統計學（一）

第七章 信賴區間估計
(Estimation Using Confidence Intervals)

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本課程內容參考書目

- **教科書**
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  - Larson, H. J. (1982). *Introduction to probability theory and statistical inference*, 3rd Edition. New York: Wiley.
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  - 林惠玲、陳正倉（2009），「統計學：方法與應用」，第四版，雙葉書廊有限公司。
  - 唐麗英、王春和（2013），「從範例學MINITAB統計分析與應用」，博碩文化公司。
  - 唐麗英、王春和（2008），「SPSS 統計分析 14.0 中文版」，儒林圖書公司。
  - 唐麗英、王春和（2007），「Excel 2007統計分析」，第二版，儒林圖書公司。
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點估計
(Point Estimators and their Properties)

Types of Estimation

- **Recall : The Objective of Statistics**
  - To make inference about a **population** based on information contained in a **sample**.

- **Two Types of Estimation:**
  1) Point Estimation (點估計)
  2) Interval Estimation (區間估計)

Point Estimation

- **What is a Point Estimation (點估計)**
  - **Def :** A point estimator of a population parameter is a **rule** (or **formula**) that tells you how to calculate a **single number** based on sample data. The resulting number is called a **point estimate** of the parameter.
  - **Example:** What are the point estimators for the following population parameters ?
    - μ ： $\overline{X}$
    - σ ： s
    - P ： $\hat{p}$

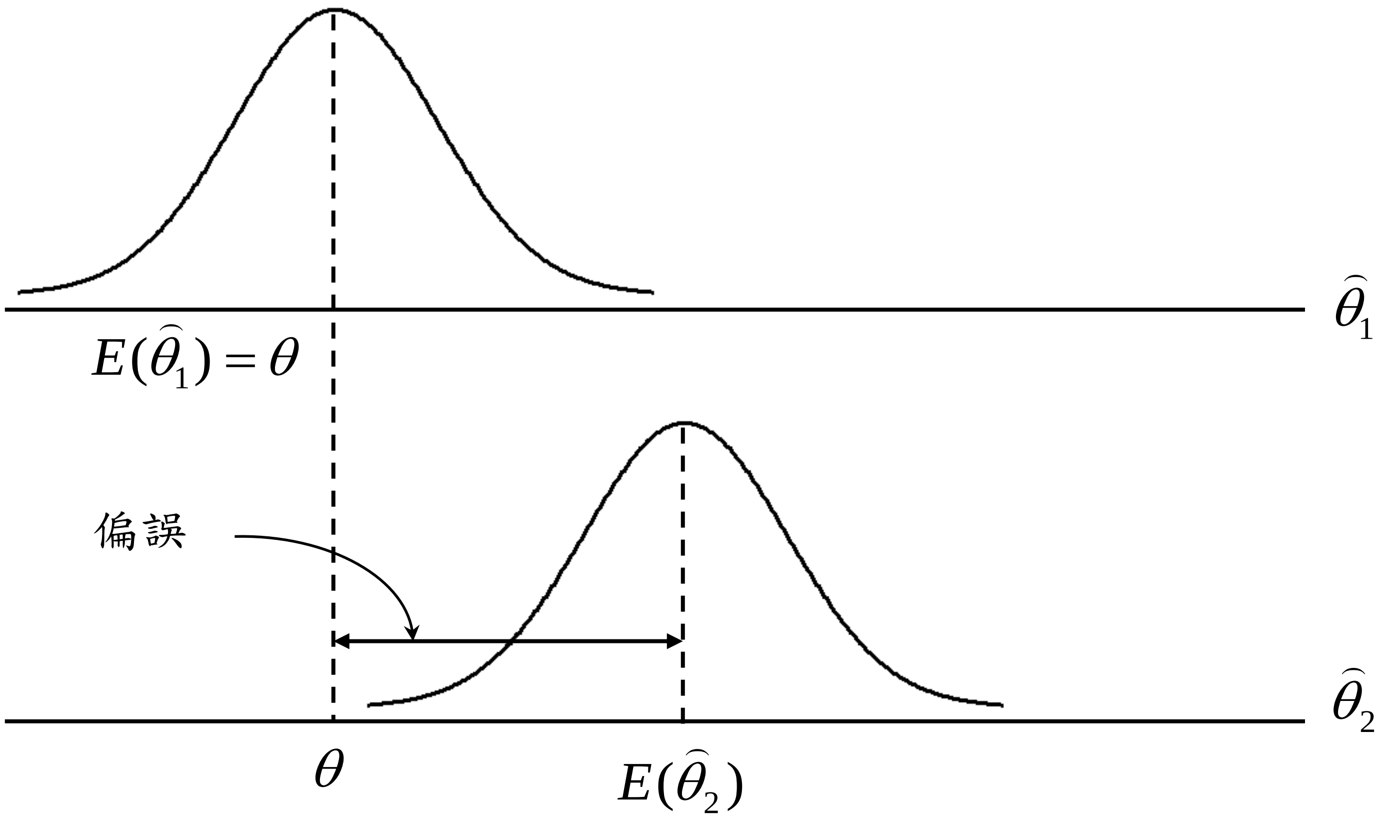
Point Estimation

- **How to Evaluate (評價) the Goodness of a Point Estimator**
  - Unbiasedness (不偏性)
  - Efficiency (有效性)
  - Consistency (一致性)
  - Sufficiency (充份性)
- **What is an Unbiased Estimator (不偏估計量)**
  - **An estimator** of a population **parameter** is said to be ***unbiased*(不偏)** if the **mean** of its sampling distribution is equal to the **parameter**. Otherwise the estimator is said to be **biased (偏頗)** .
  - That is, an estimator is unbiased if

    $$\boxed{E(\textit{Sample estimator}) = \textit{Population parameter}}$$

  - **Example :** $\overline{X}$ **and** S **and** $\hat{p}$ are **unbiased** estimators

Unbiasedness (不偏性)



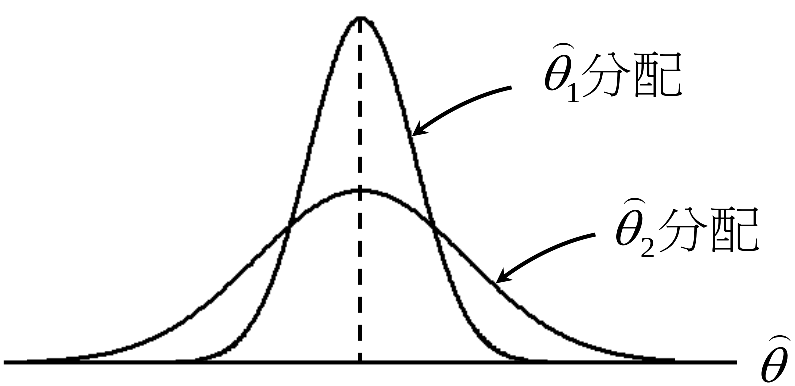
不偏估計量 $\hat{\theta}_1$ 與具有偏誤估計量 $\hat{\theta}_2$ 之比較

Efficiency (有效性)

- **What is an Efficient Estimator (有效估計量)?**
  - A point estimator $\hat{\theta}_1$ **is said to be a more efficient unbiased estimate of** θ **than** $\hat{\theta}_2$ if

