MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Fall 2007**

Turn in at your table during class labeled with your name and group (e.g. L02 7A)

Problem Set 9

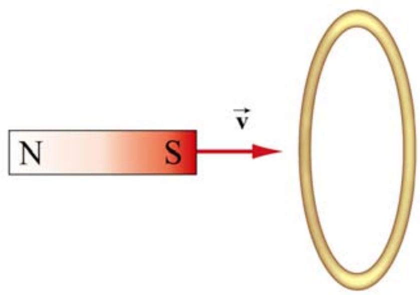
Due: Wednesday, November 7th at class beginning (before 10:15/12:15)

This problem set focuses on material from classes 21-23 (week 9). Textbook references can be found at the top of the summaries from these days. **WARNING**: Start early, this is a very long problem set.

<u>Warm Up</u>

Problem 1: Magnet Moving Through a Coil of Wire

Suppose a bar magnet is pulled through a stationary conducting loop of wire at constant speed, as shown in the figure below.



Assume that the south pole of the magnet enters the loop of wire first, and that the center of the magnet is at the center of the loop at time $t = 0$.

(a) Sketch qualitatively the magnetic flux Φ_B through the loop as a function of time.

(b) Sketch qualitatively the current I in the loop as a function of time. Take the direction of positive current to be clockwise in the loop as viewed from the left.

(c) What is the direction of the force on the permanent magnet due to the current in the coil of wire just before the magnet enters the loop?

(d) What is the direction of the force on the magnet just after it has exited the loop?

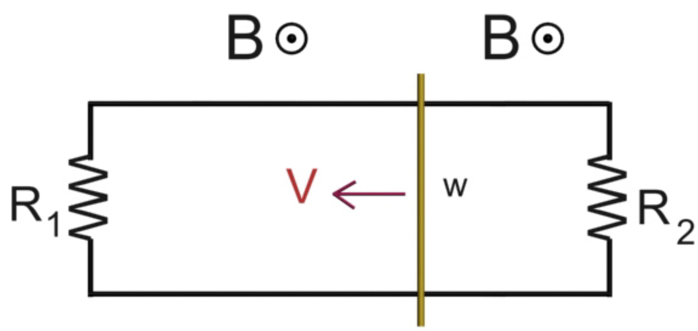
(e) Do your answers in (c) and (d) agree with Lenz's law?

(f) Where does the energy come from that is dissipated in ohmic heating in the wire?

Sample Exam Problem…

Problem 2: Faraday's Law

A conducting rod with zero resistance and length *w* slides without friction on two parallel perfectly conducting wires. Resistors R_1 and R_2 are connected across the ends of the wires to form a circuit, as shown. A constant magnetic field **B** is directed out of the page. In computing magnetic flux through any surface, take the surface normal to be out of the page, parallel to **B**.



(a) The magnetic flux in the right loop of the circuit shown is (circle one)
   1) decreasing
   2) increasing.

What is the magnitude of the rate of change of the magnetic flux through the right loop?

(b) What is the current flowing through the resistor R_2 in the right hand loop of the circuit shown? Gives its magnitude and indicate its direction on the figure.

(c) The magnetic flux in the left loop of the circuit shown is (circle one)
   1) decreasing
   2) increasing.

What is the magnitude of the rate of change of the magnetic flux through the right loop?

(d) What is the current flowing through the resistor R_1 in the left hand loop of the circuit shown? Gives its magnitude and indicate its direction on the figure.

