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# Challenging a Newtonian prediction through Gaia wide binaries

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Under Newtonian dynamics, the relative motion of the components of a binary star should follow a Keplerian scaling with separation. Once orientation effects and a distribution of ellipticities are accounted for, dynamical evolution can be modelled to include the effects of Galactic tides and stellar mass perturbers, over the lifetime of the solar neighbourhood. This furnishes a prediction for the relative velocity between the components of a binary and their projected separation. Taking a carefully selected small sample of 81 solar neighbourhood wide binaries from the *Hipparcos* catalogue, we identify these same stars in the recent Gaia DR2, to test the prediction mentioned using the latest and most accurate astrometry available. The results are consistent with the Newtonian prediction for projected separations below 7000 AU, but inconsistent with it at larger separations, where accelerations are expected to be lower than the critical $a_0 = 1.2 \times 10^{-10}$ m s$^{-2}$ value of MONDian gravity. This result challenges Newtonian gravity at low accelerations and shows clearly the appearance of gravitational anomalies of the type usually attributed to dark matter at galactic scales, now at much smaller stellar scales.

*Keywords*: Gravitation; star binary; general.

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## 1. Introduction

In galactic dynamics, the range of systems over which gravitational anomalies appear in the low acceleration regime extends across vast astronomical scales. Ultra faint dwarf galaxies with scale radii of order a few tens of parsecs show stellar velocity dispersions implying Newtonian mass to light ratios in the hundreds or even thousands (e.g. Ref. 1). The classical dwarfs have sizes of order a kpc and mass to light ratios derived from observed stellar kinematics inconsistent with those of naked stellar populations under standard gravity by well over an order of magnitude (e.g. Ref. 2). This reflects what is observed in spiral galaxies at tens of kpc, where rotation curves (e.g. Ref. 3) again yield dynamics not corresponding to Newtonian dynamics given the empirically determined matter content. The trend has been extended to include elliptical galaxies observed out to their external low acceleration regions recently by Ref. 4, and even for the case of Galactic globular clusters where velocity dispersion profiles suggest a change away from Newtonian dynamics for low accelerations e.g. (Ref. 5, Ref. 6).

Empirically, the above gravitational anomalies can be described by MONDian dynamics (Ref. 7, Ref. 8), where below an acceleration threshold of $a_0 = 1.2 \times 10^{-10}$ m s$^{-2}$ kinematics stop falling along Newtonian expectations of $v \propto R^{-1/2}$ to flatten out at the Tully-Fisher values of $V_{TF} = (GMa_0)^{1/4}$ for centrifugal equilibrium velocities or corresponding velocity dispersions for pressure supported systems, where $M$ is the total baryonic mass of the system in question. The standard interpretation of this being the presence of dominant halos of a yet undetected hypothetical dark matter component surrounding the astrophysical systems being observed.

Wide binary pairs in the solar neighbourhood offer an opportunity to probe dynamics at low accelerations on the smallest astrophysical scales. In principle these can yield crucial restrictions on the structure of gravity at low accelerations and lengths where the presence of dark matter is not expected. For a solar mass binary, at separations of above $3.4 \times 10^{-2}$pc, 7000 AU, accelerations will fall below $a_0$ under Newtonian expectations. A first attempt in this direction was made by two of us in Ref. 9 where we used the Ref. 10 -henceforth SO11- carefully selected sample of *Hipparcos* wide binaries. This catalogue includes a full Bayesian model and use of local 5-dimensional phase space density to identify wide binary candidates and rigorously assign a probability that each candidate forms a physical system, rather than being the result of chance associations.

Retaining a sample of binaries from SO11 where contamination was limited to less than 10%, in Ref. 9 results indicated relative velocities for the binary pairs studied above the Newtonian expectations for accelerations below $a_0$. This remains true even after accounting for projection effects, ellipticity distributions and disruption and evolution of ionised binaries due to the Galactic tidal field and encounters with field stars and stellar remnants, as modelled by Ref. 11, albeit the relatively large errors in proper motions present in the *Hipparcos* catalogue.

One of us, in Ref. 12, explored the problem using a small sample of 60 candidate wide binaries with projected separations between 0.004 and 1.0 pc. That study found that a number of wide binaries are capable of surviving the galactic tides and stellar encounters of the solar neighbourhood dynamical environment, with a small sub-sample of the widest pairs showing kinematics more consistent with MONDian dynamics than Newtonian ones. More recently, theoretical studies by Ref. 13 and Ref. 14 confirmed that Gaia data, in terms of expected number of detected wide binaries and confidence intervals for the relevant proper motions and parallaxes, are sufficient to detect MONDian deviations from Newtonian dynamics in the low acceleration regime probed by these systems, should they be present.

The obvious next step is to reproduce the careful and detailed procedure presented in SO11, but using this time the Gaia DR2 catalogue. This painstaking approach will ultimately furnish a definitive answer regarding the presence of gravitational anomalies at stellar scales in the low acceleration regime, but is currently hampered by our incomplete understanding of the problems still present in the data of the very novel Gaia DR2. For example, only about two thirds of the Hipparcos2 sources have Gaia DR2 counterparts (Ref. 15). Also, Gaia treats all binaries closer than about 1" (depending on the magnitude difference) as single sources, which may give anomalous parallaxes, and the parallax solutions may be more sensitive to duplicity in certain areas, etc. (see Ref. 16 A1 for some of the known issues.)

A first order sampling of the issue can be more directly probed by taking advantage of the correspondence between the Gaia and the *Hipparcos* catalogues. One can take the sample selection from the accurate Bayesian analysis of SO11, and the actual astrometric data from the Gaia DR2. Here we present results of such an approach, yielding a small sample of 81 wide binaries from the original SO11 catalogue having a $< 10\%$ probability of being chance associations, Gaia DR2 data consistent with the original *Hipparcos* reported quantities, and consistent parallaxes for both components in the Gaia DR2. Although it is only a reduced sample, the superior quality of the Gaia satellite allows us to infer relative velocities for the binaries in question to a much higher degree of accuracy than what was available to Ref. 9. Interestingly, our results show a departure from Newtonian predictions as projected separations grow beyond the critical 0.034 pc. Indeed, our new results are consistent with what was reported in Ref. 9: the mean values of the inferred relative velocities are essentially unchanged, with the error bars showing a dramatic reduction. This effectively rules out the Newtonian prediction of Ref. 11 and provides solid evidence for a gravitational anomaly in the low acceleration regime, this time at stellar scales.

## 2. Sample selection

As outlined in the introduction, the sample selection is based on the wide binary catalogue of SO11, which was constructed using a very detailed Bayesian procedure. This identifies and quantifies the probability of each binary pair being an actual physical system, rather than the result of projection effects or chance associations, including also the Tycho-2 and the *Tycho* double star catalogues (Ref. 17 and Ref. 18). To that end a 5-dimensional space of spatial positions and proper motions was cross-correlated with a galactic phase-space density library, explicitly excluding the largest known local star clusters, to identify binary candidates as significant local over-densities in phase space. Corrections due to spherical projections effects were also considered, to yield a catalogue of 840 wide binaries with projected separations of between 0.003 and 10 pc and, crucially, a well determined probability of chance association, $P_{ch}$. Taking only those pairs where this probability satisfies $P_{ch} < 10\%$, reduces the original SO11 sample to 359 wide binaries. This catalogue is also narrowly restricted in spectral type for both primaries and companions of each binary, yielding stars in a narrow range of masses centred on $0.5 M_{\odot}$. This last is important as it allows a clean comparison to the fixed mass binary simulations of Ref. 11, see below. Following the original SO11 terminology, the brightest star in each binary system is termed the primary.

The SO11 search criteria ensure the absence of near neighbours, and results in binary candidates with separations which are always many times smaller than the typical interstellar separations at the location of the binaries in question. Extensive testing with synthetic samples in SO11 guarantees the catalogue includes very few multiple systems with undetected extra companions and is highly complete in the 6 to 100 pc distance range from the Sun.

We now take advantage of the *Hipparcos* to Gaia DR2 correspondence availability to search for the updated astrometry of the 359 $P_{ch} < 10\%$ wide binary SO11 sample in the Gaia DR2. The search returns only 151 pairs where each component of the SO11 binaries appears in the Gaia DR2 with two proper motion parameters and measured parallax. This is not surprising, since only about two thirds of the Hipparcos2 sources have Gaia DR2 counterparts (Ref. 15). It is not yet clear why there are so many sources missing. According to the above authors a combination of effects may be present. As each binary is excluded if either component is absent from the DR2, the fraction we obtain is typical. Next, the SO11 catalogue returns a few systems where more than one secondary is identified as the companion to a given primary, and also cases where a single secondary is identified as the companion to more than one primary. We remove all these cases of multiple identifications, bringing the sample down to 131 binaries.

