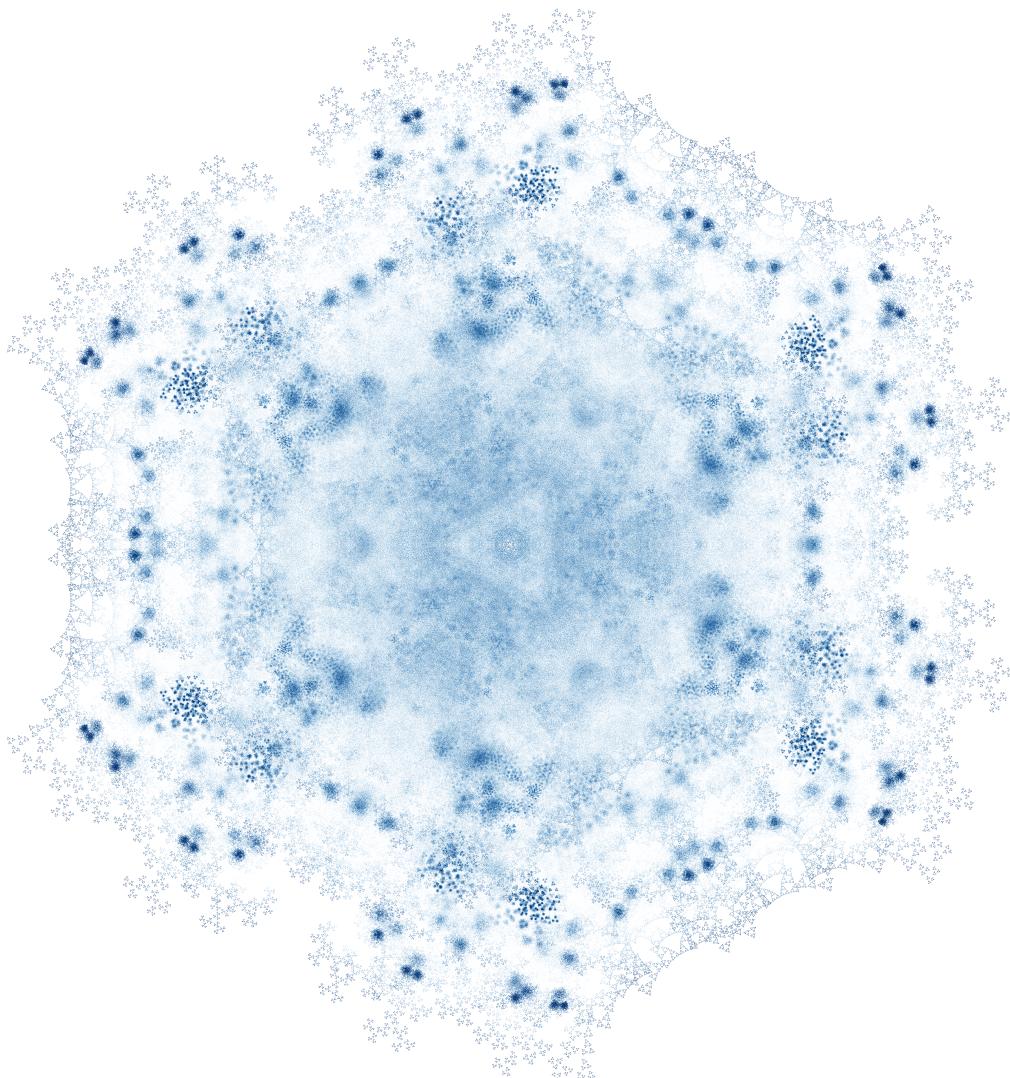


# Bohemian Matrices 2023 Calendar

Images constructed in Maple™ or Python  
using methods described in the SIAM book  
“Computational Discovery on Jupyter”

by Neil J. Calkin, Eunice Y.S. Chan, and Robert M. Corless

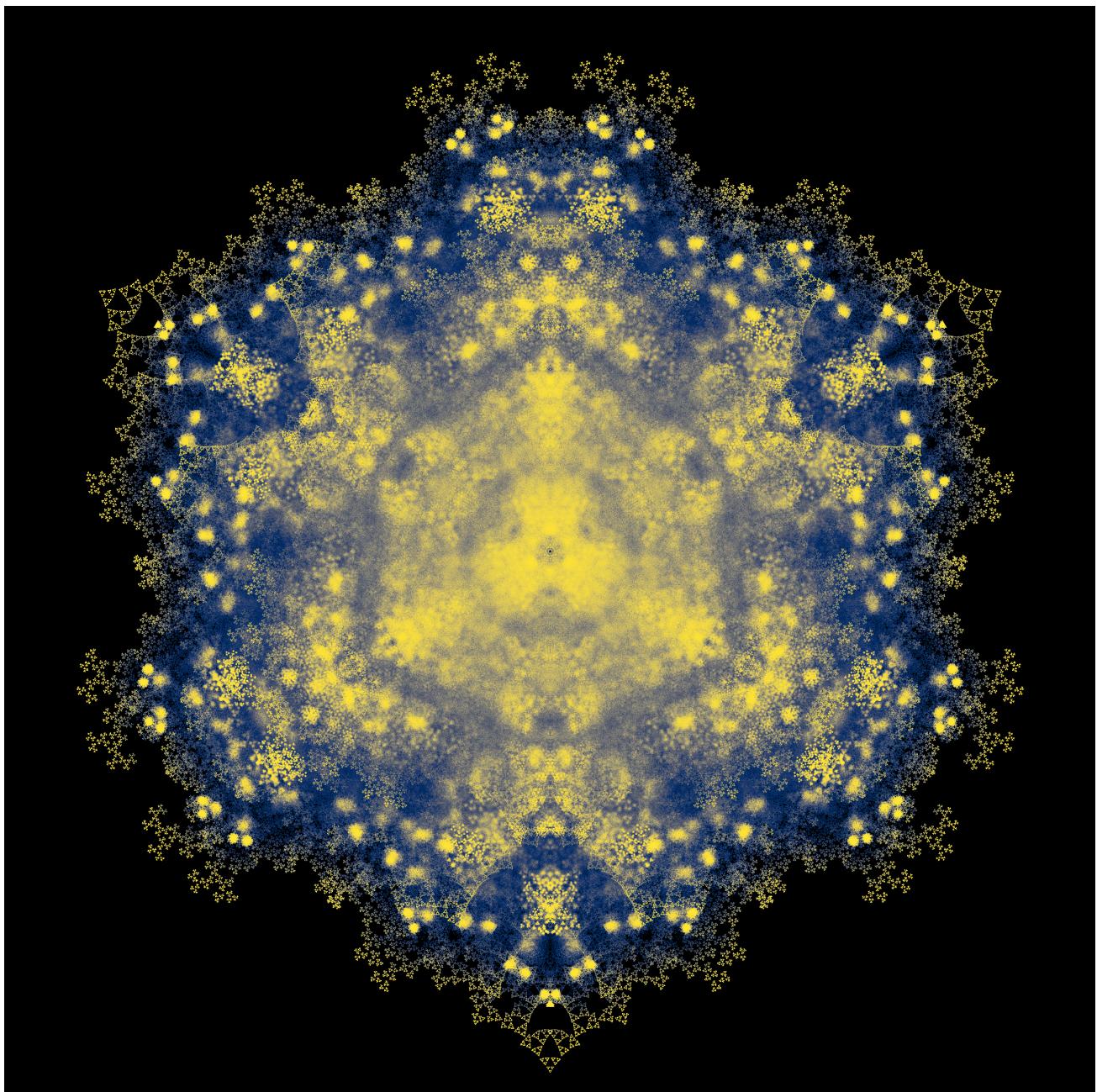
Calendar compiled using `cdcalendar.cls 1.4.2` by LianTze Lim



