* Regressão Logística

$$\frac{d e^{ax}}{dx} = a e^{ax}$$

$$p_i = \frac{1}{1 + e^{-M}}$$

$$M = \sum_{i=1}^{\|D\|} x_{ij} w_i$$

$$\mathcal{L} = \sum_{i=1}^{\|D\|} y_i \log p_i + (1 - y_i) \log(1 - p_i)$$

$$\frac{\partial \mathcal{L}}{\partial w_j} = ?$$

$$\frac{\partial}{\partial w_j} \log(p_i) = \frac{1}{p_i} \frac{\partial p_i}{\partial w_j} = \boxed{\frac{1}{p_i} p_i (1 - p_i) x_{ij}} = (1 - p_i) x_{ij}$$

$$\frac{\partial}{\partial w_i} \log(1 - p_i) = \frac{1}{1 - p_i} \frac{\partial (1 - p_i)}{\partial w_j} = \frac{1}{1 - p_i} (-p_i (1 - p_i)) x_{ij}$$

$$= -p_i x_{ij}$$

$$\frac{\partial p_i}{\partial w_j} = \frac{\partial p_i}{\partial M} \frac{\partial M}{\partial w_j}$$

$$= \frac{\partial (1 + e^{-M})^{-1}}{\partial M} \cdot \frac{\partial M}{\partial w_j}$$

$$= -(1 + e^{-M})^{-2} \cdot -e^{-M} \cdot \frac{\partial M}{\partial w_j}$$

$$= \frac{e^{-M}}{(1 + e^{-M})^2} \cdot x_{ij}$$

$$= p_i (1 - p_i) x_{ij}$$

$$\frac{e^{-M} + 1 - 1}{(1 + e^{-M})^2}$$

$$\frac{e^{-M} + 1}{(1 + e^{-M})^2} - \frac{1}{(1 + e^{-M})^2}$$

$$\left(\frac{1}{1 + e^{-M}}\right)\left(\frac{\cancel{e^{-M} + 1}}{\cancel{e^{-M} + 1}} - \frac{1}{1 + e^{-M}}\right)$$

$$p_i (1 - p_i)$$

$$\frac{\partial(1-p_i)}{\partial w_j} = \frac{\partial(1-p_i)}{\partial M} \cdot \frac{\partial M}{\partial w_j}$$

$$= \frac{\partial\left(1-(1+e^{-M})^{-1}\right)}{\partial M} \cdot \frac{\partial M}{\partial w_i}$$

$$= -\frac{e^{-M}}{(1-e^{-M})^2} \cdot x_{ij}$$

$$= -p_i(1-p_i)\,x_{ij}$$