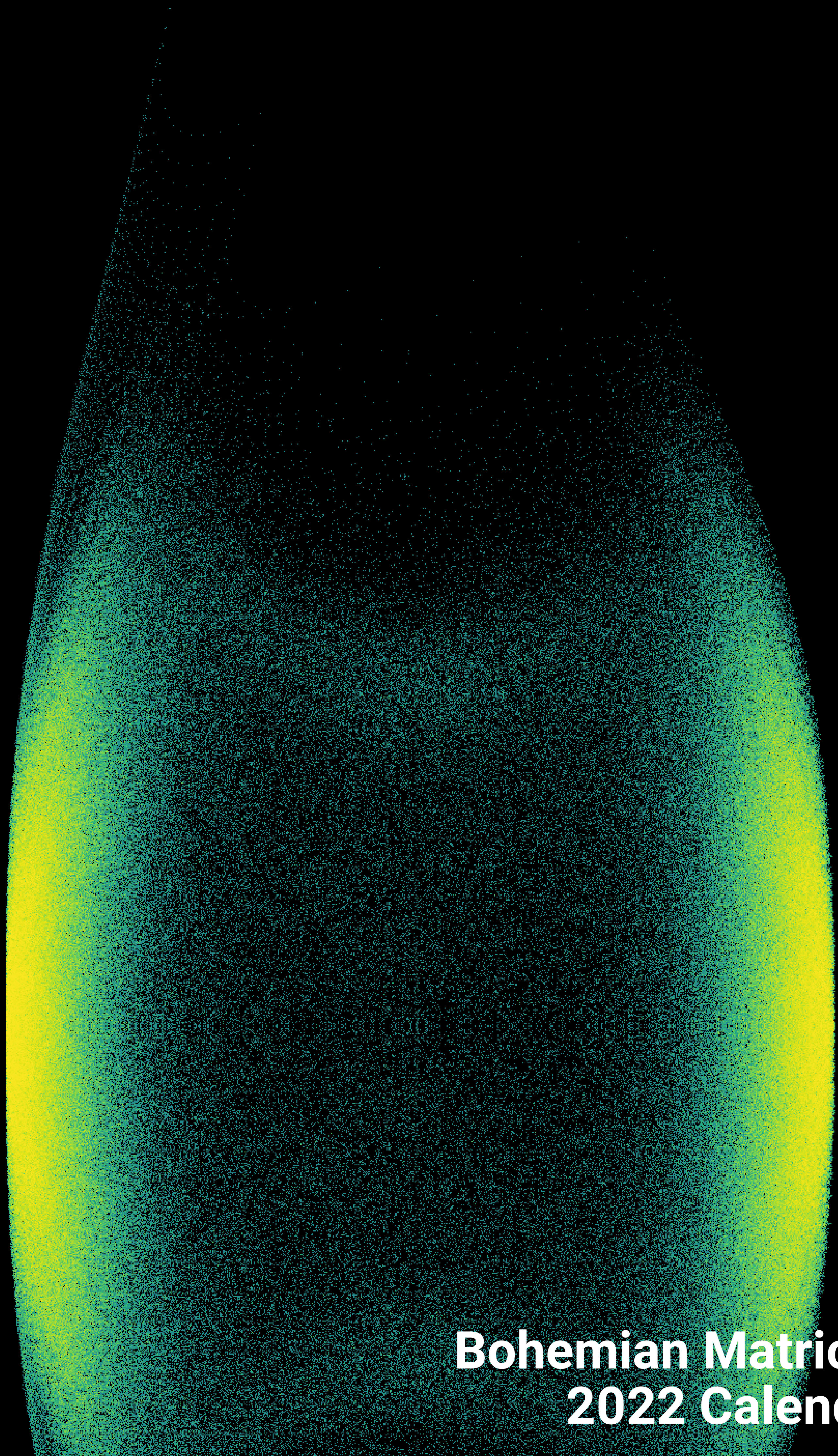
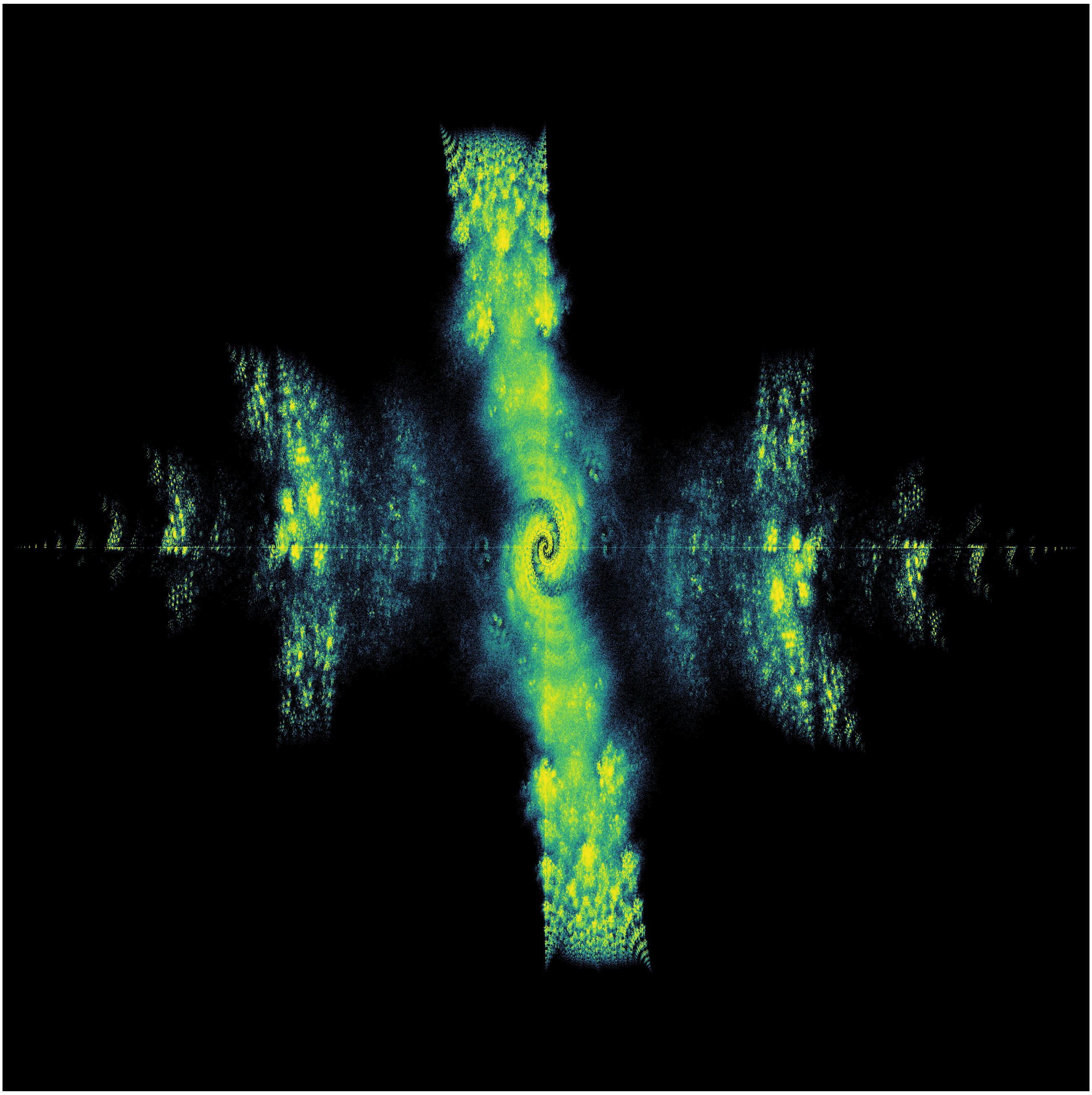


