MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Fall 2007**

Turn in to boxes at classroom entrance labeled with your name and group (e.g. L01 6B)

Problem Set 5

Due: Wednesday, October 10th at beginning of class (before 10:15/1:15)

SAMPLE EXAM

The following exam is similar to what you should expect to find on Thursday evening, October 11th (7:30 – 9:30 pm in 54-100). Please note that the exam is closed notes and that calculators are not allowed. I strongly recommend that you take this sample exam under the same conditions.

The exam is out of 100 points. It consists of the following:

(1) 5 multiple choice, concept questions (4 pts each, 20 pts total)
(2) 1 back of the envelope calculation (5 pts)
(3) 3 analytic problems (25 pts each, 75 pts total)

Some notes about format:
In the multiple choice questions there will often be an answer "I don't know." You will get one point for answering this – we'd like to discourage random guessing, so if you don't know be honest about it (on the other hand, if you can figure it out so much the better). Aside from this there is no partial credit. If you don't circle any answer you will receive a 0. You must circle some answer. Make your circle clear.

In the back of the envelope calculation you must provide both a conceptual explanation of what you are doing (write some words and equations so we know what you are thinking) AND some numerical values that lead to a numerical answer. Remember, no calculators are allowed, so make your numbers nice. Don't leave π or any physical constants (like ε_0) in your answer. If you don't have the appropriate units it will be considered wrong.

Finally, the analytic problems will be graded as per usual for exam problems, with partial credit awarded if you make clear what you are doing (and part of it is correct). One of the problems will involve a number of intermediate steps that we will not lead you through (for example, "Calculate the capacitance of this spherical capacitor."). In this case you need to show us that you understand the steps that are necessary to do this. USE WORDS. A quick outline at the beginning or an outline as you go will help us know that you know what you are doing. A correct answer without appropriate explanation will receive no credit. You don't need to write a book (or even as much as I do in the solutions), but make the process clear.

$\vec{\mathbf{E}} = \frac{q}{4\pi\varepsilon_o r^2}\hat{\mathbf{r}} = \frac{q}{4\pi\varepsilon_o r^3}\vec{\mathbf{r}}$

$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r}$ *points* **from** *source* q **to** *observer*

$\oiint_{closed\ surface} \kappa\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q_{inside}}{\varepsilon_o}$

$d\vec{\mathbf{A}}$ points from inside to outside

$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$

$\Delta V_{moving\ from\ a\ to\ b} = V_b - V_a = -\int_a^b \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$

$\oint_{closed\ path} \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = 0$

$W = \Delta U = q\Delta V$

$V_{\text{point charge}} = \frac{q}{4\pi\varepsilon_o r}$

$U = \sum_{all\ pairs} \frac{q_i q_j}{4\pi\varepsilon_o |\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j|}$

$U = \oiiint \left[\frac{1}{2}\varepsilon_o E^2\right] dV_{vol}$

$E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$

$E_r = -\frac{\partial V}{\partial r}$ for spherical symmetry

$C = \frac{|Q|}{|\Delta V|}$ $U = \frac{1}{2} C \Delta V^2 = \frac{Q^2}{2C}$

$C_{parallel} = C_1 + C_2$ $C_{series} = \frac{C_1 C_2}{C_1 + C_2}$

Circumferences, Areas, Volumes:

Circle of radius r:

Area = πr^2

Circumference = $2\pi r$

Sphere of radius r

Surface Area = $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$

Cylinder of radius r and height h

Side surface area = $2\pi r h$

End cap surface area = $2\pi r^2$

Volume = $\pi r^2 h$

Definition of trig functions

sin is opposite/hypotenuse;

cos is adjacent/hypotenuse;

tangent is opposite over adjacent;

Properties of 30, 45, and 60 degrees

(π/6, π/4, and π/3 radians)**:**

sin(π/6) = cos(π/3) = 1/2,

sin(π/3) = cos(π/6) = $\sqrt{3}/2$;

sin(π/4) = cos(π/4) = $1/\sqrt{2}$;