Density plot of all eigenvalues of all 1,594,323 dimension 14 upper Hessenberg Toeplitz matrices with population the cube roots of unity. Grid is 4800 by 4800 and the colour scheme is "Blues." Compare with the August image, which has a different population, dimension, and colour scheme, and the December image, which is more like the August image. Image © (2022) by Robert M. Corless, generated using the code from <https://computational-discovery-on-jupyter.github.io/Computational-Discovery-on-Jupyter/>.

							December						
S	M	T	W	T	F	S	S	M	T	W	T	F	S
<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	4	5	6	7	8	9	10
<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	11	12	13	14	15	16	17
<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	18	19	20	21	22	23	24
<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>	25	26	27	28	29	30	31
<b>29</b>	<b>30</b>	<b>31</b>					February						
							S	M	T	W	T	F	S
							1	2	3	4			
							5	6	7	8	9	10	11
							12	13	14	15	16	17	18
							19	20	21	22	23	24	25
							26	27	28				

# January 2023



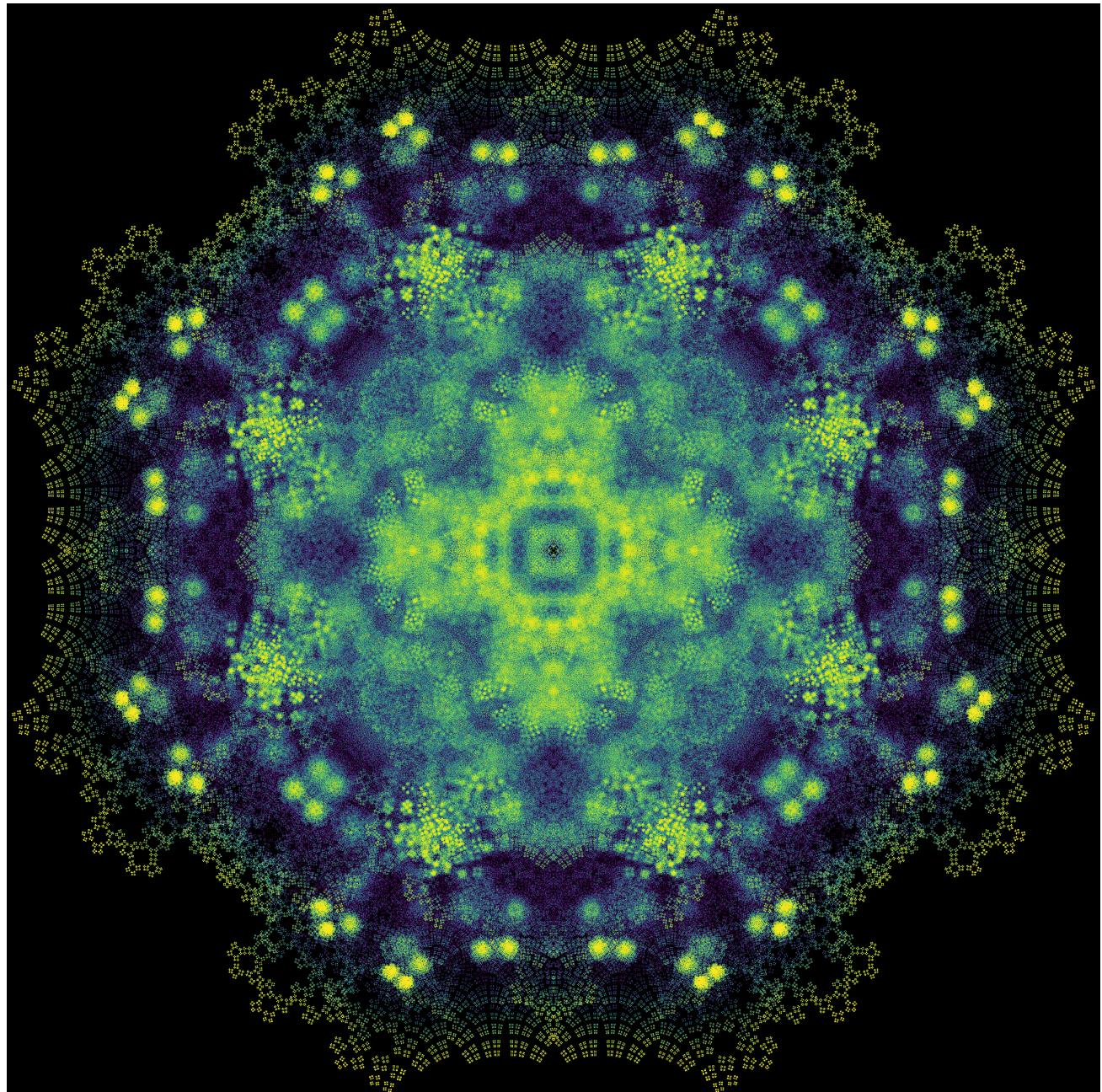
Density plot of all 4,782,969 dimension  $m = 15$  Upper Hessenberg Toeplitz zero-diagonal matrices with population  $P$  being the cube roots of  $-1$ . The colour scheme is 'cividis'. The density plot is rotated by 30 degrees to make the "hamster in the middle" inescapable! See the paper by Corless, Labahn, Piponi, and Rafiee Sevyeri <https://arxiv.org/abs/2202.07769>. Image © (2022) Robert M. Corless Generated using the code from <https://computational-discovery-on-jupyter.github.io/Computational-Discovery-on-Jupyter/>.

