

Variable Time Step Reinforcement Learning for Robotic Applications

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Abstract—Traditional reinforcement learning (RL) generates discrete control policies, assigning one action per cycle. These policies are usually implemented as in a fixed-frequency control loop. This rigidity presents challenges as optimal control frequency is task-dependent; suboptimal frequencies increase computational demands and reduce exploration efficiency. Variable Time Step Reinforcement Learning (VTS-RL) addresses these issues with adaptive control frequencies, executing actions only when necessary, thus reducing computational load and extending the action space to include action durations. In this paper we introduce the Multi-Objective Soft Elastic Actor-Critic (MOSEAC) method to perform VTS-RL, validating it through theoretical analysis and experimentation in simulation and on real robots. Results show faster convergence, better training results, and reduced energy consumption with respect to other variable- or fixed-frequency approaches.

Index Terms—Variable Time Step Reinforcement Learning (VTS-RL), Deep Learning in Robotics and Automation, Learning and Adaptive Systems, Optimization and Optimal Control

I. INTRODUCTION

DEEP reinforcement learning (DRL) algorithms have achieved significant success in gaming [1], [2] and robotic control [3], [4]. Traditional DRL employs a fixed control loop at set intervals (e.g., every 0.1 seconds). This fixed-rate control can cause stability issues and high computational demands, particularly in dynamic environments where the optimal action frequency varies.

Variable Time Step Reinforcement Learning (VTS-RL) has been recently proposed to address these issues. Based on reactive programming principles [5], [6], VTS-RL executes control actions only when necessary, reducing computational load and expanding the action space to include variable action durations. For example, in robotic manipulation, VTS-RL allows a robot arm to dynamically adjust its control frequency, using lower frequencies during simple, repetitive tasks and higher frequencies during complex maneuvers or when handling delicate objects [7].

Two notable VTS-RL algorithms are Soft Elastic Actor-Critic (SEAC) [8] and Continuous-Time Continuous-Options (CTCO) [7]. CTCO employs continuous-time decision-making with flexible option durations. CTCO requires tuning several hyperparameters, such as its radial basis functions (RBFs) and

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the time-related hyperparameters τ for its adaptive discount factor γ , complicating tuning in some environments.

SEAC incorporates reward terms for task energy (number of actions) and task time, making it effective in time-restricted environments like racing video games [9]. However, it also requires careful hyperparameter tuning (balancing task, energy, and time costs) to maintain performance. The sensitivity of SEAC and CTCO to hyperparameter settings challenges users to fully leverage their potential.

We introduce the Multi-Objective Soft Elastic Actor-Critic (MOSEAC) algorithm [10]. MOSEAC integrates action durations into the action space and adjusts hyperparameters based on observed trends in task rewards during training, reducing the need to set multiple hyperparameters. Additionally, our hyperparameter setting approach can be broadly applied to any continuous action reinforcement learning algorithm. This adaptability facilitates the transition from fixed-time step to variable-time step reinforcement learning, significantly expanding its practical application.

In this paper, we provide an in-depth analysis of MOSEAC's theoretical performance, covering its framework, implementation, performance guarantees, convergence and complexity analysis. We also deploy the MOSEAC model as the navigation policy in simulation and on a physical AgileX Limo [11]. Our evaluation includes statistical performance analysis, trajectory similarity analysis between simulated and real environments, and a comparison of average computational resource consumption on both CPU and GPU.

MOSEAC allows the optimization of the control frequency for RL policies, as well as a reduction in computational load on the robots' onboard computer. The resources saved can be redirected to other critical tasks such as environmental sensing [12] and communication [13]. In addition, the MOSEAC principles can be applied to a variety of RL algorithms, making it a useful tool for the deployment of RL policies on physical robotics systems, reducing the real-to-sim gap.

The paper is organized as follows. Section II reviews the current research status of VTS-RL. Section III details the MOSEAC framework with its pseudocode. Section IV presents the theoretical analysis of MOSEAC. Section V describes the implementation of our validation environment in the real world and the method to build the simulation environment. Section VI presents the experiment results on both the simulation and the real Agilex Limo. Finally, Section VII concludes the paper.

II. RELATED WORK

The importance of action duration in reinforcement learning (RL) has been significantly underestimated, yet it is crucial for applying RL algorithms in real-world scenarios, impacting an agent's exploration capabilities. For instance, high frequencies may reduce exploration efficiency but are essential in delayed environments [14]. Recent studies by Amin et al. [15], and Park et al. [16] have highlighted this issue, showing that variable action durations can significantly affect learning performance. Building on these insights, Wang & Beltrame [9] and Karimi et al. [7] further explored how different frequencies impact learning, noting that excessively high frequencies can impede convergence. Their findings suggest that dynamic control frequencies, which adjust in real-time based on reactive principles, could enhance performance and adaptability.

Expanding on the idea of adaptive control, Sharma et al. [17] introduced a related concept of repetitive action reinforcement learning, where an agent performs identical actions over successive states, combining them into a larger action. This concept was further explored by Metelli et al. [18] and Lee et al. [19] in gaming contexts. However, despite its potential, this method does not fully address physical properties or computational demands, limiting its practical application.

Additionally, Chen et al. [20] proposed modifying the traditional control rate by integrating actions such as "sleep" to reduce activity periods. However, this approach still required fixed-frequency system checks, ultimately failing to reduce computational load as intended.

In the domain of traffic signal control, research focused on hybrid action reinforcement learning. The study highlighted the importance of synchronously optimizing stage specifications and green interval durations, underscoring the potential of variable action durations to improve decision-making processes in complex, real-time environments [21].

Furthermore, research on variable damping control for wearable robotic limbs has showcased the application of reinforcement learning to adjust control parameters adaptively in response to changing states. This study illustrated the effectiveness of RL in maintaining system stability and performance under varying conditions, reinforcing the value of dynamic control frequencies in practical applications [22].

These studies collectively highlight the ongoing efforts to integrate variable action durations and adaptive control mechanisms into RL. They also underscore the nascent stage of effectively integrating variable control frequencies and repetitive behaviors into practical applications, emphasizing their critical importance for enhancing algorithm performance and applicability in real-world scenarios.

III. ALGORITHM FRAMEWORK

Soft Elastic Actor-Critic (SEAC) is an extension of the Soft Actor-Critic (SAC) algorithm that addresses the limitations of fixed control rates in reinforcement learning (RL) [8]. Traditional Markov Decision Processes (MDPs) in RL do not account for the duration of actions, assuming that all actions are executed over uniform time steps. This can lead to inefficiencies, as the time between two actions can vary

widely, requiring a fixed-frequency control rate in practical deployments. SEAC breaks this assumption by dynamically adjusting the duration of each action based on the state and environmental conditions, following the principles of reactive programming. By incorporating action duration D into the action set, SEAC can decide on both the action and its duration to optimize energy consumption and computational efficiency.

MOSEAC extends SEAC [8] and combines its hyperparameters for balancing task, energy, and time rewards, and provides a method for automatically adjust its other hyperparameters during training. Similar to SEAC, MOSEAC reward includes components for:

- Quality of task execution (the standard RL reward),
- Time required to complete a task (important for varying action durations), and
- Energy (the number of time steps, a.k.a. the number of actions taken, which we aim to minimize).

Definition 1: The reward associated with the state space is:

$$R = \alpha_m R_t R_\tau - \alpha_\varepsilon \quad (1)$$

where R_t is the task reward and R_τ is a time-dependent term.

α_m is a weighting factor used to modulate the magnitude of the reward: its primary function is to prevent the reward from being too small, which could lead to task failure, or too large, which could cause reward explosion [23], ensuring stable learning.

α_ε is a penalty parameter applied at each time step to impose a cost on the agent's actions. This parameter gives a fixed cost to the execution of an action, thereby discouraging unnecessary ones. In practice, α_ε promotes the completion of a task using fewer time steps (remember that time steps have variable duration), i.e., it reduces the energy used by the control loop of the agent.

We determine the optimal policy π^* , which maximizes the reward R .

The reward is designed to minimize both energy cost (number of steps) and the total time to complete the task through R_τ . By scaling the task-specific reward based on action duration with α_m , agents are motivated to complete tasks using fewer actions:

Definition 2: The remap relationship between action duration and reward is:

$$R_\tau = D_{min}/D, \quad R_\tau \in [D_{min}/D_{max}, 1] \quad (2)$$

where t is the duration of the current action, D_{min} is the minimum duration of an action (strictly greater than 0), and D_{max} is the maximum duration of an action.

We automatically set α_m and α_ε during training. Based on previous results [9], we bind the increase of α_m to a decrease α_ε using a sigmoid function to mitigate convergence issues, specifically the problem of sparse rewards caused a large α_ε and a small α_m . This adjustment ensures that rewards are appropriately balanced, facilitating learning and convergence.

Definition 3: The relationship between α_ε and α_m is:

$$\alpha_\varepsilon = 0.2 \cdot \left(1 - \frac{1}{1 + e^{-\alpha_m+1}} \right) \quad (3)$$

Based on [8]’s experience, we establish a mapping relationship between the two parameters: when the initial value of α_m is 1.0, the initial value of α_ε is 0.1. As α_m increases, α_ε decreases, but never falls below 0.

Overall, we update α_m based on the current trend in average reward during training. To guarantee stability, we ensure the change is monotonic, forcing a uniform sweep of the parameter space, and a maximum value of α_m , namely α_{max} . To determine the trend in the average reward, we perform a linear regression across the current training episode and compute the slope of the resulting line: if it is negative, the reward is declining.

Definition 4: The slope of the average reward (k_R) is:

$$k_R(R_a) = \frac{n \sum_{i=1}^n (i \cdot R_{ai}) - (\sum_{i=1}^n i)(\sum_{i=1}^n R_{ai})}{n \sum_{i=1}^n i^2 - (\sum_{i=1}^n i)^2} \quad (4)$$

where n is the total number of data points collected across the update interval (k_{update} in Algorithm 1). The update interval is a hyperparameter that determines the frequency of updates for these neural networks used in the actor and critic policies, occurring after every n episode [24]. R_a represents the list of average rewards ($R_{a1}, R_{a2}, \dots, R_{an}$) during training. Here, an average reward R_{ai} is calculated across one episode.