Integrals that may be useful

$\int_a^b dr = b - a$

$\int_a^b \frac{1}{r} dr = \ln(b/a)$

$\int_a^b \frac{1}{r^2} dr = \left(\frac{1}{a} - \frac{1}{b}\right)$

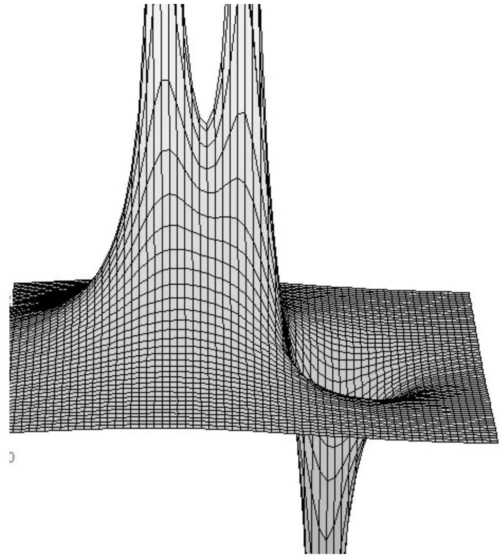
Some potentially useful numbers

$k_e = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \frac{\text{N m}^2}{\text{C}^2}$

Breakdown of air	E ~ 3 x 10^6 V/m
Speed of light	c = 3 x 10^8 m/s
Electron charge	e = 1.6 x 10^{-19} C
Avagadro's number	N_A = 6.02 x 10^{23} mol^{-1}

Problem 1: Five Short Questions. Circle your choice for the correct answer.

Question A (4 points out of 20 points):

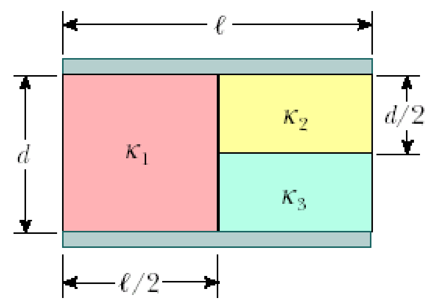


Three charges lie on the x-axis a distance *a* apart. The charges are numbered from 1 to 3 moving from left to right. A representation of the electric potential *V* of these three charges is shown above. Which one of the following statements is true?

The electric field is zero…

1. …at some point between charges 1 and 2 and also at some point between charges 2 and 3
2. …at some point between charges 1 and 2 but never between charges 2 and 3
3. …never between charges 1 and 2 but at some point between charges 2 and 3
4. …nowhere between charges 1 and 2 or between charges 2 and 3
5. I don't know (this answer is worth 1 point)

Question B (4 points out of 20 points):



Consider the capacitor at right, with $d \ll \ell$, filled with three different dielectrics, as pictured. What is the capacitance of this capacitor?

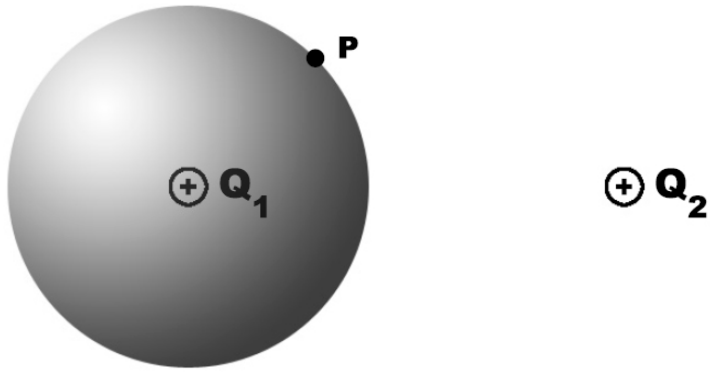
1) $C = \dfrac{d}{\varepsilon_0 A}\left(\dfrac{\kappa_1}{2} + \dfrac{\kappa_2\kappa_3}{\kappa_2+\kappa_3}\right)$ 2) $C = \dfrac{\varepsilon_0 A}{d}\left(\dfrac{\kappa_1}{2} + \dfrac{\kappa_2\kappa_3}{\kappa_2+\kappa_3}\right)$

3) $C = \dfrac{\varepsilon_0 A}{d}\left(\kappa_1 + \dfrac{\kappa_2\kappa_3}{\kappa_2+\kappa_3}\right)$ 4) $C = \dfrac{\varepsilon_0 A}{d}\left(\kappa_2 + \dfrac{\kappa_1\kappa_3}{\kappa_1+\kappa_3}\right)$

