one below, indicate where you will attach the leads to the resistor, the battery, the voltage sensor \overrightarrow{V} , and the current sensor \overrightarrow{A} . For the battery and sensors make sure that you indicate which color lead goes where, using the convention that red is "high" (or the positive input) and black is "ground." When you draw a resistor or other circuit element it should go between two pads with each end touching one of the spring clips. Do NOT just draw a typical circuit diagram. You need to think about how you will actually wire this board during the lab. RECALL: ammeters must be in series with the element they are measuring current through, while voltmeters must be in parallel.

Problem 6: Measuring Voltage and Current continued...

Make sure that you understand this (ask someone if you don't). You will not be able to do the lab if you don't.

Here is one possibility. There are many. What is important is that the battery, ammeter and resistor (R1) are in series and that the voltmeter is in parallel with R1. It is also important that the red leads of the meters are on the "uphill" side as determined by the positive (red) terminal of the battery.



Problem 7: Resistors in Parallel

In Part 2 you will add a second resistor in parallel with the first. Show where you would attach this second resistor in the diagram you drew for question 1, making sure that the ammeter continues to measure the current through the first resistor and the voltmeter measures the voltage across the first resistor. In this configuration how does the potential drop across R2 compare to that across R1?

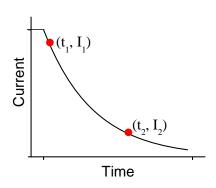
R2 is drawn in the circuit above. Note that it is in parallel with R1 and the ammeter in series, which is fine since the ammeter is ideally a zero resistance element. Since

Problem 8: Measuring the Time Constant τ

As you have seen, current always decays exponentially in RC circuits with a time constant τ : $I = I_0 \exp(-t/\tau)$.

We will measure this time constant in two different ways.

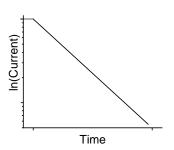
(a) After measuring the current as a function of time we choose a point on the curve (t_1,I_1) . We then look for a point (t_2, I_2) where the current I_2 is related to I_1 such that the time constant can be determined by subtraction: $\tau = t_2 - t_1$. What I_2 do you look for?



 I_2 must satisfy $I_2 = I_1/e$. We should, in theory, be able to find this for any t_1 , as long as we don't switch the battery off (or on) before enough time has passed. In practice the current will eventually get low enough that we won't be able to accurately measure it.

Problem 8: Measuring the Time Constant τ continued...

(b) We can also plot the natural log of the current vs. time, as shown at right. If we fit a line to this curve we will obtain a slope m and a y-intercept y_0 . From these fitting parameters, how can we calculate the time constant?



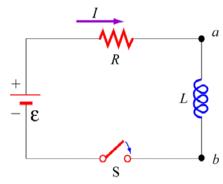
Since $I(t) = I_0 e^{-t/\tau}$, the slope of the plot at right is given by

$$\begin{split} m &= \frac{\text{rise}}{\text{run}} = \frac{\ln\left(I\left(t_{2}\right)\right) - \ln\left(I\left(t_{1}\right)\right)}{t_{2} - t_{1}} = \frac{1}{t_{2} - t_{1}} \ln\left(\frac{I\left(t_{2}\right)}{I\left(t_{1}\right)}\right) \\ &= \frac{1}{t_{2} - t_{1}} \ln\left(\frac{I_{0}e^{-t_{2}/\tau}}{I_{0}e^{-t_{1}/\tau}}\right) = \frac{1}{t_{2} - t_{1}} \ln\left(e^{-(t_{2} - t_{1})/\tau}\right) = \frac{1}{t_{2} - t_{1}} \left(\frac{-\left(t_{2} - t_{1}\right)}{\tau}\right) = -\frac{1}{\tau} \end{split}$$

(c) Which of these two methods is more likely to help us obtain an accurate measurement of the time constant? Why?

Probably the fitting will yield a more accurate result because rather than relying on only two points it takes advantage of a large number of points along the curve.

Problem 9: RL Circuits



Consider the circuit at left, consisting of a battery (emf a ϵ), an inductor L, resistor R and switch S.

For times t<0 the switch is open and there is no current in the circuit. At t=0 the switch is closed.