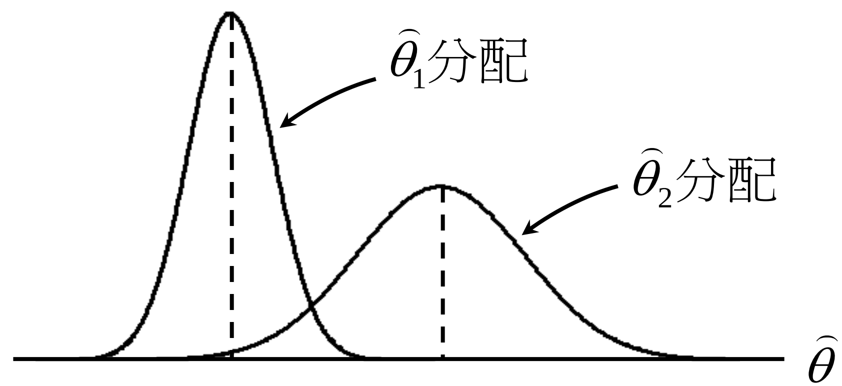
    1) $\hat{\theta}_1$ and $\hat{\theta}_2$ are both **unbiased** estimates of θ .

    2) **the variance** of the sampling distribution of $\hat{\theta}_1$ is less than that of $\hat{\theta}_2$.



$E(\hat{\theta}_1) = E(\hat{\theta}_2) = \theta$

$\hat{\theta}_1$ 為 θ 之不偏，有效估計量



$E(\hat{\theta}_1)$ $\quad$ $E(\hat{\theta}_2)$

$\hat{\theta}_1$ 為 θ 之有效估計量

Maximum Error of Estimate for μ

- **What is the Maximum Error of Estimate for μ**
  - When we use $\overline{X}$ to estimate μ, the **maximum error of estimate (最大可能估計誤差)** can be expressed as follows:

$$E = Max\left|\overline{X} - \mu\right| = Z_{\frac{1-\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right) \text{ or } Z_{\frac{1-\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$ for **large** sample

$$= T_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$ for **small** sample

Maximum Error of Estimate for μ

- **Example 1 :**
  - An industrial engineer intends to use the mean of a random sample of size **150** to estimate the average mechanical aptitude of assembly line workers in a large industry. If, on the basis of experience, the engineer can assume that **σ= 6.2** for such data, what can he assert with probability **0.99** about the **maximum** size of the estimation **error**?

Solution :

$$E = Max\left|\overline{X} - \mu\right| = Z_{\frac{1-\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right) = Z_{0.005}\left(\frac{6.2}{\sqrt{150}}\right) = 2.58 \times 0.506 = 1.306$$

Maximum Error of Estimate for μ

- **Example 2 :**
  - In six determinations of the melting point of tin, a chemist obtained a mean of 232.26 degrees Celsius with a standard deviation of 0.14 degree. If he uses this mean as the actual melting point of tin, what can the chemist assert with 98% confidence about the maximum error?

Solution :

$$E = Max\left|\overline{X} - \mu\right| = T_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) = T_{0.01}\left(\frac{0.14}{\sqrt{6}}\right) = 3.365 \times 0.057 = 0.1923$$

估計群體之平均數
(Estimation of a Population Mean)

Interval Estimation

- **Interval Estimation**
  - Def: An interval estimator of a **population parameter** is a **rule** that tells you how to calculate **two numbers** based on sample data.

- **Confidence Coefficient 信賴係數**
  - Def: The **probability** that a **confidence interval** will enclose the estimated parameter is called the **confidence coefficient**.

Interval Estimation of μ

- **Interval Estimation of μ :**
  - **Case 1: Large-Sample** (1-α)100% Confidence Interval for the Population Mean, **μ**

$$\bar{y} \pm z_{\frac{1-\alpha}{2}} \sigma_{\bar{y}} = \bar{y} \pm z_{\frac{1-\alpha}{2}} \left(\frac{\sigma}{\sqrt{n}}\right) \approx \bar{y} \pm z_{\frac{1-\alpha}{2}} \left(\frac{s}{\sqrt{n}}\right)$$

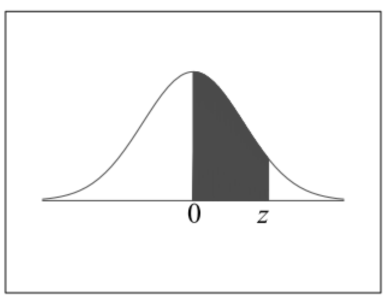
- **Assumptions: None.**
  - The CLT guarantees that ȳ is approximately normally distributed **regardless of** the distribution of the sampled population.
- **Note**
  - The **value of z** selected for constructing such a confidence interval is called the **critical value (臨界值)**of the distribution.
  - (1-α) is called the **confidence coefficient. 信賴係數**
  - (1-α)100% is called the **confidence level. 信賴水準**

Interval Estimation of μ

- **Some Confidence Coefficients for Estimating μ**

1-α	$Z_{\frac{1-\alpha}{2}}$
.90	1.645
.95	1.96
.98	2.33
.99	2.58

Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Interval Estimation of μ

- **Example 1 :**
  - The quality control manager at a light bulb factory needs to estimate the **average** life of a large shipment of light bulbs. The process standard deviation is known to be 100 hours. A random **sample** of 50 light bulbs indicated a sample average life of 350 hours. Find a 95% confidence interval estimate of the true average life of light bulbs in this shipment.
    Interpret your result.

Solution:

$$\bar{y} \pm z_{\frac{1-\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right) = 350 \pm z_{\frac{1-0.05}{2}}\left(\frac{100}{\sqrt{50}}\right) = 350 \pm 1.96 \times 14.14$$

A 95% confidence interval for average life of light bulbs = (322.3 , 377.7)

ANS: We can feel __95%__ confident that the **true average life** of light bulbs in this shipment lies between __322.3__ hours and _377.7_ hours.

Interval Estimation of μ

- **Example 2: 請參考課本281頁的Example7.7**
  - Suppose a PC manufacturer wants to evaluate the performance of its hard disk memory system. One measure of performance is the average time between failures of the disk drive. To estimate this value, a quality control engineer recorded the time between failures for a random sample of 45 disk-drive failures. The following sample statistics were computed:

$$\bar{y} = 1{,}726 \text{ hours} \qquad s = 215 \text{ hours}$$

a) Estimate the true mean time between failures with a 90% confidence interval.

b) If the hard disk memory system is running properly, the true mean time between failures will exceed 1,700 hours. Based on the interval, part **a**, what can you infer about the disk memory system.

Interval Estimation of μ

Solution

a) A 90% confidence interval for is **μ** given by

$$\bar{y} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) = \bar{y} \pm z_{0.05}\left(\frac{\sigma}{\sqrt{n}}\right) \approx \bar{y} \pm z_{0.05}\left(\frac{s}{\sqrt{n}}\right) = 1{,}762 \pm z_{0.05}\left(\frac{215}{\sqrt{45}}\right)$$

$$= 1{,}762 \pm 1.645\left(\frac{215}{\sqrt{45}}\right) = 1{,}762 \pm 52.7$$

where $z_{0.05}$ is the z value corresponding to an upper-tail area of 0.05. Thus, $z_{0.05}$=1.645.

we are 90% confidence that the interval (1,709.3, 1,814.7) enclose **μ** , the true mean time between disk failures.

b) Since all values within the 90% confidence exceed 1,700 hours, we can infer (with 90% confidence) that the hard disk memory system is running properly.