Bohemian Matrices
2022 Calendar



Skew-symmetric tridiagonals with population $(2, 5i, 2+i)$, on $-10 \le \Re(\lambda) \le 10$, $-5 \le \Im(\lambda) \le 5$ (axes not identically scaled). Eigenvalues of all $3^{13} = 1,594,323$ dimension 14 matrices computed and plotted in Maple™. Do you think the apparent 180° rotational symmetry is accurate? Can you explain it, if so? And what can you say about that "vortex" in the middle?
See https://doi.org/10.5206/mt.v1i2.14360 Image ©(2022) Robert M. Corless

January 2022

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

December

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19	20	21	22	23	24	25
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February

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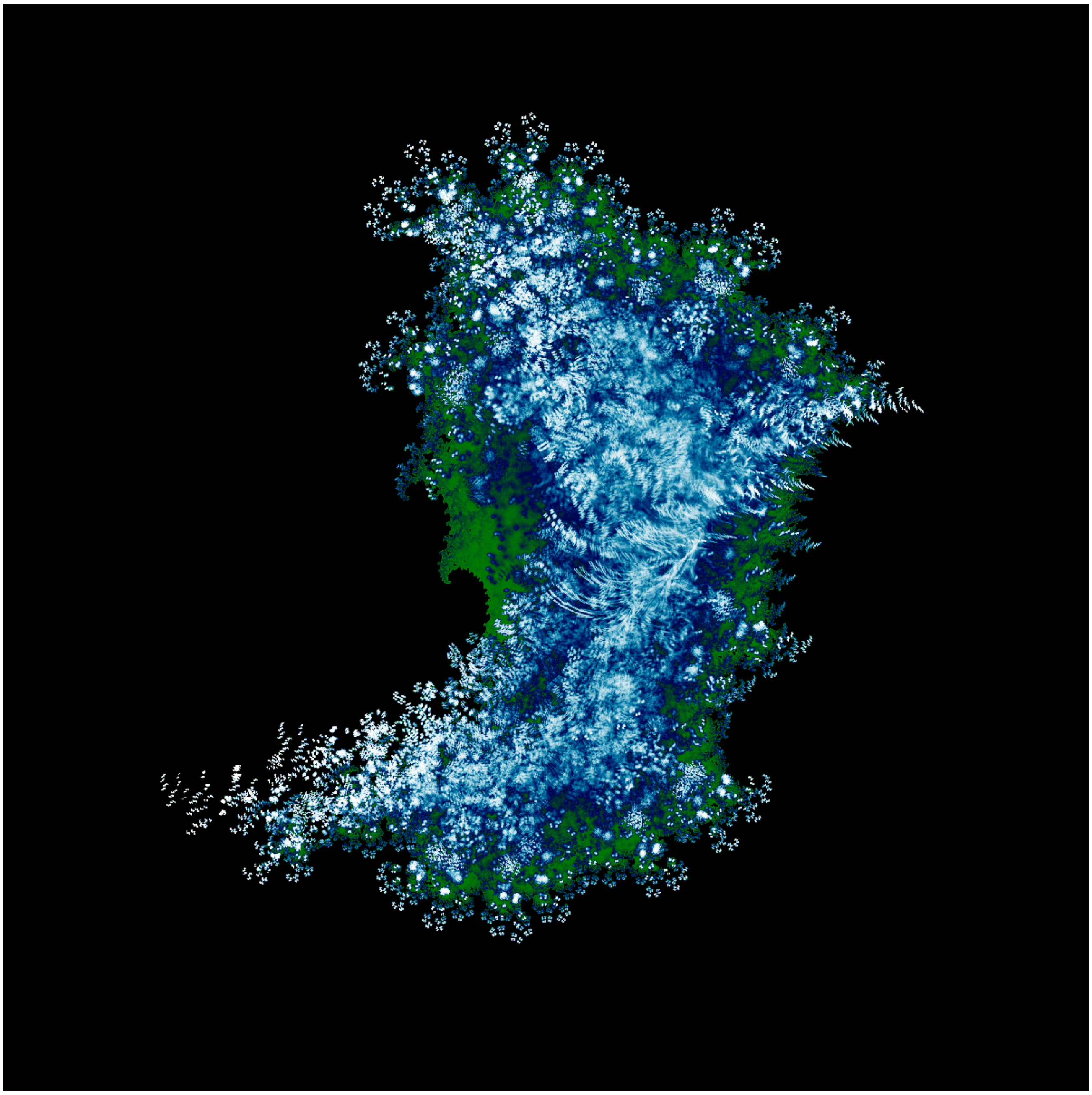

Density plot of a sample of 1.5 million dimension $m = 31$ upper Hessenberg Toeplitz (zero diagonal) matrices with population $P = (1, -i)$. Can you prove that the region containing eigenvalues is inside the disk of radius 3, regardless of the dimension of the matrix? The colour scheme is “ocean” and lighter shades indicate higher density. 1200 by 1200 grid.
See https://doi.org/10.1016/j.laa.2020.03.037 Image ©(2022) Eunice Y. S. Chan and Robert M. Corless

