統計學（一）

第五章 連續型隨機變數 (Continuous Random Variables)

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本課程內容參考書目

- **教科書**
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  - Berenson, M. L., Levine, D. M., & Krehbiel, T. C. (2009). *Basic business statistics: Concepts and applications*, 11th Edition. Upper Saddle River, N.J: Pearson Prentice Hall.
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  - Montgomery, D. C., & Runger, G. C. (2011). *Applied statistics and probability for engineers*, 5th Edition. Hoboken, NJ: Wiley.
  - Watson, C. J. (1997). *Statistics for management and economics*, 5th Edition. Englewood Cliffs, N.J: Prentice Hall.
  - 林惠玲、陳正倉（2009），「統計學：方法與應用」，第四版，雙葉書廊有限公司。
  - 唐麗英、王春和（2013），「從範例學MINITAB統計分析與應用」，博碩文化公司。
  - 唐麗英、王春和（2008），「SPSS 統計分析 14.0 中文版」，儒林圖書公司。
  - 唐麗英、王春和（2007），「Excel 2007統計分析」，第二版，儒林圖書公司。
  - 唐麗英、王春和（2005），「STATISTICA6.0與基礎統計分析」，儒林圖書公司。
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  - 彭昭英、唐麗英（2010），「SAS123」，第七版，儒林圖書公司。

隨機變數的兩種型式

1) **定義：離散型隨機變數 (Discrete Random Variable)**
   - 離散型隨機變數為計數值的隨機變數。
   - 例：生產線上某次抽檢之不良品的數目

2) **定義：連續型隨機變數 (Continuous Random Variable)**
   - 連續型隨機變數為連續值的隨機變數。
   - 例：厚度、重量與長度

隨機變數的兩種型式

- **例1：** Each of the following experiments results in one value of the random variable (one measurement).
  1. State whether the random variable is a **discrete or continuous**.
  2. Determine, at least in principle, **all possible values** of the random variable.
     a) The number of the leaves on a tree.
     b) The time required to read the book "How to lie with Statistics"
     c) The number of women in a jury of 12.
     d) The speed of a passing car.
     e) The number of heads observed when flip a coin two times.
     f) The sum of the two numbers that occur when roll a pair of fair dice one time.

(Cumulative) Distribution Function (c.d.f.)

- **Def: The (Cumulative) Distribution Function (簡稱c.d.f. or d.f.)累加函數**
  - The distribution function (c.d.f.) of a random variable X is defined to be

$$\boxed{F_X(t) = P(X \leq t) \quad for \quad -\infty \leq t \leq \infty}$$

Remark:

- If X is a discrete R.V., then $\boxed{F_X(t) = \sum_{x \leq t} P(X = t)}$

  (i.e. $F_X(t)$是一累積機率函數)

Example

FIGURE 5.1 ▶
Probability distribution for a binomial random variable ($n = 5$, $p = .5$); shaded area corresponds to $F(3)$