Definition 5: We adaptively adjust the reward every k_{update} episodes when $k_R < 0$:

$$\begin{cases} \alpha_m = \alpha_m + \psi & \text{if } \alpha_m < \alpha_{max} \\ \alpha_m = \alpha_{max} & \text{otherwise} \end{cases} \quad (5)$$

where ψ serves as the sole additional MOSEAC hyperparameter necessary for adjusting the reward equation during training. The parameter α_{max} represents the upper limit of α_m , ensuring algorithmic convergence and preventing reward explosion. Furthermore, α_ε is adjusted in accordance with Definition 3.

Algorithm 1 shows the pseudocode of MOSEAC. In short, MOSEAC extends the SAC algorithm by incorporating action duration D into the action policy set. This expansion allows the algorithm to predict the action and its duration simultaneously. The reward is calculated using 1, and its changes are continuously monitored. If the reward trend declines, α_m increases linearly at a rate of ψ , without exceeding α_{max} . The action and critic networks are periodically updated like the SAC algorithm based on these preprocessed rewards.

Here, t_{max} represents the maximum number of training steps [24]; k_{length} is the maximum number of exploration steps per episode [24]; k_{init} is the number of steps in the initial random exploration phase [24]. The reward $R(S_i, A_i, D_i)$ depends on state (S_i), action (A_i) and duration $D_i \in [D_{min}, D_{max}]$.

Overall, our reward scalarizes a multi-objective optimization problem including rewards for task, time, and energy. Unlike Hierarchical Reinforcement Learning (HRL) [25], [26], which seeks Pareto optimality [27], [28] with layered reward policies, our method simplifies the approach, making our strategy easily adaptable to various algorithms. While our goal is to avoid the complexity of Pareto optimization, dynamically adjusting the reward structure inevitably places us on the Pareto optimization curve. For a theoretical analysis of the Pareto curve

Algorithm 1: Multi-Objective Soft Elastic Actor and Critic (MOSEAC)

Require: a policy π with a set of parameters θ, θ' , critic parameters ϕ, ϕ' , variable time step environment model Ω , learning-rate λ_p, λ_q , reward buffer β_r , replay buffer β .

1: Initialization $i = 0, t_i = 0, \beta_r = 0$, observe S_0
2: **while** $t_i \leq t_{max}$ **do**
3: **for** $i \leq k_{length} \vee \text{Not Done}$ **do**
4: $A_i, D_i = \pi_\theta(S_i)$
5: → simple action and its duration
6: $S_{i+1}, R_i = \Omega(A_i, D_i)$
7: → compute reward with Definition 1 and 2
8: $i \leftarrow i + 1$
9: **end**
10: $\beta_r \leftarrow 1/i \times \sum_0^i R_i$
11: → collect the average reward for one episode
12: $\beta \leftarrow S_{0 \sim i}, A_{0 \sim i}, D_{0 \sim i}, R_{0 \sim i}, S_{1 \sim i+1}$
13: $i = 0$
14: $t_i \leftarrow t_i + 1$
15: **if** $t_i \geq k_{init} \wedge t_i | k_{update}$ **then**
16: Sample S, A, D, R, S' from (β)
17: $\phi \leftarrow \phi - \lambda_q \nabla_\phi \mathcal{L}_Q(\phi, S, A, D, R, S')$
18: → critic update
19: $\theta \leftarrow \theta - \lambda_p \nabla_\theta \mathcal{L}_\pi(\theta, S, A, D, \phi)$
20: → actor update
21: **if** $k_R(\beta_r)$ **then**
22: $\alpha_m = \alpha_m + \psi$ $\text{ifa}_{max} < \alpha_{max}$
23: Or $\alpha_m = \alpha_{max}$ otherwise
24: → see Definition 4 for k_R
25: $\alpha_\varepsilon \leftarrow F_{update}(\alpha_m)$
26: → update $\alpha_m, \alpha_\varepsilon$ followed Definition 3
27: **end**
28: $\beta_r = 0$
29: → Re-record average reward values under new hyperparameters
30: **end**
31: Perform soft-update of ϕ' and θ'
32: **end**

and how α_{max} ensures algorithmic stability, please refer to Appendix A.

Apart from a series of hyperparameters inherent to RL that need adjustment, such as learning rate, t_{max} , k_{update} , etc., ψ is the primary hyperparameter that requires tuning in MOSEAC. Determining the appropriate ψ value remains a critical optimization point. We recommend using the pre-set ψ value provided in our implementation to reduce the need for additional adjustments. If training performance is poor, a large ψ may cause the reward signal’s gradient to change too rapidly, leading to instability, suggesting a reduction in ψ . Conversely, if training is slow, a small ψ may result in a weak reward signal, affecting convergence, suggesting an increase in ψ .

IV. THEORETICAL ANALYSIS

We have analyzed the theoretical performance of MOSEAC through various aspects, including performance guarantees, convergence and complexity analysis. Table I provides the notation for the following.

A. Performance Guarantees

In the standard SAC algorithm [29], the policy $\pi_\theta(a|s)$ selects action a , and updates the policy parameters θ and value function parameters ϕ . The objective function is:

$$J(\pi_\theta) = \mathbb{E}_{(s,a) \sim \pi_\theta} [Q^\pi(s, a) + \alpha \mathcal{H}(\pi_\theta(\cdot|s))] \quad (6)$$

where $J(\pi_\theta)$ is the objective function, $Q^\pi(s, a)$ is the state-action value function, α is a temperature parameter controlling the entropy term, and $\mathcal{H}(\pi_\theta(\cdot|s))$ is the entropy of the policy.

When the action space is extended to include the action duration D and the reward function is modified to $R = \alpha_m R_t R_\tau - \alpha_\varepsilon$, where $\alpha_m \geq 0$, $0 < R_\tau \leq 1$, and α_ε is a small positive constant, the new objective function becomes:

$$J(\pi_\theta) = \mathbb{E}_{(s,a,D) \sim \pi_\theta} [Q^\pi(s, a, D) + \alpha \mathcal{H}(\pi_\theta(\cdot|s))] \quad (7)$$

where $Q^\pi(s, a, D)$ is the extended state-action value function with time dimension D , and R_t and R_τ are components of the reward function, see Definition 1.

Replacing Q^π the objective function for MOSEAC, incorporating the extended action space and the adaptive reward function, becomes:

$$J(\pi) = \mathbb{E}_{(s_t, a_t, D_t) \sim \rho_\pi} \left[\sum_{t=0}^{\infty} \gamma^t \left(\alpha_m(t) R_t \frac{D_{\min}}{D_t} - \alpha_\varepsilon(t) + \alpha \mathcal{H}(\pi(\cdot|s_t)) \right) \right] \quad (8)$$

where:

- $\alpha_m(t)$ is the adaptive parameter that increases monotonically when the average reward decreases, with an upper limit of α_{max} .
- $\alpha_\varepsilon(t)$ is the adaptive parameter that decreases monotonically when the average reward decreases.
- $\mathcal{H}(\pi(\cdot|s_t)) = -\mathbb{E}_{(a_t, D_t) \sim \pi(\cdot|s_t)} [\log \pi(a_t, D_t | s_t)]$ is the entropy of the policy.

The SAC policy improvement theorem [29] ensures that

$$Q^{\pi_k}(s, a, D) \geq Q^{\pi_{k-1}}(s, a, D) \quad (9)$$

for any iteration $k \geq 0$.

The soft Bellman equation in MOSEAC is:

$$Q^\pi(s, a, D) = \alpha_m(t) R_t \frac{D_{\min}}{D} - \alpha_\varepsilon(t) + \gamma \mathbb{E}_{s' \sim P(\cdot|s, a, D)} [V^\pi(s')] \quad (10)$$

where the value function $V^\pi(s)$ is defined as:

$$V^\pi(s) = \mathbb{E}_{(a,D) \sim \pi(\cdot|s)} \left[Q^\pi(s, a, D) - \alpha \log \pi(a, D | s) \right] \quad (11)$$

To prove convergence, we need to show that the soft Bellman operator is a contraction mapping. Let \mathcal{T} be the soft Bellman operator. For any two Q-functions Q_1 and Q_2 :

$$\begin{aligned} \|\mathcal{T}Q_1 - \mathcal{T}Q_2\|_\infty &= \sup_{s,a,D} \left| \alpha_m(t) R_t \frac{D_{\min}}{D} - \alpha_\varepsilon(t) \right. \\ &\quad \left. + \gamma \mathbb{E}_{s' \sim P(\cdot|s, a, D)} [V_1(s')] \right. \\ &\quad \left. - \left(\alpha_m(t) R_t \frac{D_{\min}}{D} - \alpha_\varepsilon(t) + \gamma \mathbb{E}_{s' \sim P(\cdot|s, a, D)} [V_2(s')] \right) \right| \\ &\leq \sup_{s,a,D} \gamma \left| \mathbb{E}_{s' \sim P(\cdot|s, a, D)} [V_1(s') - V_2(s')] \right| \\ &\leq \gamma \|V_1 - V_2\|_\infty \\ &\leq \gamma \|Q_1 - Q_2\|_\infty \end{aligned} \quad (12)$$

Since $\gamma < 1$, the soft Bellman operator \mathcal{T} is a contraction mapping. By Banach's fixed-point theorem, there exists a unique fixed point Q^* such that $Q^* = \mathcal{T}Q^*$ [30].

Let π^* be the optimal policy and π_k be the policy at iteration k . The error bound between the value functions of the optimal and learned policies is

$$\|V^{\pi^*} - V^{\pi_k}\| \leq \frac{2\alpha\gamma \log |\mathcal{A} \times [D_{\min}, D_{\max}]|}{(1-\gamma)^2} \quad (13)$$

With $|\mathcal{A} \times [D_{\min}, D_{\max}]|$ the size of the extended action space, the contraction property of the soft Bellman operator ensures that

$$\|V^{\pi^*} - V^{\pi_k}\|_\infty \leq \frac{2\alpha\gamma \log |\mathcal{A} \times [D_{\min}, D_{\max}]|}{(1-\gamma)^2} \quad (14)$$

where α is the coefficient of the entropy term and γ is the discount factor.

B. Convergence Analysis

With our reward function, the policy gradient is:

$$\nabla_\theta J(\pi_\theta) = \mathbb{E}_{\pi_\theta} \left[\nabla_\theta \log \pi_\theta(a, D | s) \cdot (Q^\pi(s, a, D) \cdot (\alpha_m \cdot R_\tau) - \alpha_\varepsilon) \right] \quad (15)$$

where $\nabla_\theta J(\pi_\theta)$ is the gradient of the objective function with respect to the policy parameters θ .

