Theory Assignment-5: ADA Winter-2024

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1 Assumptions

- A box B_2 can only be placed inside a box B_1 if dimensions of B_2 can be rearranged to be **strictly smaller** than dimensions of B_1. If not, some trivial minor changes can be made to the implementation.

2 Formulation as a Network Flow problem

Let there be n boxes $B_1, B_2, \cdots, B_n$. with dimensions $d_1 = (x_1, y_1, z_1), d_2 = (x_2, y_2, z_2), \cdots, d_n$.

We want to minimise $k =$ number of visible boxes.

k can also be expressed as $n - v$ where v is the number of invisible boxes.

Thus, we have an equivalent problem of **maximising v, the number of invisible boxes.**

3 Why Max Flow corresponds to answer

4 Complexity Analysis (Using Ford-Fulkerson's Algorithm)

To solve this Max-Flow problem, we are running the Ford-Fulkerson Algorithm on our auxiliary bipartite graph construction $G = (V, E)$ where $|V| = O(n)$ and $|E| = O(n^2)$ (see Proof of Correctness section for cardinality of E).

As will be proved shortly, the maximum possible flow in this graph is also the maximum number of invisible boxes, which is $value(flow) = n - 1 = O(n)$.

Ford-Fulkerson's Algorithm runs in $O(value(flow)(|V| + |E|))$ time, **therefore our algorithm runs in**

$$\begin{aligned} & O\Big(value(flow)\,(|V| + |E|)\Big) \\ &= O(value(flow))O(|V| + |E|) \\ &= O(n)O(n + n^2) \\ &= O(n)O(n^2) \\ &= O(n^3) \end{aligned}$$

$O(n^3)$ time where n is the number of boxes.

5 Proof of Correctness

Addendum 1: C++ implementation

Addendum 2: Sample Tests

Addendum 3: People discussed with

Swapnil Panigrahi, Sahil Gupta and Yash Bhardwaj.

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Thank you.