A first test of the reliability with which the *Hipparcos* binaries have been identified in the Gaia DR2 comes from comparing the proper motion measurements reported in SO11 with the corresponding measurements reported in the Gaia DR2. This is shown in Figures 1 and 2, where the ranges shown in the axes were chosen so as to display clearly most of our sample; a handful of very discordant systems do not appear in the plots, as they are very far from the identity line shown. It is clear that in most cases, the proper motion values reported by both satellites are in agreement with each other, to within their respective confidence intervals, the *Hipparcos* error ranges being much larger than the Gaia DR2 ones. Still, we introduce a cut to remove from consideration any binary candidate where for any component the *Hipparcos* and Gaia DR2 data are more than $3\sigma$ from each other. Our final results are not sensitive to this threshold, provided the few discordant misidentified binaries are removed. This cut leaves us with only 117 stellar pairs. Next, as shown in Figure 3, we check that the Gaia DR2 reported parallax measurements for each of the primaries and companions are not discordant, and remove from the sample any binary where the distances to each component are not within 13% of each other. This exclusion criterion identifies 17 candidates, thus reducing the sample to 100 pairs. Notice that the test shown in Figures 1 and 2 effectively gives us a 10 year baseline which, on top of the robustness in the SO11 catalogue towards multiple systems, where no radial velocities are involved, removes any remaining binary where a third component might be altering proper motions with timescales shorter than 10 years (see Ref. 14).

Finally, we take advantage of the Gaia radial velocity measurements (when available) and remove any binaries where the radial velocity difference, $\Delta V_r$, between both components is larger than 4 km s$^{-1}$. The resulting cut is not very sensitive to this velocity threshold, as the removed binary pairs typically have much larger and discordant $\Delta V_r$ values, with an average value for those removed of $\Delta V_r = 28$ km s$^{-1}$. Our final sample comprises 81 binary pairs.

Thus, we have prefered very strict cuts to our final sample which leave us with modest numbers, but guarantee the exclusion of misidentified stars in going from *Hipparcos* data to Gaia DR2 and chance alignment contamination in the original SO11 catalogue, all of which become conspicuous in the comparisons presented in this section. Table 1 summarises the Gaia DR2 properties used and catalogue numbers from both *Hipparcos* and Gaia for the primaries and companions of all the binaries used, together with the results of the exclusion criteria described.

## 3. Gaia wide binary projected kinematic results

In Ref. 9 we calculated the projected separation in the plane of the sky using only the parallax to the primary of each binary, but given the higher quality Gaia DR2 data, we now compute the projected separation between the components of each binary using explicitly the observed Gaia positions and parallaxes to each component of the binary. The average parallax to both components is used to gauge the distance to each pair. Using reported Gaia proper motions, the relative velocity difference in one dimension is calculated for each binary twice, once considering only right ascension proper motions, and once considering only declination proper motions. In all cases the relative physical motion is calculated including full spherical geometric effects (e.g. Ref. 19), and not under the more standard small angle approximation. This requires the radial velocity of at least one component, which we have for 71 of our 81 final pairs. For the remaining 10, a $V_r = 0$ was assumed for the effects of the above correction (e.g. SO11). The effects of the above refinement are only relevant for the nearest and widest of pairs, in our sample only a few individual systems, as can be readilly cheched from the data used and shown in table 1.

Figure 4 gives the two $\Delta v_{1D}$ measurements for each binary pair in the final sample, with corresponding $1\sigma$ error bars. A clear flat upper envelope is evident. In Figure 5 we show the rms. value for the one dimensional velocity differences described above, plotted against projected separations in a binned logarithmic scale, circles and triangles for right ascension and declination data, respectively. The horizontal error bars give the bin sizes, while the vertical ones show the contribution of Gaia reported errors and error propagation, to which a Poisson contribution has been added, and which given the small numbers of binaries in each bin (21, 23, 17, 8 and 12, from left to right), actually dominates the error budget in most cases. The dashed vertical line appears at 7000 AU, the approximate scale where acceleration is expected to drop below $a_0$.

Also shown in Figure 5 are the Newtonian predictions for this same quantity from Ref. 11, where large collections of 50,000 simulated binaries are modelled for a range of plausible distributions of ellipticities, and followed dynamically under Newtonian expectations within the local Galactic tidal field. These are also subject to the effects of field star and field stellar remnant bombardment for a 10 Gyr period. Finally, the resulting bound and un-bound stellar pairs are projected along a fiducial line of sight to yield a robust prediction for the expected $< \Delta v_{1D}^2 >^{1/2}$ as a function of $\Delta r$, solid curve. This results are easy to understand; a $\Delta v \propto \Delta r^{-1/2}$ trend is apparent, down to the tidal radius of the problem which appears at 1.7 pc. Beyond this point, ionised binaries continue to move along practically common Galactic orbits, with relative velocities which show a mild enhancement which then levels off at close to 0.1 km s$^{-1}$.

It is clear that to the left of the $a_0$ dashed line, our results are consistent within confidence intervals with the Newtonian expectations. However, in going to separations larger than 7000AU, the observed points stop following the expected trend and actually level off at a $\Delta v$ amplitude close to the values seen at the $a_0$ point, reproducing qualitatively the phenomenology seen in galactic rotation curves.

This result for the binary sample presented is extremely challenging to a Newtonian point of view, where the relative velocities are expected to be much lower than observed. Given the construction of the SO11 sample, binaries with small velocity differences would appear as stronger local over densities in phase space, and hence, selection criteria, if anything, are biased against binaries with large velocity differences, not small ones. Thus, from a Newtonian point of view, bound binaries with separations smaller that the tidal radius of 1.7 pc and larger than 7000 AU are unexpectedly missing. Also, as already mentioned in Ref. 20, a population of nonchance associated binaries appears at scales above 7000AU having relative velocities over an order of magnitude above bound expectations, with relative velocities of $\sim$ 1 km s$^{-1}$ and separations of a few pc, the dynamical ages of un-bound systems are of only a few $10^6$ years. What sustains and replenishes these populations under a Newtonian framework? At separations below the 1.7 pc tidal radius of the problem, bound binaries should appear, under a Newtonian framework.

Furthermore, the results shown in figure 5 confirm what was presented in Ref. 9, though the much coarser *Hipparcos* data of that first study yielded significantly larger error bars. That those first results might have been the result of missed biases or simply the error structure of the *Hipparcos* data now appears very unlikely, as we see two consistent results coming from data obtained by two completely independent satellites. Indeed, the error bars have significantly shrunk, with central results changing little. Note also that the two $\Delta v_{1D}$ estimates we obtain, using only right ascension or only declination data, are consistent with each other.

A potential caveat on the interpretation presented comes from the possible presence of a population of misidentified binaries being actually parts of loose, dissolving moving groups. Ref. 20 recently showed that although isolated wide binaries dominate, a search algorithm of the type used in SO11 could also pick up a fraction of misidentified binaries being part of larger moving groups, many of the smallest of which probably remain undiscovered. A full answer and validation or otherwise of our results, necessarily awaits a much more extensive study through the full Gaia catalogue, and a fuller exploration and understanding of the phase-space structure and over-densities of the Solar Neighbourhood and local Milky Way disk environment.

Although the gravitational anomaly detected appears on crossing the $a_0$ threshold of MOND, in MOND as such, the results are equally unexpected as the external field effect of MOND (or AQUAL e.g. Ref. 8 should dominate. Given that the orbital acceleration of the solar neighbourhood is higher than the internal acceleration of the binaries in question, in MOND as such, only a very modest enhancement of the effective value of G would be expected (e.g. Ref. 13). Thus, within a MONDian frame our results imply not the most well studied version, but rather a variant where the external field effect does not appear, or is much suppressed e.g. as in Ref. 21. In terms of covariant extensions to GR having a MONDian low velocity limit, it is hard to know to what extent an external field effect might be present in many of the recently developed options (e.g. the f(R) variants reviewed in Ref. 22, the emergent gravity of Ref. 23 or the F(R,L) proposal of Ref. 24, so our results then serve as constraints in terms of requiring a minimal external field effect, at least for the sub-parsec scales in the solar neighbourhood explored.

## 4. Final remarks

We have presented a sample of 81 wide binaries which were very carefully selected against chance associations or projection effects through the cross correlation of the

*Hipparcos*, Tycho-2 and the *Tycho* double star catalogues, amongst others, with the detailed 5-dimensional phase space structure of the solar neighbourhood by SO11. By taking advantage of the cross-identification of the *Hipparcos* catalogue and the Gaia DR2 data, we updated the parallax and proper motion observations of SO11 to use exclusively Gaia DR2 astrometry.

These binaries are then compared to Newtonian predictions for the expected one dimensional rms. relative velocity between the components of each binary and their projected separations, including modeling orientation effects, a number of plausible distributions of ellipticities and, crucially, the effects of Galactic tides and stellar and stellar remnant perturbers over a 10 Gyr period, by Jiang & Tremaine (2010).

For separations below 7000 AU, where accelerations are expected to be above the $a_0 = 1.2 \times 10^{-10}$ m s$^{-2}$ of MOND, we find the data to be consistent with the Newtonian predictions. For projected separations above 7000 AU however, the data are inconsistent with Newtonian predictions. This challenges the validity of Newtonian dynamics at the low acceleration regime, and shows the existence of gravitational anomalies of the type generally attributed to the presence of a hypothetical and dominant dark matter component, this time down to the relatively tiny sub-parsec stellar scales.