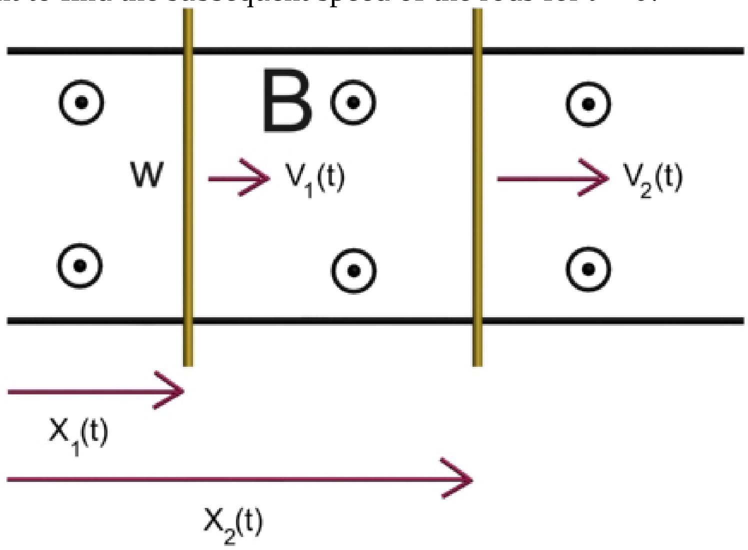
(e) What is the magnitude **and direction** of the magnetic force exerted on this rod?

Analytic Problems…

Problem 3: Coupled Rods

Two rods slide without friction on two conducting rails separated by a distance *w*, in the presence of a constant background magnetic field **B**, as shown in the diagram. The mass of each rod is *m* and each rod has a resistance *R*. The rails have no resistance. At time *t*, the rod on the left is a distance $x_1(t)$ from the origin, and the rod on the right is a distance $x_2(t)$ from the origin. The respective speeds of the rods are given by $V_1(t) = \frac{dx_1(t)}{dt}$ and $V_2(t) = \frac{dx_2(t)}{dt}$.

At $t = 0$ the rod ***on the right is moving to the right with speed*** V_o, and the ***rod on the left is at rest.*** We want to find the subsequent speed of the rods for $t > 0$.



(a) At time t, what is the magnetic flux through the circuit consisting of the two rods and the rails connecting them?

(b) At time t, what is the current through the circuit consisting of the two rods and the rails connecting them. Give its magnitude and also give its sense (clockwise or counterclockwise) using a Lenz's Law argument. You can make this argument at $t = 0$ if you wish, when $V_1(t)\big|_{t=0} = 0$ and $V_2(t)\big|_{t=0} = V_0$.

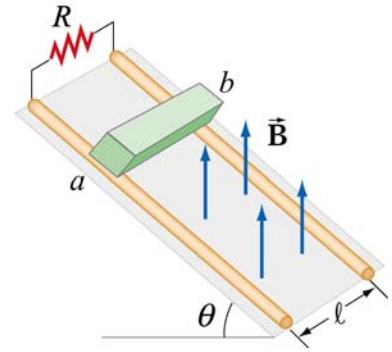
(c) At time t, what is the magnetic force $\vec{\mathbf{F}}_{right}$ on the rod on the right, and what is the magnetic force $\vec{\mathbf{F}}_{left}$ on the rod on the left?

(d) Using $F_{right} = m\frac{d}{dt}V_2(t)$, $F_{left} = m\frac{d}{dt}V_1(t)$, and the results of (c), derive equations for $\frac{d}{dt}V_1(t)$ and $\frac{d}{dt}V_2(t)$. Using these equations, derive differential equations for the two new functions defined by $P(t) = V_1(t) + V_2(t)$ and $M(t) = V_2(t) - V_1(t)$.

(e) What are the solutions for $P(t) = V_1(t) + V_2(t)$ and $M(t) = V_2(t) - V_1(t)$ given the initial conditions? Remember that $V_1(t)\big|_{t=0} = 0$ and $V_2(t)\big|_{t=0} = V_0$. (Note: if the function $D(t)$ satisfies the equation $\frac{d}{dt}D(t) = -\frac{D(t)}{\tau}$, then $D(t) = D_o e^{-t/\tau}$; if the function $E(t)$ satisfies the equation $\frac{d}{dt}E(t) = 0$, then $E(t) = E_o$).

(f) Solve for the speeds of rods 1 and 2 at time t using the time dependence of P and M. After a very long time, what are the speeds of rods 1 and 2?