5) $C = \dfrac{3\varepsilon_0 A}{d}\left(\dfrac{\kappa_1\kappa_2\kappa_3}{\kappa_1+\kappa_2+\kappa_3}\right)$ 6) $C = \dfrac{3d}{\varepsilon_0 A}\left(\dfrac{\kappa_1\kappa_2\kappa_3}{\kappa_1+\kappa_2+\kappa_3}\right)$

7) I don't know (this answer is worth 1 point)

Question C (4 points out of 20 points):
In the figure below, a point charge $+Q_1$ is at the center of an imaginary spherical Gaussian surface and another point charge $+Q_2$ is outside of the Gaussian surface. Point *P* is on the surface of the sphere. Which <u>one</u> of the following statements is true?

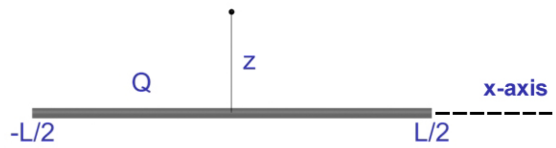


1. Both charges $+Q_1$ and $+Q_2$ contribute to the net electric flux through the sphere but only charge $+Q_1$ contributes to the electric field at point *P* on the sphere.
2. Both charges $+Q_1$ and $+Q_2$ contribute to the net electric flux through the sphere but only charge $+Q_2$ contributes to the electric field at point *P* on the sphere.
3. Only the charge $+Q_1$ contributes to the net electric flux through the sphere but both charges $+Q_1$ and $+Q_2$ contribute to the electric field at point *P*.
4. Only the charge $+Q_2$ contributes to the net electric flux through the sphere but both charges $+Q_1$ and $+Q_2$ contribute to the electric field at point *P*.
5. Only the charge $+Q_1$ contributes to the net electric flux through the sphere and to the electric field at point *P* on the sphere.
6. Only the charge $+Q_2$ contributes to the net electric flux through the sphere and to the electric field at point *P* on the sphere.
7. I don't know (this answer is worth 1 point)

Question D (4 points out of 20 points):
Consider an electric dipole immersed in a uniform electric field. Which of the following statements regarding the force $\vec{\mathbf{F}}$ and the torque $\vec{\boldsymbol{\tau}}$ is strictly true regardless of the orientation of the dipole:

1. $\vec{\mathbf{F}} \neq 0$ and $\vec{\boldsymbol{\tau}} \neq 0$.
2. $\vec{\mathbf{F}} = 0$ and $\vec{\boldsymbol{\tau}} \neq 0$.
3. $\vec{\mathbf{F}} \neq 0$ and $\vec{\boldsymbol{\tau}} = 0$.
4. $\vec{\mathbf{F}} = 0$ and $\vec{\boldsymbol{\tau}} = 0$.
5. $\vec{\mathbf{F}} = 0$ but $\vec{\boldsymbol{\tau}}$ may or may not be zero.
6. $\vec{\mathbf{F}}$ may or may not be zero but $\vec{\boldsymbol{\tau}} = 0$.
7. I don't know (this answer is worth 1 point)

Question E (4 points out of 20 points):



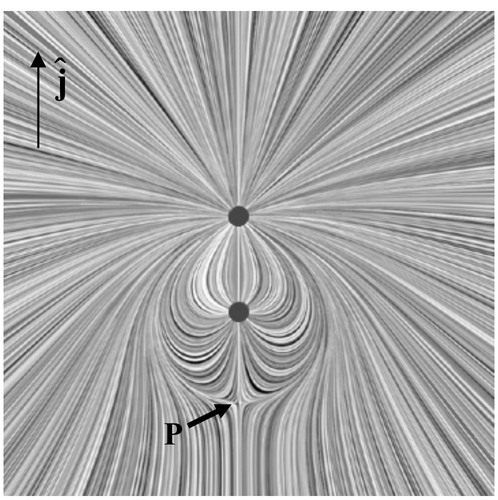
A rod of length L carries a charge per unit length λ uniformly distributed over its length. The magnitude of the electric field a distance z from the center of the rod along its perpendicular bisector (see sketch above) is given by the expression