February 2022

S	M	T	W	T	F	S
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January

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March

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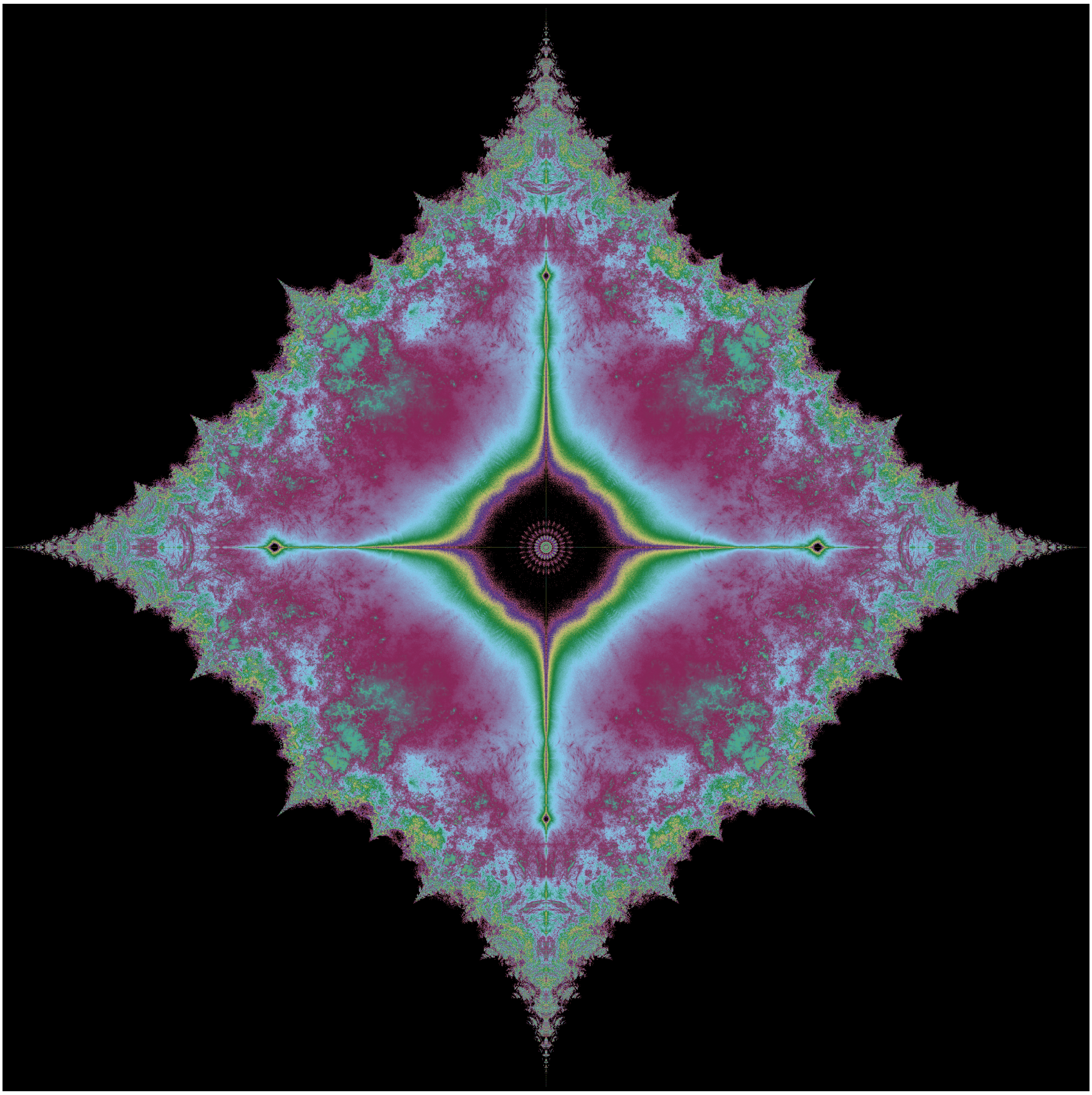
March 2022



Density plot of all eigenvalues of dimension $m = 31$ skew-symmetric tridiagonal matrices with popultaion $P = (1, i)$. Computation of the eigenvalues of all 2^{30} (more than a billion) matrices took 55 hours on the machine scg.uwaterloo.ca. Can you prove that there are exactly 16 nilpotent matrices in this Bohemian family at this dimension?
See https://doi.org/10.5206/mt.v1i2.14360. Image ©(2021) Aaron Asner and Robert M. Corless

S	M	T	W	T	F	S
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27	28	29	30	31		

February

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April

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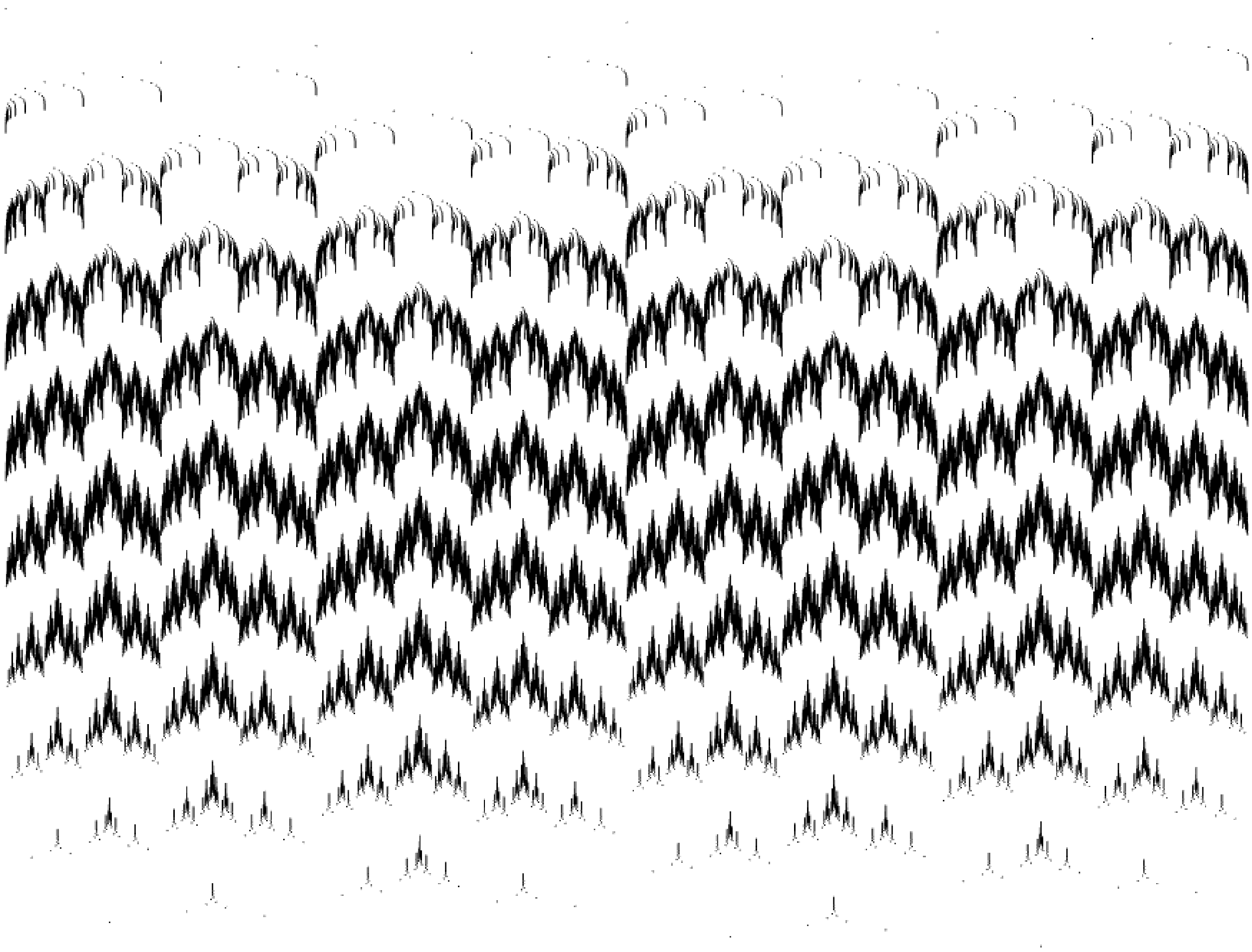
April 2022



