Série de Taylor

$f(x) : \mathbb{R} \to \mathbb{R}$

$p(x) \sim f(x) \qquad x = 0$

$p(x) = C_0 + C_1 x + C_2 x^2 + \boxed{C_3 x^3}$

$x = 0 \to p(0) = f(0)$

$\hookrightarrow p(x) = C_0$

$\hookrightarrow C_0 = f(0)$

$$\frac{\partial p(x)}{\partial x} = \frac{\partial f}{\partial x} \to x = 0$$

$$\frac{\partial p}{\partial x} = C_1 + \cancel{2C_2 x}^{\,0}$$

$$C_1 = \frac{\partial f}{\partial x}$$

$$\frac{\partial^2 p(x)}{\partial x^2} = \frac{\partial^2 f}{\partial x^2} , \quad x = 0$$

$$\frac{\partial^2 p}{\partial x^2} = 2C_2 = \frac{\partial^2 f}{\partial x^2}$$

$$C_2 = \frac{1}{2}\frac{\partial^2 f}{\partial x^2}$$

→ Fazer para polinômio de 4º grau

$$P_k = f(0) + \sum_{i=1}^{k} \frac{1}{i!} \frac{d^i f(0)}{dx^i} x^i$$

$x = a$?

$$p(x) = C_0 + C_1 x + C_2 x^2$$

$\Downarrow$

$$p(x) = C_0 + C_1 (x-a) + C_2 (x-a)^2$$

$$C_0 = f(a)$$

$$\frac{\partial p}{\partial x} = C_1 + 2C_2 \cancel{(x-a)} = \frac{\partial f}{\partial x}$$

$$C_1 = \frac{\partial f}{\partial x}$$

$$\frac{\partial^2 p}{\partial x^2} = 2C_2 = \frac{\partial f}{\partial x}$$

$$C_2 = \frac{1}{2}\frac{\partial f}{\partial x}$$

$$P_k(x) = f(a) + \sum_{i=1}^{k} \frac{1}{i!} \frac{d^i f(a)}{dx^i} (x-a)^i$$

$f(x, y)$

$\cancel{p(x,y) = x^2 + y^2 + xy + x + y}$

$$p(x,y) = c_0 + c_1(x-a) + c_2(y-b) + c_3(x-a)^2 + c_4(x-a)(y-b) + c_5(y-b)^2$$

$c_0 = f(a,b)$

$\frac{\partial p}{\partial x} = \frac{\partial f}{\partial x} = c_1$ $\quad \frac{\partial^2 p}{\partial x \partial y}$

$\frac{\partial p}{\partial y} = \frac{\partial f}{\partial y} = c_2$ $\quad \frac{\partial^2 p}{x^2}$ $\quad \partial^2$

$\left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}\right] \begin{bmatrix} x_1 - p \\ x_2 - p \end{bmatrix}$

$$p(x) = f(p) + \nabla f(p)^T (x-p)$$

$$p_2(x) = f(p) + \nabla f(p)^T (x-p) + \frac{1}{2}(x-p)^T H_f(p)(x-p)$$