1. $\dfrac{\lambda}{4\pi\varepsilon_o}\displaystyle\int_{-L/2}^{L/2}\frac{x\,dx}{\left[x^2+z^2\right]^{3/2}}$
2. $\dfrac{\lambda}{4\pi\varepsilon_o}\displaystyle\int_{-L/2}^{L/2}\frac{x\,dx}{\left[x^2+z^2\right]}$
3. $\dfrac{\lambda}{4\pi\varepsilon_o}\displaystyle\int_{-L/2}^{L/2}\frac{z\,dx}{\left[x^2+z^2\right]^{3/2}}$
4. $\dfrac{\lambda}{4\pi\varepsilon_o}\displaystyle\int_{-L/2}^{L/2}\frac{z\,dx}{\left[x^2+z^2\right]}$
5. $\dfrac{\lambda}{4\pi\varepsilon_o}\displaystyle\int_{-L/2}^{L/2}\frac{dx}{\left[x^2+z^2\right]}$
6. I don't know (this answer is worth 1 point)

Problem 2: Back of the Envelope Calculation (5 Pts)
As always, you are not given enough information to exactly determine the answer to this question. Make your best estimates for unknowns, clearly indicating what your estimates are (e.g. Radius R ~ ….) NO CREDIT will be given for simply guessing a final numerical answer from scratch. It must be properly motivated (i.e. write equations!)

In class I used a small Van de Graff generator that when fully charged was crackling. About how much charge was on it?

Problem 3: Charges (25 pts)



The grass seeds diagram at left represents the electric field created by two charges, each located on the y-axis at y = 0 and y = *d*. The charge at y = 0 is +*Q*.

(a) What is the sign of the charge at y = *d*?

POSITIVE NEGATIVE

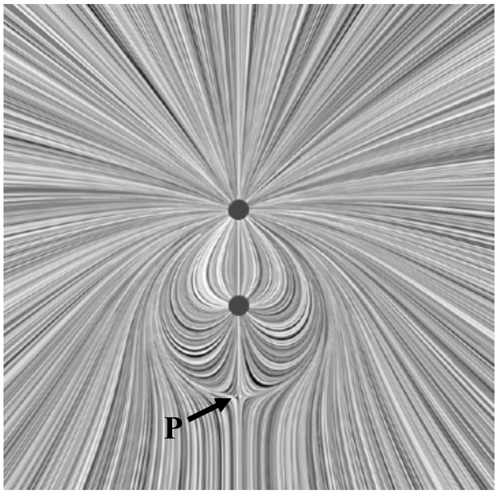
(b) In which direction does the electric field point at the bottom of the figure on the y-axis?

UP DOWN

(c) Now consider the electric field at the point P (on the axis at y = -*d*). Is the electric field there

POSITIVE NEGATIVE or ZERO

Problem 3 *continued*: Charges



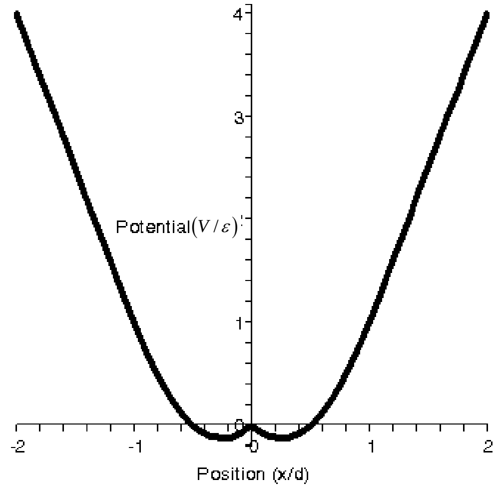
(d) Calculate the sign and magnitude Q_d of the charge located at y = d.

(e) Now consider a third point charge q presently located a very large distance away on the positive x-axis. How much work does it take to move this charge from its current location to point P? (NOTE: If you were unable to do part (d) but you need this value, simply refer to is as Q_d.)

Problem 4: Planar Potential (25 pts)

The electric potential *V(x,y,z)* for a planar charge distribution is given by:

$$V(x,y,z)=\begin{cases} -\varepsilon\left(3\dfrac{x}{d}+2\right) & \text{for } x<-d \\ \varepsilon\left(2\left(\dfrac{x}{d}\right)^2+\dfrac{x}{d}\right) & \text{for } -d\le x<0 \\ \varepsilon\left(2\left(\dfrac{x}{d}\right)^2-\dfrac{x}{d}\right) & \text{for } 0\le x<d \\ \varepsilon\left(3\dfrac{x}{d}-2\right) & \text{for } x>d \end{cases}$$



where d is the distance from the origin along the x axis where $V=\varepsilon$.