							January
S	M	T	W	T	F	S	S M T W T F S
							1 2 3 4 5 6 7
							8 9 10 11 12 13 14
							15 16 17 18 19 20 21
							22 23 24 25 26 27 28
							29 30 31
<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	
<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	
<b>19</b>	<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	
<b>26</b>	<b>27</b>	<b>28</b>					

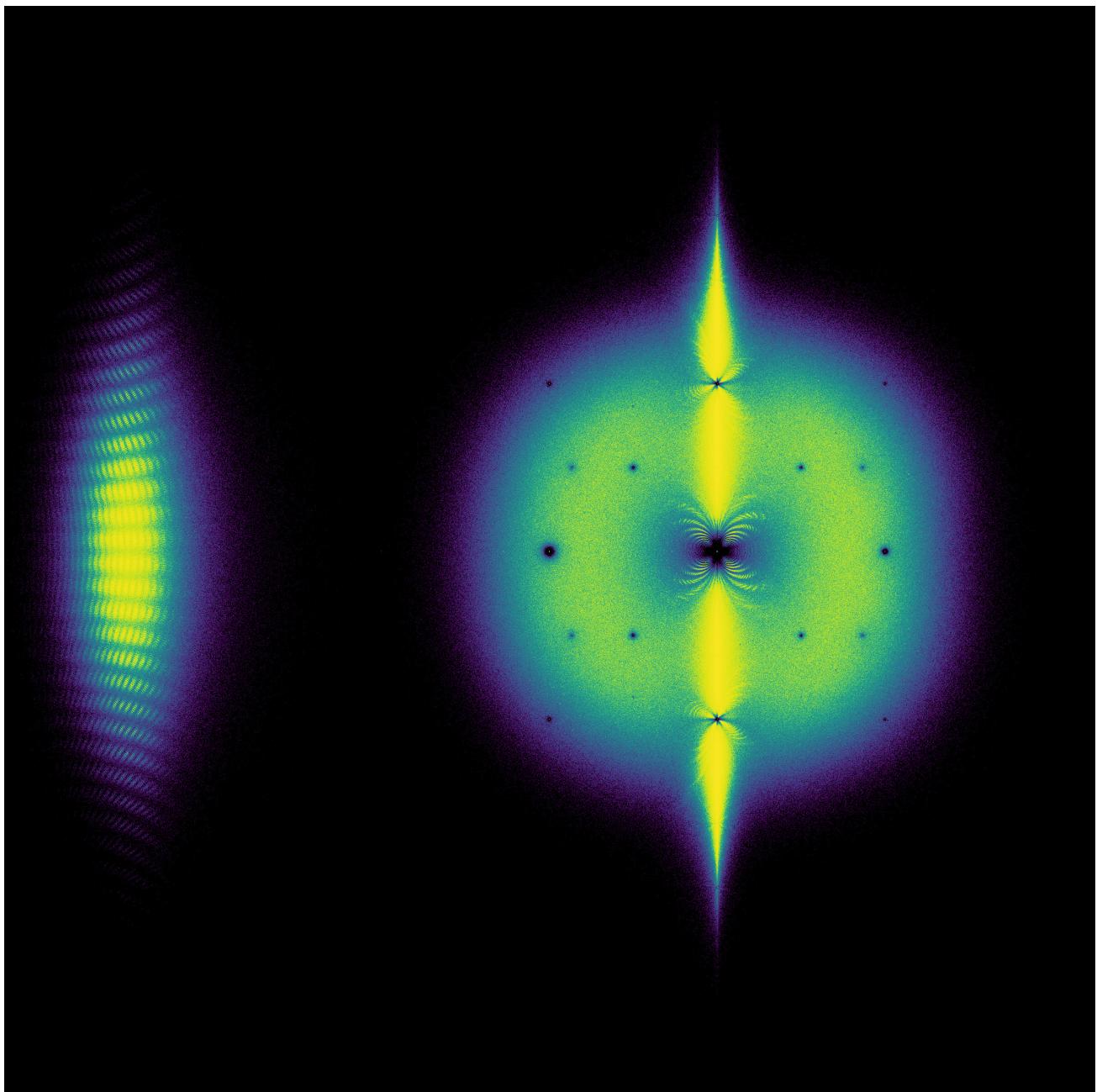
							March
S	M	T	W	T	F	S	S M T W T F S
							1 2 3 4
							5 6 7 8 9 10 11
							12 13 14 15 16 17 18
							19 20 21 22 23 24 25
							26 27 28 29 30 31

# February 2023



Upper Hessenberg Toeplitz zero diagonal matrices of dimension 11 with population  $\pm 1 \pm i$ . The eigenvalues of all 1,048,576 matrices were computed and their density plotted on a 2400 by 2400 grid. The colour scheme is "viridis". Generated using the code from <https://computational-discovery-on-jupyter.github.io/Computational-Discovery-on-Jupyter/>.

March 2023



Density plot of the eigenvalues of a sample of 20 million dimension  $m = 7$  dense matrices with population  $-1 \pm i$ . The grid is 2400 by 2400 and the colour scheme is "viridis." We see a clear separation into two regions, together with a high-density cloud of eigenvalues near the imaginary axis. Many of these features are as yet unexplained mathematically.  
 Image © Robert M. Corless, generated in Maple™

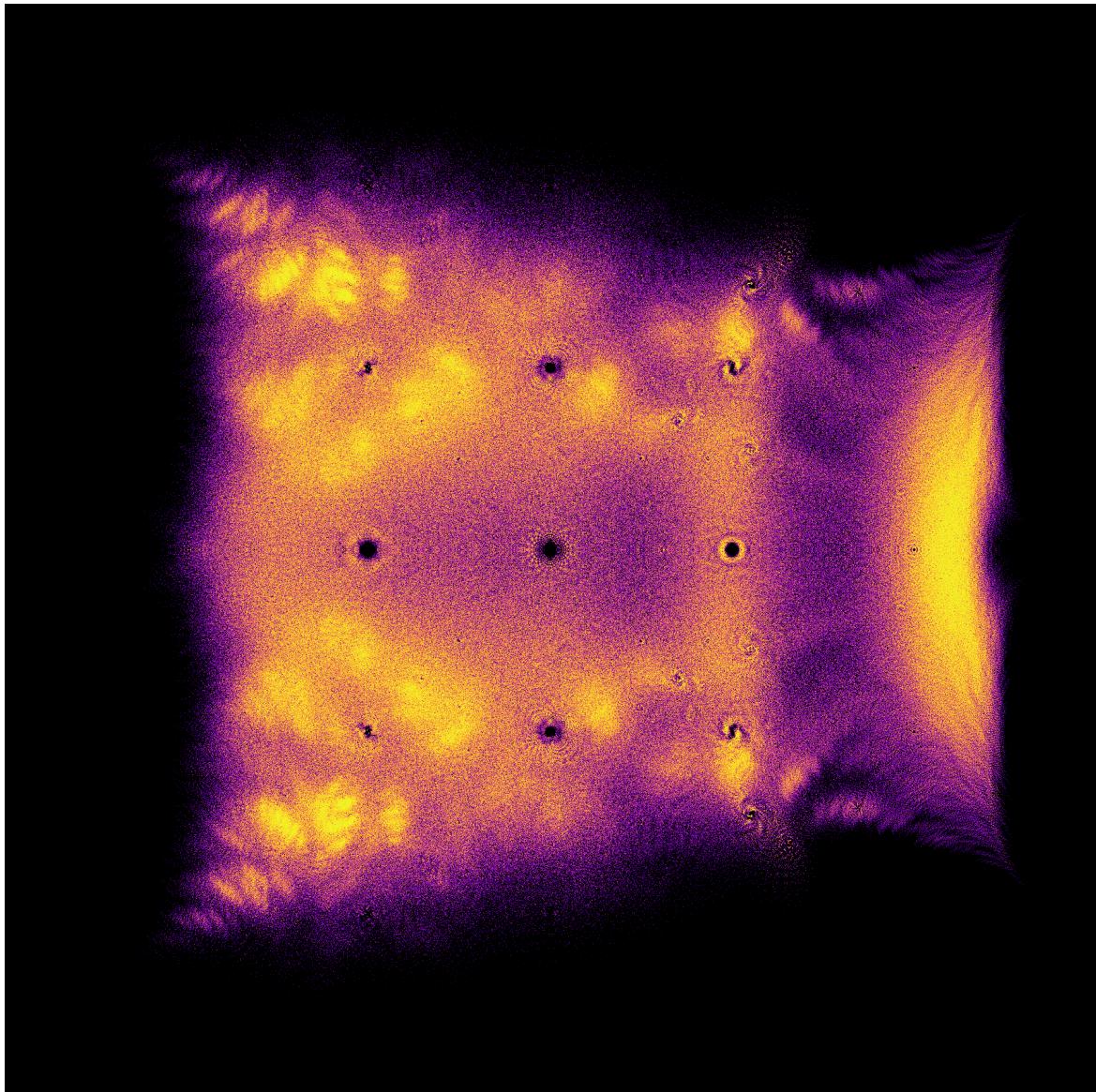
Image © Robert M. Corless, generated in Maple™.