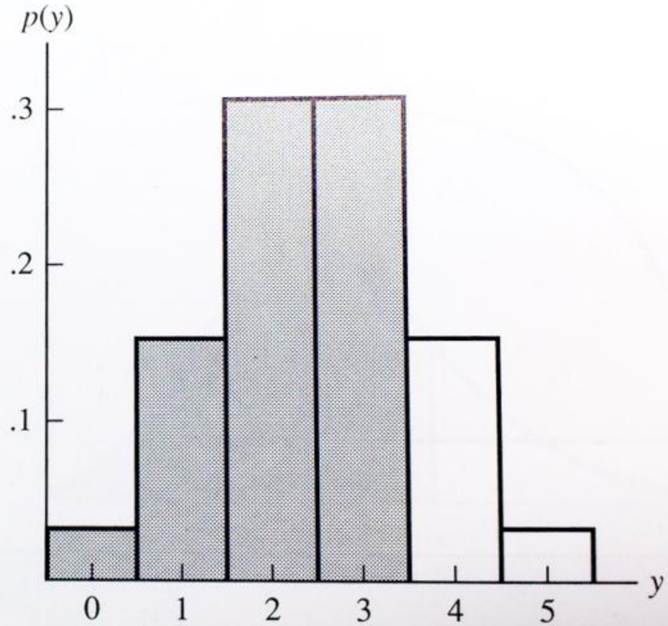
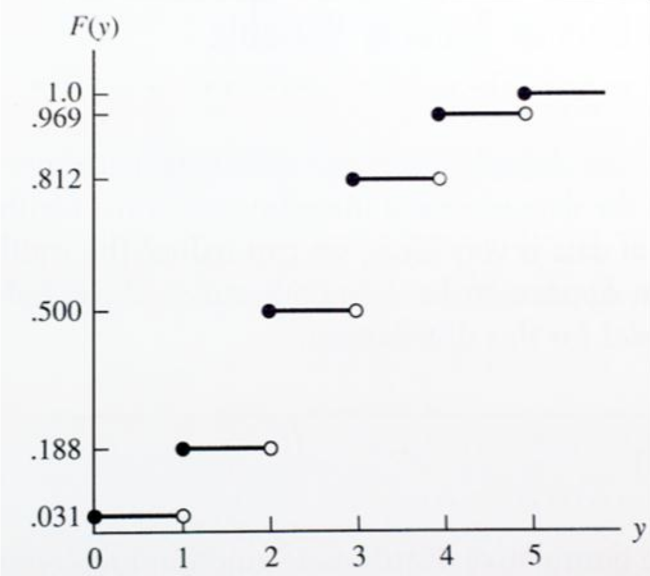


FIGURE 5.2 ▼
Cumulative distribution function $F(y)$ for a binomial random variable ($n = 5$, $p = .5$)



$$F(0) = P(y \le 0) = \sum_{y=0}^{0} p(y) = p(0) = .031$$

$$F(1) = P(y \le 1) = \sum_{y=0}^{1} p(y) = .188$$

$$F(2) = P(y \le 2) = \sum_{y=0}^{2} p(y) = .500$$

$$F(3) = P(y \le 3) = .812$$

$$F(4) = P(y \le 4) = .969$$

$$F(5) = P(y \le 5) = 1$$

(Cumulative) Distribution Function (c.d.f.)

- **例2：**
  - Suppose a hat contains four slips of paper; each slip bears the number 1, 2, 3 and 4. One slip is drawn from the hat without looking. Let X be the number on the slip that is drawn.

1) What is the **probability function of X**?
2) What is the **distribution function of X**?
3) Graph the **distribution function of X**.

(Cumulative) Distribution Function (c.d.f.)

- **Properties or Requirements of F(x)**

1) $If\ a < b, then\ F(a) \leq F(b)$

   (i.e. F(x) is a monotone nondecreasing function.單調非減函數)

2) $\lim_{t \to -\infty} F_X(t) = 0$

3) $\lim_{t \to \infty} F_X(t) = 1$

4) $F_X(t)$ is a right continuous function. (右連續函數)

   (i.e., $\lim_{h \to 0} F(t+h) = F(t)\ for\ any\ t\ and\ h > 0$)

Remarks: We may use the c.d.f. , $\mathbf{F_X(t)}$, to evaluate the probability that **X** lies in a particular **interval**.

(Cumulative) Distribution Function (c.d.f.)

- **例3 : Show that** $P(a < X \leq b) = F_X(b) - F_X(a)$

  Proof:

- **例4 : Show that** $P(X < b) = \lim_{h \to 0} F(b - h)$.

  Proof:

- **Note:**

  If X is a discrete R.V., then $P(X \leq b) \neq P(X < b)$. Why ?

(Cumulative) Distribution Function (c.d.f.)

- 例5 : Verify that

$$F_y(t) = \begin{cases} 0 & t < -2 \\ 1/3 & -2 \le t < 1 \\ 7/12 & 1 \le t < 5 \\ 11/12 & 5 \le t < 11 \\ 1 & 11 \le t \end{cases}$$

is a distribution function and specify the probability function for Y.