This function is plotted to the right, with the x-axis in units of d and the y-axis in units of ε.

(a) What is the electric field $\vec{\mathbf{E}}(x)$ for this problem?

Region I: -d > x

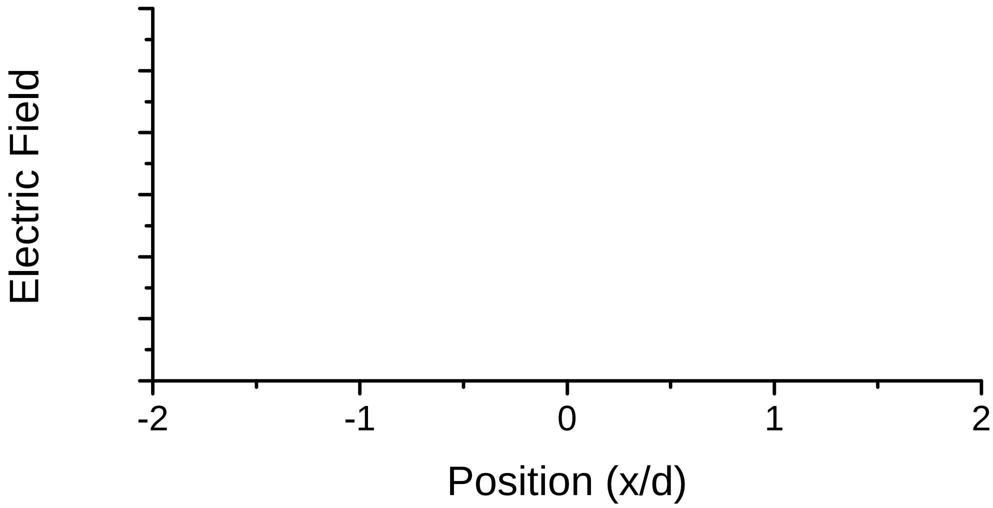
Region II: $-d \le x < 0$

Region III: $0 \le x < d$

Region IV: $x > d$

Problem 4 *continued*: Planar Potential

(b) Plot the electric field that you just calculated on the graph below. Be sure to properly label the y-axis (top and bottom) to indicate the limits of the magnitude of the E field!



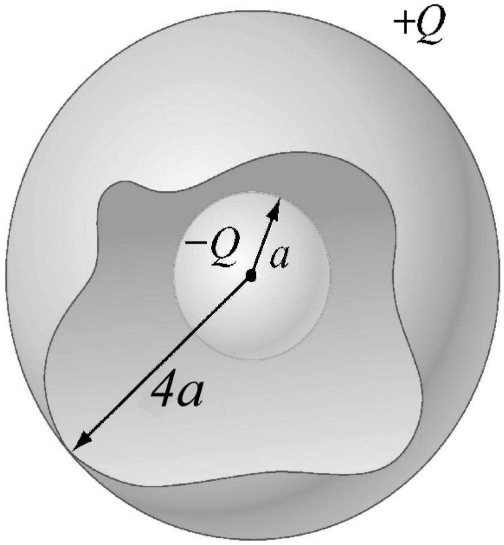
(c) Consider a cylinder of cross-sectional area A and length $3d$ lying along (coaxial with) the x-axis, and centered at the origin (i.e. lying between $x = -3d/2$ and $x = 3d/2$). How much net charge does this cylinder contain?

(d) Now shift this cylinder to the right so that it lies between $x = d$ to $x = 3d$. Now how much charge does it contain?

Problem 4 *continued*: Planar Potential

(e) Finally, we think about the distribution of charges that produces the above electric potential and electric field – a single sheet of charge (charge density σ) and a single slab of charge (charge density ρ). Using Gauss's Law arguments determine where this slab and sheet are and what their charge densities are. Make sure to state the Gaussian surface(s) that you use and what you conclude about the charge distribution by applying Gauss's Law to those surfaces.

Problem 5: Capacitor (25 pts)



Consider two nested spherical shell conductors, of radii a and $4a$. The inner conductor has a charge -Q placed on it. An equal and opposite charge +Q is placed on the outer conductor.

What is the capacitance of this configuration?

(Remember: You must show your work for credit – a correct answer without reasonable explanation will earn no points)