Interval Estimation of μ

- **Interval Estimation of μ :**
  - **Case 2: Small-Sample** (1-α)100% Confidence Interval for the Population Mean, **μ**

$$\bar{y} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

- ***Assumptions*:**
  - The **population** from which the sample is selected has an approximate **normal** distribution.
- **Note**
  - $t_{\alpha/2}$ is obtained from the Student's *T* distribution with **(*n*-1)** degrees of freedom.

Interval Estimation of μ

- **Example 3 :**
  - A sample of 25 employees from a company was selected and the annual salary for each was recorded. The **mean** and **standard deviation** of this sample were found to be **7750** and **900**, respectively.
  - Construct the **95 %C.I.** for the **population average salary**. **Interpret your result.**

Solution:

$$\bar{y} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) = 7750 \pm t_{0.05/2}\left(\frac{900}{\sqrt{25}}\right) = 7750 \pm 2.064 \times 180$$

A 95% confidence interval for the average salary = (7378.48 , 8121.52)

ANS: We are 95% confident that the true average salary lies between 7378.48 and 8121.52 .

Interval Estimation of μ

- **Example 4: 請參考課本283頁的Example7.8**
  - The Geothermal Loop Experimental Facility, located in the Salton Sea in southern California, is a U.S. Department of Energy operation for studying the feasibility of generating electricity from the hot, highly saline water of the Salton Sea. Operating experience has shown that these brines(濃鹽水) leave silica scale deposits (矽垢沈積) on metallic plant piping, causing excessive plant outages. Jacobsen et al. (*Journal of Testing and Evaluation*, Mar. 1981) have found that scaling can be reduced somewhat by adding chemical solutions to the brine. In on screening experiment, each of five antiscalants(阻垢劑) was added to an aliquot of brine, and the solutions were filtered. A silica determination (parts per million of silicon dioxide) was made on each filtered sample after a holding time of 24 hours, with the results shown in Table 7.2. Estimate the mean amount of silicon dioxide present in the five antiscalant solutions. Use a 99% confidence interval.

Table 7.2 Silica measurements for Example 7.8				
229	255	280	203	229

Interval Estimation of μ

- **Solution**
  - Step 1 : compute the sample statistics

$$n = 5 \qquad \bar{y} = 239.2 \qquad s = 29.3$$

  - Step 2 : find 99% confidence interval for **μ** (small-sample)

$$\bar{y} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) = 239.2 \pm t_{0.005}\left(\frac{29.3}{\sqrt{5}}\right)$$

$$= 239.2 \pm (4.604)\left(\frac{29.3}{\sqrt{5}}\right) = 239.2 \pm 60.3$$

where $t_{0.05}$ is the value corresponding to an upper-tail area of 0.005 in the Student's T distribution based on (n-1)=4 degrees of freedom. Thus, $t_{0.005}$=4.604.

  - Step 3 : Interpret the result. If the distribution of silicon dioxide amounts is approximately normal, we can be 99% confident that the interval (178.9, 299.5) encloses **μ**, the true mean amount of silicon dioxide present in an antiscalant solution.

Interpretation of the confidence interval

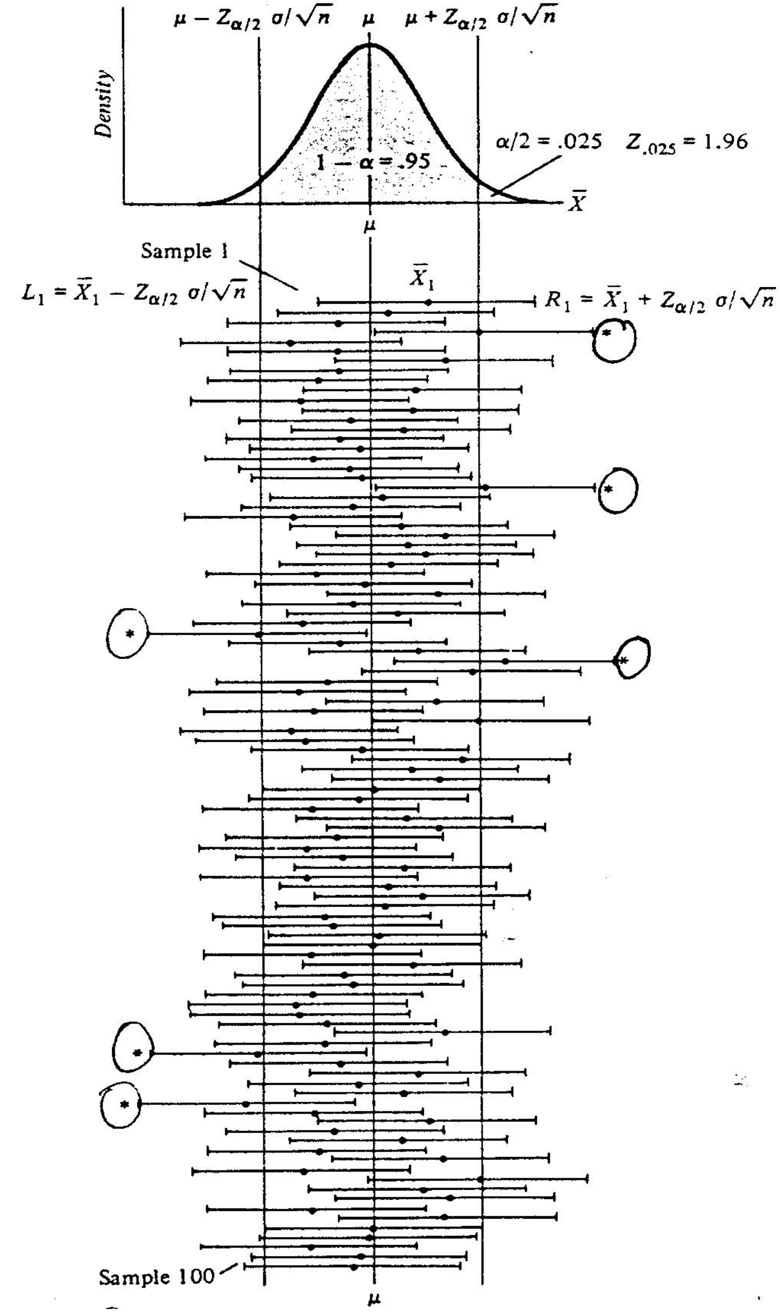
- **Interpretation of the confidence interval for population mean**
  - The **confidence** we have in the limits _322.3_ hours and _377.7_ hours in example **1** derives from our confidence in the statistical procedure that gave rise to them. The procedure gives random variables L and R that have a **95 %** chance of enclosing the **true** but **unknown** mean μ; whether their specific values _____ and enclose μ we have **no way of knowing**.
  - The reason that **"we can feel 95% confident"** is as follows: If we were to take 100 different samples from the sample population and calculate the confidence limits for each sample, then we would expect that about **95** of these **100** intervals would contain the true value of μ, and **5** would not contain the true value of μ. Since we usually only have one sample and hence one confidence interval, we do not know whether our interval is one of the 95 or one of the 5. In this sense, then, **we are 95% confident.**

Interpretation of the confidence interval

– The meaning of being **95% confidence** is shown in the results of a sampling exercise. Statistic students took 100 different random samples of size n=10 each from a normal population with a mean μ and a standard deviation σ= 1.66. One hundred sample means, X's, and corresponding confidence intervals were then computed, and the results are presented in the following Figure (see next page).