Table 1. Comparison of acoustic for frequencies for piston-cylinder problem.

| Piston mass | Analytical frequency (Rad/s) | TRIA6-$S_1$ model (Rad/s) | % Error |
|---|---|---|---|
| 1.0 | 281.0 | 280.81 | 0.07 |
| 0.1 | 876.0 | 875.74 | 0.03 |
| 0.01 | 2441.0 | 2441.0 | 0.0 |
| 0.001 | 4130.0 | 4129.3 | 0.16 |

If tables need to extend over to a second page, the continuation of the table should be preceded by a caption, e.g. "*Table 2.* (*Continued*)".

| Index | HIP2 | GDR2 | $\alpha$ | $\delta$ | $\mu_\alpha$ | $\mu_\delta$ | $v_r$ | $d$ | $\log_{10}(\Delta R)$ | Exclusion |
|---|---|---|---|---|---|---|---|---|---|---|
| SO11 | | | (deg) | | (mas/yr) | | (km/s) | (pc) | | Test |
| 16 | 15371 | 4722135642226356736 | 49.565826 | -62.503574 | 1331.151±0.355 | 648.523±0.431 | 12.01± 0.32 | 12.046±0.027 | | |
| | 15330 | 4722111590409480064 | 49.454851 | -62.572523 | 1337.591±0.142 | 649.930±0.154 | 12.21± 0.17 | 12.039±0.011 | -1.744±0.002 | |
| 17 | 17414 | 43335880716390784 | 55.969727 | 16.670667 | 157.945±0.086 | -316.326±0.052 | 34.18± 0.15 | 17.209±0.012 | | |
| | 17405 | 43335537119008896 | 55.939232 | 16.665933 | 156.215±0.089 | -310.291±0.064 | 34.22± 1.08 | 17.239±0.014 | -2.051±0.001 | |
| 21 | 19859 | 3285218186904332288 | 63.869527 | 6.186399 | -109.700±0.095 | -107.368±0.079 | -7.21± 0.15 | 22.087±0.026 | | |
| | 19855 | 3285218255623808640 | 63.857006 | 6.199170 | -101.763±0.080 | -111.982±0.055 | -7.93± 0.16 | 22.104±0.022 | -2.163±0.001 | |
| 22 | 23693 | 4763906879239461632 | 76.377477 | -57.472197 | -32.140±0.276 | 117.417±0.310 | -1.15± 0.22 | 11.625±0.020 | | |
| | 23708 | 4763897739549071744 | 76.447122 | -57.553318 | -32.784±0.049 | 119.633±0.054 | -0.88± 0.16 | 11.698±0.003 | -1.740±0.001 | |
| 25 | 25278 | 3400292798990117888 | 81.107227 | 17.383504 | 250.765±0.316 | -7.332±0.209 | 37.67± 0.24 | 14.585±0.037 | | |
| | 25220 | 3394298532176344960 | 80.911047 | 17.324092 | 251.000±0.092 | -5.778±0.068 | 37.94± 0.13 | 14.565±0.011 | -1.301±0.001 | |
| 28 | 26690 | 3395863205942142976 | 85.075658 | 15.350146 | 2.551±0.100 | -36.058±0.080 | | 167.953±2.110 | | |
| | 26844 | 3347826784173590656 | 85.495633 | 15.337041 | 79.115±0.087 | -43.071±0.068 | | 22.423±0.025 | -0.172±0.010 | c |
| 29 | 26779 | 263916708025623680 | 85.334752 | 53.478804 | 2.784±0.075 | -523.602±0.072 | 1.07± 0.18 | 12.280±0.007 | | |
| | 26801 | 263916742385357056 | 85.378072 | 53.487581 | 3.915±0.078 | -515.938±0.078 | 1.92± 0.27 | 12.278±0.008 | -2.234±0.001 | |
| 65 | 62229 | 6060965699625586176 | 191.308450 | -57.358567 | -201.033±0.052 | -131.548±0.046 | 14.76± 0.16 | 19.808±0.018 | | |
| | 69570 | 6092573252981419776 | 213.602784 | -48.147162 | -102.218±0.095 | -121.897±0.120 | -23.92± 0.13 | 37.842±0.087 | 0.910±0.002 | c,d |
| 73 | 70529 | 1254695603704323712 | 216.434870 | 23.612274 | 792.548±0.092 | -1116.601±0.111 | 8.78± 0.21 | 16.346±0.014 | | |
| | 70536 | 1254694882149817728 | 216.448113 | 23.615657 | 793.487±0.085 | -1119.010±0.095 | | 16.339±0.014 | -2.445±0.004 | |
| 80 | 79755 | 1642641410934267008 | 244.172569 | 67.239204 | -498.018±0.050 | 84.110±0.053 | -19.86± 0.19 | 10.768±0.004 | | |
| | 79762 | 1642642957122493824 | 244.183431 | 67.256628 | -483.168±0.066 | 89.266±0.072 | | 10.765±0.005 | -2.473±0.001 | |
| 81 | 80337 | 6018047019138644480 | 246.005789 | -39.192966 | 74.146±0.306 | 3.666±0.226 | 12.89± 0.13 | 12.908±0.023 | | |
| | 80300 | 6018034958869558912 | 245.891422 | -39.229487 | 77.135±0.147 | 0.334±0.110 | | 12.914±0.012 | -1.666±0.001 | |
| 85 | 83591 | 4364527594192166400 | 256.260183 | -5.071414 | -916.562±0.155 | -1138.804±0.104 | 34.14± 0.15 | 10.466±0.007 | | |
| | 83599 | 4364480521350598144 | 256.303450 | -5.099110 | -917.276±0.098 | -1131.947±0.065 | 34.44± 0.44 | 10.456±0.006 | -2.029±0.001 | |
| 87 | 84405 | 4109030160308317312 | 258.835189 | -26.607739 | -466.541±0.646 | -1142.063±0.451 | | 5.960±0.008 | | |
| | 84478 | 4109034455276324608 | 259.053369 | -26.550990 | -479.850±0.101 | -1124.545±0.068 | -0.04± 0.22 | 5.950±0.003 | -1.675±0.001 | b |
| 112 | 493 | 2797111130991722240 | 1.477436 | 18.234370 | -150.936±0.121 | -147.985±0.089 | -45.70± 0.16 | 37.429±0.102 | | |
| | 495 | 2773086595766697856 | 1.480470 | 18.075245 | -147.614±0.094 | -145.798±0.079 | -45.00± 0.74 | 37.683±0.085 | -0.982±0.002 | |
| 130 | 7699 | 4911275281704066048 | 24.782478 | -56.429512 | 91.146±0.041 | -31.652±0.041 | 9.48± 0.31 | 46.855±0.061 | | |
| | 6485 | 4909846500703006976 | 20.839305 | -57.480902 | 92.790±0.045 | -36.082±0.038 | 8.62± 0.21 | 45.315±0.053 | 0.284±0.001 | |
| 132 | 9487 | 2517584007848935808 | 30.511899 | 2.763759 | 32.692±0.940 | -2.895±0.821 | | 50.506±1.707 | | b |
| | 9519 | 2517585927699042944 | 30.611532 | 2.815808 | 36.368±0.126 | -10.811±0.098 | 4.58± 0.16 | 48.404±0.137 | -1.014±0.016 | |
| 140 | 11477 | 4967177781457918976 | 37.007194 | -33.811016 | 18.526±0.121 | 5.163±0.193 | | 47.258±0.315 | | |
| | 11448 | 4967153630858709120 | 36.926542 | -33.896542 | 18.383±0.030 | 4.481±0.052 | 12.26± 0.17 | 47.708±0.075 | -1.046±0.004 | b |
| 155 | 15304 | 10584899657116672 | 49.360666 | 7.655784 | 166.360±0.071 | -6.237±0.062 | 31.35± 0.20 | 47.288±0.092 | | |
| | 15310 | 10608573516849536 | 49.387317 | 7.690127 | 168.749±0.073 | -6.314±0.063 | 31.90± 0.15 | 47.269±0.088 | -1.447±0.002 | |
| 157 | 15527 | 5060104351007433472 | 50.016623 | -28.854353 | 349.057±0.036 | -65.299±0.041 | 39.89± 0.13 | 36.024±0.041 | | |
| | 15526 | 5060105892897388288 | 50.013976 | -28.784118 | 348.847±0.065 | -66.663±0.076 | 40.31± 0.14 | 36.009±0.052 | -1.355±0.001 | |
| 173 | 22534 | 4777112872882315264 | 72.734825 | -53.459330 | -80.769±0.929 | 85.658±0.968 | 6.44± 3.13 | 38.970±0.624 | | |
| | 22562 | 4777119126354782592 | 72.835038 | -53.405518 | -82.413±0.043 | 83.684±0.052 | 10.70± 0.49 | 38.006±0.032 | -1.268±0.008 | d |
| 175 | 24046 | 3422042582096699520 | 77.517088 | 27.556119 | 197.860±0.093 | -89.579±0.056 | 15.64± 0.16 | 40.470±0.074 | | |
| | 24035 | 3422047495539178496 | 77.504564 | 27.642799 | 198.179±0.596 | -88.591±0.357 | 21.24± 0.43 | 40.295±0.457 | -1.210±0.006 | a,b,d |
| 187 | 33705 | 5607190344506642432 | 105.040814 | -31.141532 | 18.789±0.043 | 35.596±0.049 | 16.61± 0.17 | 38.123±0.043 | | |
| | 33691 | 5607189485513198208 | 105.011544 | -31.227929 | 18.893±0.061 | 36.119±0.065 | 16.83± 0.21 | 38.356±0.061 | -1.222±0.001 | |
| 190 | 34714 | 890422213103244544 | 107.830615 | 33.111833 | -110.510±0.075 | -7.495±0.066 | 3.05± 0.24 | 45.125±0.095 | | |
| | 34700 | 890346243721923968 | 107.810770 | 32.614987 | -110.866±0.102 | -6.789±0.088 | 3.02± 0.33 | 44.748±0.143 | -0.409±0.002 | |
| 195 | 37718 | 5493209501673364736 | 116.051359 | -50.456108 | -114.435±0.047 | 143.459±0.042 | 8.69± 0.37 | 30.084±0.022 | | |
| | 37727 | 5493209437253410432 | 116.067894 | -50.466001 | -111.783±0.060 | 142.603±0.053 | 9.16± 0.30 | 30.080±0.027 | -2.120±0.002 | |
| 201 | 42401 | 5746824674801810816 | 129.688303 | -13.256527 | -63.858±0.051 | 38.374±0.041 | 20.53± 0.25 | 31.596±0.032 | | |
| | 41662 | 5751951182125903872 | 127.418304 | -9.976270 | -56.556±0.064 | 34.122±0.050 | 2.30± 0.24 | 35.328±0.055 | 0.364±0.001 | d |
| 204 | 43970 | 610526719204475136 | 134.312562 | 15.322854 | 60.337±0.292 | 19.748±0.221 | | 48.401±0.415 | | |