- **Use it to compute P(-1 < Y < 2)**

The Density Function for a Continuous Random Variable

Continuous Random Variable

- **Def: Continuous Random Variable**
  - X is a continuous random variable if its distribution function, **$F_X(t)$, is a continuous function of X, for $-\infty < t < \infty$.**
  - For a continuous R.V. X, the role of the probability function is taken by a **probability density function, f(x).**

The Density Function for a Continuous R.V.

- **Def: The Probability Density Function of a Continuous R.V.**
  - Let X be a continuous random variable with distribution function,
  - $F(x) = P(X \leq x)$. The probability density function for X is

$$\boxed{f(x) = \frac{dF(x)}{dx} = F'(x)}$$

  - The range for a continuous R.V. X is $R_x = \{x | f(x) \geq 0\}$.

Example

FIGURE 5.4 ▶
Density function $f(y)$ for a continuous random variable

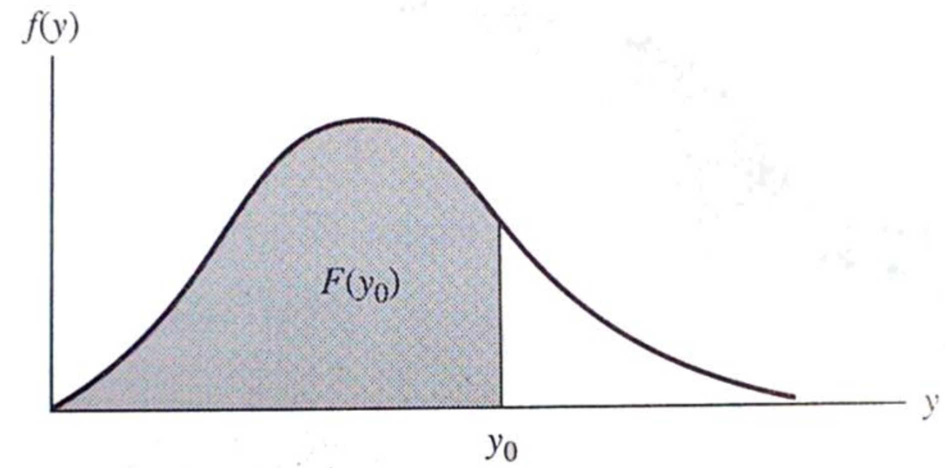
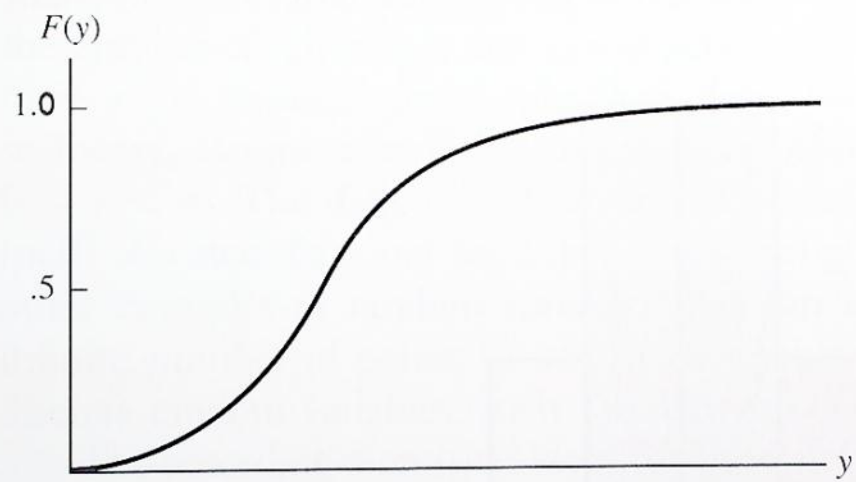


FIGURE 5.3 ▶
Cumulative distribution function for a continuous random variable



The Density Function for a Continuous R.V.