Components u_k of the left singular vector $\mathbf{u}$ corresponding to the largest magnitude eigenvalue of the Mandelbrot matrix $\mathbf{M}_{20}$ of dimension $2^{20} - 1 = 1,048,575$. This is also the Perron vector of the matrix $\mathbf{S}_{20} = \mathbf{M}_{20}\mathbf{J}$, where $\mathbf{J}$ is the self-involutory permutation matrix (a.k.a. the anti-identity). Horizontal axis is linear, running from 0 to 10^6, while the vertical axis is logarithmic (base 2) from 2^0 at the top to about 2^{-36} (about 10^{-11}) at the bottom. Why do you think there seem to be vertical divisions in this graph? How many do you see? Why do you think similar-looking structures appear in different parts of the graph? From a paper, with permission, to appear in the American Mathematical Monthly. See https://arxiv.org/abs/2104.01116.

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March

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		1	2	3	4	5
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27	28	29	30	31		

May

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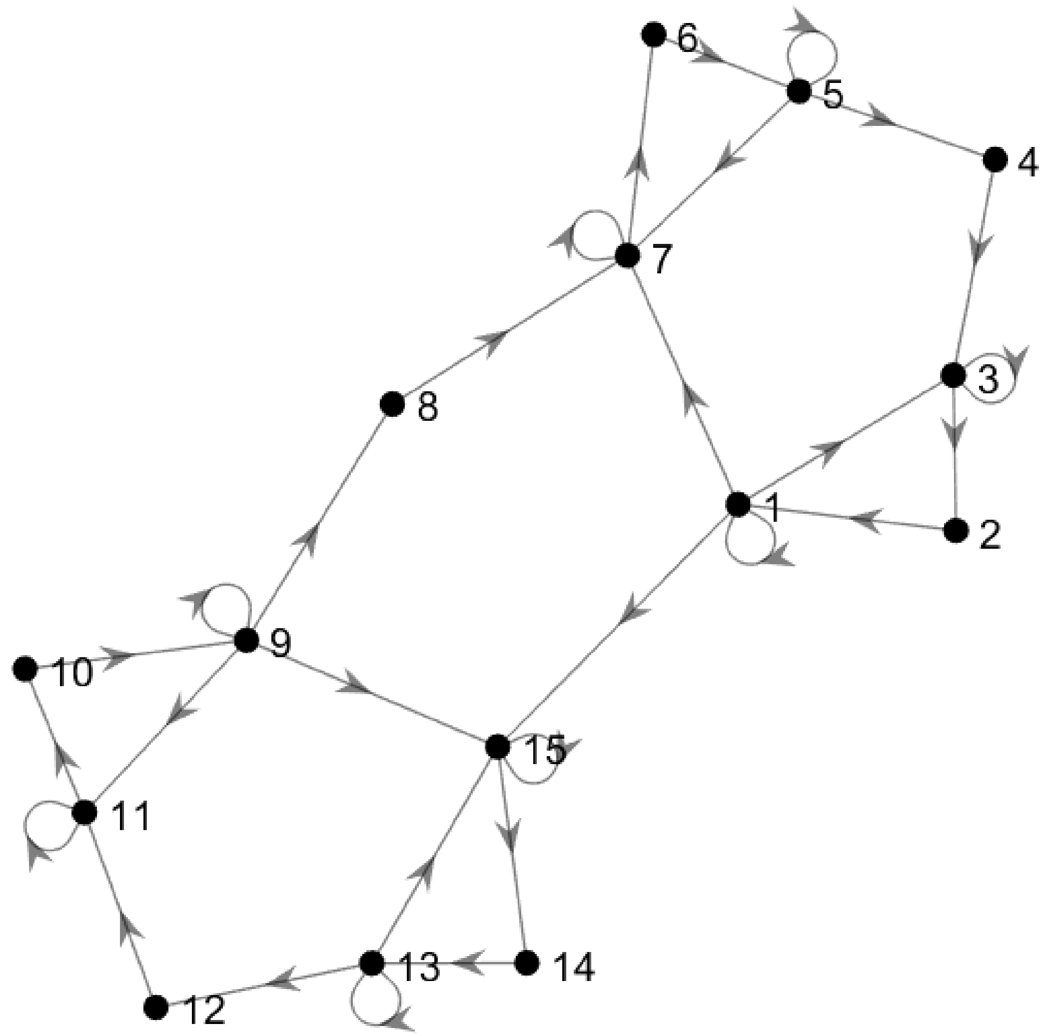

A directed graph G_5 for the Mandelbrot polynomial $p_5(z) = zp_4^2 + 1$ and its minimal height companion matrix. To fix notation, here $p_0 = 0$, so $p_1 = 1$, $p_2 = z + 1$ (whose digraph just has a single loop), $p_3 = z^3 + 2z^2 + z + 1$ (whose digraph has three nodes and two loops), p_4 is degree 7 (seven nodes and four loops) and so p_5 is degree 15 (fifteen nodes and eight loops). The recursive construction is as follows. Put G_2 to be a loop with one vertex. To construct G_{k+1}, start with two copies of G_k and join them, once with just one edge and once with an edge-vertex-edge. Can you see the four copies of G_3 in the above? The two copies of G_4? By permision, from a paper to appear in the American Mathematical Monthly. See https://arxiv.org/abs/2104.01116.