– The mean μ was contained in **94** of the 100 intervals, as shown in the figure. This result conforms to our expectation that about 95 of our 100 intervals should encompass the mean μ.

∗These six intervals do not contain the mean μ. Of the 100 confidence intervals. 94 contain the mean μ



Source: Watson, C., Billingsley, P., Croft, J., Huntsberger, O. (1986) *Statistics for Management and Economics* 4th Ed. Allyn and Bacon, Newton, MA, p 328.

估計群體之比率值
(Estimation of a Population Proportion)

Estimation of a Population Proportion

- **Estimation of a Population Proportion**
  - **Large-Sample (1-α)100% Confidence Interval for the Population Proportion, *p***

$$\widehat{p} \pm z_{\alpha/2}\,\sigma_{\widehat{p}} \approx \widehat{p} \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}\widehat{q}}{n}}$$

- ***Assumptions*:**
  - The sample size **n** is sufficiently large so that the approximation is valid. As a rule of thumb, the condition of a "**sufficiently large**" sample size will be satisfied if $\boldsymbol{n\widehat{p} \geq 4 \text{ and } n\widehat{q} \geq 4}$.

Estimation of a Population Proportion

- **Example 5:**
  - In a random sample of 400 industrial accidents, it was found that 231 were due at least partially to unsafe working conditions. Construct a 99% confidence interval for the corresponding true proportion. **<u>Explain your result.</u>**

Solution: $\hat{p} = \dfrac{231}{400} = 0.5775 \quad \hat{q} = 1 - \hat{p} = 0.4225$

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.5775 \pm z_{0.01/2}\sqrt{\frac{0.5775 \times 0.4225}{400}} = 0.5775 \pm 0.0637$$

A 99% confidence interval for the true proportion = (0.5138 , 0.6412)

- **<u>ANS:</u>** We are <u>99%</u> confident that the true proportion of accidents due at least partially to unsafe working conditions lies between <u>0.5138</u> and <u>0.6412</u>.

Estimation of a Population Proportion

- **Example 6: 請參考課本300頁的Example7.12**
  - Stainless steels are frequently used in chemical plants to handle corrosive fluids(腐蝕性液體). However, these steels are especially susceptible to stress corrosion cracking(易受應力腐蝕裂開) in certain environments. In a sample of 295 steel alloy(合金) failures that occurred in oil refineries(煉油廠) and petrochemical plants(石化廠) in Japan, 118 were caused by stress corrosion cracking and corrosion fatigue(腐蝕疲勞) (*Materials performance*, June 1981). Construct a 95% confidence interval for the true proportion of alloy failures caused by stress corrosion cracking.

Estimation of a Population Proportion

- **Solution**

$$\hat{p} = \frac{118}{295} = 0.4 \quad and \quad \hat{q} = 1 - 0.4 = 0.6$$

  – The approximate 95% confidence interval is then

$$\hat{p} \pm z_{0.025}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.4 \pm 1.96\sqrt{\frac{(0.4)(0.6)}{295}} = 0.4 \pm 0.056$$

(Note that the approximation is valid since $n\hat{p} = 118$ and $n\hat{q} = 177$ both exceed 4)

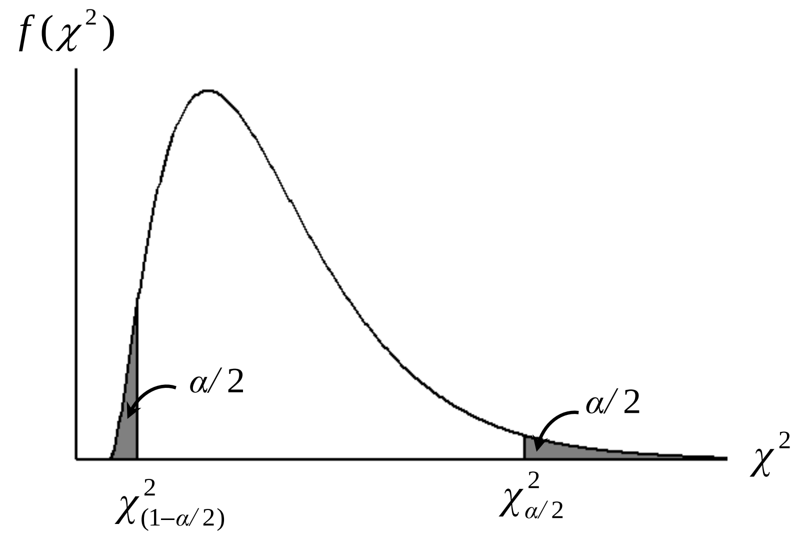
  – We are 95% confident that the interval from 0.344 to 0.456 encloses the true proportion of alloy failures that were caused by corrosion.

估計群體之變異數
(Estimation of a Population Variance)

Confidence Interval for σ²

- **A (1-α)100% Confidence Interval for a Population Variance, σ²**

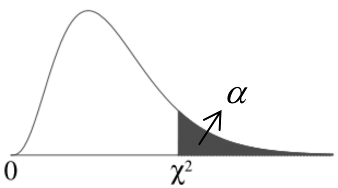
$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}}$$



- **Assumptions:**
  - The Population from which the sample is selected has an approximate **normal** distribution.

- **Note:**
  - $\boldsymbol{\chi^2_{\alpha/2}}$ and $\boldsymbol{\chi^2_{(1-\alpha/2)}}$ can be obtained from Table 8.

Chi-Square Distribution Table



df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928

Confidence Interval for σ²

- **Example 7:**
  - While performing a strenuous (激烈的) task, the pulse rate (脈搏跳動率) of **25** workers increased on the average by **18.4** beats per minute with a standard deviation of **4.9** beats per minute. Find a 95% confidence interval for the corresponding **population standard deviation.**

Solution:

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}} \Rightarrow \frac{(25-1)(4.9)^2}{\chi^2_{0.05/2}} \le \sigma^2 \le \frac{(25-1)(4.9)^2}{\chi^2_{(1-0.05/2)}}$$

$$\Rightarrow \frac{576.24}{39.3641} \le \sigma^2 \le \frac{576.24}{12.4011} \Rightarrow 14.64 \le \sigma^2 \le 46.47 \Rightarrow 3.83 \le \sigma \le 6.82$$

A 95% confidence interval for the population standard deviation = (3.83 , 6.82)

Confidence Interval for σ^2

- **Example 8: 請參考課本307頁的Example7.14**
  - A quality control supervisor in a cannery knows that the exact amount each can contains will vary, since there are certain uncontrollable factors that affect the amount of fill. The mean fill per can is important, but equally important is the variation, $\boldsymbol{\sigma}^2$, of the amount of fill. If σ^2 is large, some cans will contain too little and others too much. To estimate the variation of fill at the cannery, the supervisor randomly selects 10 cans and weights the contents of each. The weights (in ounces) are listed in Table 7.8. Construct a 90% confidence interval for the true variation in fill of cans at the cannery.