Problem 4: Sliding Bar on Wedges



A conducting bar of mass m and negligible resistance slides down two frictionless conducting rails which make an angle θ with the horizontal, separated by a distance ℓ and connected at the top by a resistor R, as shown in the figure. In addition, a uniform magnetic field $\vec{\mathbf{B}}$ is applied vertically upward. The bar is released from rest and slides down. At time t the bar is moving along the rails at speed $v(t)$.

(a) Find the induced current in the bar at time t. Which way does the current flow, from a to b or b to a?

(b) Find the terminal speed v_T of the bar.

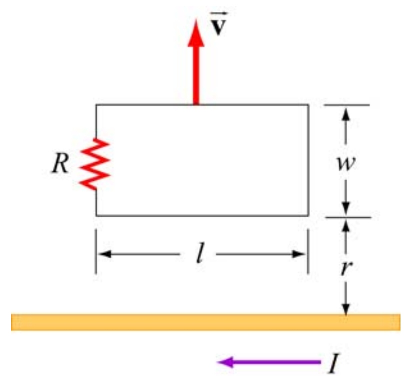
After the terminal speed has been reached,

(c) What is the induced current in the bar?

(d) What is the rate at which electrical energy is being dissipated through the resistor?

(e) What is the rate of work done by gravity on the bar? The rate at which work is done is $\vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$. How does this compare to your answer in (d)? Why?

Problem 5: Moving Loop



A rectangular loop of dimensions l and w moves with a constant velocity $\vec{\mathbf{v}}$ away from an infinitely long straight wire carrying a current I in the plane of the loop, as shown in the figure. The total resistance of the loop is R.

(a) Using Ampere's law, find the magnetic field at a distance s away from the straight current-carrying wire.

(b) What is the magnetic flux through the rectangular loop at the instant when the lower side with length l is at a distance r away from the straight current-carrying wire, as shown in the figure?

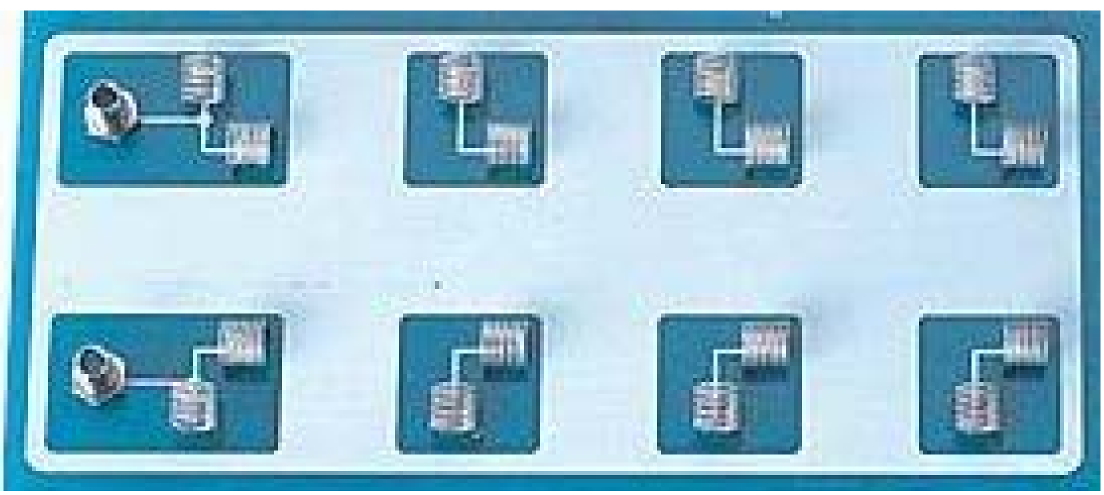
(c) At the instant the lower side is a distance r from the wire, find the induced EMF and the corresponding induced current in the rectangular loop. Which direction does the induced current flow?

Experiment 6: Ohm’s Law, RC and RL Circuits
Record your answers to these questions in your notes – you will need them for the lab.