May 2022

S	M	T	W	T	F	S
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29	30	31				

April

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June

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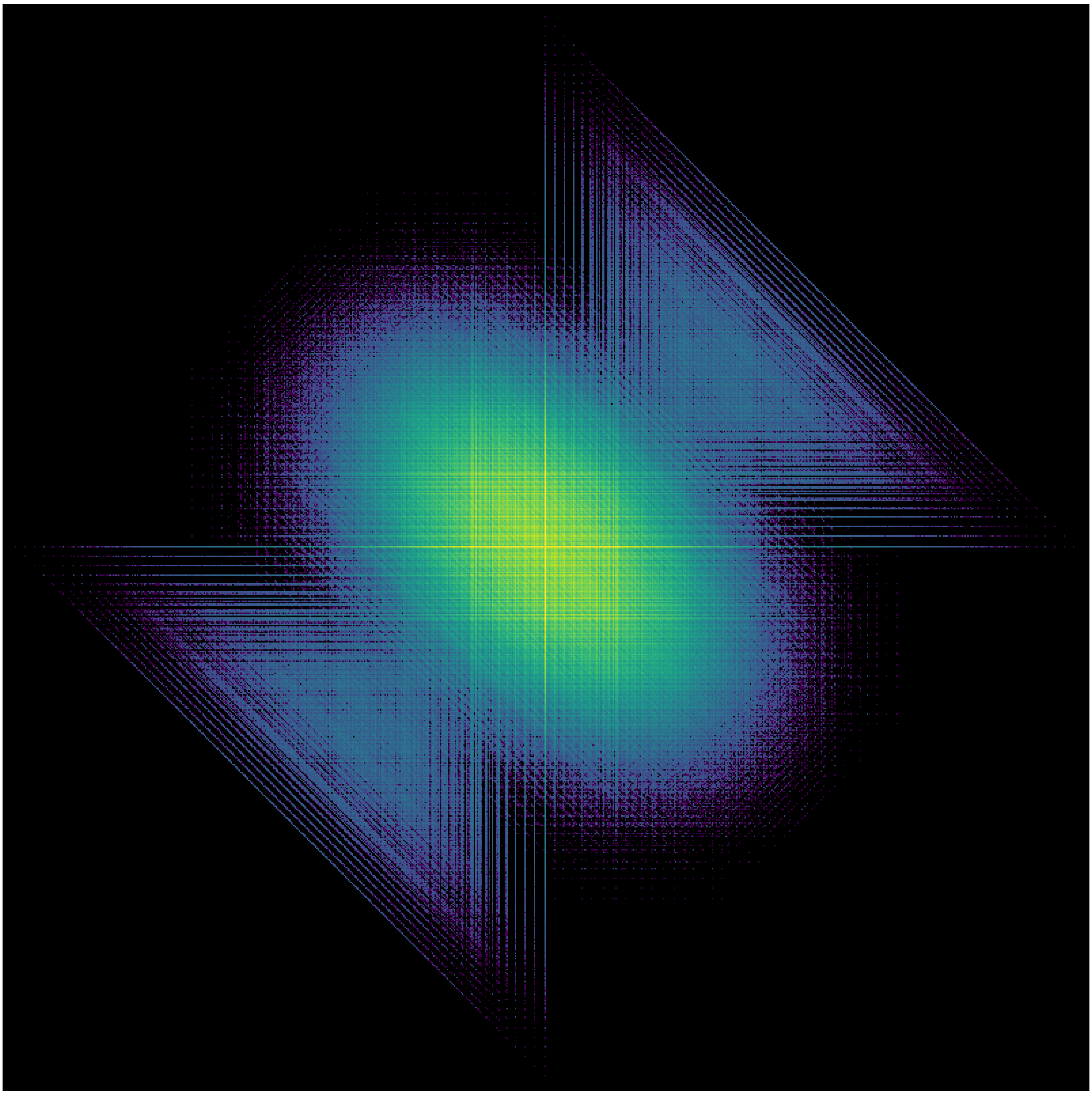

This mysterious picture starts with a recipe for constructing magic squares, but plays a variation. Take the population $(0, 1, i)$ and make circulant matrices in the following way: choose an integer ($m = 11$ in this case) and put $n = 2m + 1 = 23$. Write out all possible n-vectors of the form $(0, u_m, -u_1, u_{m-1}, -u_2, \ldots, u_1, -u_m)$. Use each vector as the first row of a circulant matrix. Make a density plot of the eigenvalues of all $3^{11} = 177,147$ of these dimension 23 matrices. Is the result as symmetric as it seems?
See https://doi.org/10.1016/j.laa.2014.06.054. Image ©(2019) Robert M. Corless, generated in Maple™.

June 2022

S	M	T	W	T	F	S
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5	6	7	8	9	10	11
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19	20	21	22	23	24	25
26	27	28	29	30		

May

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29	30	31				

July

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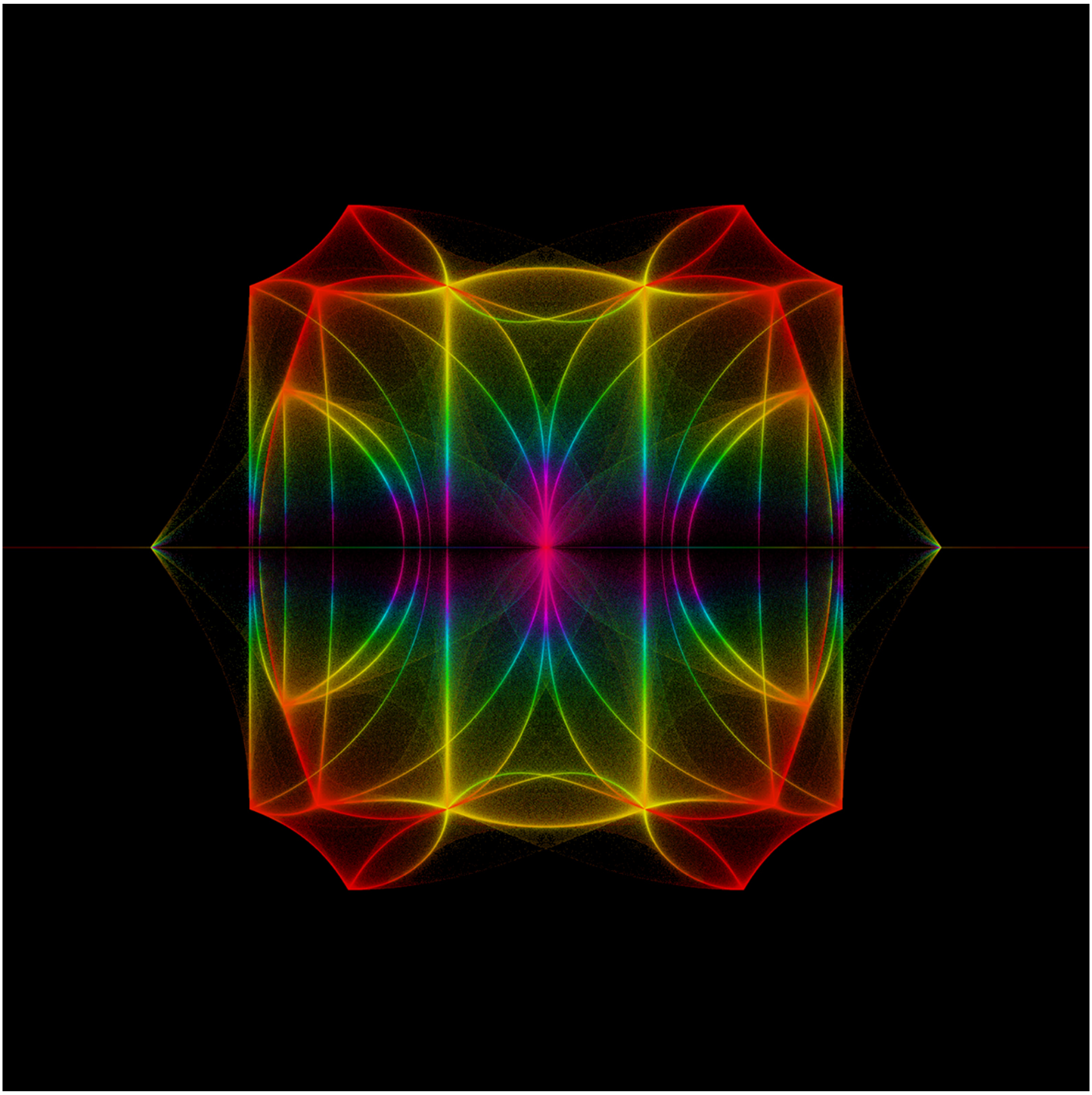