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

March						
S	M	T	W	T	F	S
				1	2	3
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

May						
S	M	T	W	T	F	S
1	2	3	4	5	6	
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

April 2023



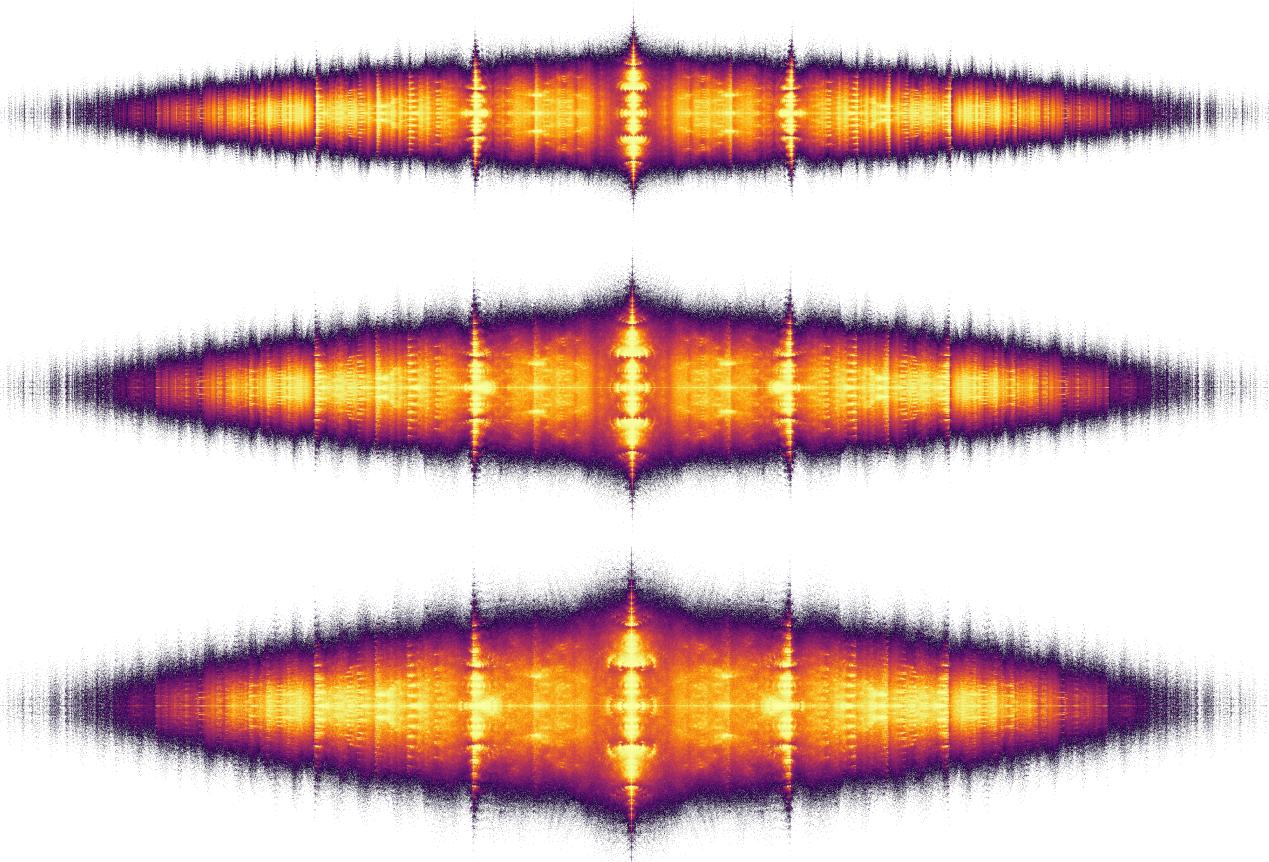
Density plot of all eigenvalues of dimension  $m = 6$  unit upper Hessenberg matrices with population  $P = 1 \pm i$ . The colour scheme is "plasma". The grid is 1800 by 1800. Image © (2022) Robert M. Corless, generated in Maple™.

April						
S	M	T	W	T	F	S
						1
					2	3
					4	5
					6	7
					8	9
					10	11
					12	13
					14	15
					16	17
					18	19
					20	21
					22	23
					24	25
					26	27
					28	29
					30	31

June						
S	M	T	W	T	F	S
					1	2
					3	
					4	5
					6	7
					8	9
					10	11
					12	13
					14	15
					16	17
					18	19
					20	21
					22	23
					24	25
					26	27
					28	29
					30	

May 2023



The population is  $-1, 0, 1, \alpha i, -\alpha i$ , the dimension is  $m = 5$ , and this plot shows three separate density plots of 20 million complex symmetric matrices chosen uniformly at random from the set of  $5^{20} = 30,517,578,125$  such matrices, for  $\alpha = 0.15$  (top graph),  $\alpha = 0.2$  (middle graph), and  $\alpha = 0.25$  (bottom graph). The horizontal axis is from  $-4$  to  $4$ , and the vertical axis (for all three plots) is from  $-1$  to  $1$ . The grids are 1000 vertical by 4000 horizontal and the colour scheme for all three is "inferno." Image © (2022) Robert M. Corless