Table 7.8 Fill Weights of Cans									
7.96	7.90	7.98	8.01	7.97	7.96	8.03	8.02	8.04	8.02

Confidence Interval for σ²

- **Solution**

  Assume that the sample of observations (amounts of fill) is selected from a normal population.

  – Sample standard deviation : $s = 0.043$

  – Construct a 90% confidence interval for the true variation

  $(n = 10, \quad \alpha = 0.1)$

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}} \Rightarrow \frac{(10-1)(0.043)^2}{\chi^2_{0.1/2}} \le \sigma^2 \le \frac{(10-1)(0.043)^2}{\chi^2_{(1-0.1/2)}}$$

$$\Rightarrow \frac{0.016641}{16.9190} \le \sigma^2 \le \frac{0.016641}{3.32511} \Rightarrow 0.00098 \le \sigma^2 \le 0.00500$$

  – We are 90% confident that the true variance in amount of fill of cans at the cannery falls between 0.00098 and 0.00500.

選擇樣本大小以估計群體參數值
(Choosing the Sample Size)

Choosing the Sample Size, n

1) **Choosing n for estimating μ correct to within E units with confidence (1-α)**

$$n = \left[\frac{z_{(1-\alpha)/2} \cdot \sigma}{E} \right]^2$$

- **Note:**
  - The **population standard deviation** σ will usually have to be approximated by S.

Choosing the Sample Size, n

2) Choosing n for estimating P correct to within E units with confidence (1-α)

$$n = p \cdot q \cdot \left[\frac{z_{(1-\alpha)/2}}{E} \right]^2 \qquad \text{where } \; q = 1\text{-}p$$

- **Note:**
  - This technique requires previous estimates of **p** and **q.** If none are available, use p=q=0.5 for a conservative choice of **n.**

Choosing the Sample Size, n

- **Example 9:**
  - Suppose that we wish to estimate the average daily yield **μ** of a chemical product, and we know the **estimated population standard deviation** is **21** tons. We wish the **error** of estimation to be within **±4** tons with **95** % confidence. **How large a sample** should be collected?

Solution

$$n = \left[\frac{z_{(1-\alpha)/2} \cdot \sigma}{E}\right]^2 = \left[\frac{z_{(1-0.05)/2} \cdot 21}{4}\right]^2 = 105.9 \approx 106$$

Ans: Using a sample size **n = 106** we would be reasonably certain (with confidence = 95%) that our estimate lies within ±4 tons of the true average daily yield.

補充內容：信賴區間的證明
(Proofs of Confidence Intervals)

A (1-α)100% Confidence Interval for μ

Following are the most frequently used confidence intervals for the mean μ and the variance σ^2 of a normal random variable

1) A (1-α)100% Confidence Interval for μ

– **Theorem 1:** Assume $\mathbf{X_1, X_2, \ldots, X_n}$ is a random sample of a **normal** random variable **X** with **unknown mean μ** and **unknown variance σ^2,** then a **(1-α)100% confidence interval** for **μ**, is

$$\bar{y} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

where **t** $_{\alpha/2}$ denotes the **t** value for which the area under the **T**-distribution to its right is equal to α/2

Proof :

A (1-α)100% Confidence Interval for σ^2

2) A (1-α)100% Confidence Interval for σ^2

– **Theorem 2:** Assume $X_1, X_2, \ldots, X_n$ is a random sample of a normal random variable **X** with **unknown mean μ** and **unknown variance σ^2,** then a **(1-α)100% confidence interval** for σ^2, is

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}}$$

where $\chi^2_{\alpha/2}$ denotes the χ^2 value for which the area under this χ^2-density to its right is equal to $\alpha/2$

Proof :

兩獨立母體平均數差之估計
(Estimation of the Difference Between Two Population Means: Independent Samples)

Interval Estimation of $(\mu_1 - \mu_2)$

- **Case 1:** **Large-Sample (1-α)100% Confidence Interval for $\mu_1 - \mu_2$:** **Independent Samples**

$$\left(\bar{y}_1 - \bar{y}_2\right) \pm z_{\alpha/2}\sigma_{\bar{y}_1-\bar{y}_2} = \left(\bar{y}_1 - \bar{y}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \approx \left(\bar{y}_1 - \bar{y}_2\right) \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

- **Note:**

Recall : **Point Estimation of** $(\mu_1-\mu_2) = \bar{y}_1 - \bar{y}_2$

  – We have used the sample variance $\mathbf{s_1^2}$ and $\mathbf{s_2^2}$ as approximations to the corresponding population parameters.

- **Assumptions:**
  – The two random samples are selected in an **independent** manner from the target populations. That is, the choice of elements in one sample does not affect, and is not affected by, the choice of elements in the other sample.
  – The sample size n_1 and n_2 are sufficiently large for the central limit theorem to apply. (We recommend $\mathbf{n_1}\geq 30$ and $\mathbf{n_2}\geq 30$.)
  (即兩樣本是獨立地抽自兩獨立母體，且$n_1\geq 30$ and $n_2\geq 30$)

Interval Estimation of $(\mu_1 - \mu_2)$

- **Example: 請參考課本288頁的Example7.9**
  - We want to estimate the difference between the mean starting salaries for recent graduates with mechanical engineering and electrical engineering degrees from the University of Michigan (UM). The following information is available:

1) A random sample of 59 starting salaries for UM mechanical engineering graduates produced a sample mean of \$48,736 and a standard deviation of \$4,430.

2) A random sample of 30 starting salaries for UM electrical engineering graduates produced a sample mean of \$53,208 and a standard deviation of \$4,286.

Interval Estimation of $(\mu_1 - \mu_2)$

- **Solution (1/2)**
  - μ_1: Population mean starting salary of all recent UM mechanical engineering graduates
  - μ_2: Population mean starting salary of all recent UM electrical engineering graduates

Summary information for Example 7.9

	Mechanical Engineers	**Electrical Engineers**
Sample Size	$n_1 = 59$	$n_1 = 30$
Sample Mean	$\bar{y}_1 = 48{,}736$	$\bar{y}_2 = 53{,}208$
Sample Standard Deviation	$s_1 = 4{,}430$	$s_2 = 4{,}286$

Interval Estimation of $(\mu_1 - \mu_2)$

- **Solution (2/2)**

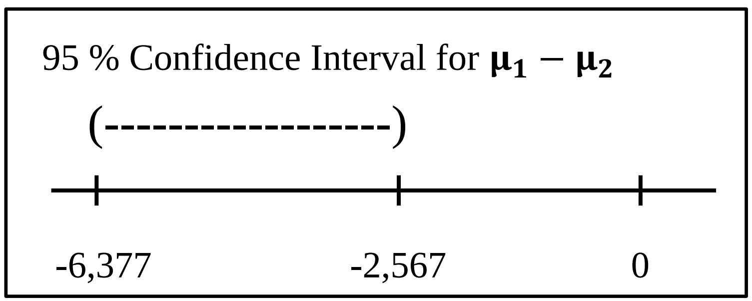
95 % Confidence Interval for $\mu_1 - \mu_2$:

$$(\bar{y}_1 - \bar{y}_2) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \approx (\bar{y}_1 - \bar{y}_2) \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$= (48{,}736 - 53{,}208) \pm z_{0.025}\sqrt{\frac{(4{,}430)^2}{59} + \frac{(4{,}286)^2}{30}}$$

$$= -4{,}472 \pm 1{,}905$$

$$\Rightarrow (-6{,}377, -2{,}567)$$



Since the interval includes only negative differences, we can be reasonably confident that the mean starting salary of mechanical engineering graduates of UM is between \$2,567 and \$6,377 lower than the mean starting salary of electrical engineering graduates.

