Regressão Linear

$$y_i = w_0 + w_1 x_1 + \cdots + w_p x_p$$

$$\bar{y} = X\bar{w} + \varepsilon$$

$$\bar{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \qquad X = \begin{bmatrix} \bar{x}_1^T \\ \bar{x}_2^T \\ \vdots \\ \bar{x}_n^T \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & \cdots & \cdots & x_{np} \end{bmatrix}$$

$$\bar{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_p \end{bmatrix} \qquad \bar{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

$$\cancel{\bar{y} = X}$$

$$\bar{y} = \bar{X}\bar{w} + \varepsilon$$

$$\mathcal{L}(D, \bar{w}) = \| \bar{y} - X\bar{w} \|^2 = \sum_{i=1}^{n} \left(y_i - \sum_{j=1}^{p} x_{ij} w_j \right)$$

$$= (\bar{y} - X\bar{w})^T (\bar{y} - X\bar{w})$$

$$= \bar{y}^T \bar{y} - \bar{y}^T X \bar{w} - \bar{w}^T X^T \bar{y} + \bar{w}^T X^T X \bar{w}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{w}} = -2X^T\bar{y} + 2X^TX\bar{w} = 0$$

$$\bar{w} = (X^TX)^{-1} X^T \bar{y}$$

Chain Rule

$$\frac{\partial \mathcal{L}}{\partial \bar{w}} = (\bar{y} - X\bar{w})^T (\bar{y} - X\bar{w})$$

$$\cancel{= -2(\bar{y} - X\bar{w})^T X}$$

$$\cancel{= -2(\bar{y}^T X - \bar{w}^T X^T X) = 0}$$

$$\cancel{\bar{w} = (X^TX)^{-1} \bar{y}^T X}$$

$$= -2X^T(y - X\bar{w})$$

$$= -2X^Ty + 2X^TX\bar{w} = 0$$

$$\bar{w} = (X^TX)^{-1} X^T y$$

+ Propriedades de matrizes transpostas

1. $(A^T)^T = A$

2. $(A+B)^T = A^T + B^T$

3. $(AB)^T = B^T A^T$

4. $(cA)^T = c A^T$

5. $(det(A^T)) = det(A)$

6. $[\bar{a} \cdot \bar{b}] = \bar{a}^T \bar{b}$

7. Se A possui apenas valores reais $A^T A$ é uma matriz semidefinida positiva.

8. $(A^T)^{-1} = (A^{-1})^T$

9. $\bar{X}^T A = (A^T \bar{x})^T$