**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**
**Department of Physics**

**8.02** **Fall 2007**

# Problem Set 8 Solutions

## Problem 1: Charges in a Uniform Field

An electron with velocity $v_x\hat{\mathbf{i}}+v_y\hat{\mathbf{j}}$ moves through a uniform magnetic field $B_x\hat{\mathbf{i}}+B_y\hat{\mathbf{j}}$

(a) Find the magnitude of the force on the electron. (b) Repeat your calculation for a proton having the same velocity.

This is just a question of calculating a cross-product. Although it wasn't introduced in class, the way to do this is with a determinant:

$$\vec{\mathbf{F}}=q\vec{\mathbf{v}}\times\vec{\mathbf{B}}=q\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ v_x & v_y & 0\\ B_x & B_y & 0\end{vmatrix}=q\left(v_xB_y-v_yB_x\right)\hat{\mathbf{k}}$$

So the magnitude of the force is $F=e\left(v_xB_y-v_yB_x\right)$ for both an electron and proton, the only difference is the direction is reversed ($q$=-$e$ for an electron).

## Problem 2: Power Line

A horizontal power line carries a current of $I$ from south to north. Earth's magnetic field ($B_{\text{Earth}}$ = 62.1 μT) is directed toward the north and is inclined downward at an angle $\theta$ to the horizontal. Find the magnitude of the magnetic force on a length $L$ of the line due to Earth's field. Please also calculate a value numerically for $I$ = 3000 A, $\theta$ = 78.0º and $L$ = 100 m, since it's useful to think about real world numbers.

Since the line is running south to north, it has an angle θ relative to the Earth's field. Thus:

$$\vec{\mathbf{F}}=I\vec{\mathbf{L}}\times\vec{\mathbf{B}}\Rightarrow F=ILB\sin\left(\theta\right)$$

For the numbers given, $F=ILB\sin\left(\theta\right)=\left(3000\text{ A}\right)\left(100\text{ m}\right)\left(62.1\ \mu T\right)\sin\left(78^{\circ}\right)=18\text{ N}$, which is a small force. For comparison, if the cable has a mass of 300 kg (about what a one cm radius copper wire of that length will weigh) it will feel a force of about 3000 N due to gravity. Of course most power lines are AC so this will also be an oscillating, not unidirectional force.

## Problem 3: Triangular Loop

A current loop, carrying a current $I$, is in the shape of a right triangle with sides 30, 40, and 50 cm. The loop is in a uniform magnetic field $B$ whose direction is parallel to the current in the 50 cm side of the loop. Find the magnitude of **(a)** the magnetic dipole moment of the loop in amperes-square meters and **(b)** the torque on the loop.

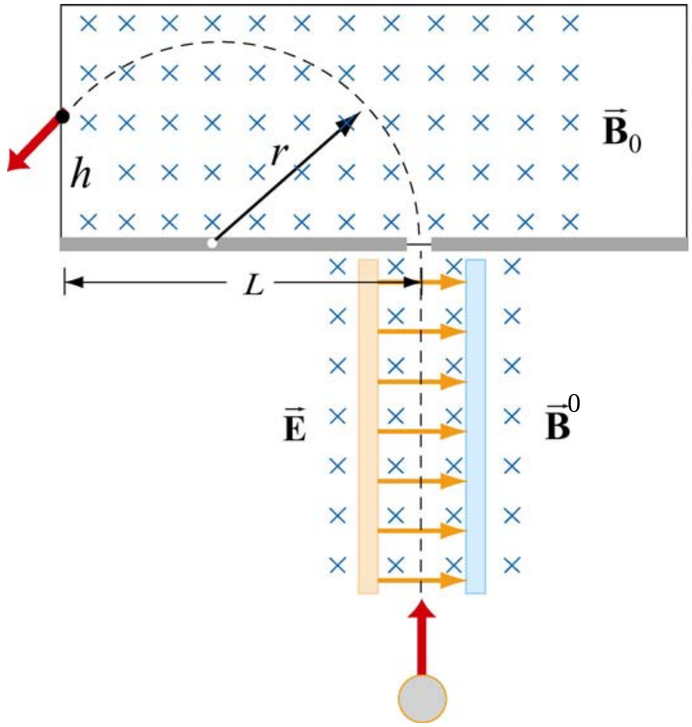
The loop has an area of $A=\frac{1}{2}\text{base}\cdot\text{height}=600\text{ cm}^2$ so its dipole moment is