Continued on next page

| Index | HIP2 | GDR2 | $\alpha$ | $\delta$ | $\mu_\alpha$ | $\mu_\delta$ | $v_r$ | $d$ | $\log_{10}(\Delta R)$ | Exclusion |
|---|---|---|---|---|---|---|---|---|---|---|
| SO11 | | | (deg) | | (mas/yr) | | (km/s) | (pc) | | Test |
| | 44001 | 610549499710989440 | 134.396934 | 15.581374 | 60.440±0.161 | 20.509±0.106 | | 48.801±0.233 | -0.638±0.006 | |
| 207 | 44858 | 692119656035933568 | 137.099198 | 27.535750 | -53.239±0.128 | 71.659±0.096 | 30.02± 0.22 | 48.891±0.237 | | |
| | 44864 | 692120029700390912 | 137.112900 | 27.543447 | -51.819±0.113 | 73.524±0.074 | 30.31± 0.14 | 49.124±0.230 | -1.910±0.007 | |
| 215 | 51312 | 749786562715192320 | 157.213545 | 34.885408 | -111.186±0.069 | -62.017±0.069 | 4.11± 0.17 | 49.344±0.126 | | |
| | 52140 | 748360706587700352 | 159.782027 | 32.832960 | -110.338±0.106 | -58.939±0.087 | -0.60± 0.17 | 50.382±0.138 | 0.411±0.002 | d |
| 218 | 52787 | 3550081879381593728 | 161.879229 | -22.348154 | -124.690±0.115 | -28.341±0.084 | 23.85± 0.25 | 33.612±0.085 | | |
| | 52776 | 3550084490721711872 | 161.855204 | -22.286843 | -124.539±0.112 | -29.837±0.070 | 23.24± 0.52 | 33.603±0.079 | -1.417±0.002 | |
| 224 | 55765 | 3967618155853506304 | 171.400912 | 16.456513 | -142.347±0.204 | -5.645±0.179 | 18.88± 0.67 | 47.198±0.296 | | |
| | 55262 | 3965063921622777856 | 169.776912 | 13.390754 | -146.589±0.080 | -0.006±0.069 | 8.33± 0.21 | 47.472±0.093 | 0.454±0.004 | d |
| 229 | 58067 | 3975129194660883328 | 178.631559 | 19.411157 | -450.502±0.095 | -16.554±0.069 | 5.94± 0.29 | 39.622±0.082 | | |
| | 58073 | 3975223065466473216 | 178.644111 | 19.427759 | -450.600±0.083 | -15.497±0.058 | 5.76± 0.22 | 39.601±0.071 | -1.851±0.002 | |
| 230 | 58085 | 5236197322996128128 | 178.686141 | -66.376482 | 13.792±0.037 | -65.816±0.036 | -6.80± 0.37 | 44.078±0.047 | | b |
| | 58121 | 5236196498362394112 | 178.814952 | -66.409354 | 12.724±0.048 | -63.279±0.045 | -7.63± 0.13 | 44.016±0.063 | -1.328±0.001 | |
| 245 | 64057 | 3945118265299248128 | 196.914217 | 24.010449 | -261.638±0.077 | 148.003±0.059 | -1.68± 0.15 | 37.442±0.053 | | |
| | 64059 | 3945118643256370688 | 196.918496 | 24.020558 | -262.455±0.078 | 146.042±0.054 | -1.70± 0.13 | 37.433±0.051 | -2.150±0.003 | |
| 253 | 67246 | 3721126409323324416 | 206.735799 | 6.349899 | -510.447±0.071 | -110.225±0.064 | -30.42± 0.20 | 31.489±0.039 | | |
| | 67291 | 3721114933170707328 | 206.867792 | 6.315179 | -509.440±0.083 | -111.022±0.061 | -30.67± 0.15 | 31.332±0.045 | -1.128±0.001 | |
| 264 | 73365 | 1586977844504488576 | 224.886978 | 45.464606 | -33.680±0.043 | 100.046±0.044 | -18.94± 0.14 | 33.745±0.027 | | |
| | 73360 | 1586977737129182848 | 224.877247 | 45.448463 | -35.321±0.037 | 101.074±0.039 | -18.99± 0.26 | 33.768±0.024 | -1.986±0.002 | |
| 270 | 74666 | 1278391075716738560 | 228.876113 | 33.314351 | 82.549±0.554 | -111.909±0.560 | | 37.342±0.531 | | a |
| | 74674 | 1278392381386793856 | 228.910214 | 33.320448 | 82.776±0.040 | -110.040±0.044 | -11.83± 0.18 | 36.948±0.042 | -1.724±0.009 | |
| 271 | 75104 | 1277115023753077248 | 230.207752 | 31.480707 | -179.339±0.036 | 139.058±0.039 | -26.82± 0.17 | 45.885±0.055 | | |
| | 75011 | 1277185633015465856 | 229.916339 | 31.843109 | -181.021±0.033 | 139.573±0.042 | -26.32± 0.19 | 45.433±0.057 | -0.456±0.001 | |
| 284 | 79958 | 5931674608438449792 | 244.816099 | -55.504655 | 5.669±0.067 | 18.412±0.051 | -0.31± 0.52 | 28.113±0.029 | | |
| | 80365 | 5831891213733627264 | 246.077754 | -59.344070 | -3.205±0.063 | -5.572±0.052 | | 7649.524±2501.324 | 2.417±0.283 | c |
| 298 | 85620 | 1440518669436791296 | 262.435226 | 63.851870 | 1.533±0.049 | -182.061±0.053 | -34.02± 0.16 | 45.738±0.051 | | |
| | 85575 | 1440425863783337856 | 262.319088 | 63.868353 | 0.864±0.053 | -181.967±0.060 | -33.72± 0.18 | 45.663±0.060 | -1.368±0.001 | |
| 309 | 92388 | 4505477838068264064 | 282.408880 | 13.217640 | -199.313±0.096 | -223.979±0.102 | -54.54± 0.21 | 37.760±0.068 | | |
| | 92638 | 4312046495498949888 | 283.121483 | 11.209184 | -189.830±0.070 | -224.477±0.079 | -22.45± 0.90 | 38.302±0.065 | 0.150±0.002 | d |
| 314 | 94150 | 6421542154150684160 | 287.472100 | -68.424635 | 155.088±0.122 | -42.976±0.141 | -9.61± 0.12 | 36.982±0.122 | | |
| | 94154 | 6421555485729063424 | 287.479128 | -68.299872 | 155.710±0.036 | -41.937±0.046 | -9.28± 0.13 | 36.655±0.041 | -1.096±0.002 | |
| 322 | 97384 | 4240508901699614976 | 296.888713 | 1.087910 | -28.713±0.067 | -226.754±0.046 | | 47.116±0.093 | | |
| | 95829 | 4287506873404614784 | 292.359656 | 0.527119 | -25.932±0.072 | -237.555±0.070 | | 44.067±0.083 | 0.560±0.002 | |
| 323 | 97940 | 4240626686883261184 | 298.562299 | 1.942265 | -1.562±0.085 | -270.206±0.052 | 9.55± 0.16 | 40.151±0.097 | | |
| | 97950 | 4240625896609242624 | 298.607267 | 1.940219 | -0.186±0.077 | -269.831±0.047 | 10.20± 0.20 | 40.047±0.087 | -1.502±0.002 | |
| 325 | 99171 | 4236276194243977344 | 302.008090 | -0.678476 | 115.520±0.095 | -67.593±0.060 | -3.27± 0.14 | 46.678±0.137 | | |
| | 99100 | 4235895935019949952 | 301.788725 | -0.874544 | 115.276±0.104 | -67.915±0.050 | -3.08± 0.14 | 46.633±0.144 | -0.621±0.003 | |
| 326 | 99572 | 6879764552737781888 | 303.108646 | -12.618340 | 193.068±0.111 | -195.497±0.064 | 27.30± 0.16 | 28.249±0.054 | | |
| | 99550 | 6879662263797806976 | 303.040242 | -12.893999 | 192.627±0.068 | -193.957±0.045 | 27.53± 0.25 | 28.248±0.033 | -0.854±0.001 | |
| 331 | 100896 | 4228891667990334976 | 306.864400 | -2.103364 | -64.779±0.352 | -67.523±0.216 | | 49.464±0.479 | | |
| | 100895 | 4228891221313732864 | 306.861958 | -2.119817 | -61.630±0.099 | -69.039±0.057 | -14.70± 0.17 | 46.078±0.108 | -1.858±0.006 | |
| 342 | 106353 | 6830027182179257472 | 323.096671 | -20.957957 | -279.074±0.103 | -124.241±0.086 | 32.57± 0.16 | 28.740±0.061 | | |
| | 106350 | 6830027143525634432 | 323.086142 | -20.970035 | -284.665±0.091 | -123.311±0.075 | 33.62± 0.28 | 28.738±0.053 | -2.107±0.004 | |
| 344 | 107299 | 6458951765971500672 | 325.995032 | -57.325475 | -116.563±0.062 | -52.575±0.054 | 36.95± 0.20 | 44.462±0.089 | | |
| | 107300 | 6458952345790198144 | 326.000646 | -57.283282 | -115.298±0.049 | -52.865±0.045 | 37.16± 0.16 | 44.373±0.066 | -1.484±0.002 | |
| 352 | 110084 | 6613555642140332672 | 334.478622 | -32.479717 | 163.485±0.077 | -203.690±0.078 | -77.34± 0.20 | 47.692±0.111 | | |
| | 111520 | 6620882890706789248 | 338.918255 | -28.133824 | 198.181±1.765 | -163.762±1.749 | 1.78± 0.54 | 51.467±2.529 | 0.700±0.023 | d |
| 359 | 112724 | 2211820991079869312 | 342.419369 | 66.199867 | -63.663±0.625 | -125.772±0.633 | | 37.163±0.525 | | |
| | 112324 | 2007431640735691776 | 341.264616 | 58.146467 | -62.565±0.062 | -135.767±0.059 | -10.25± 0.18 | 35.353±0.046 | 0.708±0.007 | |
| 366 | 114131 | 6541802406664428672 | 346.719425 | -43.520412 | -49.037±0.621 | -13.454±0.651 | | 36.046±0.634 | | |
| | 114112 | 6541802578463122688 | 346.662879 | -43.503830 | -47.921±0.088 | -22.151±0.100 | 15.37± 0.23 | 40.237±0.143 | -1.531±0.010 | |