Practical Interpretation of a Confidence Interval for $(\theta_1 - \theta_2)$

- **Let (LCL, UCL) represent a $(1 - \alpha)$100% confidence interval for $(\theta_1 - \theta_2)$.**

1) If **LCL > 0** and **UCL > 0**, conclude $\boldsymbol{\theta_1 > \theta_2}$

2) If **LCL < 0** and **UCL < 0**, conclude $\boldsymbol{\theta_1 < \theta_2}$

3) If **LCL < 0** and **UCL > 0**, (i.e., the interval includes 0), conclude **no evidence of a difference between $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_2}$**

Interval Estimation of $(\mu_1 - \mu_2)$

- **Case 2: Small-Sample (1-α)100% Confidence Interval for $\mu_1 - \mu_2$: Independent Samples and $\sigma_1^2 = \sigma_2^2$ (note: both $\sigma_1^2 = \sigma_2^2$ are unknown)**

$$(\bar{y}_1 - \bar{y}_2) \pm t_{\alpha/2}\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \qquad \text{where} \quad s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

(s_p^2 represents the **pooled variances**合併之變異數)

and the value of $t_{\alpha/2}$ is based on $(n_1 + n_2 - 2)$ degrees of freedom.

- **Note:**
  - We have used the sample variance $\mathbf{s_1^2}$ and $\mathbf{s_2^2}$ as approximations to the corresponding population parameters.
- **Assumptions:**
  - Both of the populations from which the samples are selected have relative frequency distributions that are approximately normal.
  - The variance σ_1^2 and σ_2^2 of the two populations are equal.
  - The random samples are selected in an **independent** manners from the two populations.

Interval Estimation of $(\mu_1 - \mu_2)$

- **Example:請參考課本289頁的Example7.10**
  - The *Journal of Testing and Evaluation* (July 1981) reported on the results of laboratory tests conducted to investigate the stability and permeability(可透性) of open-graded asphalt(瀝青) concrete(混凝土). In one part of the experiment, four concrete specimens(樣品) were prepared for asphalt contents of 3% and 7% by total weight of mix. The water permeability of each concrete specimen was determined by flowing deaerated water(除氧水) across the specimen and measuring the amount of water loss. The permeability measurements (recorded in inches per hour) for the eight concrete specimens are shown in Table 7.4. Find a 95% confidence interval for the difference between the mean permeabilities of concrete mad with asphalt contents of 3% and 7%. Interpret the interval.

Asphalt Content	3%	1,189	840	1,020	980
	7%	853	900	733	785

Interval Estimation of $(\mu_1 - \mu_2)$

Solution (Assume two samples are independently selected from normal populations)

- Calculate the means and variances of the two samples.

Asphalt Content	**3%**	**7%**
Sample Size	$n_1 = 4$	$n_1 = 4$
Sample Mean	$\bar{y}_1 = 1{,}007.25$	$\bar{y}_2 = 817.75$
Sample Standard Deviation	$s_1 = 143.66$	$s_2 = 73.63$

$$(\bar{y}_1 - \bar{y}_2) \pm t_{\alpha/2}\sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})} = (1{,}007.25 - 817.75) \pm t_{0.025}\sqrt{s_p^2(\frac{1}{4} + \frac{1}{4})}$$

$$= 189.5 \pm 197.50 \Rightarrow (-8.00, 387.00)$$

where $$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{3(143.66)^2 + 3(73.63)^2}{6} = 13{,}028.92$$

 – We are 95% confident that the interval (-8.00, 387.00) enclosed the true difference between the mean permeabilities of the two types of concrete. Since the interval includes 0, we are unable to conclude that the two means differ.

Interval Estimation of $(\mu_1 - \mu_2)$

- **Case 3:** **Small-Sample (1-α)100% Confidence Interval for $\mu_1 - \mu_2$:** **Independent Samples and $\sigma_1^2 \neq \sigma_2^2, n_1 = n_2 = n$**

$$\left(\bar{y}_1 - \bar{y}_2\right) \pm t_{\alpha/2}\sqrt{\frac{1}{n}(s_1^2 + s_2^2)}$$

and the value of $t_{\alpha/2}$ is based on (2(n - 1)) degrees of freedom.

- **Assumptions:**
  - Both of the populations from which the samples are selected have relative frequency distributions that are approximately **normal**.
  - The random samples are selected in **independent** manner from the two populations. **(即兩樣本是獨立地抽自兩常態母體)**

Interval Estimation of $(\mu_1 - \mu_2)$

- **Case 4: Small-Sample (1-α)100% Confidence Interval for $\mu_1 - \mu_2$:**
  Independent Samples and $\sigma_1^2 \neq \sigma_2^2$, $n_1 \neq n_2$, $n_1 < 30$, $n_2 < 30$

$$(\bar{y}_1 - \bar{y}_2) \pm t_{\alpha/2}\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}$$

and the value of $t_{\alpha/2}$ is based on (ν) degrees of freedom.

where $$\nu = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$$

- **Assumptions:**
  - Both of the populations from which the samples are selected have relative frequency distributions that are approximately **normal**.
  - The random samples are selected in **independent** manner from the two populations. **(即兩樣本是獨立地抽自兩常態母體)**

兩配對母體平均數差之估計
(Estimation of the Difference Between Two Population Means: Matched Pairs)

Interval Estimation of $\mu_d = (\mu_1 - \mu_2)$: Matched Pairs

- **$(1-\alpha)100\%$ Confidence Interval for $\mu_d = (\mu_1 - \mu_2)$: Matched Pairs**
  - Let $d_1, d_2, d_3, \dots, d_n$ represent the differences between the pairwise observations in a random sample of n matched pairs, $\bar{\mathbf{d}}$ = mean of the n sample differences, and $\mathbf{s_d}$ = standard deviation of the n sample differences.

Large Sample	**Small Sample**
$\bar{\mathbf{d}} \pm z_{\alpha/2}\left(\frac{\sigma_d}{\sqrt{n}}\right)$	$\bar{\mathbf{d}} \pm t_{\alpha/2}\left(\frac{s_d}{\sqrt{n}}\right)$
where $\boldsymbol{\sigma_d}$ is the population deviation of differences.	where $t_{\alpha/2}$ is based on (n-1) degrees of freedom.
Assumption: $\mathbf{n \geq 30}$	Assumption: The population of paired differences is normally distributed.