**(a)** $\boxed{\mu=1200\cdot I\ \text{cm}^2}$

**(b)** The torque is

$\boxed{\vec{\boldsymbol{\tau}}=\vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}}=600IB\text{ cm}^2}$ (in the plane of the loop, perpendicular to the hypotenuse)

**Problem 4:**
A Bainbridge mass spectrometer is shown in the figure. A charged particle with mass $m$, charge $q$ and speed $V$ enters from the bottom of the figure and traces out the trajectory shown in the fields shown. The magnetic field is everywhere $\mathbf{B_0}$ as pictured. The only electric field is in the region where the trajectory of the charge is a straight line.




(a) Is the sign of the charge positive or negative? To get credit for your answer you must give a logical reason to justify it.

The charge is positive. We can tell that by looking at the way it curves when it enters the upper region. Since the field is into the sheet, $\vec{\mathbf{V}} \times \vec{\mathbf{B}}_o$ is to the left and the charge moves in that direction. Therefore the charge must be positive.

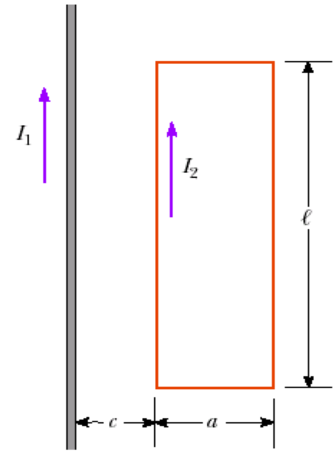
(b) What must the electric field have been set to in order to keep the particle moving in a straight line? Justify your answer.

For the charge to move in a straight line the total electromagnetic force must be zero. This means that $\vec{\mathbf{E}} + \vec{\mathbf{V}} \times \vec{\mathbf{B}}_0 = 0$ or $V = \dfrac{E}{B_0}$.

(c) When the charge is moving through the second (curved) segment of its trajectory, *derive* a formula for the radius $r$ of the arc through which it moves. To get credit for your answer, you must show the details of your derivation.

Balancing force, $qVB_0 = m\dfrac{V^2}{r}$ so we must have that $r = \dfrac{mV}{qB_0}$

**Problem 5: Force on the Rectangular Loop**




In the figure, the current in the long, straight wire is $I_1$ and the wire lies in the plane of the rectangular loop of length $\ell$ and width $a$, that carries the current $I_2$. The distance between the left side of the rectangular loop and the straight wire is $c$. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.

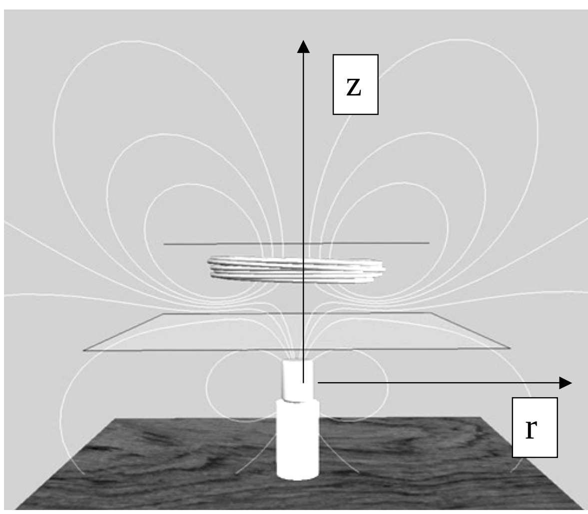
By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{\mathbf{i}} = \boxed{-\frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{a}{c(c+a)}\right)\hat{\mathbf{i}}}\,.$$

The force is attractive because the close (parallel) wire dominates over the far (anti-parallel) one.

**Problem 6: Levitating Coil**

A coil of wire has radius $R$ has $N$ turns and total mass $m$. It sits on a platform a distance $z$ above a magnetic dipole of dipole moment $M_{dipole}$ (see sketch). The north pole of the magnet is on top. The coil is held down to the platform on which it sits by a clamp.




(a) Using a battery attached to the coil, you run a current $I$ in the coil. Viewed from above, in what direction do you want the current in the coil to flow (clockwise or counterclockwise) so that the force on the coil has an upward component?

Since the closest pole of the magnet is the North pole, we want our dipole's North pole pointing down in order repel and lead to levitation. This means that we want a clockwise current (when viewed from above).