Problem 6: Measuring Voltage and Current

In Part 1 of this experiment you will measure the potential drop across and current through a single resistor attached to the “variable battery.” Below is a picture of the circuit board you will use for wiring up the experiment. Each pad (dark green area) has two spring clips (the little slinky shaped objects) connected by a wire (indicated by the white lines), aside from the left most pads, which also have a third connection post. The pads themselves are isolated. If you want to connect two of them you need to run a wire between them.

On a diagram similar to the one below, indicate where you will attach the leads to the resistor, the battery, the voltage sensor V, and the current sensor A. For the battery and sensors make sure that you indicate which color lead goes where, using the convention that red is “high” (or the positive input) and black is “ground.” When you draw a resistor or other circuit element it should go between two pads with each end touching one of the spring clips. Do NOT just draw a typical circuit diagram. You need to think about how you will actually wire this board during the lab. RECALL: ammeters must be in series with the element they are measuring current through, while voltmeters must be in parallel.
Make sure that you understand this (ask someone if you don’t). You will not be able to do the lab if you don’t.



Problem 7: Resistors in Parallel

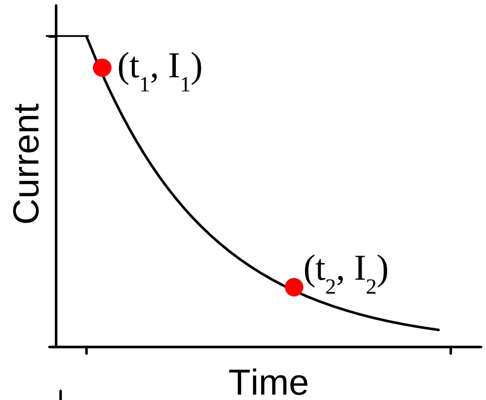
In Part 2 you will add a second resistor in parallel with the first. Show where you would attach this second resistor in the diagram you drew for question 1, making sure that the ammeter continues to measure the current through the first resistor and the voltmeter measures the voltage across the first resistor.

Problem 8: Measuring the Time Constant τ

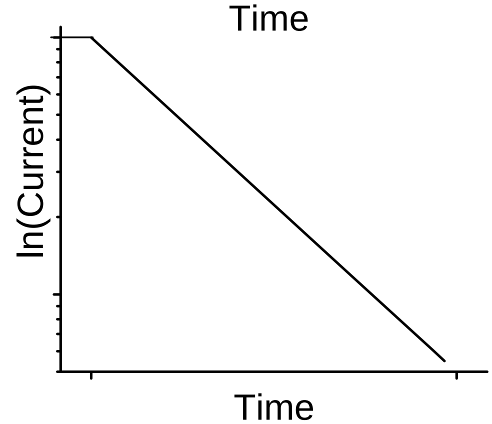
As you have seen, current decays exponentially in RC circuits with a time constant τ: $I = I_0 \exp(-t/\tau)$.

We measure this time constant in 2 different ways.

(a) After measuring the current as a function of time we choose a point on the curve (t_1,I_1). We then look for a point (t_2, I_2) where the current I_2 is related to I_1 such that the time constant can be determined by subtraction: $\tau = t_2 - t_1$. What I_2 do you look for?

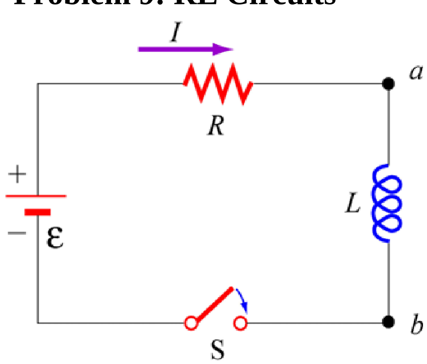


(b) We can also plot the natural log of the current vs. time, as shown at right. If we fit a line to this curve we will obtain a slope *m* and a y-intercept y_0. From these fitting parameters, how can we calculate the time constant?



(c) Which of these two methods is more likely to help us obtain an accurate measurement of the time constant? Why?