The value function update, incorporating the time dimension D and our reward function, is:

$$L(\phi) = \mathbb{E}_{(s,a,D,r,s')} \left[(Q_\phi(s, a, D) - (r + \gamma \mathbb{E}_{(a',D') \sim \pi_\theta} [V_\phi(s') - \alpha \log \pi_\theta(a', D' | s')]))^2 \right] \quad (16)$$

where $L(\phi)$ is the loss function for the value function update, r is the reward, γ is the discount factor, and $V_\phi(s')$ is the target value function.

The new policy parameter θ update rule is:

$$\theta_{k+1} = \theta_k + \beta_k \mathbb{E}_{s \sim D, (a,D) \sim \pi_\theta} \left[\nabla_\theta \log \pi_\theta(a, D | s) \cdot (Q_\phi(s, a, D) \cdot (\alpha_m \cdot R_\tau) - \alpha_\varepsilon - V_\phi(s) + \alpha \log \pi_\theta(a, D | s)) \right] \quad (17)$$

where β_k is the learning rate at step k .

To analyze the impact of dynamically adjusting α_m and α_ε , we assume:

- 1) **Dynamic Adjustment Rules:** α_m increases monotonically by a small increment ψ if the reward trend decreases over consecutive episodes, and its upper limit is α_{max} , which guarantees algorithmic convergence and prevents reward explosion. α_ε decreases as defined.

- 2) **Learning Rate Conditions:** α_k and β_k must satisfy [31]:

$$\sum_{k=0}^{\infty} \alpha_k = \infty, \quad \sum_{k=0}^{\infty} \alpha_k^2 < \infty \quad (18)$$

$$\sum_{k=0}^{\infty} \beta_k = \infty, \quad \sum_{k=0}^{\infty} \beta_k^2 < \infty \quad (19)$$

Assuming the critic estimates are unbiased:

$$\mathbb{E}[Q_\phi(s, a, D) \cdot (\alpha_m \cdot R_\tau) - \alpha_\varepsilon] = Q^\pi(s, a, D) \cdot (\alpha_m \cdot R_\tau) - \alpha_\varepsilon \quad (20)$$

Since R_τ is a positive number within $[0, 1]$, its effect on $Q^\pi(s, a, D)$ is linear and does not affect the consistency of the policy gradient.

- 1) **Positive Scaling:** As $0 \leq R_\tau \leq 1$ and $\alpha_m \geq 0$, α_m only scales the reward without altering its sign. This scaling does not change the direction of the policy gradient but affects its magnitude.
- 2) **Small Offset:** α_ε is a small constant used to accelerate training. This small offset does not affect the direction of the policy gradient but introduces a minor shift in the value function, which does not alter the overall policy update direction.

Under these conditions, MOSEAC will converge to a local optimum [24]:

$$\lim_{k \rightarrow \infty} \nabla_\theta J(\pi_\theta) = 0 \quad (21)$$

C. Complexity Analysis

As MOSEAC closely follows SAC [29], but with an expanded action set that includes durations, its time complexity is similar to SAC, replacing the action set A with the expanded one $A \times D$. SAC and MOSEAC have 3 computational components:

- **Policy Evaluation:** consists of computing the value function $V^\pi(s)$ and the Q-value function $Q^\pi(s, a)$ for each state-action pair, i.e. $O(|\mathcal{S}| \cdot |\mathcal{A} \times D|)$.
- **Policy Improvement:** consists updating the policy parameters θ based on the policy gradient, involving the Q-value function and the entropy term. The time complexity is $O(|\mathcal{S}| \cdot |\mathcal{A} \times D|)$.
- **Value Function Update:** consists of computing the target value using the Bellman backup equation, with a time complexity of $O(|\mathcal{S}| \cdot |\mathcal{A} \times D|)$.

The overall time complexity for one iteration of the SAC algorithm is therefore

$$O(|\mathcal{S}| \cdot |\mathcal{A} \times D|) \quad (22)$$

The space complexity of MOSEAC is determined by the storage requirements for the policy, value functions, and

other necessary data structures. As for the time complexity, MOSEAC follows SAC with a different action set:

- **Policy Storage:** Requires $O(|\theta|)$ space for the policy parameters.
- **Value Function Storage:** Requires $O(|\mathcal{S}| \cdot |\mathcal{A}| \cdot |D|)$ space for the value functions.

The overall space complexity is:

$$O(|\theta| + |\mathcal{S}| \cdot |\mathcal{A}| \cdot |D|) \quad (23)$$

The dynamic adjustment of α_m and α_ε adds a small computational overhead of $O(1)$ per update due to simple arithmetic operations and comparisons.

In summary, MOSEAC has higher computational complexity than SAC, but its dynamic adjustment mechanism and expanded action space provide greater adaptability and potential for improved performance in multi-objective optimization scenarios.

TABLE I
COMPLEXITY ANALYSIS NOTATION

Symbol	Description
\mathcal{S}	Set of states
\mathcal{A}	Set of actions
D	Action durations set
π_θ	Policy parameters θ
$ \theta $	Policy parameter count
$V^\pi(s)$	Value function
$Q^\pi(s, a, D)$	Q-value function with D
α	Entropy temperature param.
α_m	Reward scaling param.
α_ε	Reward offset param.
R_t	Reward at time t
R_τ	Time-based reward
γ	Discount factor
ϕ	Value function params.
β_k	Learning rate at step k
\mathcal{T}	Soft Bellman operator
$\mathcal{H}(\pi(\cdot s))$	Policy entropy
ρ_π	State-action distribution
O	Complexity upper bound

V. EXPERIMENTS

We conducted a systematic evaluation and validation of MOSEAC's performance on task completion ability (e.g., navigating to target locations, avoiding yellow lines), resource consumption (measured by the number of steps required to complete the task), and time cost. Additionally, we assessed the trajectory and control output similarity to report the differences between simulation and reality. Finally, we compared the computing resource usage between variable and fixed RL models.

Figure 1 illustrates our workflow. Our simulation to reality approach involves a process to ensure the reliability of MOSEAC when applied to a real-world Limo. The workflow includes the following steps:

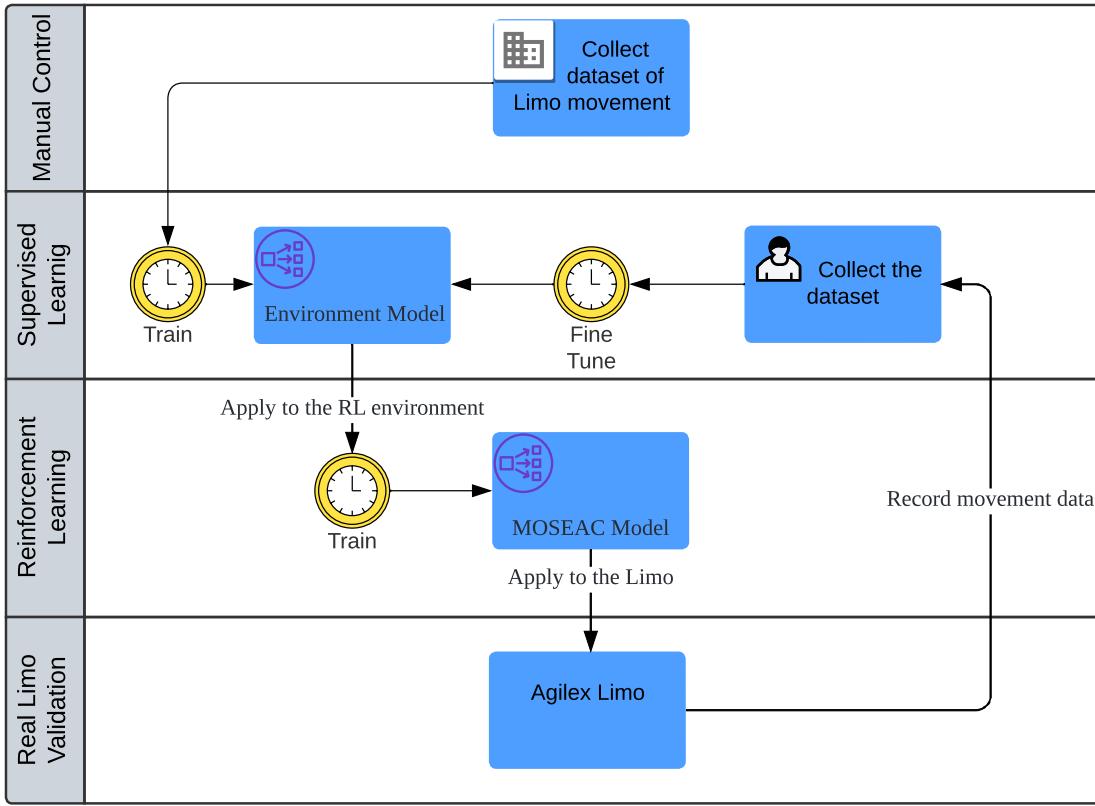


Fig. 1. The workflow for our MOSEAC implementaion to the Agilex Limo, we use a joystick to control the Limo movement for the initial environmental data collection (top).

- 1) **Manual Control and Data Collection:** Initially, we manually control the Limo to collect a dataset of its movements.
- 2) **Supervised Learning:** This dataset is then used to train an environment model in a supervised learning manner.
- 3) **Reinforcement Learning:** The trained environment model is applied in the reinforcement learning environment to train the MOSEAC model.
- 4) **Validation and Fine-Tuning:** The MOSEAC model is applied to the real Limo. Fine-tuning is performed based on the recorded movement data to ensure accurate translation from simulation to real-world application.
- 5) **Application:** Finally, the refined MOSEAC model is deployed for practical use and validated through real-world tasks.

A. Environment Setup

Figure 2 shows Limo's real-world validation environment. We used an OptiTrack [32] system for real-time positioning. We use a 3x3 meter global coordinate frame with its center aligned with the center of the 2D map, recording the positions of yellow lines within the map. The Limo's navigable area is confined within the map boundaries, excluding four enclosed



Fig. 2. This photo depicts the real-world environment used to validate the performance of MOSEAC on the Agilex Limo. The cameras on the left, right, and middle stands are three of the four cameras comprising the OptiTrack positioning system.

regions. Specific values of these positions, the specifications of the Agilex Limo, and other metrics can be found in Appendix B.