Beta gallery 3 from bohemianmatrices.com. The lines are algebraic curves traced out by eigenvalues as a parameter varies continuously. The colours reflect the eigenvalue conditioning. Why are the hottest colours near the origin?

July 2022

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					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

June

S	M	T	W	T	F	S
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19	20	21	22	23	24	25
26	27	28	29	30		

August

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	1	2	3	4	5	6
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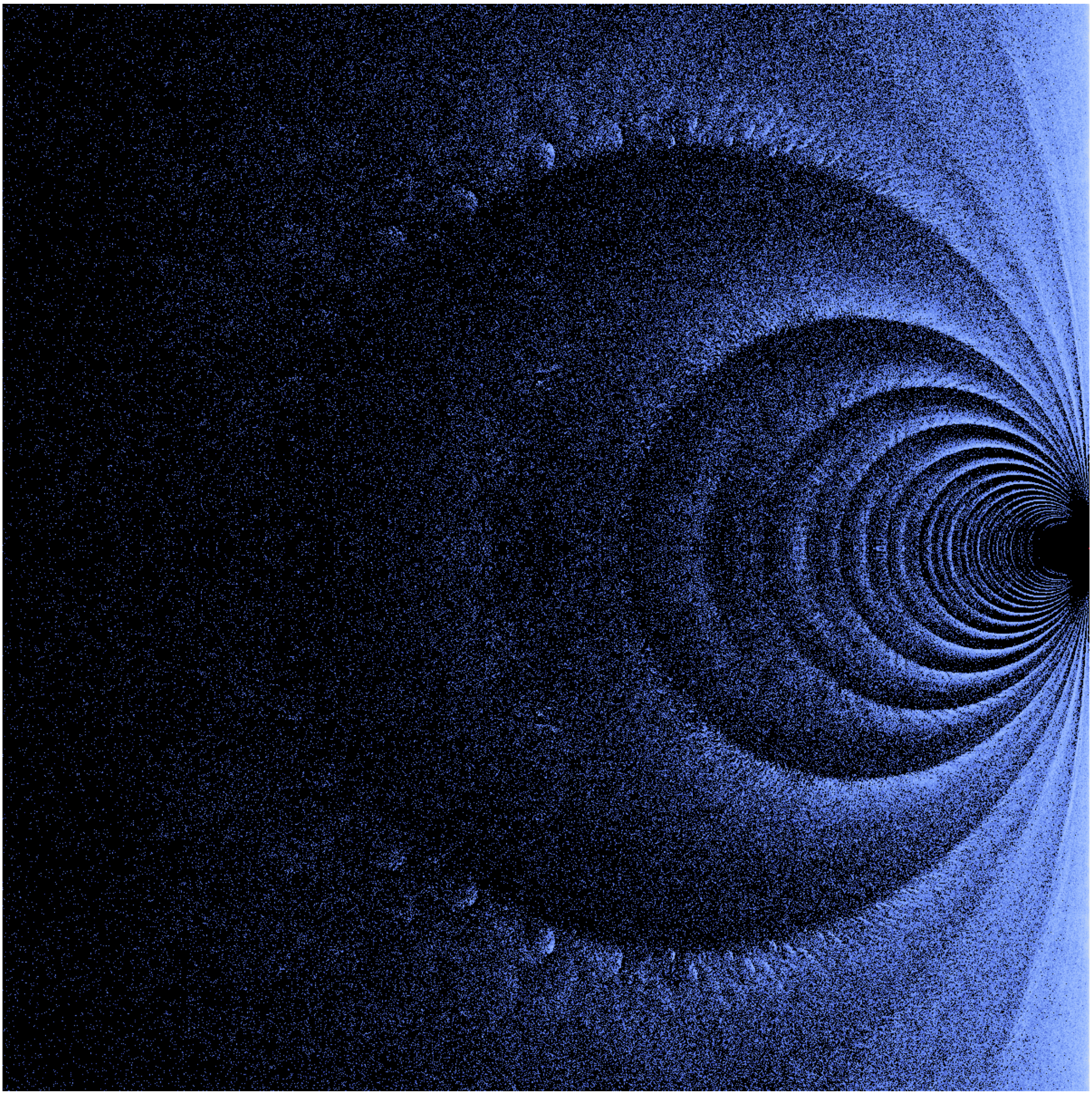

Symmetric matrices with population $-1 \pm i$, and dimension $m = 8$. We have zoomed to look at the region $-0.5 \le \Re(\lambda) \le 0$, $-0.25 \le \Im(\lambda) \le 0.25$. Computed a sample of 10^8 matrices from the over 68 billion possible. Are those really circles?

August 2022

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
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21	22	23	24	25	26	27
28	29	30	31			

July

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
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17	18	19	20	21	22	23
24	25	26	27	28	29	30
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September

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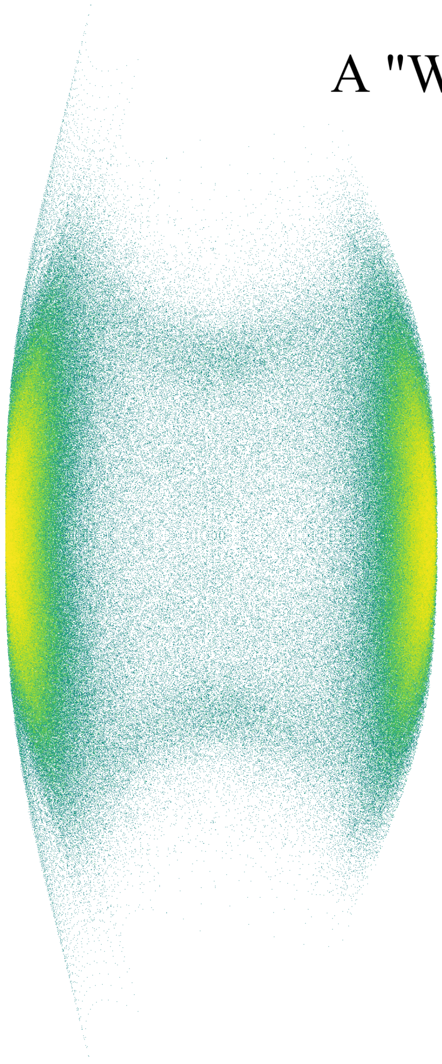
A "Wedge Hankel" matrix of order 5



$$\begin{bmatrix} E & F & G & H & 0 \\ F & G & H & 0 & -1 \\ G & H & 0 & -1 & 0 \\ H & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