(a) Using Kirchhoff's loop rules (really Faraday's law now), write an equation relating the emf on the battery, the current in the circuit and the time derivative of the current in the circuit.

Walking in the direction of current, starting at the switch

$$\varepsilon - IR - L\frac{dI}{dt} = 0$$

In class we stated that this equation was solved by an exponential. In other words:

$$I = A(X - \exp(-t/\tau))$$

(b) Plug this expression into the differential equation you obtained in (a) in order to confirm that it indeed is a solution and to determine what the time constant τ and the constants A and X are. What would be a better label for A? (HINT: You will also need to use the initial condition for current. What is I(t=0)?)

Problem 9: RL Circuits continued...

$$0 = \varepsilon - A\left(X - e^{-t/\tau}\right)R - L\frac{Ae^{-t/\tau}}{\tau} = \left(\varepsilon - ARX\right) + \left(AR - L\frac{A}{\tau}\right)e^{-t/\tau}$$

Both the constant and time dependent part must equal zero, giving us two equations. The third (because there are three unknowns) we can get from initial conditions:

$$I(t=0) = A(X-1) = 0 \qquad \Rightarrow X = 1$$

$$\varepsilon - ARX = 0 \qquad \Rightarrow A = \frac{\varepsilon}{RX} = \frac{\varepsilon}{R}$$

$$\left(AR - L\frac{A}{\tau}\right)e^{-t/\tau} = 0 \qquad \Rightarrow \tau = \frac{L}{R}$$

A better label for A would be I_f, the final current.

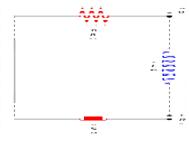
(c) Now that you know the time dependence for the current I in the circuit you can also determine the voltage drop V_R across resistor and the EMF generated by the inductor.

We find:

$$\begin{split} I\left(t\right) &= A\left(X - e^{-t/\tau}\right) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) \\ V_R\left(t\right) &= IR = \mathcal{E}\left(1 - e^{-t/\tau}\right) \\ \mathcal{E}_L\left(t\right) &= -L\frac{dI}{dt} = -L\frac{\mathcal{E}}{R\tau}e^{-t/\tau} = -\mathcal{E}e^{-t/\tau} \end{split}$$

Looking at the EMF from the inductor you see that it starts the same as the battery (but in the opposite direction) which explains why no current initially flows. Then as time goes on it relaxes.

Problem 10: 'Discharging' an Inductor



After a long time T the current will reach an equilibrium value and inductor will be "fully charged." At this point we turn off the battery (ε =0), allowing the inductor to 'discharge,' as pictured at left. Repeat each of the steps a-c in problem 4, noting that instead of $\exp(-t/\tau)$, our expression for current will now contain $\exp(-(t-T)/\tau)$.

(a) Faraday's law:

Walking in the direction of current, starting at the switch

$$-IR - L\frac{dI}{dt} = 0$$

Problem 10: 'Discharging' an Inductor continued...

(b) Confirm solution:

$$0 = -A\left(X - e^{-(t-T)/\tau}\right)R - L\frac{Ae^{-(t-T)/\tau}}{\tau} = \left(-ARX\right) + \left(AR - L\frac{A}{\tau}\right)e^{-(t-T)/\tau}$$

Both the constant and time dependent part must equal zero, giving us two equations. The third (because there are three unknowns) we can get from initial conditions:

$$-ARX = 0 \Rightarrow X = 0$$

$$\left(AR - L\frac{A}{\tau}\right)e^{-t/\tau} = 0 \Rightarrow \tau = \frac{L}{R}$$

$$I(t = T) = A(X - 1) = \frac{\varepsilon}{R} \Rightarrow A = -\frac{\varepsilon}{R}$$

A better label for A would be I_0 , the initial current.