B. Simulation Environment

For our kinematic modeling, we use an Ackerman model:

Definition 6: The Ackerman kinematic model.

Table II gives the symbols and descriptions.

TABLE II
LIST OF SYMBOLS AND DESCRIPTIONS

Symbol	Description
X and Y	Current positions of Limo
V	Current velocity of Limo
θ	Current angular velocity of Limo
Δt	Control duration
V_{target}	Control linear velocity
δ	Control angular velocity
μ_k	Coefficient of kinetic friction
P	Power factor (which can be negative) of Limo
$g = 9.81 \text{ m/s}^2$	Gravitational acceleration
$M = 4.2 \text{ kg}$	Mass of the Limo (measured)
$L = 0.204 \text{ m}$	Distance between the centers of the front and rear wheels (measured)

The forces and accelerations are calculated as:

$$\begin{aligned} F_{friction} &= -\mu_k Mg \\ a_{friction} &= \frac{F_{friction}}{M} = -\mu_k g \\ a_{power} &= \frac{P}{M} \\ a_{net} &= a_{power} + \frac{V_{target} - V}{\Delta t} + a_{friction} \end{aligned} \quad (24)$$

The updates for velocity and position are:

$$\begin{aligned} V_{new} &= V + a_{net} \Delta t \\ X_{new} &= X + V_{new} \cos(\theta_{new}) \Delta t \\ Y_{new} &= Y + V_{new} \sin(\theta_{new}) \Delta t \end{aligned} \quad (25)$$

This formulation accounts for the dynamics of the Limo vehicle, considering friction, power, and vehicle mass, ultimately predicting the new position. This kinematic model is referred to as $M_{Ackerman}$.

We employ supervised learning [33], [34] to model the motion dynamics from the real-world environment to the simulation environment. This model predicts kinematic and physical information, μ_k and P , across different regions within the environment. We select a Transformer Model ¹ [35] to predict these data. The input, output, and its loss function are defined in Definition 7.

Definition 7:

The input \mathbf{X} is:

$$\mathbf{X} = (X, Y, V, \theta, \Delta t, V_{target}, \delta)$$

The output \mathbf{O} is:

$$\mathbf{Y} = (\mu_k, P)$$

Using $M_{Ackerman}$ from Definition 6, the loss function of the Transformer model T is computed as:

¹Our code is publicly available on GitHub with the shape and related hyperparameters of this Transformer Model inside.

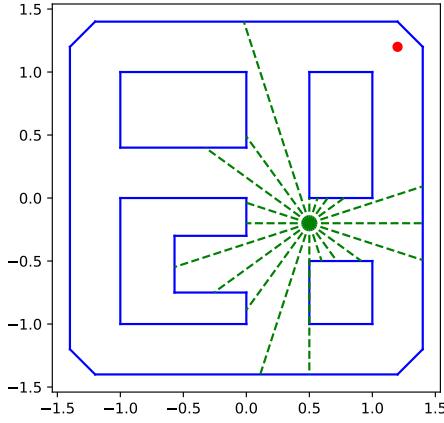


Fig. 3. The simulated lidar system generates 20 rays from Limo's position, calculates their intersections with environment enclosed regions, and returns the nearest intersection points for each ray.

$$\begin{aligned} \text{predict} &= M_{Ackerman}(\mathbf{X}, \mathbf{Y}, g) \\ \text{loss} &= \frac{1}{N} \sum_{i=1}^N (\text{predict}_i - \text{target}_i)^2 \end{aligned} \quad (26)$$

The loss function is defined as the Mean Squared Error (MSE) between the predicted positions and the target positions recorded in the dataset:

where:

- predict_i are the predicted positions for the i -th data point
- target_i are the actual positions for the i -th data point

After learning the kinematic model, we need to address the state definition of MOSEAC. It should include the necessary environmental information to enable the model to understand the positions of obstacles in the environment. Although we have recorded all the positions of these yellow lines, it is not reasonable to input them directly as state values. This would compromise the generality of our approach, and a large amount of invariant fixed position information might cause the neural network processing units to struggle with capturing the critical information steps or lead to overfitting [36], [37].

We developed a simulated lidar system centered on Limo's current position. This system generates 20 rays, similar to lidar operation, to detect intersections with enclosed regions in the environment. These intersections provide information about restricted areas, as shown in Figure 3.

Since the enclosed regions data in our simulation are based on real-world measurements, the gap between the virtual and real world is negligible. We designated eight turning points on this map as navigation endpoints, with a specific starting point for Limo.

The state dimensions for our MOSEAC model include:

- robot current position, • goal position (chosen among 8 random locations), • linear velocity, • steering angle, • 20 lidar points, and • previous control duration, linear and angular velocity. The action dimensions include control duration, linear velocity, and angular velocity.

Let R_t represent the task reward and T denote the termination flag (where 1 indicates termination and 0 indicates continuation). The function $d(\mathbf{p}_1, \mathbf{p}_2)$ measures the distance between positions \mathbf{p}_1 and \mathbf{p}_2 . The variables \mathbf{a}_{new} and \mathbf{t} denote the new position of the agent and the target position, respectively. The initial distance from the starting position to the target position is d_0 , and δ specifies the threshold distance determining whether Limo is sufficiently close to a point. Additionally, C_{inner} indicates a collision with enclosed regions, while C_{outer} indicates a collision with the map boundary.

Implementing a penalty mechanism instead of a termination mechanism in our reward definition offers significant advantages: the penalty mechanism provides incremental feedback, allowing the model to continue learning even after errors, thereby avoiding frequent resets that could disrupt the training process. This approach reduces overall training time and encourages the agent to explore a broader range of strategies, leading to a more comprehensive understanding of the environment's dynamics. However, we still terminate an experiment if the robot wonders outside the map boundary to stop meaningless exploration. The reward R_t is:

$$R_t = \begin{cases} R_{\text{boundary_penalty}}, & \text{if } C_{\text{inner}}(\mathbf{a}_{\text{new}}) \\ R_{\text{success_reward}}, & \text{if } d(\mathbf{a}_{\text{new}}, \mathbf{t}) \leq \delta \\ R_{\text{failure_penalty}}, & \text{if } C_{\text{outer}}(\mathbf{a}_{\text{new}}) \\ R_{\text{distance_reward}} = d_0 - d(\mathbf{a}_{\text{new}}, \mathbf{t}), & \text{otherwise} \end{cases} \quad (27)$$

The termination flag T is determined as:

$$T = \begin{cases} 0, & \text{if } C_{\text{inner}}(\mathbf{a}_{\text{new}}) \\ 1, & \text{if } d(\mathbf{a}_{\text{new}}, \mathbf{t}) \leq \delta \\ 1, & \text{if } C_{\text{outer}}(\mathbf{a}_{\text{new}}) \\ 0, & \text{otherwise} \end{cases} \quad (28)$$

The initial data collected using remote control was skewed for short action durations, causing some discrepancies between the physical robot behaviour and the behaviour in simulation. To address these discrepancies we fine-tuned a Transformer model that approximates Limo's kinematic model by collecting new real-world movement data.

We begin by freezing all the layers of the Transformer model except the final fully connected layer. This ensures that the initial generalized features are preserved while adapting the model's outputs to the specific characteristics of the Limo dataset. The training process is:

- 1) **Freeze Initial Layers:** Initially, all layers except the final fully connected layer are frozen. θ_i represents the parameters of the i -th layer:

$$\text{if } i < N_{\text{unfrozen}}, \quad \nabla_{\theta_i} L = 0 \quad (29)$$

where L is the loss function and N_{unfrozen} is the number of layers that are not frozen.

- 2) **Gradual Unfreezing:** After training the final layer, we incrementally unfreeze the preceding layers and fine-tune the model further. This is done iteratively, where at each stage, one additional layer is unfrozen and the model is re-trained. The number of unfrozen layers increases until

all layers are eventually fine-tuned:

$$\text{if } i \geq N_{\text{unfrozen}}, \quad \nabla_{\theta_i} L \neq 0 \quad (30)$$

- 3) **Early Stopping:** To prevent overfitting, we employ early stopping. If the validation loss does not improve for a specified number of epochs, training is halted:

$$\text{if } \Delta L_{\text{valid}} > 0 \text{ for } n \text{ epochs, stop training} \quad (31)$$

where ΔL_{valid} is the change in validation loss and n is the patience parameter.

- 4) **Evaluation:** The model's performance is evaluated on a test dataset after each training phase. The final model is selected based on the lowest validation loss.

The total training process can be described by the following optimization problem:

$$\min_{\theta} \sum_{(x,y) \in \mathcal{D}_{\text{train}}} \mathcal{L}(f_{\theta}(x), y) \quad (32)$$

where $\mathcal{D}_{\text{train}}$ is the training dataset, f_{θ} is the model with parameters θ , and \mathcal{L} is the loss function 7. The fine-tuning process specifically involves updating the parameters layer by layer, ensuring that the generalized pre-trained knowledge is gradually adapted to the specific nuances of the new Limo dataset.

This fine-tuning strategy allows the use of pre-trained models, adapting them to new tasks with potentially smaller datasets while maintaining robust performance and preventing overfitting.

For more detailed information please refer to Appendix C.

VI. RESULTS

We conducted six experiments involving MOSEAC, MOSEAC without the α_{\max} limitation, SEAC, CTCO, and SAC (at 20 Hz and 60 Hz, SAC20 and SAC60, respectively) within the simulation environment and applied these trained models to the real Limo. These training tests were performed on a computer equipped with an Intel Core i5-13600K CPU and an Nvidia RTX 4070 GPU running Ubuntu 22.04 LTS. The deployment tests were conducted on an AgileX Limo equipped with a Jetson Nano [38] (with a Cortex-A57 CPU and Maxwell 128 cores GPU) running Ubuntu 18.04 LTS.

We trained our RL model using PyTorch [39] within a Gymnasium environment [40]. Subsequently, we converted the model parameters to ONNX format [39]. Finally, we utilized TensorRT [41] to build an inference engine from the ONNX model in the AgileX Limo local environment and deployed it for RL navigation tasks². More system information details can be found in Appendix D.