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| Index SO11 | HIP2 | GDR2 | $\alpha$ (deg) | $\delta$ (deg) | $\mu_\alpha$ (mas/yr) | $\mu_\delta$ (mas/yr) | $v_r$ (km/s) | $d$ (pc) | $\log_{10}(\Delta R)$ | Exclusion Test |
|---|---|---|---|---|---|---|---|---|---|---|
| 381 | 1266 | 2428524528072046464 | 3.966260 | -8.876220 | 42.692±0.099 | 10.121±0.050 | 7.21± 0.26 | 98.401±0.465 | | |
| | 118 | 2428948114926672128 | 0.386803 | -9.430027 | 46.899±0.117 | 1.430±0.085 | 1.66± 0.26 | 78.579±0.471 | 0.742±0.005 | c,d |
| 384 | 2292 | 2526899001640197248 | 7.316650 | -5.911442 | -110.351±0.145 | -221.185±0.071 | 9.54± 0.18 | 54.877±0.174 | | |
| | 2350 | 2526925389919277056 | 7.499004 | -5.764960 | -107.019±0.094 | -223.039±0.062 | 9.77± 0.27 | 55.049±0.155 | -0.650±0.003 | |
| 392 | 4702 | 2356290256259997696 | 15.117303 | -19.389626 | 128.469±0.101 | -54.504±0.062 | 5.31± 0.16 | 82.225±0.309 | | |
| | 4833 | 2356080043380354816 | 15.529355 | -19.669909 | 129.340±0.085 | -54.529±0.061 | 5.62± 0.19 | 81.436±0.274 | -0.165±0.003 | |
| 402 | 6668 | 315635261093206656 | 21.393618 | 31.550204 | 156.296±0.086 | -48.602±0.076 | 18.27± 0.20 | 63.089±0.207 | | |
| | 6675 | 315635192373731328 | 21.411410 | 31.545948 | 155.319±0.086 | -50.508±0.075 | | 63.343±0.183 | -1.760±0.004 | |
| 404 | 6772 | 4916890556306664192 | 21.776124 | -51.965451 | 294.867±0.051 | 75.327±0.049 | 49.99± 0.18 | 56.705±0.096 | | |
| | 6804 | 4916887395210737024 | 21.895937 | -52.044359 | 294.046±0.032 | 75.068±0.033 | 50.23± 0.29 | 56.764±0.068 | -0.971±0.001 | |
| 405 | 7189 | 4929377881661762944 | 23.151342 | -49.728032 | -28.252±0.047 | -73.686±0.044 | 3.37± 0.37 | 81.153±0.209 | | |
| | 7086 | 4929369360446613248 | 22.818624 | -49.904326 | -28.369±0.040 | -75.405±0.044 | | 81.000±0.207 | -0.406±0.002 | b |
| 417 | 11137 | 76300510625993344 | 35.831622 | 15.419357 | 112.396±0.074 | 184.300±0.061 | 27.02± 0.15 | 60.120±0.132 | | |
| | 11134 | 76300476266255488 | 35.821737 | 15.417794 | 111.098±0.064 | 183.174±0.050 | 27.30± 0.24 | 60.105±0.118 | -1.994±0.004 | |
| 422 | 11736 | 2500621360930896640 | 37.871913 | 1.091259 | -29.750±0.103 | -23.012±0.082 | | 107.030±0.560 | | |
| | 12728 | 2498487861696380672 | 40.903989 | -0.344632 | -26.869±0.069 | -25.355±0.073 | 9.73± 0.31 | 90.724±0.345 | 0.763±0.004 | c |
| 426 | 13223 | 31986240656172800 | 42.554350 | 13.710430 | 59.150±0.093 | -8.651±0.090 | -14.72± 15.28 | 94.865±0.546 | | |
| | 13122 | 33216147491735808 | 42.195278 | 14.613596 | 58.013±0.088 | -6.609±0.085 | 30.25± 0.21 | 76.807±0.280 | 0.161±0.004 | c,d |
| 427 | 13271 | 4949158198924394496 | 42.699429 | -39.932267 | 41.760±0.052 | -9.937±0.057 | | 96.694±0.329 | | |
| | 13499 | 4949081572411575552 | 43.466162 | -40.194081 | 42.102±0.043 | -6.409±0.047 | 18.12± 0.91 | 96.453±0.291 | 0.035±0.003 | |
| 441 | 15432 | 5047006006423053440 | 49.720802 | -36.127117 | 37.017±0.040 | -5.105±0.059 | | 105.680±0.336 | | |
| | 15152 | 5046487311812732544 | 48.821500 | -37.041717 | 38.972±0.039 | -5.318±0.057 | 17.59± 0.97 | 100.649±0.266 | 0.322±0.003 | |
| 445 | 16410 | 117916235464382336 | 52.833080 | 27.571643 | 38.561±0.106 | -31.966±0.079 | | 104.541±0.602 | | |
| | 16742 | 69733883588579840 | 53.838995 | 26.852557 | 28.033±0.088 | -31.248±0.069 | | 195.915±1.768 | 0.478±0.007 | c |
| 448 | 16959 | 4728825002249947904 | 54.542882 | -59.776194 | 26.094±0.042 | 43.356±0.047 | | 76.949±0.139 | | |
| | 16942 | 4728825036609672576 | 54.511235 | -59.775713 | 25.698±0.199 | 44.655±0.207 | 9.03± 2.40 | 77.615±0.735 | -1.668±0.008 | |
| 450 | 17486 | 4666907551119833984 | 56.195067 | -70.027068 | -10.234±0.043 | -97.151±0.049 | 27.65± 0.14 | 55.118±0.069 | | |
| | 17515 | 4666907516760096512 | 56.256345 | -70.024519 | -9.249±0.048 | -97.358±0.050 | 27.94± 0.19 | 54.988±0.074 | -1.693±0.002 | |
| 462 | 20109 | 4780193841901310336 | 64.667115 | -52.859782 | 47.538±0.086 | 72.662±0.154 | | 75.921±0.271 | | a |
| | 20074 | 4780194185498710144 | 64.564161 | -52.901991 | 49.705±0.061 | 73.646±0.100 | 21.51± 0.25 | 74.962±0.192 | -1.005±0.003 | |
| 465 | 21537 | 3230677565443833088 | 69.361411 | 0.553169 | 16.653±0.102 | 11.603±0.057 | 38.60± 0.17 | 64.132±0.214 | | |
| | 21534 | 3230677870385455232 | 69.358778 | 0.574687 | 15.953±0.100 | 12.163±0.054 | 38.97± 0.17 | 64.119±0.204 | -1.615±0.003 | |
| 467 | 21704 | 2976981131534077056 | 69.904334 | -21.247450 | 12.719±0.049 | 22.037±0.077 | 9.57± 0.16 | 83.375±0.300 | | |
| | 21702 | 2976981337692506240 | 69.895710 | -21.239607 | 12.411±0.042 | 22.892±0.046 | 7.96± 0.23 | 83.051±0.224 | -1.788±0.005 | |
| 478 | 25453 | 3238066592819780608 | 81.661830 | 6.868519 | 9.167±0.097 | -32.465±0.080 | | 101.834±0.560 | | |
| | 25483 | 3334161160308637312 | 81.750020 | 7.170142 | 9.341±0.102 | -33.164±0.089 | | 98.459±0.715 | -0.260±0.006 | |
| 481 | 25711 | 3447142233536837376 | 82.352673 | 31.426028 | -0.561±0.099 | -46.080±0.069 | | 75.565±0.284 | | |
| | 25614 | 3447107495841475712 | 82.083455 | 31.048402 | -2.399±0.088 | -53.567±0.066 | -21.34± 0.30 | 132.969±1.180 | -0.094±0.006 | c |
| 484 | 26309 | 2907397747897070336 | 84.043022 | -28.708027 | 25.325±0.054 | -3.082±0.073 | | 58.897±0.154 | | |
| | 26453 | 2907308172059100544 | 84.415231 | -28.626307 | 24.080±0.034 | -3.100±0.041 | 26.05± 0.45 | 59.114±0.083 | -0.460±0.002 | |
| 487 | 26680 | 3339560999352744960 | 85.048812 | 10.252564 | -7.541±0.119 | -28.727±0.092 | 29.53± 0.54 | 82.925±0.369 | | |
| | 26646 | 3339565461821955712 | 84.946518 | 10.339045 | -8.312±0.085 | -29.008±0.069 | 30.69± 0.68 | 82.508±0.328 | -0.718±0.004 | |
| 489 | 27069 | 3023084272561678976 | 86.106147 | -5.464855 | 14.670±0.080 | -23.036±0.085 | | 87.312±0.462 | | |
| | 26588 | 3017087261266087552 | 84.773120 | -5.897631 | 15.729±0.086 | -21.779±0.080 | 25.40± 0.74 | 89.594±0.410 | 0.333±0.004 | |
| 492 | 27791 | 3454470100579668992 | 88.219656 | 34.444815 | -8.834±0.110 | -71.574±0.089 | -37.59± 5.29 | 89.428±0.675 | | a,b |
| | 28872 | 3453690993510788480 | 91.436317 | 36.934039 | -8.260±0.080 | -70.593±0.066 | 25.30± 0.50 | 109.959±0.579 | 0.798±0.005 | c,d |
| 499 | 31323 | 3382205557837836288 | 98.529539 | 22.123988 | -8.301±0.087 | -35.555±0.073 | | 94.444±0.524 | | a |
| | 31316 | 3382205592197580928 | 98.515108 | 22.117513 | -7.316±0.092 | -34.348±0.076 | -26.28± 0.30 | 94.856±0.692 | -1.610±0.008 | |
| 510 | 35341 | 948509515477490560 | 109.509153 | 40.883448 | -12.784±0.165 | 12.065±0.155 | | 95.725±1.022 | | |
| | 35799 | 974093501788080384 | 110.764196 | 45.000203 | -11.563±0.064 | -5.904±0.039 | 32.57± 0.27 | 1029.404±39.123 | 1.617±0.031 | c |
| 525 | 41081 | 5322206718812246656 | 125.729666 | -52.123645 | -23.039±0.140 | 19.548±0.117 | | 73.249±0.382 | | |