(c) Determine V_R across resistor and the EMF generated by the inductor. Everything is exponentially decaying with time:

$$\begin{split} I\left(t\right) &= A\left(X - e^{-t/\tau}\right) = \frac{\varepsilon}{R} e^{-t/\tau} \\ V_R\left(t\right) &= IR = \varepsilon e^{-t/\tau} \\ \varepsilon_L\left(t\right) &= -L\frac{dI}{dt} = L\frac{\varepsilon}{R\tau} e^{-t/\tau} = \varepsilon e^{-t/\tau} \end{split}$$

Problem 11: The Coil

The coil you will be measuring has is made of thin copper wire (radius ~ 0.25 mm) and has about 600 turns of average diameter 25 mm over a length of 25 mm. What approximately should the resistance and inductance of the coil be? The resistivity of copper at room temperature is around 20 n Ω -m. Note that your calculations can only be approximate because this is not at all an ideal solenoid (where length >> diameter).

The resistance

$$R = \frac{\rho L}{A} = \frac{\rho \cdot N\pi d}{\pi a^2} \approx \frac{(20 \text{ n}\Omega \text{ m}) \cdot (600)(25 \text{ mm})}{(0.25 \text{ mm})^2} \approx 4.8 \Omega$$

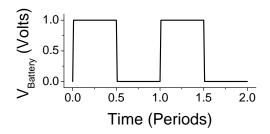
The inductance of a solenoid we calculated in class to be:

$$L = \mu_0 n^2 \pi R^2 l \approx \left(4\pi \times 10^{-7} \text{ T m A}^{-1}\right) \left(\frac{600}{25 \text{ mm}}\right)^2 \pi \left(\frac{25 \text{ mm}}{2}\right)^2 (20 \text{ mm}) \approx 7 \text{ mH}$$

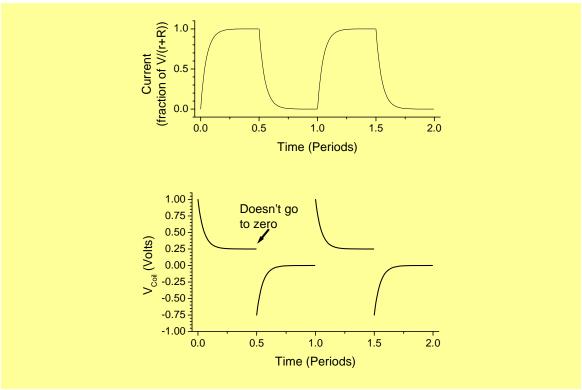
Problem 12: A Real Inductor

As mentioned above, in this lab you will work with a coil that does not behave as an ideal inductor, but rather as an ideal inductor in series with a resistor. For this reason you have no way to independently measure the voltage drop across the resistor or the EMF induced by the inductor, but instead must measure them together. None-the-less, you want to get information about both. In this problem you will figure out how.

(a) In the lab you will hook up the circuit of problem 4 with the ideal inductor L of that problem now replaced by a coil that is a non-ideal inductor – an inductor L and resistor r in series. The battery will periodically turn on and off, displaying a voltage as shown here:



Sketch the current through the battery as well as what a voltmeter hooked across the coil would show versus time for the two periods shown above. Assume that the period of the battery turning off and on is comparable to but longer than several time constants of the circuit.



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(b) How can you tell from your plot of the voltmeter across the coil that the coil is not an ideal inductor? Indicate the relevant feature clearly on the plot. Can you determine the resistance of the coil, *r*, from this feature?

The voltage measured across the coil doesn't go to zero because even when the inductor is "off" the coil resistance still has a voltage drop across it. You can determine r from this voltage -r = V/I (in this case I made $r \frac{1}{4}$ of the total resistance, that is, $\frac{1}{3}$ of R).

(c) In the lab you will find it easier to make measurements if you do NOT use an additional resistor *R*, but instead simply hook the battery directly to the coil. (Why? Because the time constant is difficult to measure with extra resistance in the circuit). Plot the current through the battery and the reading on a voltmeter across the coil for this case. We will only bother to measure the current. Why?

The current is the same as the current above (although the time constant will be longer because of the lower resistance). The voltage measured across the coil will be the same as the voltage measured across the battery because they are the only two things in the circuit, so there is no need to measure it.

(d) For this case (only a battery & coil) how will you determine the resistance of the coil, *r*? How will you determine its inductance *L*?

In this case we can determine the resistance from the final current (r = V/I) and the inductance from the time constant $(L = \tau r)$