Figure 4 and Figure 5 illustrate the results of the training process. Several key insights can be drawn from these figures:

Figure 4 indicates that MOSEAC consistently improves over time compared to the other algorithms, suggesting that MOSEAC adapts well to the environment and maintains a

²Our code is publicly available on GitHub with the hyperparameters of the MOSEAC model inside.

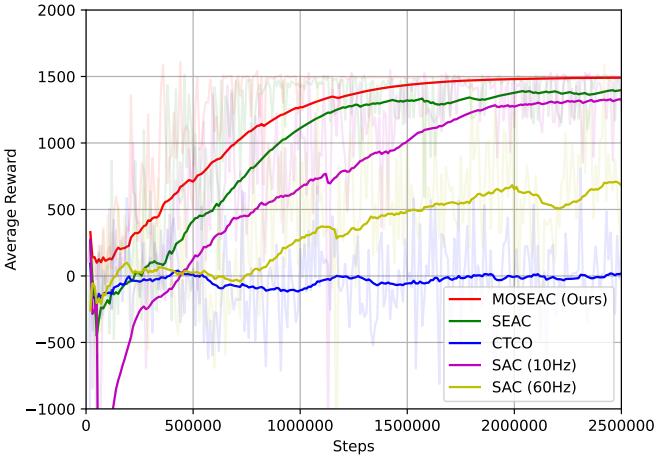


Fig. 4. Average returns of 5 reinforcement learning algorithms over 2.5M steps during training.

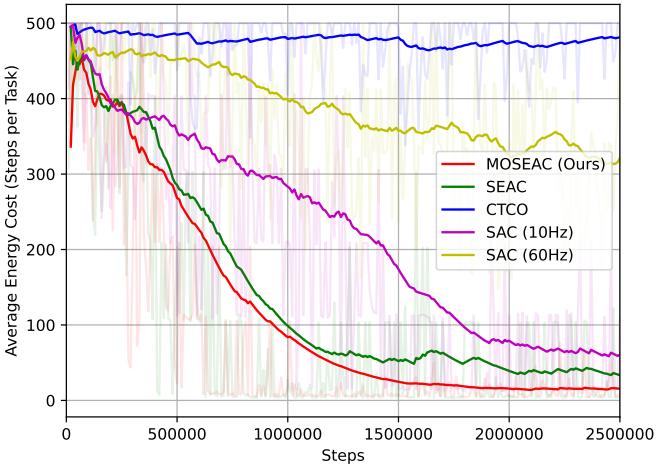


Fig. 5. Average energy costs of 5 reinforcement learning algorithms over 2.5M steps during training.

stable learning curve. While SEAC also shows stable performance, it converges slightly slower than MOSEAC. Additionally, Figure 4 illustrates that MOSEAC exhibits higher action duration robustness than CTCO. MOSEAC utilizes a fixed discount factor (γ), ensuring stable long-term planning capabilities without the negative impact of varying γ . In contrast, CTCO's performance is susceptible to the choice of action duration range, significantly affecting its γ . As training progresses, CTCO tends to favor smaller γ values, placing greater demands on its τ parameter that controls the range of γ . This sensitivity makes CTCO less adaptable to diverse environments, whereas MOSEAC maintains consistent performance. Furthermore, Figure 5 provides a comparative analysis of energy costs among the different algorithms. MOSEAC demonstrates lower energy costs per task, indicating higher energy efficiency during training. This efficiency is critical for practical applications where compute resource consumption is a concern.

We compared the effects of imposing an upper limit on the parameter α_m versus allowing it to increase without restriction

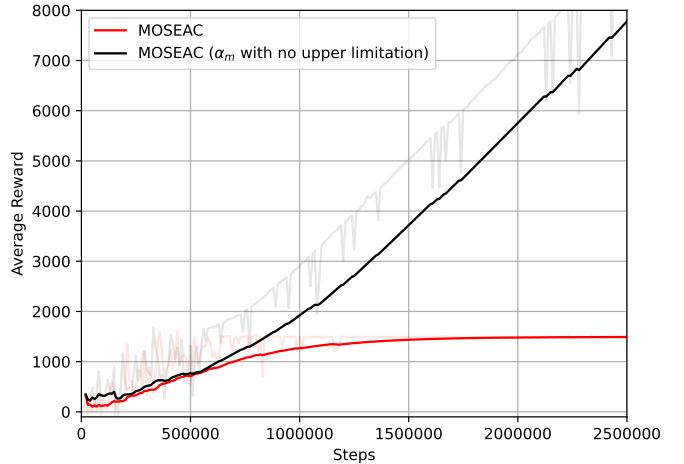


Fig. 6. Average returns of MOSEAC and MOSEAC (without α_m limitation) in 2.5M steps during the training.

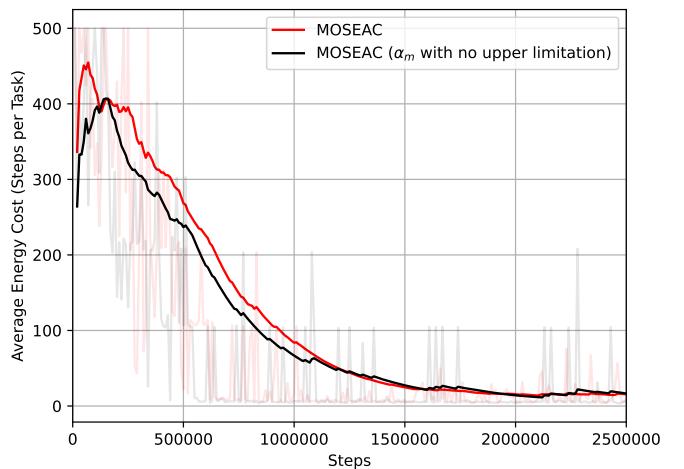


Fig. 7. Average Energy cost of MOSEAC and MOSEAC (without α_m limitation) in 2.5M steps during the training.

in the context of our MOSEAC algorithm. Our findings indicate significant differences in performance stability and energy efficiency between the two approaches.

Introducing an upper limit on α_m (denoted as α_{max}) is required to prevent reward explosion. Figure 6 shows that MOSEAC has consistent and stable improvement in average reward. In contrast, MOSEAC without the upper limit initially followed a similar trend but eventually diverged, leading to instability and potential reward explosion. This divergence suggests that without the upper limit, α_m may increase uncontrollably, destabilizing the reward structure.

However, as shown in Figure 7, MOSEAC without an upper limit on α_m generally exhibited lower average energy costs compared to MOSEAC with the upper limit. This suggests that while the absence of an upper limit on α_m may lead to reward explosion, it does not significantly impair task performance in practice.

We update α_m based on the declining reward trend. However, in random multi-objective tasks with varying target locations, the average reward can fluctuate significantly, causing

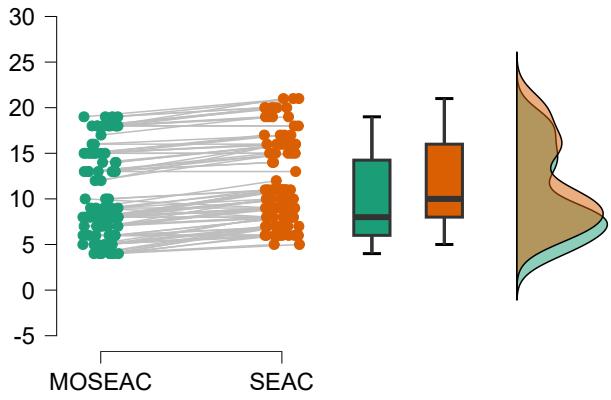


Fig. 8. Energy cost (as number of time steps) for 100 random tasks. We use the same seed for MOSEAC and SEAC to ensure that the tasks are the same for the two algorithms.

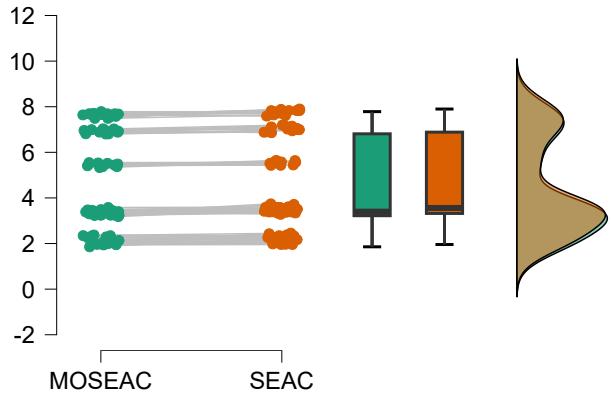


Fig. 9. Time cost for 100 random tasks. We use the same seed for MOSEAC and SEAC to ensure that the tasks are the same for the two algorithms.

α_m to be updated continuously. This can lead to an unbounded increase in reward during training when α_m is unrestricted, amplifying the reward signal. The increased gradient variability results in rapid strategy adjustments, enhancing short-term energy efficiency. Despite the potential for reward explosion, the impact on task performance is minimal, with notable improvements in energy efficiency.

We compared MOSEAC's task performance with the best-performing SEAC after training. Due to the poor performance of CTCO and SAC, we excluded them from the analysis.

Figure 8 and Figure 9 illustrate the energy and time distributions for MOSEAC and SEAC methods. Given the lack of normality in the data distribution, we use the Wilcoxon signed-rank test to compare the paired samples. For energy consumption (Figure 8), MOSEAC demonstrates significantly lower median and overall energy usage than SEAC ($W = 35.0, z = -7.904, p < .001$). For time efficiency (Figure 9), MOSEAC also shows a significantly lower median time, indicating quicker task completion than SEAC ($W = 502.0, z = -6.956, p < .001$). The descriptive statistics and test results are summarized in Appendix E.

The improved performance of MOSEAC over SEAC can be attributed to its reward function. While SEAC's reward function is linear, combining task reward, energy penalty, and time penalty independently, MOSEAC introduces a multiplicative relationship between task reward and time-related reward. This non-linear interaction enhances the reward signal, particularly when both task performance and time efficiency are high, and naturally balances these factors. By keeping the energy penalty separate, MOSEAC maintains flexibility in tuning without complicating the relationship between time and task rewards. This design allows MOSEAC to more effectively guide the agent's decisions, resulting in better energy efficiency and task completion speed in practical applications.