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| Index | HIP2 | GDR2 | $\alpha$ | $\delta$ | $\mu_\alpha$ | $\mu_\delta$ | $v_r$ | $d$ | $\log_{10}(\Delta R)$ | Exclusion |
|---|---|---|---|---|---|---|---|---|---|---|
| SO11 | | | (deg) | | (mas/yr) | | (km/s) | (pc) | | Test |
| | 40916 | 5320716335102626048 | 125.251760 | -52.227884 | -22.095±0.056 | 19.644±0.059 | | 72.713±0.138 | -0.402±0.003 | |
| 527 | 41880 | 3076534861386561536 | 128.092411 | -0.937638 | -133.683±0.073 | 83.519±0.053 | 15.31± 0.29 | 69.099±0.201 | | |
| | 41881 | 3076535243639085440 | 128.096341 | -0.927400 | -135.608±0.076 | 81.968±0.056 | 14.85± 0.35 | 69.356±0.222 | -1.878±0.003 | |
| 540 | 45811 | 5742852762061629056 | 140.120838 | -9.555820 | -12.896±0.462 | -27.707±0.536 | 25.14± 0.16 | 68.539±1.707 | | |
| | 45802 | 5742848123496954112 | 140.087475 | -9.610300 | -13.982±0.178 | -29.110±0.155 | 31.99± 1.32 | 66.917±0.523 | -1.124±0.015 | d |
| 541 | 45937 | 5310444491348428544 | 140.506149 | -54.543106 | 18.112±0.055 | 120.947±0.055 | 91.12± 0.16 | 67.899±0.139 | | |
| | 45952 | 5310446621652246144 | 140.579963 | -54.464723 | 22.511±0.108 | 114.544±0.121 | | 67.493±0.238 | -0.977±0.002 | |
| 542 | 46298 | 1131772432806417280 | 141.620478 | 78.437919 | 30.488±0.071 | 26.021±0.069 | -10.71± 0.18 | 53.853±0.117 | | |
| | 46299 | 1131771646828736000 | 141.625549 | 78.429867 | 34.502±0.053 | 25.967±0.066 | -11.18± 0.21 | 53.877±0.096 | -2.117±0.005 | |
| 546 | 47017 | 5249310751464090112 | 143.734891 | -64.999265 | -34.698±0.058 | 43.970±0.051 | 44.93± 14.31 | 87.442±0.228 | | |
| | 47351 | 5249282164160875136 | 144.725079 | -64.990582 | -34.905±0.051 | 41.369±0.050 | 22.62± 1.01 | 90.548±0.222 | -0.187±0.002 | d |
| 549 | 47231 | 1050821477623210368 | 144.406188 | 60.213545 | -11.880±0.065 | -32.046±0.053 | 9.19± 0.13 | 112.655±0.488 | | |
| | 46170 | 1038770314786277248 | 141.234404 | 59.993030 | -17.608±0.054 | -20.520±0.055 | -19.63± 0.28 | 103.250±0.378 | 0.478±0.003 | d |
| 550 | 47335 | 5247416705243609088 | 144.688027 | -66.858881 | -36.613±0.052 | 45.089±0.053 | | 85.534±0.230 | | |
| | 47017 | 5249310751464090112 | 143.734891 | -64.999265 | -34.698±0.058 | 43.970±0.051 | 44.93± 14.31 | 87.442±0.228 | 0.458±0.002 | |
| 551 | 47399 | 813906786608830464 | 144.864275 | 42.285845 | -6.119±0.063 | -6.587±0.065 | -3.36± 0.48 | 74.017±0.272 | | |
| | 47851 | 801608096216800000 | 146.304399 | 41.155079 | -1.902±0.089 | -3.864±0.100 | | 370.751±9.574 | 0.782±0.019 | c |
| 557 | 51536 | 828983049535772928 | 157.894852 | 43.264808 | -20.822±0.059 | 15.860±0.084 | | 99.483±0.437 | | |
| | 51726 | 828895294768816000 | 158.531822 | 43.464793 | -11.095±0.080 | 8.714±0.105 | -11.98± 0.33 | 142.610±1.172 | 0.028±0.006 | c |
| 558 | 51608 | 5229614856076359168 | 158.134154 | -72.428414 | -56.798±0.047 | -45.861±0.046 | 0.37± 0.20 | 70.927±0.136 | | |
| | 51611 | 5229612622693362048 | 158.145528 | -72.441652 | -56.698±0.043 | -46.489±0.037 | 0.40± 0.54 | 70.918±0.112 | -1.771±0.002 | |
| 582 | 57950 | 5343610331876174336 | 178.283059 | -56.727293 | -39.410±0.058 | -10.227±0.052 | 13.36± 0.66 | 102.128±0.460 | | |
| | 59716 | 6075548419252906368 | 183.711072 | -55.789916 | -35.722±0.050 | -11.041±0.042 | 14.22± 1.23 | 114.852±0.518 | 0.776±0.004 | |
| 588 | 58751 | 3575541998835891456 | 180.749457 | -10.752474 | 33.041±0.105 | -18.763±0.077 | -12.39± 0.20 | 57.262±0.183 | | |
| | 58722 | 3575542995268304512 | 180.664538 | -10.713711 | 33.067±0.840 | -20.664±0.747 | -16.16± 0.70 | 54.426±1.115 | -1.047±0.010 | d |
| 589 | 58813 | 6147474037517738624 | 180.943487 | -44.122728 | 98.970±0.059 | 18.173±0.039 | 6.41± 0.12 | 53.089±0.131 | | |
| | 58815 | 6147474071877478144 | 180.949433 | -44.117898 | 101.361±0.055 | 15.724±0.036 | 6.14± 0.17 | 52.952±0.126 | -2.224±0.010 | b |
| 592 | 59243 | 5788355123068307328 | 182.281096 | -78.781346 | -43.364±0.062 | -7.619±0.060 | | 99.808±0.327 | | |
| | 58490 | 5224879500011370496 | 179.925399 | -76.023964 | -41.025±0.043 | -6.190±0.038 | 12.63± 1.40 | 99.746±0.231 | 0.689±0.002 | |
| 595 | 59865 | 6054216931644194304 | 184.157892 | -63.197879 | -145.407±0.046 | 6.329±0.041 | -9.93± 0.16 | 68.919±0.157 | | |
| | 59913 | 6054219165008718848 | 184.342422 | -63.226932 | -145.829±0.047 | 7.784±0.045 | -9.15± 0.19 | 69.046±0.166 | -0.974±0.002 | |
| 596 | 59898 | 5860789299912353024 | 184.275888 | -65.693002 | -38.331±0.083 | -10.073±0.077 | | 105.943±0.604 | | |
| | 60183 | 5860587917472173056 | 185.117150 | -65.842707 | -41.434±0.100 | -11.322±0.084 | | 97.363±0.587 | -0.175±0.005 | |
| 597 | 59960 | 6072902994276659200 | 184.471331 | -55.975581 | -39.014±0.057 | -12.872±0.046 | | 103.334±0.458 | | |
| | 59716 | 6075548419252906368 | 183.711072 | -55.789916 | -35.722±0.050 | -11.041±0.042 | 14.22± 1.23 | 114.852±0.518 | -0.053±0.004 | |
| 607 | 61135 | 3907637494456207360 | 187.913755 | 12.127870 | -57.344±0.128 | 3.719±0.078 | | 106.825±0.700 | | |
| | 61416 | 3931759748777206272 | 188.777443 | 12.938992 | -57.292±0.155 | 6.364±0.073 | 17.46± 0.19 | 107.151±0.850 | 0.339±0.006 | |
| 610 | 61937 | 3927438771159865856 | 190.392833 | 10.426268 | -106.518±0.092 | -0.193±0.057 | | 71.160±0.247 | | b |
| | 62350 | 3927604694336986368 | 191.641939 | 11.378495 | -112.544±0.114 | -0.472±0.059 | -17.49± 0.14 | 65.405±0.297 | 0.267±0.003 | |
| 620 | 65466 | 1446271417351852544 | 201.277774 | 23.854387 | -10.836±0.220 | -7.596±0.194 | | 90.650±0.841 | | |
| | 65508 | 1448260262087566848 | 201.413988 | 25.682150 | -10.309±0.088 | -8.666±0.071 | -0.35± 0.66 | 86.755±0.292 | 0.453±0.006 | |
| 624 | 65779 | 5863709946382545280 | 202.275373 | -66.073676 | -127.902±0.048 | 11.368±0.056 | -24.72± 0.29 | 78.909±0.228 | | |
| | 65785 | 5863709946382541952 | 202.289332 | -66.078529 | -127.318±0.041 | 11.056±0.050 | -24.52± 0.35 | 78.838±0.207 | -1.989±0.008 | |
| 626 | 65899 | 1442870284289795968 | 202.628390 | 22.512881 | 25.400±0.076 | -45.436±0.034 | -35.48± 0.18 | 83.756±0.263 | | |
| | 65884 | 1441682158897145600 | 202.579217 | 21.500051 | 22.957±0.075 | -49.366±0.032 | 58.12± 0.30 | 87.091±0.277 | 0.179±0.003 | d |
| 630 | 66749 | 6293670601102816128 | 205.211181 | -17.792721 | -36.521±0.086 | -11.333±0.079 | -8.19± 0.46 | 90.540±0.458 | | |
| | 66717 | 6293764540627556224 | 205.135238 | -17.343786 | -45.844±0.116 | -13.594±0.112 | -1.02± 0.51 | 108.416±1.021 | -0.103±0.006 | c,d |
| 633 | 67250 | 1496459690754708736 | 206.748323 | 38.542600 | -134.290±0.088 | -22.464±0.115 | -11.20± 0.13 | 92.492±0.788 | | |
| | 67041 | 1496708798857641600 | 206.083788 | 38.797635 | -141.262±0.026 | -23.293±0.038 | -32.94± 0.18 | 95.946±0.296 | -0.022±0.005 | d |
| 635 | 67470 | 3728726646011331328 | 207.399613 | 13.010410 | -147.396±0.067 | 25.352±0.059 | | 56.281±0.141 | | b |
| | 67673 | 3728039760481678592 | 207.961829 | 11.937313 | -149.496±0.077 | 25.658±0.055 | -2.80± 0.48 | 58.219±0.162 | 0.081±0.002 | |