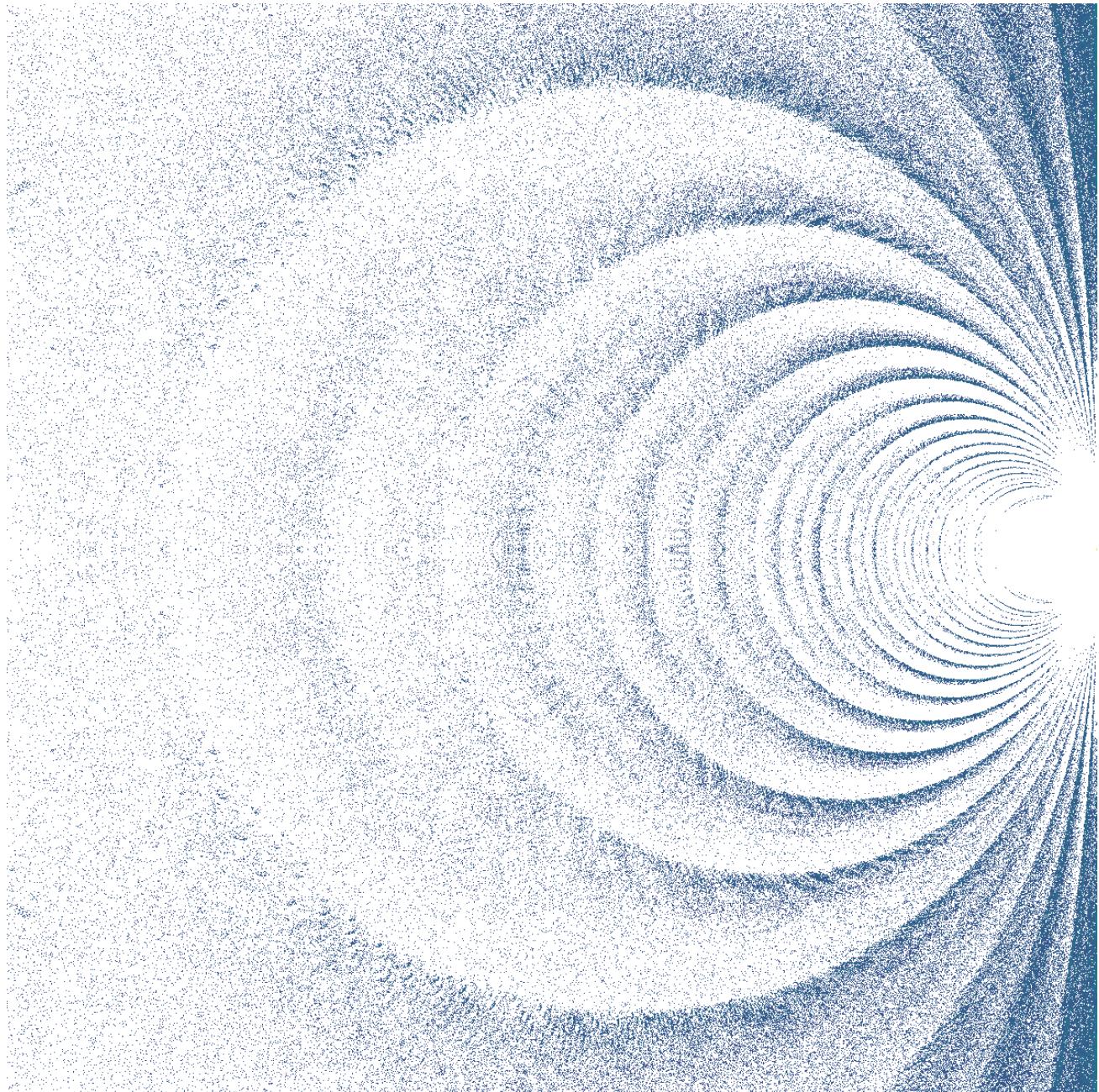
Generated using the code from <https://computational-discovery-on-jupyter.github.io/Computational-Discovery-on-Jupyter/>.

S	M	T	W	T	F	S
				1	2	3
<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>
<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>	<b>17</b>
<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>
<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>	<b>29</b>	<b>30</b>	

May						
S	M	T	W	T	F	S
1	2	3	4	5	6	
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

July						
S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

# June 2023



Symmetric matrices with population  $-1 \pm i$ , and dimension  $m = 8$ . We have zoomed to look at the region  $-0.5 \leq \Re(\lambda) \leq 0$ ,  $-0.25 \leq \Im(\lambda) \leq 0.25$ . Computed a sample of  $10^8$  matrices from the over 68 billion possible. Are those really circles?

Image © (2022) Eunice Y. S. Chan and Robert M. Corless

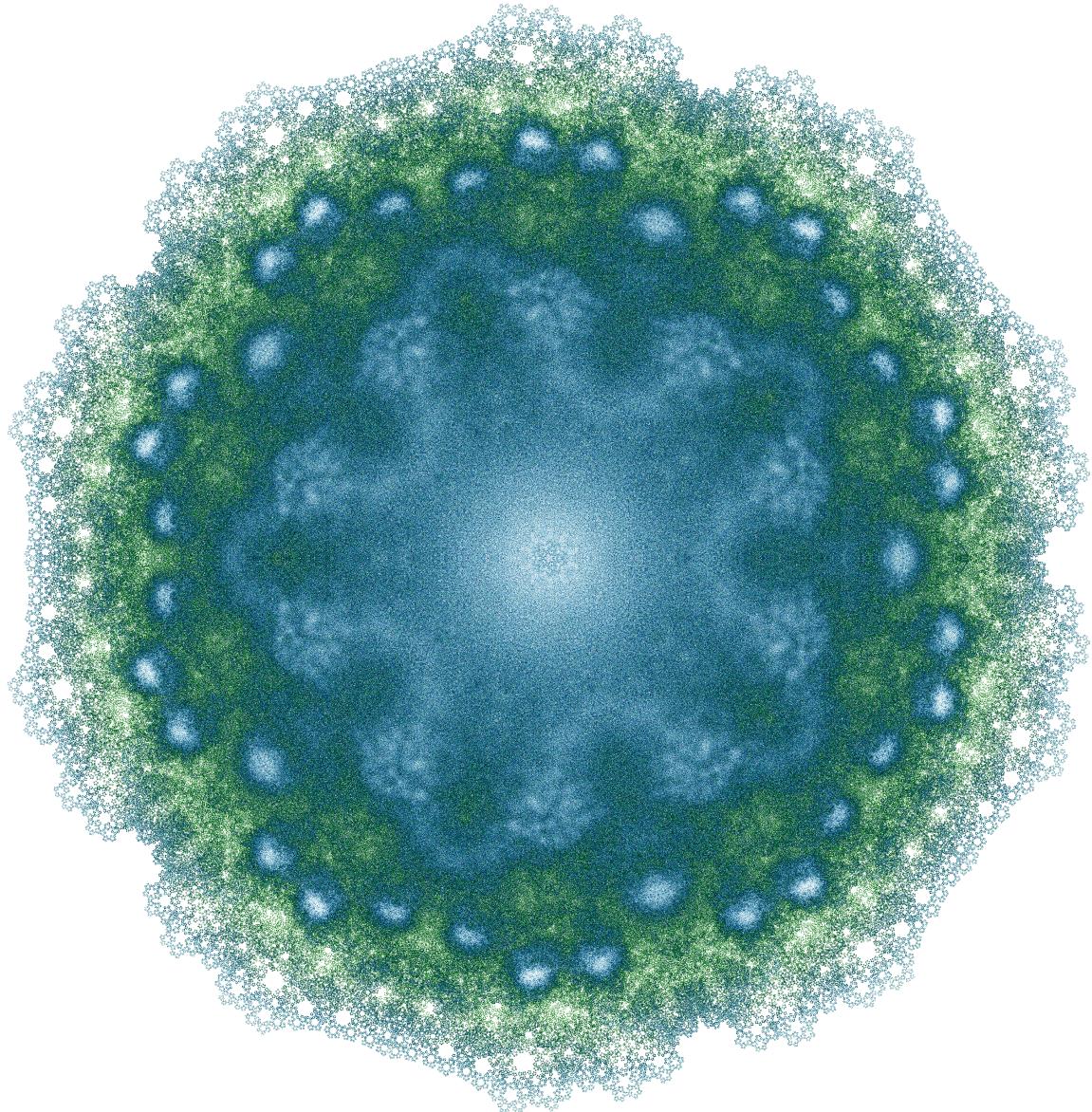
Generated using the code at <https://computational-discovery-on-jupyter.github.io/Computational-Discovery-on-Jupyter/>.