"Wedge" Hankel matrices, sub-antidiagonal all -1, zero antidiagonal, population $\pm i/4$. Eigenvalues of all $131,072$ matrices of dimension $m = 18$. The asymmetry is correct. Can you explain it?
Image ©(2022) Robert M. Corless, generated in Maple™

September 2022

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4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

August

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October

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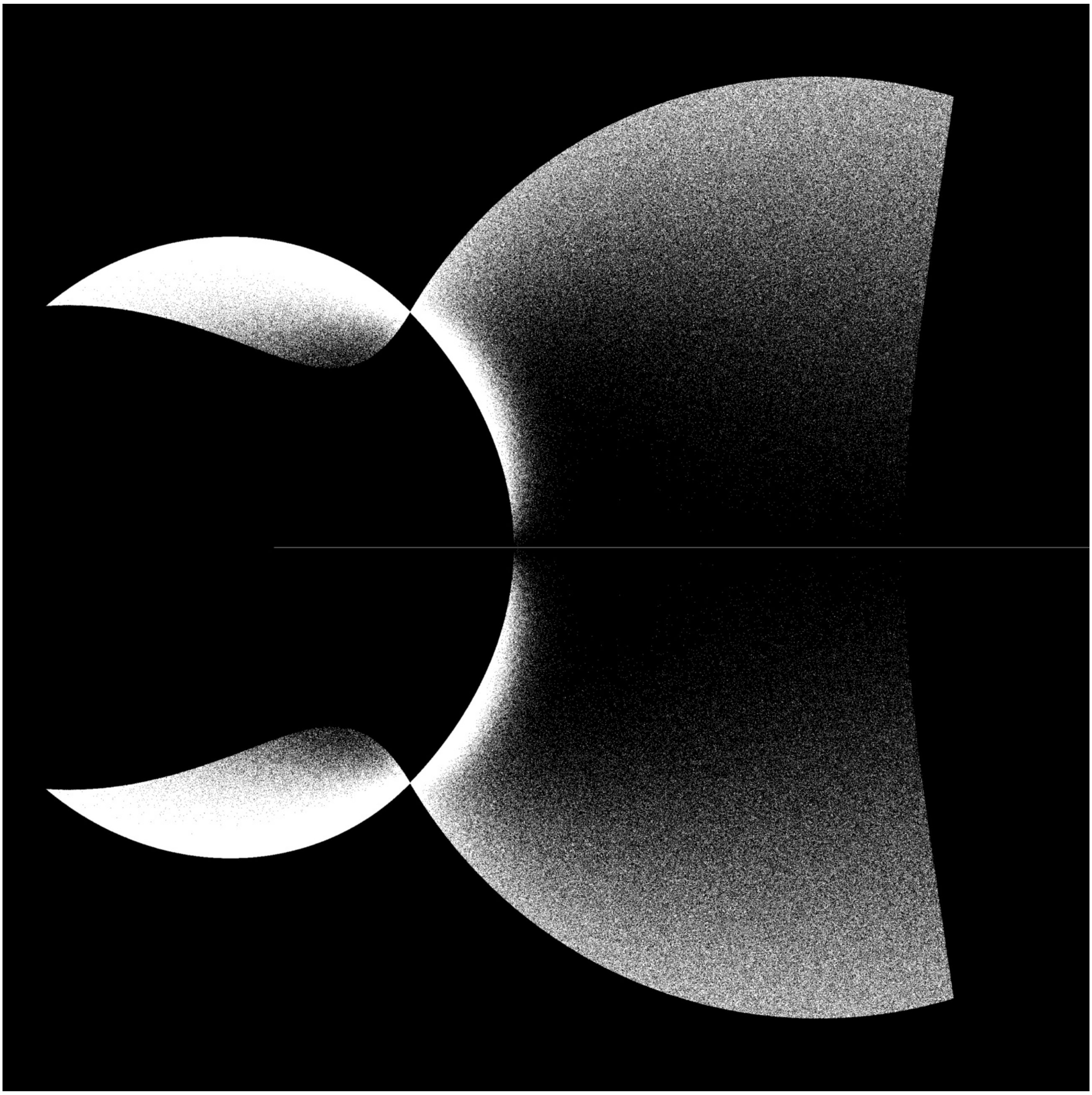

Eigenfish 2 from www.bohemianmatrices.com. "Eigenfish" are projections of the two-parameter algebraic surfaces of eigenvalues of two-parameter families of matrices, shaded by density. Can you guess what matrix this image is from?

October 2022

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						1
2	3	4	5	6	7	8
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September