Continued on next page

| Index SO11 | HIP2 | GDR2 | $\alpha$ (deg) | $\delta$ (deg) | $\mu_\alpha$ (mas/yr) | $\mu_\delta$ (mas/yr) | $v_r$ (km/s) | $d$ (pc) | $\log_{10}(\Delta R)$ | Exclusion Test |
|---|---|---|---|---|---|---|---|---|---|---|
| 636 | 67639 | 6094716308525121408 | 207.884045 | -48.293349 | -114.076±0.108 | -27.236±0.101 | 5.75± 0.18 | 68.484±0.284 | | |
| | 67645 | 6094716239805635072 | 207.893709 | -48.298861 | -113.740±0.104 | -26.793±0.102 | 4.72± 0.22 | 68.085±0.278 | -1.996±0.013 | |
| 642 | 68830 | 6291206045789315328 | 211.410852 | -18.072550 | 39.425±0.094 | -45.424±0.076 | 15.18± 12.84 | 88.382±0.436 | | |
| | 68833 | 6291205977069837696 | 211.412416 | -18.081112 | 37.270±0.094 | -44.559±0.081 | 28.05± 0.18 | 89.125±0.420 | -1.871±0.008 | d |
| 655 | 71726 | 1282815063829295360 | 220.077109 | 30.443646 | 93.410±0.064 | -44.363±0.086 | -11.80± 0.15 | 54.983±0.136 | | |
| | 71737 | 1282817022334383232 | 220.118799 | 30.520323 | 93.470±0.073 | -43.678±0.095 | -11.49± 0.14 | 54.724±0.156 | -1.091±0.002 | |
| 677 | 76046 | 4414349489701257984 | 232.953494 | -2.066574 | -79.827±0.083 | -30.371±0.071 | -9.52± 0.26 | 76.037±0.302 | | |
| | 75923 | 4416093315142832128 | 232.604610 | -1.318934 | -70.150±0.072 | -28.983±0.060 | -25.82± 1.27 | 67.463±0.203 | 0.014±0.003 | d |
| 678 | 76133 | 4416110082695309184 | 233.241330 | -1.186582 | -17.765±0.214 | -42.217±0.205 | -15.85± 0.15 | 82.944±0.887 | | |
| | 77728 | 4403070149671286656 | 238.052074 | -2.278682 | -15.408±0.093 | -43.981±0.066 | -3.50± 0.26 | 88.630±0.468 | 0.868±0.007 | d |
| 691 | 78283 | 1202709349617637632 | 239.757321 | 18.325889 | -59.445±0.065 | 2.520±0.058 | -61.82± 0.16 | 71.079±0.306 | | |
| | 78067 | 1204270110670555136 | 239.094563 | 20.418776 | -56.658±0.058 | 9.315±0.061 | -18.21± 0.71 | 80.184±0.321 | 0.460±0.004 | d |
| 704 | 80886 | 1299204074916904320 | 247.745828 | 23.985828 | -30.683±0.041 | 31.931±0.050 | -20.02± 0.23 | 74.958±0.181 | | |
| | 81875 | 1299508639637961344 | 250.861555 | 23.196612 | -30.122±0.032 | 30.284±0.045 | -40.40± 0.29 | 99.969±0.249 | 0.655±0.002 | c,d |
| 707 | 81641 | 4434301983614947072 | 250.161198 | 4.219745 | 0.353±0.175 | -10.476±0.134 | | 101.209±1.084 | | |
| | 81634 | 4434301841878055296 | 250.146462 | 4.207160 | 0.364±0.124 | -9.859±0.095 | | 99.832±0.640 | -1.469±0.008 | |
| 713 | 84054 | 4571879578631697024 | 257.763126 | 24.237827 | -11.782±0.054 | 30.655±0.056 | | 80.848±0.272 | | |
| | 84070 | 4571879475552172544 | 257.820456 | 24.252545 | -11.230±0.037 | 29.770±0.042 | | 79.667±0.190 | -1.119±0.003 | |
| 715 | 84515 | 4388409330344765056 | 259.153687 | 3.408540 | -60.249±0.072 | 19.568±0.063 | -14.44± 0.22 | 82.661±0.311 | | |
| | 85911 | 4485937214320449280 | 263.339536 | 5.700825 | -54.948±0.083 | 21.769±0.071 | -22.79± 0.29 | 73.460±0.234 | 0.812±0.003 | d |
| 726 | 85940 | 4487578544660527488 | 263.469996 | 8.103517 | -16.092±0.099 | -59.011±0.087 | -28.07± 0.18 | 90.978±0.397 | | b |
| | 85944 | 4487581018562340992 | 263.483482 | 8.165793 | -13.134±0.087 | -60.275±0.084 | | 90.133±0.337 | -0.997±0.004 | |
| 737 | 88782 | 6719536193567989120 | 271.858999 | -46.899274 | -32.617±0.086 | -106.932±0.081 | -83.59± 0.12 | 71.053±0.322 | | |
| | 87887 | 5954345541675673728 | 269.293720 | -46.122032 | -43.037±0.096 | -102.548±0.085 | 19.85± 0.25 | 84.984±0.363 | 0.419±0.004 | c,d |
| 740 | 89373 | 6721441784663730816 | 273.581281 | -43.154515 | -0.873±0.104 | -53.633±0.091 | -44.16± 0.33 | 103.306±0.605 | | |
| | 89274 | 6724728980838829952 | 273.257554 | -42.288522 | -5.729±0.101 | -51.286±0.087 | | 109.826±0.798 | 0.223±0.006 | |
| 743 | 90026 | 6649398690418063232 | 275.541876 | -55.564195 | 77.717±0.075 | 1.586±0.076 | | 96.083±0.534 | | |
| | 90028 | 6649398690418059392 | 275.542705 | -55.572839 | 79.534±0.073 | 1.748±0.068 | -5.14± 0.16 | 96.934±0.538 | -1.836±0.013 | |
| 749 | 93029 | 2133995118527034496 | 284.270902 | 51.720942 | 43.638±0.057 | -5.617±0.055 | 1.76± 0.46 | 84.236±0.185 | | |
| | 92467 | 2143913950359678464 | 282.666306 | 49.767527 | 45.000±0.042 | -1.773±0.051 | -12.52± 0.40 | 86.532±0.173 | 0.516±0.002 | d |
| 756 | 96979 | 4301163357576782464 | 295.691253 | 8.382994 | 23.989±0.081 | 53.912±0.053 | -40.83± 0.17 | 60.953±0.220 | | |
| | 96976 | 4301163323217046912 | 295.684014 | 8.380594 | 23.964±0.074 | 52.346±0.050 | -41.00± 0.16 | 60.950±0.186 | -2.095±0.005 | |