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

June						
S	M	T	W	T	F	S
						1
						2 3
						4 5 6 7 8 9 10
						11 12 13 14 15 16 17
						18 19 20 21 22 23 24
						25 26 27 28 29 30

August						
S	M	T	W	T	F	S
						1 2 3 4 5
						6 7 8 9 10 11 12
						13 14 15 16 17 18 19
						20 21 22 23 24 25 26
						27 28 29 30 31

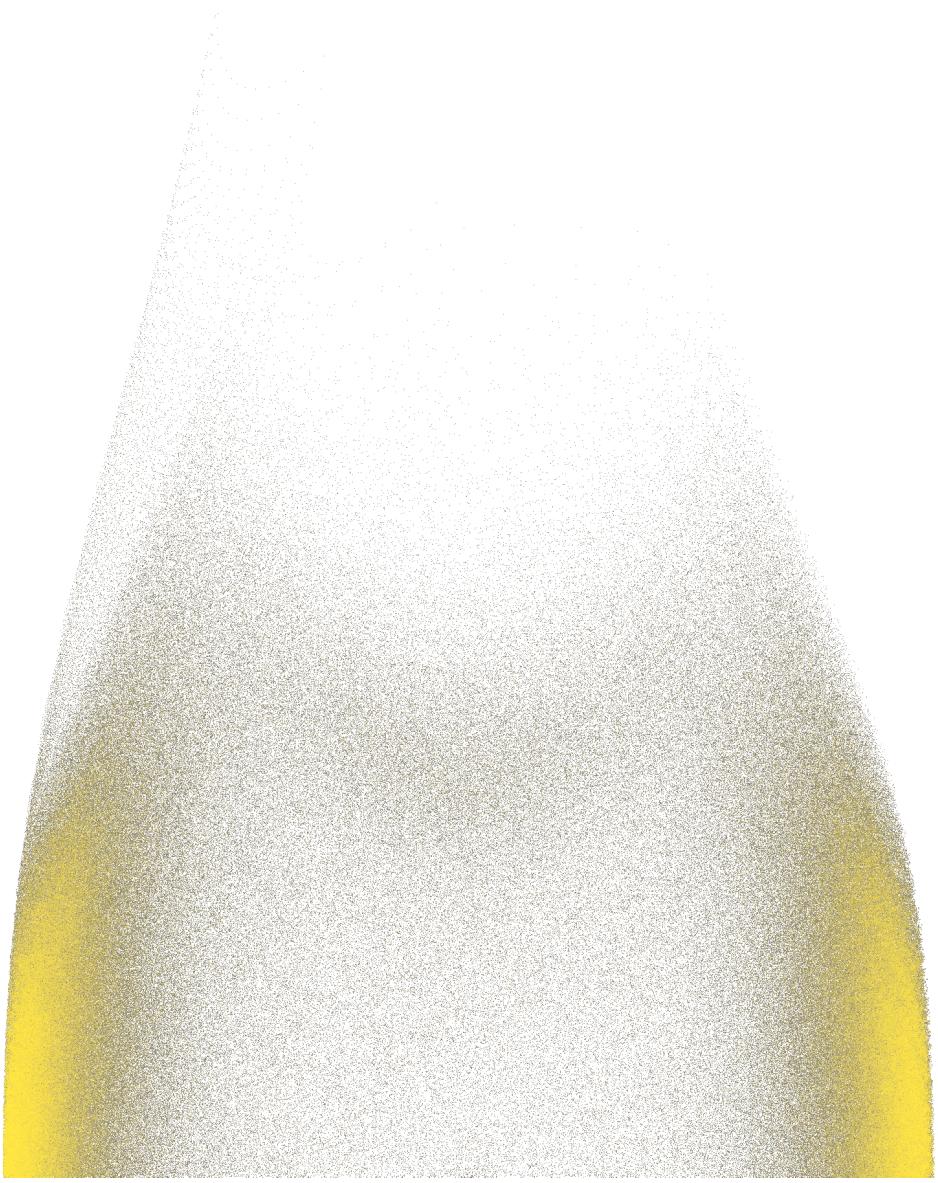
# July 2023



"A sea star in bubbles." Density plot of all eigenvalues of all 1,953,125 dimension 10 upper Hessenberg Toeplitz matrices with population the fifth roots of unity. Grid is 3600 by 3600 and the colour scheme is "ocean." Image © (2022) Robert M. Corless Generated using the code from <https://computational-discovery-on-jupyter.github.io/Computational-Discovery-on-Jupyter/>.

July						
S	M	T	W	T	F	S
			<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>
<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>
<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>
<b>27</b>	<b>28</b>	<b>29</b>	<b>30</b>	<b>31</b>		
September						
S	M	T	W	T	F	S
					<b>1</b>	<b>2</b>
					<b>3</b>	<b>4</b>
					<b>5</b>	<b>6</b>
					<b>7</b>	<b>8</b>
					<b>9</b>	
					<b>10</b>	<b>11</b>
					<b>12</b>	<b>13</b>
					<b>14</b>	<b>15</b>
					<b>16</b>	
					<b>17</b>	<b>18</b>
					<b>19</b>	<b>20</b>
					<b>21</b>	<b>22</b>
					<b>23</b>	
					<b>24</b>	<b>25</b>
					<b>26</b>	<b>27</b>
					<b>28</b>	<b>29</b>
					<b>30</b>	<b>31</b>

