* Método de Newton

$$\frac{f(x_0) - 0}{x_0 - x_1} = f'(x)$$

$$\frac{f(x)}{x_0 - x_1} = f'(x)$$

$$f'(x)(x_0 - x_1) = f(x)$$

$$\Rightarrow \quad f'(x)\,x_0 - f'(x)\,x_1 = f(x)$$

$$\Rightarrow \quad -f'(x)\,x_1 = f(x) - f'(x)\,x_0$$

$$\Rightarrow \quad x_1 = -\frac{f(x)}{f'(x)} + \frac{f'(x)\,x_0}{f'(x)}$$

$$\Rightarrow \quad x_1 = x_0 - \frac{f(x)}{f'(x)}$$

$$\Rightarrow \quad (x_1 - x_0) = -\frac{f(x)}{f'(x)}$$

$$\Rightarrow \quad f'(x)\,s = -f(x)$$

$$\Downarrow$$

$$\left[\nabla_v f(x_0)\right] s = -f(x_0)$$

* SQP

$$\min \quad x^2 - 8x - y^2 - 12y \; \cancel{+ 4y} \qquad \text{Quadratic}$$

$$\text{s.a.} \quad x + y = 8 \Big\} \quad \text{Linear}$$

$$\mathcal{L}(\bar{x},\lambda) = x^2 - 8x + y^2 - 12y + \lambda(x + y - 8)$$

$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial x} &= 2x - 8 + \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial y} &= 2y - 12 + \lambda = 0 \end{aligned}\right\} \text{linear}$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \lambda \end{bmatrix} = \begin{bmatrix} 8 \\ 12 \\ 8 \end{bmatrix}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = x + y - 8 = 0 \Big\} \text{ linear} \qquad x = 3, \; y = 5, \; \lambda = 2$$

$$\min \quad f(x)$$

$$\text{s.a.} \quad h_j(x) = 0 \qquad i = 1, \dots, m$$

$$\mathcal{L} = f(x) + h(x)^T \lambda \qquad U = \begin{bmatrix} \bar{X} \\ - \\ \lambda \end{bmatrix}$$

$$\nabla_x \mathcal{L} = \nabla f + [\nabla h]^T \lambda = 0$$

$$\nabla_\lambda \mathcal{L} = h = 0$$

$$\begin{bmatrix} \nabla^2_{xx} \mathcal{L} & \nabla h^T \\ \nabla h & 0 \end{bmatrix} \begin{bmatrix} S_x \\ S_\lambda \end{bmatrix} = \begin{bmatrix} -\nabla_x \mathcal{L} \\ -h \end{bmatrix}$$

Newton - Raphson

Set of equations

$$r(U) = 0$$

$$[\nabla_U r] \underbrace{S_U}_{\text{Step size}} = -r$$

$$S_U = \frac{-r}{\nabla_U r}$$

$$x_{k+1} = x_k + S_x$$

$$\lambda_{k+1} = \lambda_k + S_\lambda$$

$$\min \quad \frac{1}{2} x^T Q x + f^T x$$

$$\text{s.t.} \quad Ax + b = 0$$

$$\mathcal{L} = f + \lambda^T h$$

$$= \frac{1}{2} x^T Q x + f^T x + \lambda^T (Ax + b)$$

$$\nabla_x \mathcal{L} = Qx + f + A^T \lambda = 0$$

$$\nabla_\lambda \mathcal{L} = Ax + b = 0$$

$$\begin{bmatrix} Q & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} -f \\ -b \end{bmatrix}$$

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$$\min \quad \frac{1}{2} s^T \nabla_{xx} \mathcal{L}\, s + \nabla_x \mathcal{L}^T s$$

$$\text{s.t.} \quad [\nabla h]\, s + h = 0$$

$$\begin{bmatrix} \nabla^2_{xx} \mathcal{L} & \nabla h \\ \nabla h & 0 \end{bmatrix} \begin{bmatrix} s_x \\ s_\lambda \end{bmatrix} = \begin{bmatrix} -\nabla_x \mathcal{L} \\ -h \end{bmatrix}$$

$$x_{k+1} = x_k + s_x$$

$$\lambda_{k+1} = \lambda_k + s_\lambda$$