Proof that the square root of 2 is Irrational

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This is a basic number theory result on the non-rationality of $\sqrt{2}$. To proof this a *Reductio ad Absurdum* can be applied. Thus, suppose $\sqrt{2} \in \mathbb{Q}$ is true, and derive conclusions from this until we will finally arrive to get a contradiction. From this hypotheses we derive that, $\exists p, q \in \mathbb{Z}$, such as $\sqrt{2} = \frac{p}{q}$ can be write such as an irreducible fraction. Thus, taking squares on the previous expression, we can write the following equation:

$$p^2 = 2q^2 \tag{1}$$

Before to continue, it is trivial to notice that if $2|a^2 \Rightarrow 2|a$, which is trivial because if $2 \nmid a$, then $2 \nmid a^2$. (where $\nmid$ means "not divided by"). Thus, from (**??**), due to p^2 is pair (so p), $\exists k_1 \in \mathbb{N}$, such as:

$$p = 2k_1 \tag{2}$$

Therefore, we can deduce:

$$2q^2 = (2k_1)^2 = 4{k_1}^2 \Rightarrow q^2 = 2{k_1}^2 \tag{3}$$

From (**??**), as before, due to q^2 is pair, $\exists k_2 \in \mathbb{N}$, such as:

$$q = 2k_2, \tag{4}$$

From equations (**??**) and (**??**), we can rewrite $\sqrt{2}$ as a reducible fraction:

$$\sqrt{2} = \frac{p}{q} = \frac{2k_1}{2k_2} = \frac{k_1}{k_2} \tag{5}$$

Which is a contradiction $\lightning$ due to by hypothesis, $\frac{p}{q}$ is irreducibly.

$$\therefore \sqrt{2} \in \mathbb{I}$$