We conducted real-world tests on the Agilex Limo using ROS [42]³. The Limo robot navigated to random and distinct endpoints using the MOSEAC model as its control policy for Ackerman steering ⁴. We collected these data and calculated trajectory and control output similarities with respect to simulation.

For trajectory similarity, we combine all trajectory data into one trajectory, the ATE (Average Trajectory Error) [45] is 0.036 meters, indicating minimal deviations between the actual and simulated paths. Additionally, the Dynamic Time Warping (DTW) [46] value of 0.531 supports the high degree of similarity between the temporal sequences of the trajectories. These metrics suggest that our method enables the Limo to follow the planned paths with high precision, closely mirroring the simulation.

Regarding control output similarity, the Mean Absolute Error (MAE) is 0.002, and the Mean Squared Error (MSE) is $4.49E - 05$. These low error values indicate that the control outputs in the real world closely match those in the simulation. Our method effectively minimizes the discrepancies in control signals, ensuring that the robot's actions are consistent across different environments.

Overall, the empirical data supports the theoretical claims regarding MOSEAC's performance in trajectory fidelity and control output consistency.

We also recorded the computing resource usage, highlighting the efficiency of the MOSEAC algorithm in terms of energy consumption, particularly computational energy. Figure 10 provides a comparison of the average usage per second of CPU and GPU resources between MOSEAC and SAC (Soft Actor-Critic) algorithms running at different frequencies (10 Hz and 60 Hz).

The CPU usage data reveals that MOSEAC significantly reduces computational load compared to SAC at both 10 Hz and 60 Hz. Specifically, MOSEAC utilizes only $11.40\% \pm 0.12$ of the CPU resources, whereas SAC requires $16.80\% \pm 0.14$ at 10 Hz and $31.41\% \pm 1.47$ at 60 Hz. Similarly, the GPU usage data indicates that MOSEAC is more efficient, using only $2.80\% \pm 0.07$ compared to SAC's $13.79\% \pm 0.05$ at 10 Hz and $27.86\% \pm 0.19$ at 60 Hz. This reduction conserves energy (increasing battery life) and frees processing power for tasks

³Our code for ROS workspace is publicly available on GitHub, which also provides support for ROS 2 [43] and Docker [44]

⁴Video available here: <https://youtu.be/VhTa66WqxoU>

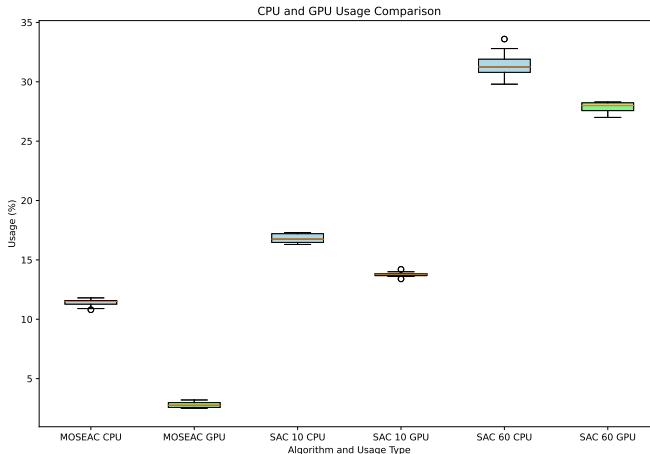


Fig. 10. Comparison of Average Compute Resource Usage Across Different Methods

like perception and communication. The descriptive statistics and test results are summarized in Appendix F.

VII. CONCLUSIONS

In this paper, we presented the Multi-Objective Soft Elastic Actor-Critic (MOSEAC) algorithm, a Variable Time Step Reinforcement Learning (VTS-RL) method designed with adaptive hyperparameters that respond to observed reward trends during training. Our analysis included theoretical performance guarantees, convergence analysis, and practical validation through simulations and real-world navigation tasks using a small rover.

We compared MOSEAC with other VTS-RL algorithms, such as SEAC [8] and CTCO [7]. While SEAC and CTCO improved over traditional fixed-time step methods, they still required extensive hyperparameter tuning and did not achieve the same level of efficiency and robustness as MOSEAC. SEAC's reward structure, though effective, was less adaptable, and CTCO's sensitivity to action duration further limited its practical application.

Additionally, the empirical data demonstrated that MOSEAC significantly outperforms traditional SAC algorithms, particularly in terms of computational resource efficiency. MOSEAC's reduced CPU and GPU usage frees up resources for other critical tasks, such as environment perception and map reconstruction, enhancing the robot's operational efficiency and extending battery life. This makes MOSEAC highly suitable for long-term and complex missions.

Our findings validate the robustness and applicability of MOSEAC in real-world scenarios, providing strong evidence of its potential to lower hardware requirements and improve data efficiency in reinforcement learning deployments.

Our future work will further refine the algorithm, particularly in the adaptive tuning of hyperparameters, to enhance its performance and applicability. We aim to apply MOSEAC to a broader range of robotic projects, including smart cars and robotic arms, to fully leverage its benefits in diverse practical settings.

VIII. ACKNOWLEDGEMENT

The authors sincerely thank Yann Bouteiller, Guillaume Ricard, and Wenqiang Du for their invaluable assistance with the sim-to-real method discussions, AgileX Limo usage, and ROS support. Their expertise and dedication were crucial in achieving high-quality sim-to-real deployment, which was essential for the success of this research. We greatly appreciate their significant time and effort invested in this project.

APPENDIX A ANALYSIS OF THE ADAPTIVE α_m ADJUSTMENT IN MOSEAC

In MOSEAC we use an adaptive adjustment scheme for α_m , specifically a simple linear increment with an upper limit α_{\max} . The inclusion of ψ and the reward adjustment strategy inherently involves Pareto optimization. Below, we provide a mathematical analysis of the stability of our scheme and its relation to the Pareto front.

A. Adaptive Adjustment Scheme for α_m

Let α_m be adjusted linearly over time with an upper limit α_{\max} :

$$\alpha_m(t) = \min(\alpha_{m,0} + kt, \alpha_{\max}) \quad (33)$$

where $\alpha_{m,0}$ is the initial value, k is the increment rate, and t represents the time or iteration index.

The stability of this linear adjustment can be analyzed by examining the impact on the reward function R and the policy updates.

B. Reward Function and ψ

The reward function R in the MOSEAC algorithm can be expressed as:

$$R = r + \psi \quad (34)$$

where r is the immediate reward and ψ is an adjustment term that influences the reward based on the adaptive α_m .

Given the linear increment of α_m , the adjustment term ψ can be represented as:

$$\psi(t) = f(\alpha_m(t)) = f(\alpha_{m,0} + kt) \text{ for } \alpha_m(t) < \alpha_{\max} \quad (35)$$

$$\psi(t) = f(\alpha_{\max}) \text{ for } \alpha_m(t) \geq \alpha_{\max} \quad (36)$$

where f is a function that defines how ψ depends on α_m .

C. Stability Analysis

To analyze the stability of the reward function under this adaptive scheme, we examine the boundedness and convergence of R . The stability is ensured if the cumulative reward remains bounded and the policy converges to an optimal policy over time.

- 1) **Boundedness:** The linear increment of α_m with an upper limit should ensure that R does not grow unbounded. This requires that $f(\alpha_m)$ grows at a controlled rate.

$$|R| = |r + f(\alpha_{m,0} + kt)| \leq M \text{ for } \alpha_m(t) < \alpha_{\max} \quad (37)$$

$$|R| = |r + f(\alpha_{\max})| \leq M \text{ for } \alpha_m(t) \geq \alpha_{\max} \quad (38)$$

where M is a constant, indicating that R remains bounded.

- 2) **Convergence:** The policy π should converge to an optimal policy π^* . The convergence is influenced by the adjustment term ψ and the learning rate α_m .

$$\lim_{t \rightarrow \infty} \pi(t) = \pi^* \quad (39)$$

If α_m is adjusted linearly with an upper limit, the learning rate should decrease over time to ensure convergence.

D. Multi-Objective Optimization

The multi-objective optimization aspect of the MOSEAC algorithm arises from the trade-off between multiple objectives in the reward function. The inclusion of ψ introduces a multi-objective optimization problem, where the algorithm aims to optimize the cumulative reward while balancing different aspects influenced by ψ .

The Pareto front represents the set of optimal policies where no single objective can be improved without degrading another [47]. The linear increment of α_m with an upper limit and the presence of ψ ensure that the algorithm explores the policy space effectively, converging to solutions on the Pareto front [48].

APPENDIX B REAL ENVIRONMENT

Table III displays the coordinate information of the enclosed regions in the real environment; Table IV shows the specifications of the Agilex Limo; Table V shows the key performance metrics of the OptiTrack system.

TABLE III

ENCLOSED REGIONS INFORMATION (YELLOW LINES ARRAYS) IN THE REAL ENVIRONMENT (IN METERS)

Zone	Coordinates
zone_left_down	$[[0.0, 0.0], [-1.0, 0.0]], [[-1.0, 0.0], [-1.0, -1.0]], [[-1.0, -1.0], [0.0, -1.0]], [[0.0, -1.0], [0.0, -0.75]], [[0.0, -0.75], [-0.57, -0.75]], [[-0.57, -0.75], [-0.57, -0.33]], [[-0.57, -0.33], [0.0, -0.33]], [[0.0, -0.33], [0.0, 0.0]]$
zone_left_up	$[[0.0, 0.4], [0.0, 1.0]], [[0.0, 1.0], [-1.0, 1.0]], [[-1.0, 1.0], [-1.0, 0.4]], [[-1.0, 0.4], [0.0, 0.4]]$
zone_right_up	$[[0.5, 0.0], [1.0, 0.0]], [[1.0, 0.0], [1.0, 1.0]], [[1.0, 1.0], [0.5, 1.0]], [[0.5, 1.0], [0.5, 0.0]]$
zone_right_down	$[[0.5, -1.0], [0.5, -0.5]], [[0.5, -0.5], [1.0, -0.5]], [[1.0, -0.5], [1.0, -1.0]], [[1.0, -1.0], [0.5, -1.0]]$

APPENDIX C SIMULATION ENVIRONMENT

The position of the goal is randomized from these eight points in each episode. Setting multiple endpoints offers several advantages. It enhances the navigation policy's generalization capability by training the agent to adapt to various goals rather than a single target. This promotes extensive exploration, prevents the agent from getting stuck in local optima, and increases robustness by preparing the agent to