**August 2023**



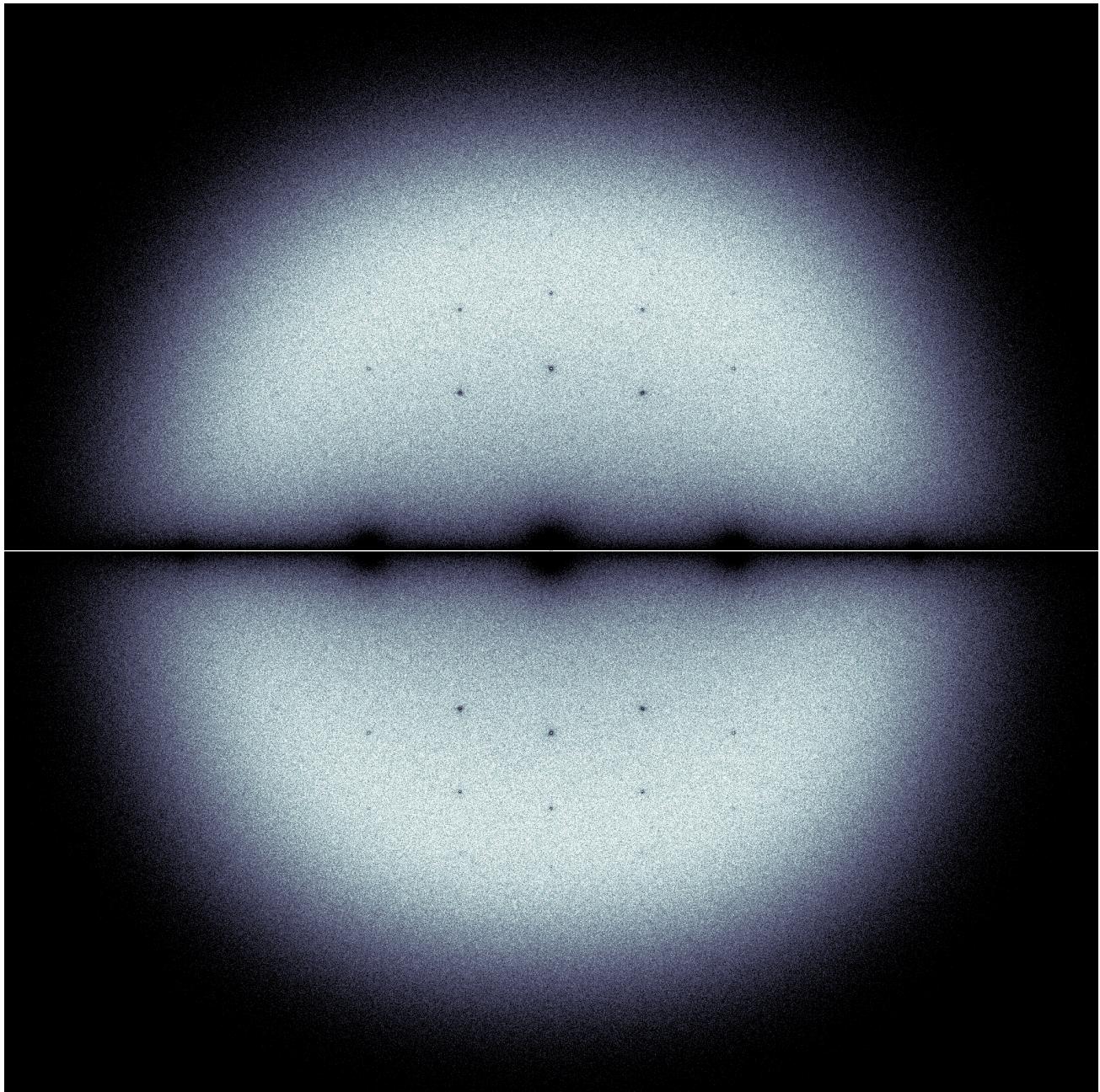
Upper half of a density plot of eigenvalues of all dimension 19 "Wedge" Hankel matrices with sub-antidiagonal all  $-1$ , zero antidiagonal, and population  $\pm i/4$ . The left-right asymmetry is correct. Can you explain the asymmetry? Image © (2022) Robert M. Corless, generated in Maple™ on an 1800 by 1800 grid. The colour scheme is "cividis."

# September 2023

S	M	T	W	T	F	S
<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>
<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>
<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>	<b>29</b>	<b>30</b>

August						
S	M	T	W	T	F	S
						1 2 3 4 5
					6 7 8 9 10 11 12	
					13 14 15 16 17 18 19	
					20 21 22 23 24 25 26	
					27 28 29 30 31	

October						
S	M	T	W	T	F	S
					1 2 3 4 5 6 7	
					8 9 10 11 12 13 14	
					15 16 17 18 19 20 21	
					22 23 24 25 26 27 28	
					29 30 31	



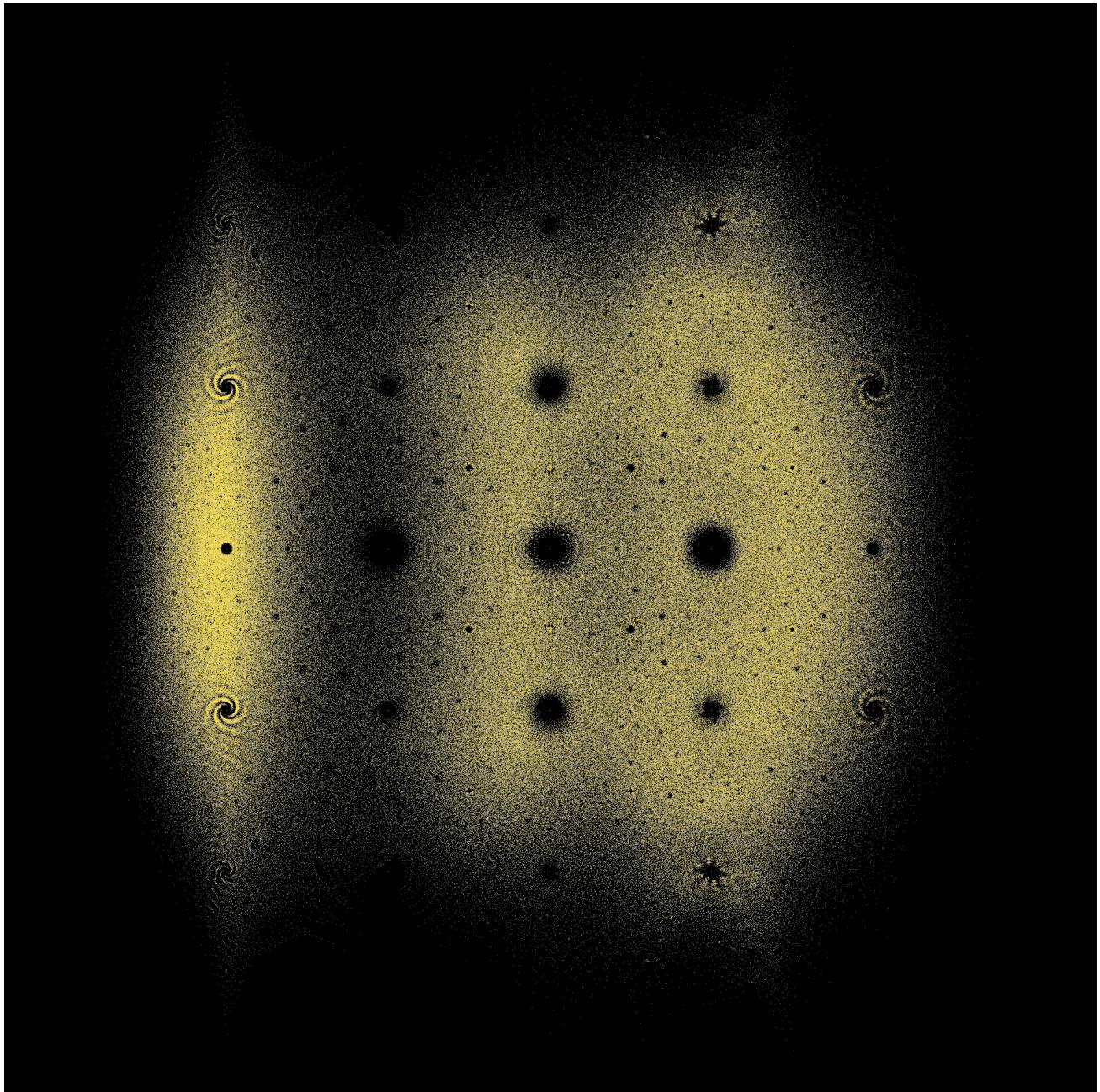
Density plot of a random sample of 20 million five by five matrices with population  $(-1, 0, 1)$ . There is a theorem by Tao and Vu which states that as the dimension goes to infinity, the eigenvalue distribution approaches a uniform one on a scaled disk. Here, with dimension  $m = 5$ , we see some effects of finite size. This image (copyright 2022 Robert M. Corless) was chosen because it is rather like a white pumpkin. Image generated using the code from <https://computational-discovery-on-jupyter.github.io/Computational-Discovery-on-Jupyter/>.