Problem 9: RL Circuits



Consider the circuit at left, consisting of a battery (EMF ε), an inductor *L*, resistor *R* and switch *S*.

For times $t<0$ the switch is open and there is no current in the circuit. At $t=0$ the switch is closed.

(a) Using Kirchhoff's loop rules (really Faraday's law now), write an equation relating the EMF on the battery, the current in the circuit and the time derivative of the current in the circuit.

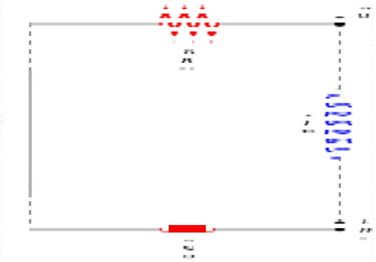
In class we stated that this equation was solved by an exponential. In other words:

$$I = A(X - \exp(-t/\tau))$$

(b) Plug this expression into the differential equation you obtained in (a) in order to confirm that it indeed is a solution and to determine what the time constant τ and the constants *A* and *X* are. What would be a better label for *A*? (HINT: You will also need to use the initial condition for current. What is $I(t=0)$?).

(c) Now that you know the time dependence for the current *I* in the circuit you can also determine the voltage drop V_R across resistor and the EMF generated by the inductor.

Problem 10: 'Discharging' an Inductor



After a long time T the current will reach an equilibrium value and inductor will be "fully charged." At this point we turn off the battery (ε=0), allowing the inductor to 'discharge,' as pictured at left. Repeat each of the steps a-c in problem 9, noting that instead of exp($-t/\tau$), our expression for current will now contain exp($-(t-T)/\tau$).

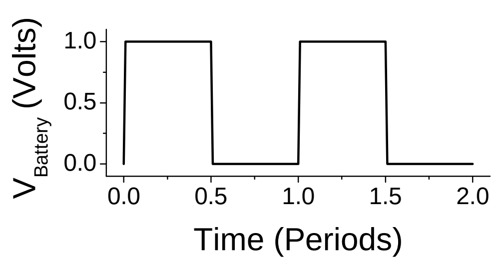
Problem 11: The Coil

The coil you will be measuring is made of thin copper wire (radius ~ 0.25 mm) and has about 600 turns of average diameter 25 mm over a length of 20 mm. What approximately should the resistance and inductance of the coil be? The resistivity of copper at room temperature is around 20 nΩ-m. Note that your calculations can only be approximate because this is not at all an ideal solenoid (where length >> diameter).

Problem 12: A Real Inductor

As mentioned above, in this lab you will work with a coil that does not behave as an ideal inductor, but rather as an ideal inductor in series with a resistor. For this reason you have no way to independently measure the voltage drop across the resistor or the EMF induced by the inductor, but instead must measure them together. None-the-less, you want to get information about both. In this problem you will figure out how.

(a) In the lab you will hook up the circuit of problem 9 with the ideal inductor L of that problem now replaced by a coil that is a non-ideal inductor – an inductor L and resistor r in series. The battery will periodically turn on and off, displaying a voltage as shown here:



Sketch the current through the battery as well as what a voltmeter hooked across the coil would show versus time for the two periods shown above. Assume that the period of the battery turning off and on is comparable to but longer than several time constants of the circuit.

Problem 12: A Real Inductor *continued…*

(b) How can you tell from your plot of the voltmeter across the coil that the coil is not an ideal inductor? Indicate the relevant feature clearly on the plot. Can you determine the resistance of the coil, r, from this feature?

(c) In the lab you will find it easier to make measurements if you do NOT use an additional resistor R, but instead simply hook the battery directly to the coil. (Why? Because the time constant is difficult to measure with extra resistance in the circuit). Plot the current through the battery and the reading on a voltmeter across the coil for this case. We will only bother to measure the current. Why?

(d) For this case (only a battery & coil) how will you determine the resistance of the coil, r? How will you determine its inductance L?

Make sure that you record your answer to 7d in your notes as you will need it for the lab.