TABLE IV
AGILEX LIMO SPECIFICATIONS [11]

Category	Specifications
Dimensions	322mm x 220mm x 251mm
Weight	4.2 kg
Maximum Speed	1 m/s
Maximum Climbing Capacity	25° (omni-wheel, differential, Ackermann steering), 40° (tracked mode)
Ground Clearance	24 mm
Battery	12V Li-ion 5600mAh
Run Time	40 minutes of continuous operation
Standby Time	2 hours
Charging Time	2 hours
Operating Temperature	-10°C to 40°C
IP Rating	IP22 (splash-proof, dust-proof)
Sensors	IMU: MPU6050 LiDAR: EAI X2L Depth Camera: ORBBEC DaBai
CPU	ARM64 Quad-Core 1.43GHz (Cortex-A57)
GPU	128-core NVIDIA Maxwell™ @ 921MHz
Communication	Wi-Fi, Bluetooth 5.0
Operating System	Ubuntu 18.04, ROS1 Melodic

TABLE V
KEY PERFORMANCE METRICS OF THE OPTITRACK SYSTEM [32]

Performance Metric	Value	Description
Camera Resolution	Up to 4096x2160	High resolution for detailed tracking
Frame Rate	Up to 360 FPS	Ensures smooth motion capture
Latency	As low as 3ms	Provides real-time feedback
Field of View (FOV)	56° to 100°	Suitable for various capture needs
Synchronization Precision	<1μs	Multi-camera synchronized capture
Marker Tracking Accuracy	<0.5mm	Precise 3D spatial positioning
Operating Range	Up to 30m	Suitable for large-scale capture environments
Optical Resolution	12 MP	High-quality image data
Ambient Light Suppression	Strong	Reduces interference from ambient light
Data Interface	Gigabit Ethernet	Fast data transmission

handle dynamic or uncertain target positions in real-world deployments [49].

Additionally, it improves data efficiency by allowing the agent to learn from diverse experiences in a single training session. Multiple endpoints provide more prosperous reward signals, accelerating learning through varied success and failure contexts. Moreover, it simulates realistic scenarios where multiple destinations are shared, thus increasing the practical value of the trained model. Lastly, this approach raises task complexity, challenging the agent to develop more sophisticated strategies and ultimately enhancing overall performance [50].

The start point is a fixed point as: $[-0.2, -0.5]$. To enhance the stability of the agent's initial position and reduce the risk of misalignment in real-world deployment, we introduce uniform noise to the starting coordinates. This approach aims to minimize data uncertainty caused by position discrepancies. By adding noise uniformly in the range $[-0.05, 0.05]$, we can achieve the desired effect. Let:

- $\mathbf{a}_{\text{location}}$ be the agent's position
- $U(a, b)$ denote a uniform distribution in the range $[a, b]$

The updated position formula is:

$$\mathbf{a}_{\text{location}} = \begin{bmatrix} -0.2 + U(-0.05, 0.05) \\ -0.5 + U(-0.05, 0.05) \end{bmatrix} \quad (40)$$

TABLE VI
LIST OF GOAL POSITIONS WITH THEIR COORDINATES

Goal Position	Coordinates
Goal Position 1	[1.2, 1.2]
Goal Position 2	[1.2, -1.2]
Goal Position 3	[-1.2, 1.2]
Goal Position 4	[-1.2, -1.2]
Goal Position 5	[1.2, 0]
Goal Position 6	[0, 1.2]
Goal Position 7	[-1.2, 0]
Goal Position 8	[0, -1.2]

This method ensures that the uniform noise maintains the desired variability without introducing bias [51].

All in all, the state dimension is 49, and their shape and space are shown in Table VII.

TABLE VII

NOTEABLE, LIMO CANNOT MAKE THE CONTROL DURATION CORRECTLY IF THE DURATION IS NOT IN THIS TIME RANGE.

State details			
Name	Shape	Space	Annotation
Limo Position	[2,]	[-1.5, 1.5]	in (X, Y), (meters)
Goal Position	[2,]	[-1.5, 1.5]	in (X, Y), (meters)
Linear Velocity	[1,]	[-1.0, 1.0]	
Steering Angle	[1,]	[-1.0, 1.0]	
Control duration	[1,]	[0.02, 0.5]	in Seconds
Previous linear velocity	[1,]	[-1.0, 1.0]	
Angular velocity	[1,]	[-1.0, 1.0]	
20 radar point positions	[40,]	[-1.5, 1.5]	reshape from [20, 2], in (X, Y)

TABLE VIII

NOTEABLE, LIMO CANNOT MAKE THE CONTROL DURATION CORRECTLY IF THE DURATION IS NOT IN THIS TIME RANGE.

Action details			
Name	Shape	Space	Annotation
Control duration	[1,]	[0.02, 0.5]	in Seconds
Linear velocity	[1,]	[-1.0, 1.0]	
Angular velocity	[1,]	[-1.0, 1.0]	

TABLE IX
REWARD VALUE SETTINGS FOR LIMO ENVIRONMENT

Reward Settings		
Name	Value	Annotation
cross_punish	-30.0	cross with the enclosed regions
success_reward	500.0	success to reach the goal
dead_punish	-100.0	go out of the map
δ	0.2	if limo close enough to a point, in meters

APPENDIX D SYSTEM DETAILS

Our experiments used the Agilex Limo equipped with a Jetson Nano running JetPack version 4.6.4. While newer

TABLE X
TRAINING PC DETAILS

PC Key Softwares' Ecosystem	
Name	Value
Nvidia Driver Version	450.67
Cuda Version	11.8
cuDNN Version	8.9.7
Python Version	3.8
Torch Version	2.1.0
ONNX Version	1.13.1
Gymnasium Version	0.29.1

TABLE XI
AGILEX LIMO DETAILS

Agilex Limo Key Softwares' Ecosystem	
Name	Value
Nvidia JetPack Version	4.6.4
Cuda Version	10.2_r440
cuDNN Version	8.2.1
TensorRT Version	8.2.1.3
Python Version	3.6
Pycuda Version	2020.1
ONNX Version	1.13.1
Limo Controller Version	2.4

versions such as 6.0+ are available, compatibility and support limitations necessitated our use of JetPack 4.6.4. Specifically, the older versions of cuDNN and TensorRT required for our setup are no longer available for direct download, although this may affect reproducibility. To assist others in replicating our results, we have provided custom-built support packages on our GitHub.

APPENDIX E

STATISTICS FOR ENERGY AND TIME CONSUMPTIONS

Statistic results for the MOSEAC and SEAC on energy and time consumptions are shown in Table XII.

TABLE XII
DESCRIPTIVES

	N	Mean	SD	SE	COV
MOSEAC_Energy	100	10.130	4.733	0.473	0.467
SEAC_Energy	100	11.660	4.740	0.474	0.407
MOSEAC_Time	100	4.341	2.038	0.204	0.469
SEAC_Time	100	4.456	2.044	0.204	0.459

APPENDIX F STATISTICS FOR COMPUTE RESOURCES REFERENCES

- [1] R. Liu, F. Nageotte, P. Zanne, M. de Mathelin, and B. Dresp-Langley, “Deep reinforcement learning for the control of robotic manipulation: a focussed mini-review,” *Robotics*, vol. 10, no. 1, p. 22, 2021.

TABLE XIII
TEST OF NORMALITY (SHAPIRO-WILK)

		W	p
MOSEAC_energy	- SEAC_energy	0.916	< 0.001
MOSEAC_time	- SEAC_time	0.993	0.894

TABLE XIV
PAIRED SAMPLES T-TEST

Measure 1	Measure 2	W	z	p
MOSEAC_Energy	- SEAC_Energy	35.000	-7.904	<0.001
MOSEAC_Time	- SEAC_Time	502.000	-6.956	<0.001

TABLE XV
DESCRIPTIVES

	N	Mean	SD	SE	COV
MOSEAC CPU usage (%)	88	11.400	0.370	0.131	0.032
SAC 10 Hz CPU usage (%)	88	16.800	0.400	0.141	0.024
SAC 60 Hz CPU usage (%)	88	31.413	1.298	0.459	0.041
MOSEAC GPU usage (%)	88	2.800	0.283	0.100	0.101
SAC 10 Hz GPU usage (%)	88	13.787	0.242	0.085	0.018
SAC 60 Hz GPU usage (%)	88	27.863	0.463	0.164	0.017

TABLE XVI
TEST OF NORMALITY (SHAPIRO-WILK)

Measure 1	Measure 2	W	p
MOSEAC CPU usage (%)	SAC 10 Hz CPU usage (%)	0.947	0.685
MOSEAC GPU usage (%)	SAC 60 Hz CPU usage (%)	0.960	0.806
	SAC 10 Hz GPU usage (%)	0.835	0.067
	SAC 60 Hz GPU usage (%)	0.901	0.296