**Problem 6: Levitating Coil *continued…***

(b) Calculate the total magnetic force on the coil given that the magnetic field from a magnetic dipole with dipole moment $M_{dipole}$ is given (in cylindrical coordinates) by:

$$\vec{\mathbf{B}}(r,z) = \frac{\mu_o M_{dipole}}{4\pi}\left[\hat{\mathbf{r}}\frac{3rz}{\left(r^2+z^2\right)^{5/2}} + \hat{\mathbf{k}}\frac{\left(2z^2-r^2\right)}{\left(r^2+z^2\right)^{5/2}}\right]$$

To see a grass seeds representation of this field, go to the [Mapping Fields] applet, go to the pull down *Examples* menu, and choose *Dipole in no field*,).

We will calculate the force by integrating the $d\vec{\mathbf{F}}$ that each part of the wire $d\vec{\mathbf{s}}$ feels in the magnetic field: $d\vec{\mathbf{F}} = I\, d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$. Since we are interested in the vertical force, and $d\vec{\mathbf{s}}$ is in the $\hat{\boldsymbol{\theta}}$ direction, the only part of the magnetic field that matters in the radial part. The $\hat{\mathbf{k}}$ term, when crossed with $\hat{\boldsymbol{\theta}}$, will lead to radial forces, which when integrated around the loop will cancel out. So, $dF_z = I\, dl\, B_r = I\, dl \frac{\mu_o M_{dipole}}{4\pi}\left[\frac{3Rz}{\left(R^2+z^2\right)^{5/2}}\right]$ (where I replaced *r* with *R* because we only care about the force on the wire, where $r = R$).

Now we just need to integrate. Everything is a constant except *dl*, which integrates to $2\pi RN$, where there are *N* turns of wire in the coil.

So the total force is:

$$\vec{\mathbf{F}} = \hat{\mathbf{k}}\, I\, 2\pi RN \frac{\mu_o M_{dipole}}{4\pi}\left[\frac{3Rz}{\left(R^2+z^2\right)^{5/2}}\right] = \boxed{\hat{\mathbf{k}}\frac{3\mu_o M_{dipole} R^2 zNI}{2\left(R^2+z^2\right)^{5/2}}}$$

(c) If you release the clamp, the coil will move upwards as long as the current *I* it carries is greater than a certain amount. What is the minimum current $I_{min}$ required for the coil to move upward when it is released?

In order to levitate, we need the upward magnetic force to at least balance gravity:

$$\frac{3\mu_o M_{dipole} R^2 zNI}{2\left(R^2+z^2\right)^{5/2}} = mg$$

Solving for I, we find the minimum current to do this: $\boxed{I_{\min} = \frac{2mg\left(R^2+z^2\right)^{5/2}}{3\mu_o M_{dipole} R^2 zN}}$

(d) Take $m = 20$ g, $M_{dipole} = 10\ \text{A}\cdot\text{m}^2$, $g = 9.8\ \text{m/s}^2$, $N = 30$ turns, $R = 50$ mm and $z = 50$ mm. What is the numerical value of $I_{min}$ in part (c) above for these parameters?

Plugging in,

$$I_{\min} = \frac{2mg\left(R^2+z^2\right)^{5/2}}{3\mu_o M_{dipole} R^2 zN} = \frac{2(20\text{ g})\left(9.8\ \tfrac{\text{m}}{\text{s}^2}\right)\left((50\text{ mm})^2+(50\text{ mm})^2\right)^{5/2}}{3\left(4\pi\times10^{-7}\text{ T m A}^{-1}\right)\left(10\text{ A m}^2\right)(50\text{ mm})^2(50\text{ mm})30} = \boxed{4.9\text{ A=I}}$$

**Problem 7: Calculating Flux from Current and Faraday's Law**

In part 1 of this lab you will move a coil from well above to well below a strong permanent magnet. You will measure the current in the loop during this motion using a current sensor. The program will also display the flux "measured" through the loop, even though this value is never directly measured. In this problem you will understand how.

