Set Covering Problem

Computação Inspirada na Natureza

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1 The problem

Problem

- You want to consume the minimum number of fruits (and vegetables) that will allow you to get the complete set of vitamins
- Each fruit/vegetable gives you a subset of all vitamins

Formulation

- Given a number of elements (e.g., cities, tasks, skills) and a collection of sets $S = \{S_1, S_2, ..., S_m\}$, where each set contains some of these elements.
- The universe $U = \bigcup_{s \in S} s_i$
- Find the smallest number of sets from S such that every element in the Universe is included in at least one selected set.
- Objective: Minimize cost (or number of sets) while ensuring full coverage.

$$\min_{|C|}\{C \subset S \mid \forall x \in U\ \exists S_k \in C : x \in S_k\}$$

Example

$U = \{1, 2, 3, 4, 5\}$

$S_1 = \{1, 2\}$

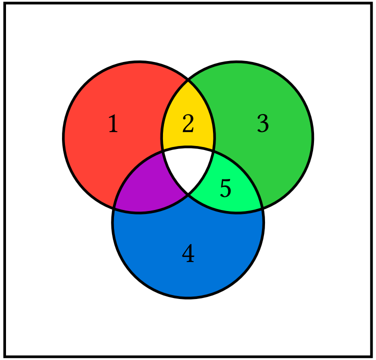
$S_2 = \{2, 3, 5\}$

$S_3 = \{1, 4\}$

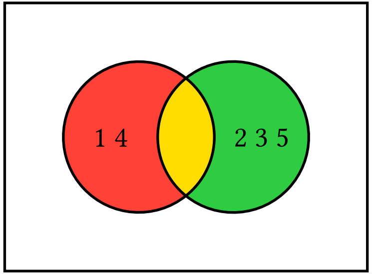
$S_4 = \{4, 5\}$

Solution	**Size**	**Elements**
$\{S_1, S_2, S_4\}$	3	$\{1, 2\} \cup \{2, 3, 5\} \cup \{4, 5\}$
$\{S_2, S_3\}$	2	$\{2, 3, 5\} \cup \{1, 4\}$

Solution using 3 sets



Solution using 2 sets



Greedy Strategy

- While not complete
- Identify currently uncovered elements
- Pick the set that covers the largest number of uncovered elements u_i
- Add it to the solution
- Repeat

2 Genetic Algorithm

Initialization

Use a randomized version of the greedy algorithm for initializing the population

1. Use roulette wheel selection where each set S_i portion of the wheel is proportional to u_i
2. Select a set randomly using the probabilities
3. For all S_i, adjust the number u_i of items that are not currently in the solution
4. Repeat until all elements are covered

Binary Representation

- Size N: Number of subsets
- Each bit encodes the presence of the subset in the solution

Repair for Binary Representation

Constraint

Should be deterministic (unless using a Lamarckian approach)

Greedy Cover

Iteratively add sets that cover the maximum of unrepresented elements

Remove Redundancy

Remove sets that are not needed

Repair for Binary Representation

Greedy Cover

- Select the first set S_i with the largest u_i
- Repeat until all elements are covered

Remove Redundancy

- Remove the first set that is not needed or
- Use a more complex algorithm

Operators for Binary Representation

- Selection: Tournament Selection
- Crossover: Uniform
- Mutation: Flip
- Repair: Use repair before evaluation
- Lamarckian: Should repair modify the individual?

Order Representation

- Size N: Number of subsets
- Chromosome indicates order of adding sets
- Only add a set if it covers previously unrepresented elements
- Remove previous sets if they are no longer needed

Operators for Order Representation

Selection

Tournament Selection

Crossover

- Order Preserving, or
- Partially Mapped

Mutation

- Insertion or
- Swap

3 Ant Colony Optimization

Procedure

- Each ant starts with no sets
- It only adds a set that has at least one unrepresented element
- Heuristic is the number of unrepresented elements u_i added by the set
- An ant finishes building the solution when it has covered the entire universe U
- Pruning may be performed a posteriori by removing superfluous sets

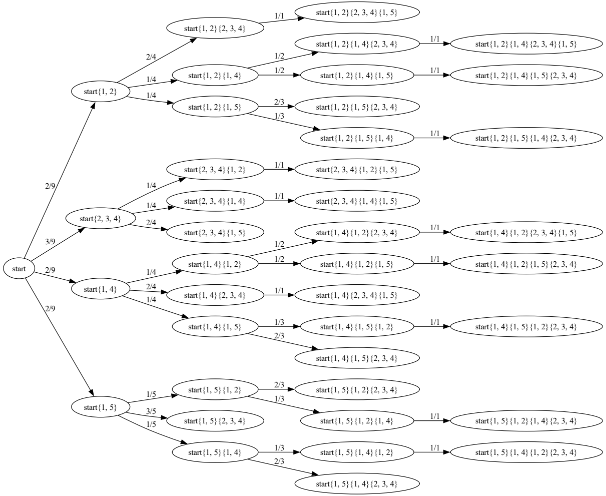


Figure 1: Graph with the heuristic value for each edge.