18.311 Pset 3

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Special

TFPb14

TFP b14

$\rho_t + q_x = 0.$

$q = q(\rho).$

$q_\ell = q(\rho_\ell)$
$q_r = q(\rho_r)$
$c_\ell = c(\rho_\ell)$
$c_r = c(\rho_r)$

$c = c(\rho) = \frac{dq}{d\rho}$

Discontinuous solution: $\rho(x,t) = \begin{cases} \rho_\ell & \text{for } x<0, \ t>0 \\ \rho_r & \text{for } x>0, \ t>0. \end{cases}$

(A) $q = \rho(2-\rho)$, with $\rho_\ell = 0.5$, $\rho_r = 1.5$; $q_\ell = 0.75$, $q_r = 0.75$; $c_\ell = 1$, $c_r = -1$.

- $s = \frac{[q]}{[\rho]}$ for a shock, $\frac{[q]}{[\rho]} = 0$ so $q_\ell = q_r$

  $\boxed{0.75 = 0.75}$ ✓.

- Entropy: $c_\ell > s > c_r$ for a shock.

  Since $s = 0$, $c_\ell = 1 > s = 0$
  $c_r = -1 < s = 0.$
  then $c_\ell > s > c_r$

YES. this candidate is actually a solution.

(B) $q = \rho(2-\rho)$ $\rho_\ell = 0$, $\rho_r = 1$; $q_\ell = 0$, $q_r = 1$; $c_\ell = 2$, $c_r = 0.$

- $s = \frac{[q]}{[\rho]} \Rightarrow s = 0. \Rightarrow q_\ell = q_r.$

  here $q_\ell \neq q_r$
  $0 \neq 1.$

- $c_\ell = 2 > s = 0 > c_r = 0.$

NO. not a solution b/c $q_\ell \neq q_r$

TFP 614, cont.

C) $q = \rho^2$ $\quad \rho_\ell = -1$, $\rho_r = 1$ $\quad q_\ell = q_r = 1$ $\quad c_\ell = -2$, $c_r = 2$

$\boxed{q_\ell = q_r}$ ✓

$\boxed{c_\ell = -2 < s = 0 < c_r = 2.}$ ✗

NO. not a soln b/c $c_\ell > s = 0 > c_r$ is not satisfied.

D) $q = 1 - \rho^2$ $\quad \rho_\ell = -1$, $\rho_r = 1$ $\quad q_\ell = q_r = 0.$ $\quad c_\ell = 2$, $c_r = -2.$

$\boxed{q_\ell = q_r}$ ✓

$\boxed{c_\ell = 2 > s = 0 > c_r = -2}$ ✓

YES. Both properties satisfied.

E) $q = \rho(2 - \rho)$ $\quad \rho_\ell = 1.25$, $\rho_r = 0.75$ $\quad q_\ell = q_r = 0.9375$ $\quad c_\ell = -0.5$, $c_r = 0.5$

$\boxed{q_\ell = q_r}$ ✓

$\boxed{c_\ell = -0.5 < s = 0.}$ ✗

NO. c_ℓ should be greater than $s = 0$.