Note: When $\boldsymbol{\sigma_d}$ is unknown (as is usually the case), use $\mathbf{s_d}$ to approximate $\boldsymbol{\sigma_d}$

Interval Estimation of $\mu_d = (\mu_1 - \mu_2)$: Matched Pairs

- **Example:請參考課本295頁的Example7.11**
  - One desirable characteristic of water pipes is that the quality of water they deliver be equal to or near the quality of water entering the system at the water treatment plant. A type of ductile iron pipe has provided an excellent water delivery system for the St. Louis County Water Company. The chlorine(氯) levels of water emerging from the South water treatment plant and at the Fire Station (Fenton Zone 13) were measured over a 12-month period, with the results shown in Table 7.7. Find a 95% confidence interval for the mean difference in monthly chlorine content between the two locations.

		Month											
		Jan.	**Feb.**	**Mar.**	**Apr.**	**May**	**June**	**July**	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**
Location	**South Plant**	2.0	2.0	2.1	1.9	1.7	1.8	1.7	1.9	2.0	2.0	2.1	2.1
	Fire Station	2.2	2.2	2.1	2.0	1.9	1.9	1.8	1.7	1.9	1.9	1.8	2.0
Difference		-0.2	-0.2	0	-0.1	-0.2	-0.1	-0.1	0.2	0.1	0.1	0.3	0.1

Interval Estimation of $\mu_d = (\mu_1 - \mu_2)$: Matched Pairs

- **Solution**
  - μ_1: Mean monthly chlorine level at the South Plant
  - μ_2: Mean monthly chlorine level at the Fire Station
  - The difference between pairs of monthly chlorine levels are computed as

    d = (South Plant level) – (Fire Station level)

$$n = 12,\ \bar{d} = -0.0083,\ S_d = 0.1676$$

$$\bar{d} \pm t_{0.025}\left(\frac{s_d}{\sqrt{n}}\right) = -0.0083 \pm 2.201\left(\frac{0.1676}{\sqrt{12}}\right) \Rightarrow (-0.115, 0.098)$$

We estimate, with 95% confidence, that the difference between the mean monthly chlorine levels of water at the two St. Louis locations falls within the interval from -0.115 to 0.098. Since 0 is within the interval, there is **insufficient evidence** to conclude there is a difference between the two means.

兩獨立母體比率差之估計
(Estimation of the Difference Between Two Population Proportions)

Interval Estimation of $(\mathbf{p_1} - \mathbf{p_2})$

- **Large – Sample $(1-\alpha)100\%$ Confidence Interval for $(\mathbf{p_1} - \mathbf{p_2})$**

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sigma_{(\hat{p}_1 - \hat{p}_2)} \approx (\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

  - where $\hat{p}_1$ and $\hat{p}_2$ are the sample proportions of observations with the characteristic of interest.

- **Note:**
  - We have followed the usual procedure of substituting the sample value $\hat{\mathbf{p}}_1$, $\hat{\mathbf{q}}_1$, $\hat{\mathbf{p}}_2$, and $\hat{\mathbf{q}}_2$ for the corresponding population values required for $\boldsymbol{\sigma}_{(\hat{\mathbf{p}}_1 - \hat{\mathbf{p}}_2)}$.

- **Assumptions:**
  - The samples are sufficiently large that the approximation is valid. As a general rule of thumb, we will require that $\mathbf{n_1}\hat{\mathbf{p}}_1 \geq 4$, $\mathbf{n_1}\hat{\mathbf{q}}_1 \geq 4$, $\mathbf{n_2}\hat{\mathbf{p}}_2 \geq 4$, and $\mathbf{n_2}\hat{\mathbf{q}}_2 \geq 4$.

Interval Estimation of $(p_1 - p_2)$

- **Example:請參考課本303頁的Example7.13**
  - A traffic engineer conducted a study of vehicular speeds on a segment of street that had the posted speed limit changed several times. When the posted speed limit on the street was 30 miles per hour, the engineer monitored the speeds of **100** randomly selected vehicles traversing the street and observed **49** violations of the speed limit. After the speed limit was raised to 35 miles per hour, the engineer again monitored the speeds of **100** randomly selected vehicles and observed **19** vehicles in violation of the speed limit. Find a 99% confidence interval for $(p_1 - p_2)$, where p_1 is the true proportion of vehicles that (under similar driving conditions) exceed the lower speed limit (30 miles per hour) and p_2 is the true proportion of vehicles that (under similar driving conditions) exceed the higher speed limit (35 miles per hour). Interpret the interval.

Interval Estimation of $(\mathbf{p_1} - \mathbf{p_2})$

- **Solution**

$$\hat{p}_1 = \frac{49}{100} = 0.49 \quad \text{and} \quad \hat{p}_2 = \frac{19}{100} = 0.19$$

Note that
$$\begin{matrix} n_1\hat{p}_1 = 49 & n_1\hat{q}_1 = 51 \\ n_2\hat{p}_2 = 19 & n_q\hat{q}_q = 81 \end{matrix}$$
all exceed 4.

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = (0.49 - 0.19) \pm 2.58\sqrt{\frac{(0.49)(0.51)}{100} + \frac{(0.19)(0.81)}{100}}$$

$$\Rightarrow (0.136, 0.464)$$

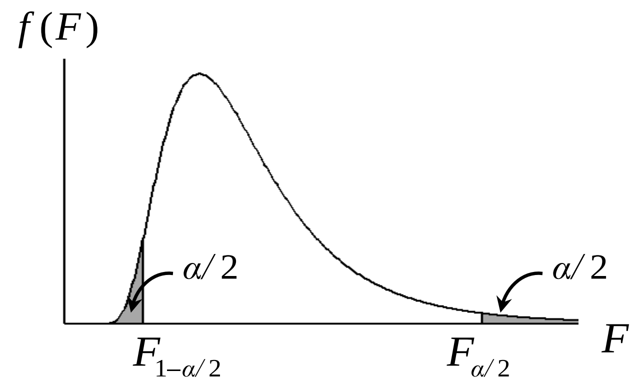
Our interpretation is that the true difference, $(p_1$-$p_2)$, falls between 0.136 and 0.464 with 99% confidence. Since the lower bound on our estimate is positive (0.136), we are fairly confident that the proportion of all vehicles in violation of the lower speed limit (30 miles per hour) exceeds the corresponding proportion in violation of the higher speed limit (35 miles per hour) by at least 0.136.

兩獨立母體變異數比之估計 (Estimation of the Ratio of Two Population Variances)

Interval Estimation of $\frac{\sigma_1^2}{\sigma_2^2}$

- **A (1−α)100% Confidence Interval for the Ratio of Two Population Variance, $\frac{\sigma_1^2}{\sigma_2^2}$**

$$\frac{s_1^2}{s_2^2} \cdot \frac{1}{F_{\alpha/2(\nu_1,\nu_2)}} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2} \cdot F_{\alpha/2(\nu_2,\nu_1)}$$



where $F_{\alpha/2(v_1,v_2)}$ is the value of F that locates an area $\alpha/2$ in the upper tail of the F distribution with $v_1 = (n_1 - 1)$ numerator and $v_2 = (n_2 – 1)$ denominator degrees of freedom, and $F_{\alpha/2(v_2,v_1)}$ is the value of F that locates an area $\alpha/2$ in the upper tail of the F distribution with $v_2 = (n_2 – 1)$ numerator and $v_1 = (n_1 - 1)$ denominator degrees of freedom.