							September														
S	M	T	W	T	F	S	S	M	T	W	T	F	S								
<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	1	2	3	4	5	6	7	8	9						
<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	10	11	12	13	14	15	16	17	18	19	20	21	22	23	
<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	24	25	26	27	28	29	30								
<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>															
<b>29</b>	<b>30</b>	<b>31</b>																			

							November																			
S	M	T	W	T	F	S	S	M	T	W	T	F	S													
							1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18		
							19	20	21	22	23	24	25	26	27	28	29	30								

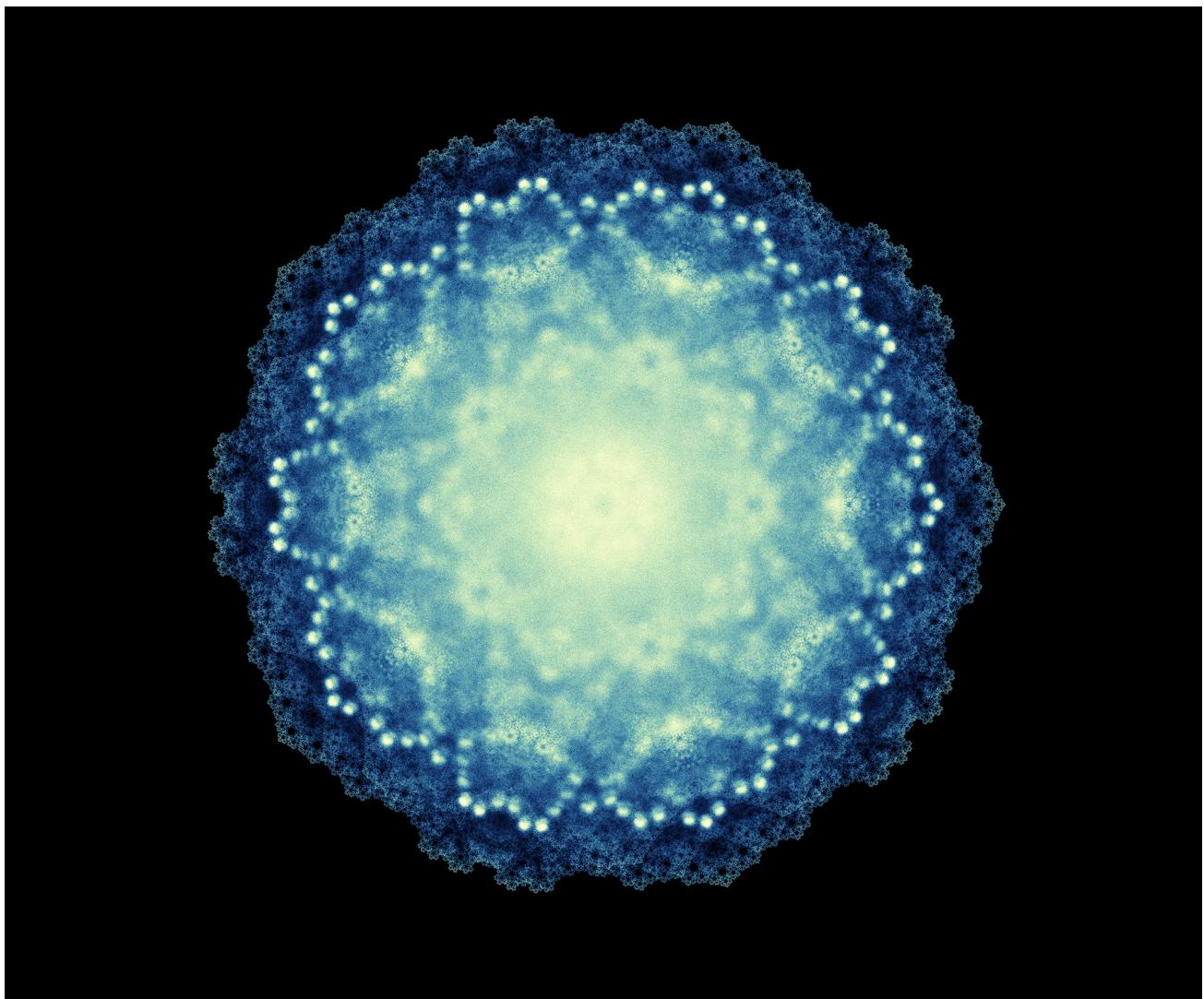
# October 2023



An 1800 by 1800 grid density plot of the eigenvalues of all upper Hessenberg matrices, population  $-1 - i, -1, -1 + i$ , and dimension  $m = 4$ . The plot is on  $-L - 1 \leq \Re(\lambda) < L - 1, -L \leq \Im(\lambda) \leq L$ , where  $L = 1 + 2 \cdot 2^{1/4}$ . The spirals are completely unexplained. See the paper by Corless, Labahn, Piponi, and Rafiee Sevyeri <https://arxiv.org/abs/2202.07769>. Image constructed in Maple™. Image © (2022) Robert M. Corless

							October
S	M	T	W	T	F	S	
				1	2	3	4
<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	
<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	
<b>19</b>	<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	
<b>26</b>	<b>27</b>	<b>28</b>	<b>29</b>	<b>30</b>			
							December
S	M	T	W	T	F	S	
							1 2
3	4	5	6	7	8	9	
10	11	12	13	14	15	16	
17	18	19	20	21	22	23	
24	25	26	27	28	29	30	
							31

# November 2023



This image also appeared on the cover of the November 2021 London Mathematical Society Newsletter, and is described in more detail therein. In brief, it shows a density plot of the eigenvalues of a sample of 10 million dimension 13 upper Hessenberg Toeplitz matrices, with zero diagonal, unit subdiagonal, and other entries the fifth roots of unity. Image copyright (2021) Steven E. Thornton.

S	M	T	W	T	F	S
<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>
<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>
<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>	<b>29</b>	<b>30</b>
<b>31</b>						

November						
S	M	T	W	T	F	S
1	2	3	4			
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

January						
S	M	T	W	T	F	S
1	2	3	4	5	6	
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

**December 2023**