18.02 Fall 2007 – Problem Set 6, Part B Solutions

1. i) does not make sense because when x is fixed we can't vary it to take a derivative with respect to x; ii) does not make sense because if y can't move, then the constraint $g(x,y)=c$ implies that x can't move either.

iii) Take the derivative of each equation with z held fixed and use the chain rule, $\left(\frac{\partial z}{\partial x}\right)_z = 0$, and $\left(\frac{\partial x}{\partial x}\right)_z = 1$ to obtain

$$g_x \cdot 1 + g_y \left(\frac{\partial y}{\partial x}\right)_z = 0; \qquad \left(\frac{\partial w}{\partial x}\right)_z = f_x \cdot 1 + f_y \left(\frac{\partial y}{\partial x}\right)_z + f_z \cdot 0.$$

The first equation implies $\left(\frac{\partial y}{\partial x}\right)_z = -g_x/g_y$, and substituting into the second equation,

$$\left(\frac{\partial w}{\partial x}\right)_z = f_x + f_y \left(\frac{\partial y}{\partial x}\right)_z = f_x + f_y(-g_x/g_y) = f_x - \frac{g_x f_y}{g_y}.$$

2. a) Differentiating with respect to t with x fixed,

$$\cos(x+y)\left(\frac{\partial y}{\partial t}\right)_x = 1; \qquad \left(\frac{\partial w}{\partial t}\right)_x = x^3 y + x^3 t \left(\frac{\partial y}{\partial t}\right)_x = x^3 y + \frac{x^3 t}{\cos(x+y)}.$$

b) $5x^4\,dx + z\,dy + y\,dz = 0$ and $(y^2+2zx)\,dx + (2xy+z^2)\,dy + (2yz+x^2)\,dz = 0$, evaluated at $(1,1,2)$, yield

$$\begin{aligned} 5dx + 2dy + dz &= 0 \\ 5dx + 6dy + 5dz &= 0 \end{aligned}$$

Multiplying the first equation by 5 and subtracting the second one to eliminate dz, we obtain $20dx + 4dy = 0$. Therefore, $dx = -\frac{1}{5}dy$ and $dx/dy = -1/5$.