- **The properties of the probability density function, f(x):**

  1) $f(x) \geq 0$

  2) $\int_{-\infty}^{\infty} f(x)dx = 1$

- **Remark:**
  – If X is a continuous R.V. with a density function f(x), then for any a < b the probability that X falls in the interval (a, b) is the area under the density function between a and b:

$$\boxed{P(a \leq X \leq b) = \int_{a}^{b} f(x)dx}$$

The Density Function for a Continuous R.V.

- **Remark:**
  - If X is a continuous R.V., then the probability that X takes on any particular value is 0:

$$\boxed{P(X = t) = 0}$$

  - If X is a continuous R.V., then

$$\boxed{P(a \leq X \leq b) = P(a \leq X < b) = P(a < X \leq b) = P(a < X < b)}$$

<u>Note : this is not true for a discrete R.V.</u>

The Density Function for a Continuous R.V.

- **Remark:**
  - The c.d.f. can also be defined as

$$F_X(t) = P(X \leq t) = \int_{-\infty}^{t} f(x)dx.$$

  - The c.d.f. can be used to evaluate the probability that X falls in an interval:

$$P(a \leq X \leq b) = F_X(b) - F_X(a) = \int_{-\infty}^{b} f(x)dx - \int_{-\infty}^{a} f(x)dx$$

The Density Function for a Continuous R.V.

- **Example 6: Verify that**

$$F_X(t) = \begin{cases} 0\ , & t < 0 \\ \sqrt{t}\ , & 0 \le t \le 1 \\ 1\ , & t > 1 \end{cases}$$

is a distribution function and derive the density function for X.

- **Use it to compute P(1/4 < X < 3/4)**

The Density Function for a Continuous R.V.

- **例7：**

Given Y has probability density function

$$f(y) = \begin{cases} 1, & 99 < y < 100 \\ 0, & otherwise \end{cases}$$

derive $F_Y(t)$.

Expected Values and Summary Measures

Expected Value

- **Measure of the Center of a probability Function (衡量機率函數"重心"之指標)**
  - ***Mean or the Expected Value***
- **Recall : The Expected Value of a Discrete Random Variable**
  - If X is a discrete R.V. with the probability mass function p(x), the Expected Value of X, denote by E(X) or μ_X (Greek letter mu), is

$$\boxed{E(X) = \mu_X = \sum_{all\ x} x \cdot p(x)}$$

provide $\sum|X| \cdot p(x) < \infty$. If the sum diverges, the expectation is undefined.

Note: $E(X) = \mu_X$ is the **balancing point** of the **probability function**

Expected Value

- **Def : The Expected Value of a Continuous Random Variable**
  - If X is a continuous R.V. with the density function f(x), the Expected Value of X, is

$$\boxed{E(X) = \mu_X = \int_{-\infty}^{\infty} x \cdot f(x)dx}$$

provided $\int |x| \cdot f(x)dx < \infty$. If the integral diverges, the expectation is undefined.

- **Remark :**
  - E(X) is **a weighted average** of all possible value of X with each value **weighted** by it **associated probability**.

Variability of a Probability Function

- **Measure of the variability of a Probability Function (衡量機率函數”變異”之指標)**
  - ***Variance or Standard Deviation***

- **Def : The Variance and Standard Deviation of any R.V. X**

$$\mathrm{Var}(X) = \sigma^2 = E[(X-\mu)^2] = E(X^2) - \mu^2$$

$$\mathrm{St.\ D.}(X) = \sigma = \sqrt{\sigma^2}$$

Definition and Theorem

- **Def :** Let X be a continuous R.V. with the density function f(x), and let g(X) be any function of X. **Then the expected value of g(X) is**

$$\boxed{E(g(X)) = \int_{-\infty}^{\infty} g(X) \cdot f(x)dx}$$

- **Theorem 5.1:** Let X be a continuous R.V., and let $g_1(X)$, $g_2(X)$,…, $g_k(X)$ be k functions of X. Then,

a) $E(C)= C$, where C is any constant.

