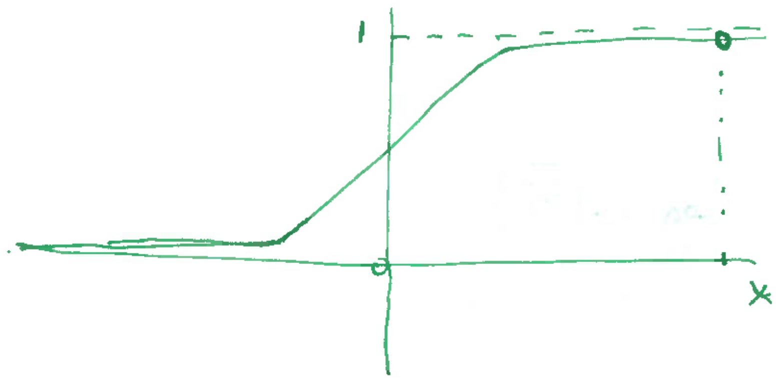
Regressão logística

$$p(y=1) = \frac{1}{1 - e^{-(w_0 + w_1 x_1 + \dots + w_m x_m)}}$$

Função Logística

$$\sigma(x) = \frac{1}{1 - e^{-x}}$$



x_0	x_1	x_2	y
1	1	4	0
1	3	8	0
1	2	1	1

$$\bar{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \hat{y} = \frac{1}{1 - e^{-(1+1+4)}}$$

- Função de Loss (Medida de qualidade do modelo)

$$\mathcal{L}(\theta \mid D) = \prod_{e \in D} P(y_e = 1)^{y_e} \left(1 - P(y_e = 1)\right)^{1 - y_e}$$

$$\log \mathcal{L}(\theta \mid D) = \sum_{e \in D} y_e \underbrace{\log(\hat{y}_e)} + (1 - y_e)\log(1 - \hat{y}_e)$$

$$\boxed{\frac{d(\log x)}{dx} = \frac{1}{x}}$$

$$\hat{y}_e = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_{e_1} + \beta_2 x_{e_2} + \dots + \beta_m x_{e_m})}}$$

$$\bar{w} = \arg\max \log \mathcal{L}(\bar{w})$$

$$\frac{\partial \log(\hat{y}_e)}{\partial \hat{y}_e} = \frac{1}{\hat{y}_e} \qquad \frac{\partial \hat{y}_e}{\partial \bar{w}_i}$$

$$\boxed{\frac{d e^{ax}}{dx} = a e^{ax}}$$

$$\frac{\partial \log(1 - \hat{y}_e)}{\partial \hat{y}_e} = -\frac{1}{1 - \hat{y}_e}$$

$$\frac{df}{dx} = \frac{df}{dg}\frac{dg}{dx}$$

$$f = \frac{1}{1 - e^{-x}}$$

$$f = \frac{1}{1 - g} \qquad g = e^{-x} \qquad = -(1-g)^{-2} \cdot -e^{-x}$$

$$= \frac{e^{-x}}{(1 - e^{-x})^2}$$

$$\mathcal{L}(\bar{w}) = \sum_{e \in D} y_e \log(\hat{y}_e) + (1 - y_e) \log(1 - \hat{y}_e)$$

$$\frac{\partial \mathcal{L}}{\partial w_i} = \sum_{e \in D} y_e \frac{\partial \log(\hat{y}_e)}{\partial w_i} + (1 - y_e) \frac{\partial \log(1 - \hat{y}_e)}{\partial w_i}$$

$$\frac{\partial \log(\hat{y}_e)}{\partial w_i} = \frac{1}{\hat{y}_e} \cdot \frac{\partial \hat{y}_e}{\partial w_i} = \frac{1}{\hat{y}_e} \hat{y}_e (1 - \hat{y}_e) x_{ei}$$

$$= (1 - \hat{y}_e) x_{ei}$$

$$\frac{\partial \log(1 - \hat{y}_e)}{\partial w_i} = \frac{1}{1 - \hat{y}_e} \frac{\partial (1 - \hat{y}_e)}{\partial w_i} = \frac{1}{1 - \hat{y}_e} (-\hat{y}_e (1 - \hat{y}_e)) x_{ei}$$

$$= -\hat{y}_e x_{ei}$$

$$\frac{\partial \mathcal{L}}{\partial w_i} = \sum_{e \in D} y_e (1 - \hat{y}_e) x_{ei} + \hat{y}_e x_{ei} (1 - \hat{y}_e)$$

$$= \sum_{e \in D} y_e (1 - \hat{y}_e) x_{ei} - (1 - y_e) \hat{y}_e x_{ei}$$

$$= \sum_{e \in D} (y_e - \hat{y}_e) x_{ej}$$

$$\mathcal{L}(w) = \sum_{e \in D} y_e \log(\hat{y}_e) + (1 - y_e)\log(1 - \hat{y}_e)$$

$$\hat{y}_e = \frac{1}{1 - e^{-mL_e}}$$

$$mL_e = \beta_0 x_{e_0} + \beta_1 x_{e_1} + \beta_2 x_{e_2} + \dots + \beta_m x_{e m}$$

$$\frac{\partial \mathcal{L}}{\partial \beta_i} = \sum_{e \in D} \frac{\partial \mathcal{L}}{\partial \hat{y}_e} \frac{\partial \hat{y}_e}{\partial mL_e} \frac{\partial mL_e}{\partial \beta_i}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{y}_e} = \frac{1}{\hat{y}_e} - \frac{1}{1 - \hat{y}_e}$$

$$e^{2x}$$

$$\frac{\partial \hat{y}_e}{\partial mL} = \frac{e^{-x}}{(1 - e^{-x})^2} \quad \cancel{\frac{1}{e^x\left(1 - 2e^{-x} + e^{-2x}\right)}}$$

$$\cancel{= \frac{1}{e^x - 2 + e^{-x}}}$$

$$\frac{e^{-x}}{(1 - e^{-x})^2} \cdot \frac{e^{2x}}{e^{2x}} = \frac{e^{x}}{e^{2x} - 2e^{x} + 1} = \frac{e^x}{(e^x - 1)^2}$$

$$\frac{\partial mL_e}{\partial \beta_i} = x_{e_i} \qquad \frac{1}{1 - e^{-T}}\left(1 - \frac{1}{1 - e^{-T}}\right)$$

$$\frac{1}{1 - e^{-T}} - \frac{1}{(1 - e^{-T})^2}$$

$$\frac{1}{1-e^{-T}} - \frac{1}{(1-e^{-T})^2}$$

$$\frac{1-e^{-T}}{(1-e^{-T})^2} - \frac{1}{(1-e^{-T})^2} = \frac{e^{-T}}{(1-e^{-T})^2}$$