(a) Starting from Faraday's Law and Ohm's law, write an equation relating the current in the loop to the time derivative of the flux through the loop.

$$\varepsilon = -\frac{d\Phi}{dt} = IR$$

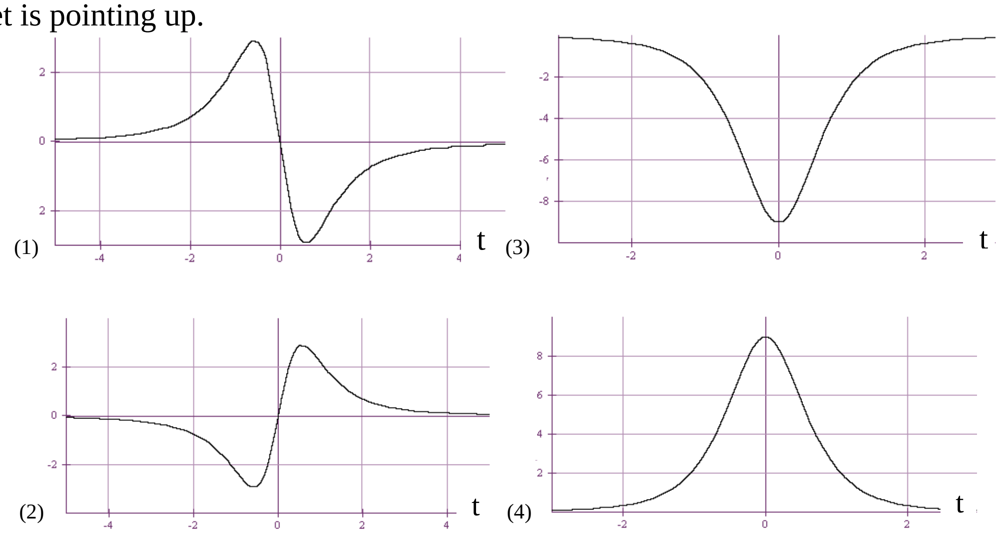
(b) Now integrate that expression to get the time dependence of the flux through the loop $\Phi(t)$ as a function of current $I(t)$. What assumption must the software make before it can plot flux vs. time?

$$d\Phi = -IR\,dt \quad \Rightarrow \quad \Phi(t) = -R\int_{t=0}^{t} I(t')\,dt'$$

The software must assume (as I did above) that the flux at time t=0 is zero.

**Problem 8: Predictions: Coil Moving Past Magnetic Dipole**

In moving the coil over the magnet, measurements of current and flux for each of several motions will look like one of the below plots. For current, counter-clockwise when viewed from above is positive. For flux, upwards is positive. The north pole of the magnet is pointing up.




Suppose you move the loop from well *above* the magnet to well *below* the magnet at a constant speed. Which graph most closely resembles the graph of:

(a) *magnetic flux through the loop* as a function of time? 4

(b) *current through the loop* as a function of time? 2

Suppose you now move the loop from well *below* the magnet to well *above* the magnet at a constant speed. Which graph most closely resembles the graph of:

(c) *magnetic flux through the loop* as a function of time? 4

(d) *current through the loop* as a function of time? 2

**Problem 9:   Predictions:  Force on Coil Moving Past Magnetic Dipole**

In part 2 of this lab you will feel the force on a conducting loop as it moves past the magnet.  For the following conditions, in what direction should the magnetic force point?

As you are moving the loop from well *above* the magnet to well *below* the magnet at a constant speed…
(a) … and the loop is *above* the magnet.
(b) … and the loop is *below* the magnet

As you are moving the loop from well *below* the magnet to well *above* the magnet at a constant speed…
(c) … and the loop is *below* the magnet.
(d) … and the loop is *above* the magnet

In all of these cases the force opposes the motion.  For (a) & (b) it points upwards, for (c) and (d) downwards.

**Problem 10: Feeling the Force**

In part 2, rather than using the same coil we use in part 1, we will instead use an aluminum cylinder to "better feel" the force.  To figure out why, answer the following.

a)  If we were to double the number of turns in the coil how would the force change?
If we were to double the number of turns we would double the total flux and hence EMF, but would also double the resistance so the current wouldn't change.  But the force would double because the number of turns doubled.

b) Using the result of (a), how should we think about the Al tube?  Why do we "better feel" the force?

Going to the cylinder basically increases many times the number of coils (you can think about it as a bunch of thin wires stacked on top of each other).  It also reduces the resistance and hence increases the current because the resistance is not through one very long wire but instead a bunch of short loops all in parallel with each other.

In case you are interested, the wire is copper, and of roughly the same diameter as the thickness of the aluminum cylinder, although this information won't necessarily help you in answering the question.