- **Assumptions:**
  - Both of the populations from which the samples are selected have relative frequency distributions that are approximately **normal**.
  - The random samples are selected in an **independent** manner from the two populations.

Interval Estimation of $\frac{\sigma_1^2}{\sigma_2^2}$

- **Example:請參考課本333頁的Example7.16**

A firm has been experimenting with two different physical arrangements of its assembly line. It has been determined that both arrangements yield approximately the same average number of finished units per day. To obtain an arrangement that produces greater process control, you suggest that the arrangement with the smaller variance in the number of finished units produced per day be permanently adopted. Two independent random samples yield the results shown in Table 7.10. Construct a 95% confidence interval for $\frac{\sigma_1^2}{\sigma_2^2}$, the ratio of the variance of the number of finished units for the two assembly line arrangements. Based on the results, which of the two arrangements would you recommend?

Line1	448	523	506	500	533	447	524	469	470	494	536
	481	492	567	492	457	497	483	533	408	453	
Line2	372	446	537	592	536	487	592	605	550	489	461
	500	430	543	459	429	494	538	540	481	484	374
	495	503	547								

Interval Estimation of $\frac{\sigma_1^2}{\sigma_2^2}$

- **Solution**

$$n_1 = 21,\ s_1^2 = 1407.89,\ n_2 = 25,\ s_2^2 = 3729.41$$

$$\frac{s_1^2}{s_2^2}\cdot\frac{1}{F_{\alpha/2(\nu_1,\nu_2)}} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2}\cdot F_{\alpha/2(\nu_2,\nu_1)} \Rightarrow \frac{(1407.89)}{(3729.41)}\cdot\frac{1}{F_{0.025(20,24)}} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{(1407.89)}{(3729.41)}\cdot F_{0.025(24,20)}$$

$$\Rightarrow \frac{(1407.89)}{(3729.41)}\cdot(\frac{1}{2.33}) \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{(1407.89)}{(3729.41)}\cdot(2.41)$$

$$\Rightarrow 0.162 \le \frac{\sigma_1^2}{\sigma_2^2} \le 0.909$$

We estimate with 95% confidence that the ration $\frac{\sigma_1^2}{\sigma_2^2}$ of the true population variance will fall between 0.162 and 0.909. Since all the values within the interval are less than 1, we can be confident that the variance in the number of units finished on line 1 (as measured by σ_1^2) is less than the corresponding variance for line 2 (as measured by σ_2^2)

選擇樣本大小以估計兩群體參數差 (Choosing the Sample Size for Estimating $(\mu_1 - \mu_2)$ and $(p_1 - p_2)$)

Choosing the Sample Size for Estimating $(\mu_1 - \mu_2)$

1) **Choosing the Sample Size for Estimating the Difference $(\mu_1 - \mu_2)$ between Two Population Means Correct to Within H units with Probability (1-α)**

$$\mathbf{n_1 = n_2 = \left(\frac{z_{\alpha/2}}{H}\right)^2(\sigma_1^2 + \sigma_2^2)}$$

where $\mathbf{n_1}$ and $\mathbf{n_2}$ are the numbers of observations sampled from each of the two populations, and $\boldsymbol{\sigma_1^2}$ and $\boldsymbol{\sigma_2^2}$ are the variances of two populations.

***Note*:** The population variances $\boldsymbol{\sigma_1^2}$ and $\boldsymbol{\sigma_2^2}$ will usually have to be approximated.

Choosing the Sample Size for Estimating $(\mu_1 - \mu_2)$

- **Example: See Applied Exercise 7.86 on page318**

High-strength aluminum alloys (鋁合金). Refer to the JOM (Jan. 2003) comparison of a new high-strength RAA aluminum alloy to the current strongest aluminum alloy, Exercise 7.37 (p.291). Suppose the researchers want to estimate the difference between the mean yield strengths of the two alloys to within 15 MPa using a 95% confidence interval. How many alloy specimens of each type must be tested in order to obtain the desired estimate ?

Data from Exercise 7.37 (p.291)

Alloy Type	**RAA**	**Current**
Number of specimens	3	3
Mean yield strength (MPa)	641.0	592.7
Standard deviation	19.3	12.4

Choosing the Sample Size for Estimating $(\mu_1 - \mu_2)$

- **Solution**

To choose the sample size for estimating the differences between two population means, we use the formula

$$n_1 = n_2 = \left(\frac{z_{\alpha/2}}{H}\right)^2 (\sigma_1^2 + \sigma_2^2)$$

For confidence coefficient .95, α = 1 − .95 = .05 and α/2 = .05/2 = .025. From Table 5 in Appendix B, $z_{.005}$ = 1.96. From Exercise 7.37, we have estimates of s_1^2 = 372.49 and s_2^2 = 153.76. $(s_1 = 19.3,\ s_2 = 12.4)$

$$n_1 = n_2 = \left(\frac{1.96}{15}\right)^2 [372.49 + 153.76] = 8.985 \Rightarrow n_1 = n_2 = 9$$

Choosing the Sample Size for Estimating $(p_1 - p_2)$

2) **Choosing the Sample Size for Estimating the Difference $(p_1 - p_2)$ Between Two Population Proportions to Within H Units with Probability (1-α)**

$$n_1 = n_2 = \left(\frac{z_{\alpha/2}}{H}\right)^2 (p_1 q_1 + p_2 q_2)$$

– Where $\mathbf{p_1}$ and $\mathbf{p_2}$ are the proportions for populations 1 and 2, respectively, and $\mathbf{n_1}$ and $\mathbf{n_2}$ are the numbers of observations to be sampled from each population.

Choosing the Sample Size for Estimating $(p_1 - p_2)$

- **Example: See Example 7.18 on page316.**

  A production supervisor suspects a difference exists between the proportions p_1 and p_2 of defective items produced by two different machines. Experience has shown that the proportion defective for each of the two machines is in the neighborhood of 0.03. If the supervisor wants to estimate the difference in the proportions correct to within 0.005 with probability 0.95, how many items must be randomly sampled from the production of each machine? (Assume that you want $n_1=n_2=n$)

Choosing the Sample Size for Estimating $(p_1 - p_2)$

- **Solution**

To choose the sample size for estimating the differences between two population proportions , we use the formula

$$n_1 = n_2 = \left(\frac{z_{\alpha/2}}{H}\right)^2 (p_1 q_1 + p_2 q_2)$$

Since $H = 0.005$ and

$p_1 = p_1 = 0.03 \Rightarrow q_1 = q_2 = 0.97$ and $Z_{\alpha/2} = Z_{0.025} = 1.96$

$$n_1 = n_2 = \left(\frac{z_{\alpha/2}}{H}\right)^2 (p_1 q_1 + p_2 q_2) = \left(\frac{1.96}{0.005}\right)^2 \left[(0.03)(0.97) + (0.03)(0.97)\right] \approx 8{,}943$$

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