| 759 | 97646 | 6447036152303430528 | 297.686877 | -59.193721 | 23.300±0.463 | -12.433±0.407 | | 85.947±2.503 | | |
| | 97581 | 6447030139349235200 | 297.514014 | -59.264087 | 24.031±0.114 | -15.899±0.082 | | 91.402±0.617 | -0.757±0.015 | |
| 761 | 98174 | 6447100091479968256 | 299.276478 | -58.901415 | 23.248±0.230 | -14.783±0.246 | | 90.149±1.382 | | |
| | 97646 | 6447036152303430528 | 297.686877 | -59.193721 | 23.300±0.463 | -12.433±0.407 | | 85.947±2.503 | 0.125±0.019 | |
| 768 | 99689 | 1808691203160200576 | 303.437164 | 16.157968 | -7.280±1.319 | 5.631±1.216 | | 124.509±12.288 | | |
| | 100451 | 1808808571723073152 | 305.555561 | 16.372994 | 1.538±0.067 | 0.497±0.060 | | 328.737±4.829 | 0.908±0.033 | c |
| 770 | 99729 | 4249652990144051840 | 303.540141 | 6.588725 | -132.528±0.101 | -58.445±0.064 | -0.18± 0.20 | 63.078±0.208 | | |
| | 99727 | 4249652783985617920 | 303.537683 | 6.576958 | -129.729±0.096 | -59.388±0.065 | -0.21± 0.19 | 63.093±0.203 | -1.878±0.004 | |
| 776 | 100941 | 6679307846231740032 | 306.999982 | -42.620131 | 94.662±0.082 | -122.858±0.057 | | 104.908±0.685 | | |
| | 100937 | 6679308018030433536 | 306.986590 | -42.620666 | 93.796±0.078 | -122.511±0.055 | -62.19± 0.23 | 105.016±0.629 | -1.743±0.009 | |
| 777 | 101082 | 2298101352139398144 | 307.366686 | 81.092228 | 66.243±0.082 | 221.381±0.105 | -14.38± 0.17 | 64.364±0.211 | | |
| | 101166 | 2298101901895214720 | 307.593111 | 81.140436 | 66.494±0.058 | 221.043±0.070 | -14.24± 0.27 | 64.049±0.124 | -1.176±0.002 | |
| 780 | 101719 | 2195226749280378368 | 309.283271 | 61.364403 | 23.834±0.051 | -42.621±0.048 | 3.21± 0.18 | 57.493±0.082 | | |
| | 102727 | 2194321816851242624 | 312.239000 | 61.118583 | 19.428±0.053 | -40.421±0.050 | -9.36± 0.18 | 65.332±0.122 | 0.189±0.001 | d |
| 782 | 102725 | 6376446646806738816 | 312.236006 | -68.791520 | 42.026±0.043 | -61.530±0.069 | | 85.988±0.270 | | |
| | 103917 | 6375793502540188800 | 315.826134 | -69.171496 | 44.104±0.053 | -59.448±0.055 | -10.04± 0.17 | 87.808±0.446 | 0.309±0.004 | |
| 784 | 103107 | 6481251098731588224 | 313.358335 | -49.383030 | 43.599±0.064 | -30.438±0.051 | -16.94± 0.28 | 89.053±0.300 | | |
| | 103139 | 6481246906845850880 | 313.465930 | -49.449714 | 43.875±0.051 | -27.428±0.044 | -17.98± 0.43 | 89.993±0.292 | -0.821±0.003 | |
| 793 | 104536 | 1839746393680655232 | 317.629011 | 22.457052 | 22.239±0.137 | -11.888±0.138 | | 116.759±1.188 | | |

Continued on next page

| Index | HIP2 | GDR2 | $\alpha$ | $\delta$ | $\mu_\alpha$ | $\mu_\delta$ | $v_r$ | $d$ | $\log_{10}(\Delta R)$ | Exclusion |
|---|---|---|---|---|---|---|---|---|---|---|
| SO11 | | | (deg) | | (mas/yr) | | (km/s) | (pc) | | Test |
| | 102981 | 1838905954482116480 | 312.957870 | 21.868618 | 20.705±0.081 | -11.991±0.072 | -11.55± 0.18 | 108.378±0.582 | 0.933±0.007 | |
| 802 | 109306 | 6611824083124944384 | 332.177751 | -33.125589 | 13.261±0.078 | 24.447±0.081 | | 78.654±0.344 | | |
| | 110433 | 6599619950733282304 | 335.519158 | -34.488798 | 11.670±0.060 | 25.950±0.066 | | 86.438±0.281 | 0.649±0.003 | |
| 815 | 112537 | 2396293134977302272 | 341.906431 | -23.172364 | 24.576±0.071 | -10.583±0.065 | -3.25± 0.37 | 78.726±0.286 | | |
| | 111596 | 6628071944405827712 | 339.129743 | -21.584986 | 31.319±0.072 | -4.837±0.062 | 11.06± 1.60 | 106.288±0.444 | 0.688±0.003 | c,d |
| 839 | 117454 | 2337899270721594752 | 357.237150 | -25.144889 | -13.177±0.093 | -42.966±0.111 | -3.30± 0.83 | 85.714±0.686 | | |
| | 117720 | 2338428513772452480 | 358.123916 | -24.165778 | -15.355±0.095 | -39.481±0.069 | 44.28± 0.67 | 113.458±0.771 | 0.343±0.006 | c,d |

Table 2: *Hipparcos* and Gaia identification numbers for the 133 binary stars used, together with Gaia DR2 data and results of the exclusion tests performed: (a) shows an individual star excluded (which results in the exclusion of the respective binary) for having an inconsistent proper motion in right ascension between *Hipparcos* and the DR2, (b) the corresponding result for declination, (c) shows a binary excluded for having discordant parallaxes to both components (distance differences larger than 13 %) in the DR2 data, and (d) gives binaries removed for having DR2 radial velocity differences $> 4$ km s$^{-1}$, with the 81 retained binaries appearing blank in this column. Upper rows give data of primaries and lower ones of secondaries.

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