- [2] J. Ibarz, J. Tan, C. Finn, M. Kalakrishnan, P. Pastor, and S. Levine, “How to train your robot with deep reinforcement learning: lessons we have learned,” *The International Journal of Robotics Research*, vol. 40, no. 4-5, pp. 698–721, 2021.
- [3] N. Akalin and A. Loutfi, “Reinforcement learning approaches in social robotics,” *Sensors*, vol. 21, no. 4, p. 1292, 2021.
- [4] B. Singh, R. Kumar, and V. P. Singh, “Reinforcement learning in robotic applications: a comprehensive survey,” *Artificial Intelligence Review*, vol. 55, no. 2, pp. 945–990, 2022.
- [5] R. Majumdar, A. Mathur, M. Pirron, L. Stegner, and D. Zufferey, “Paracosm: A test framework for autonomous driving simulations,” in *International Conference on Fundamental Approaches to Software Engineering*. Springer International Publishing Cham, 2021, pp. 172–195.
- [6] E. Bregu, N. Casamassima, D. Cantoni, L. Mottola, and K. Whitehouse, “Reactive control of autonomous drones,” in *Proceedings of the 14th Annual International Conference on Mobile Systems, Applications, and Services*, 2016, pp. 207–219.
- [7] A. Karimi, J. Jin, J. Luo, A. R. Mahmood, M. Jagersand, and S. Tosatto, “Dynamic decision frequency with continuous options,” in *2023 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2023, pp. 7545–7552.
- [8] D. Wang and G. Beltrame, “Deployable reinforcement learning with variable control rate,” *arXiv preprint arXiv:2401.09286*, 2024.
- [9] ——, “Reinforcement learning with elastic time steps,” *arXiv preprint arXiv:2402.14961*, 2024.
- [10] ——, “Moseac: Streamlined variable time step reinforcement learning,” *arXiv preprint arXiv:2406.01521*, 2024.
- [11] AgileX Robotics, “Agilex limo - multi-modal mobile robot with ai modules,” <https://www.globenewswire.com/>, accessed: [date].
- [12] P.-Y. Lajoie, B. Ramtoula, Y. Chang, L. Carbone, and G. Beltrame, “Door-slam: Distributed, online, and outlier resilient slam for robotic teams,” *IEEE Robotics and Automation Letters*, vol. 5, no. 2, pp. 1656–1663, 2020.
- [13] S. Wan, Z. Gu, and Q. Ni, “Cognitive computing and wireless communications on the edge for healthcare service robots,” *Computer Communications*, vol. 149, pp. 99–106, 2020.
- [14] Y. Bouteiller, S. Ramstedt, G. Beltrame, C. Pal, and J. Binas, “Reinforcement learning with random delays,” in *International conference on learning representations*, 2021.
- [15] S. Amin, M. Gomrokchi, H. Aboutalebi, H. Satija, and D. Precup, “Locally persistent exploration in continuous control tasks with sparse rewards,” *arXiv preprint arXiv:2012.13658*, 2020.
- [16] S. Park, J. Kim, and G. Kim, “Time discretization-invariant safe action repetition for policy gradient methods,” *Advances in Neural Information Processing Systems*, vol. 34, pp. 267–279, 2021.
- [17] S. Sharma, A. Srinivas, and B. Ravindran, “Learning to repeat: Fine grained action repetition for deep reinforcement learning,” *arXiv preprint arXiv:1702.06054*, 2017.
- [18] A. M. Metelli, F. Mazzolini, L. Bisi, L. Sabbioni, and M. Restelli, “Control frequency adaptation via action persistence in batch reinforcement learning,” in *International Conference on Machine Learning*. PMLR, 2020, pp. 6862–6873.
- [19] J. Lee, B.-J. Lee, and K.-E. Kim, “Reinforcement learning for control with multiple frequencies,” *Advances in Neural Information Processing Systems*, vol. 33, pp. 3254–3264, 2020.
- [20] Y. Chen, H. Wu, Y. Liang, and G. Lai, “Varlenmarl: A framework of variable-length time-step multi-agent reinforcement learning for cooperative charging in sensor networks,” in *2021 18th Annual IEEE International Conference on Sensing, Communication, and Networking (SECON)*. IEEE, 2021, pp. 1–9.
- [21] E. Even-Dar, S. Mannor, and Y. Mansour, “Action elimination and stopping conditions for reinforcement learning,” in *Proceedings of the 20th International Conference on Machine Learning (ICML-03)*, 2003, pp. 162–169.
- [22] S. Zhao, T. Zheng, D. Sui, J. Zhao, and Y. Zhu, “Reinforcement learning based variable damping control of wearable robotic limbs for maintaining astronaut pose during extravehicular activity,” *Frontiers in Neurorobotics*, vol. 17, p. 1093718, 2023.
- [23] N. Gottipati *et al.*, “To the max: Reinventing reward in reinforcement learning,” *arXiv preprint arXiv:2402.01361*, 2020. [Online]. Available: <https://arxiv.labs.arxiv.org/html/2402.01361>
- [24] R. S. Sutton and A. G. Barto, *Reinforcement learning: An introduction*. MIT press, 2018.
- [25] T. G. Dietterich, “Hierarchical reinforcement learning with the maxq value function decomposition,” *Journal of artificial intelligence research*, vol. 13, pp. 227–303, 2000.
- [26] S. Li, R. Wang, M. Tang, and C. Zhang, “Hierarchical reinforcement learning with advantage-based auxiliary rewards,” *Advances in Neural Information Processing Systems*, vol. 32, 2019.
- [27] I. Kacem, S. Hammadi, and P. Borne, “Pareto-optimality approach for flexible job-shop scheduling problems: hybridization of evolutionary algorithms and fuzzy logic,” *Mathematics and computers in simulation*, vol. 60, no. 3-5, pp. 245–276, 2002.
- [28] M. S. Monfared, S. E. Monabbati, and A. R. Kafshgar, “Pareto-optimal equilibrium points in non-cooperative multi-objective optimization problems,” *Expert Systems with Applications*, vol. 178, p. 114995, 2021.
- [29] T. Haarnoja, A. Zhou, K. Hartikainen, G. Tucker, S. Ha, J. Tan, V. Kumar, H. Zhu, A. Gupta, P. Abbeel *et al.*, “Soft actor-critic algorithms and applications,” *arXiv preprint arXiv:1812.05905*, 2018.
- [30] S. Banach, “Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales,” *Fundamenta mathematicae*, vol. 3, no. 1, pp. 133–181, 1922.
- [31] V. Konda and J. Tsitsiklis, “Actor-critic algorithms,” *Advances in neural information processing systems*, vol. 12, 1999.
- [32] OptiTrack, “Optitrack - motion capture systems,” <https://www.optitrack.com>, accessed: [date].
- [33] M. Rybczak, N. Popowniak, and A. Lazarowska, “A survey of machine learning approaches for mobile robot control,” *Robotics*, vol. 13, no. 1, p. 12, 2024.
- [34] Authors, “Supervised and unsupervised deep learning applications for visual slam in robotics,” in *IMECE*. ASME, 2022, p. V003T04A010.
- [35] A. Vaswani, N. Shazeer, N. Parmar, J. Uszkoreit, L. Jones, A. N. Gomez, L. Kaiser, and I. Polosukhin, “Attention is all you need,” 2023.

TABLE XVII
PAIRED SAMPLES T-TEST

Measure 1		Measure 2	W	z	p
MOSEAC CPU usage (%)	- SAC 10 Hz CPU usage (%)	-	-2.521	0.004	
	- SAC 60 Hz CPU usage (%)	-	-2.521	0.004	
MOSEAC GPU usage (%)	- SAC 10 Hz GPU usage (%)	-	-2.521	0.007	
	- SAC 60 Hz GPU usage (%)	-	-2.521	0.007	



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- [36] Authors, "Assessing the impact of distribution shift on reinforcement learning performance," *arXiv preprint arXiv:2402.03590*, 2024. [Online]. Available: <https://arxiv.org/abs/2402.03590>
- [37] M. Katakura *et al.*, "Reinforcement learning model with dynamic state space tested on target search tasks for monkeys: Extension to learning task events," *Frontiers in Robotics and AI*, 2023. [Online]. Available: <https://www.frontiersin.org/articles/10.3389/frobt.2023.00856/full>
- [38] "Jetson nano module," <https://developer.nvidia.com/embedded/jetson-nano>, 2019.
- [39] A. Paszke, S. Gross, F. Massa, A. Lerer, J. Bradbury, G. Chanan, T. Killeen, Z. Lin, N. Gimelshein, L. Antiga, A. Desmaison, A. Kopf, E. Yang, Z. DeVito, M. Raison, A. Tejani, S. Chilamkurthy, B. Steiner, L. Fang, J. Bai, and S. Chintala, "Pytorch: An imperative style, high-performance deep learning library," in *Advances in Neural Information Processing Systems*, vol. 32, 2019, pp. 8024–8035. [Online]. Available: <https://proceedings.neurips.cc/paper/2019/hash/bdbca288fee7f92f2bfa9f7012727740-Abstract.html>
- [40] M. Towers, J. K. Terry, A. Kwiatkowski, J. U. Balis, G. d. Cola, T. Deleu, M. Goulão, A. Kallinteris, A. KG, M. Krimmel, R. Perez-Vicente, A. Pierré, S. Schulhoff, J. J. Tai, A. T. J. Shen, and O. G. Younis, "Gymnasium," Mar. 2023. [Online]. Available: <https://zenodo.org/record/8127025>
- [41] "Tensorrt," <https://developer.nvidia.com/tensorrt>, 2021.
- [42] M. Quigley, K. Conley, B. Gerkey, J. Faust, T. Foote, J. Leibs, R. Wheeler, and A. Y. Ng, "Ros: an open-source robot operating system," in *ICRA workshop on open source software*, vol. 3, no. 3.2. Kobe, Japan, 2009, p. 5.
- [43] S. Macenski, T. Foote, B. Gerkey, C. Lalancette, and W. Woodall, "Ros 2: Towards a performance-centric and real-time robotics framework," in *Robotics: Science and Systems (RSS) Workshop on Real-time and Performance in Robotic Systems*, Online, 2020.
- [44] D. Inc., *Docker: Open Platform for Developing, Shipping, and Running Applications*, 2013, available at <https://docs.docker.com/>.
- [45] H. Ryan, M. Paglione, and S. Green, "Review of trajectory accuracy methodology and comparison of error measurement metrics," in *AIAA Guidance, Navigation, and Control Conference and Exhibit*, 2004, p. 4787.
- [46] M. Müller, "Dynamic time warping," *Information retrieval for music and motion*, pp. 69–84, 2007.
- [47] V. Pareto, *Manual of Political Economy*. New York: Augustus M. Kelley, 1971.
- [48] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: Nsga-ii," *IEEE transactions on evolutionary computation*, vol. 6, no. 2, pp. 182–197, 2002.
- [49] M. Riedmiller, R. Hafner, T. Lampe, M. Neunert, J. Degrave, T. van de Wiele, V. Mnih, and N. Heess, "Learning by playing - solving sparse reward tasks from scratch," in *Proceedings of the 35th International Conference on Machine Learning*, 2018. [Online]. Available: <https://arxiv.org/abs/1802.09464>
- [50] M. G. Bellemare, S. Srinivasan, G. Ostrovski, T. Schaul, D. Saxton, and R. Munos, "Unifying count-based exploration and intrinsic motivation," *arXiv preprint arXiv:1704.08302*, 2017. [Online]. Available: <https://arxiv.org/abs/1704.08302>
- [51] S. Levine, C. Finn, T. Darrell, and P. Abbeel, "End-to-end training of deep visuomotor policies," *The Journal of Machine Learning Research*, vol. 17, no. 1, pp. 1334–1373, 2016. [Online]. Available: <https://arxiv.org/abs/1604.07316>



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