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November

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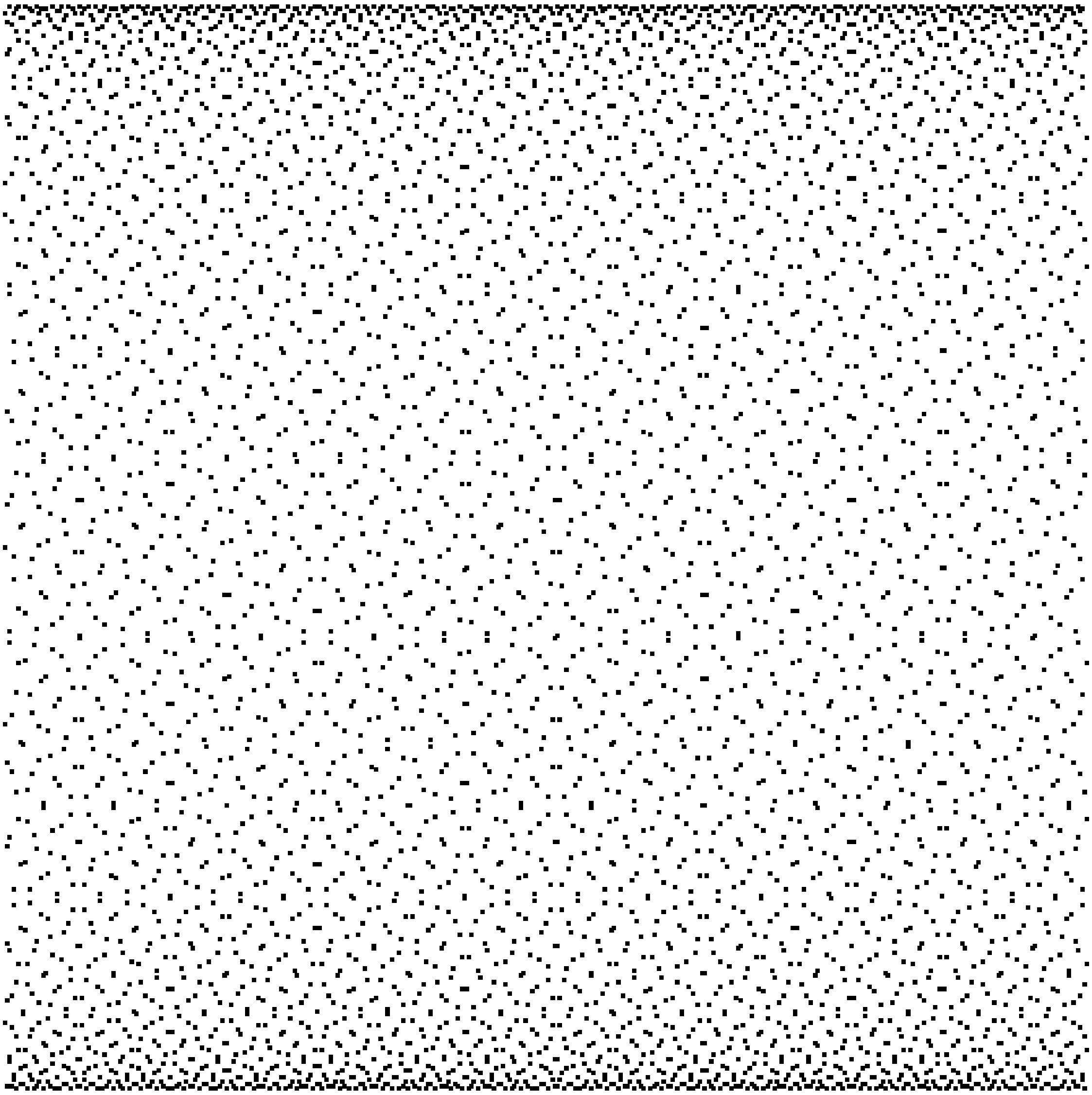

Model of the detailed relative error behaviour $n(H_n - A_n)/H_n$ in maximum characteristic height for skew-symmetric tridiagonal matrices with population $(1, i)$.
Also, a plot of $\sin 4\pi n/(5+\sqrt{5})$ discretely sampled at integers n in the range $0 \le n \le 5000$. Vertical axis is $-1 \le y \le 1$.
Can you guess the connection? See https://doi.org/10.5206/mt.v1i2.14360.

November 2022

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20	21	22	23	24	25	26
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October

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December

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				1	2	3
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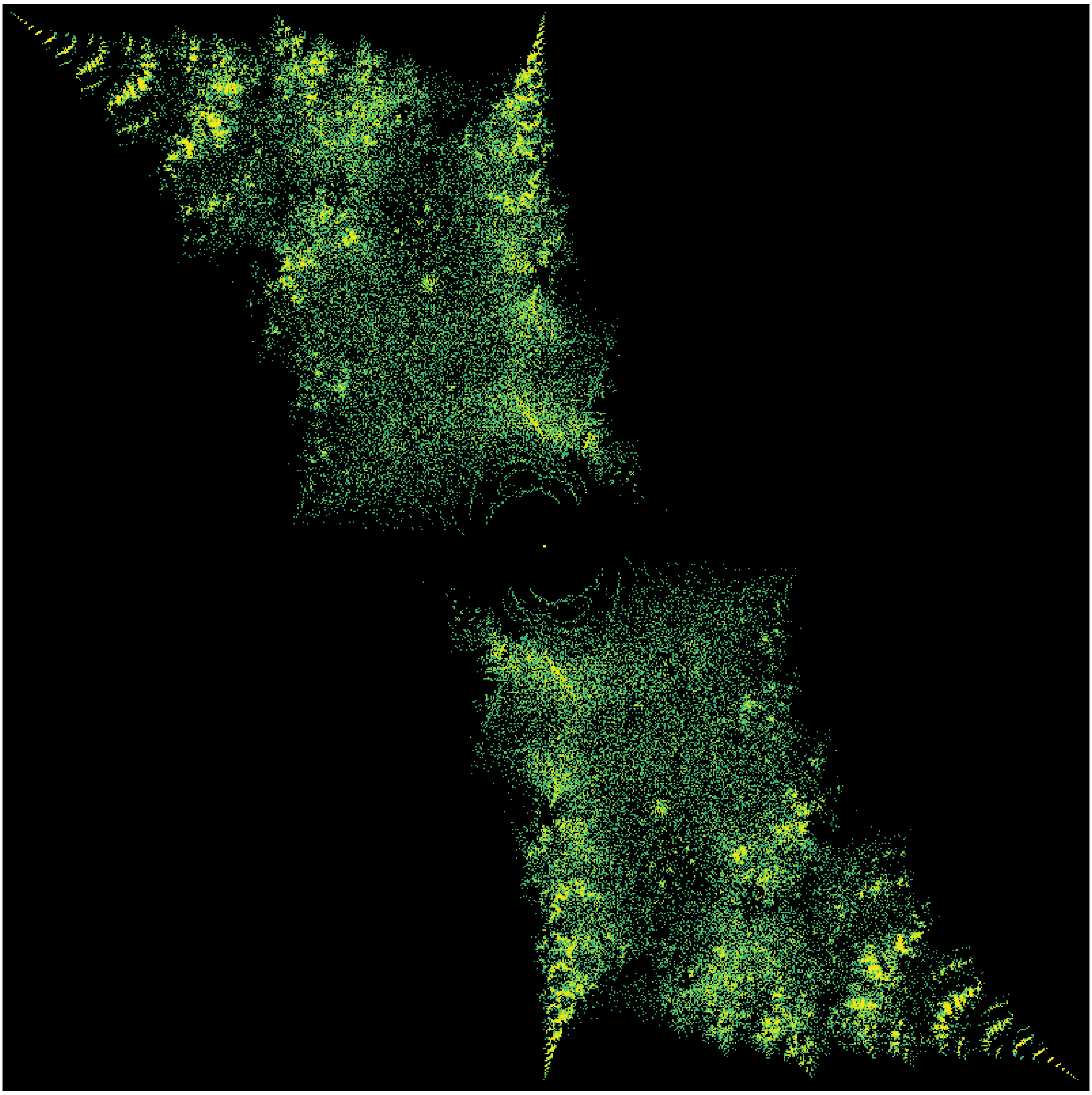

Skew-symmetric tridiagonal matrices with population $1, 1-i$; Eigenvalues of all $16,384$ dimension 15 such matrices. Can you bound the gap around the zero eigenvalue? See https://arxiv.org/abs/2109.03765. Image ©(2021) Robert M. Corless, generated in Maple™

December 2022

S	M	T	W	T	F	S
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November

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January

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22	23	24	25	26	27	28
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