b) $E[C \cdot X]=C \cdot E[X]$

c) $E[g_1(X)+g_2(X)+\ldots+g_k(X)] = E[g_1(X)]+ E[g_2(X)]+\ldots+ E[g_k(X)]$

Definition and Theorem

- **Theorem 5.2 :** Let X be a continuous R.V. with E(X)=m. Then,

$$\boxed{\sigma^2 = E[(X-\mu)^2] = E(X^2) - \mu^2}$$

- **Theorem 5.3 :** Let X be a continuous R.V. (discrete or continuous) with $E(X)=\mu_X$ and $Var(X)=\sigma_X^2$. If **Y=aX+b**, where a and b are any constants, then

a) $\mu_Y = a\mu_X + b$

b) $\sigma_Y^2 = a^2\sigma_X^2$

c) $\sigma_Y = |a| \cdot \sigma_X$

Expected Values and Summary Measures for Continuous R.V.

- **Example 1 : Suppose E(X)=5, Var(X)=10, Find**
  a) E(3X-5)
  b) Var(3X-5)

Expected Values and Summary Measures for Continuous R.V.

- **Example 2:**
  - Suppose you agree to meet a friend (who is generally late) at a specified time. Assume that you arrive on time and let the random variable **T be the length of time you must wait for you friend**. If the density function for T is assume to be

$$f(t) = \frac{1}{t^2}\ ,\ \ t \geqq 1\ \ (\text{minute})$$

Find the **time** you should **expect** to wait.

Expected Values and Summary Measures for Continuous R.V.

- **Example 3:**
  - Let X be a continuous R.V. with the **density function**

$$f(x) = 2x, \quad 0 < x < 1$$

Find

a) $E(X)$

b) $E(X^2)$, $Var(X)$, $St.D.(X)$

Expected Values and Summary Measures for Continuous R.V.

- **Example 4:**
  - If T is a continuous R.V. with c.d.f.

$$F_T(t) = \begin{cases} 0 & t < 0 \\ \sqrt{t} & 0 \leqq t \leqq 1 \\ 1 & t > 1 \end{cases}$$

Find

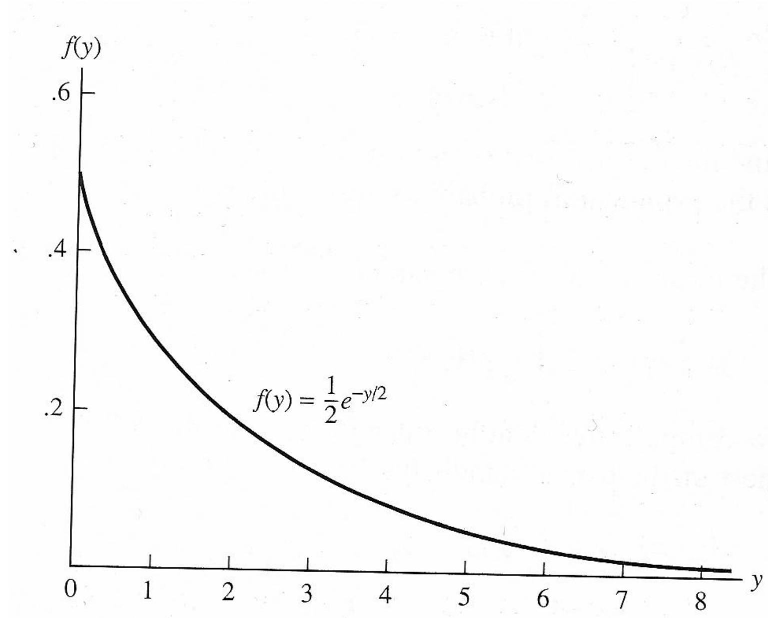
a) mean, E(T)

b) Var(T) and St.D.(T)

課本例 Example 5.5

- **Let Y be a continuous random variable with probability density function**

$$f(y)=\begin{cases}\dfrac{e^{-y/2}}{2} & \text{if } 0\le y\le\infty \\ 0 & \text{elsewhere}\end{cases}$$



Find the mean, variance and standard deviation of Y

課本例 Example 5.5

Solution :

$$\mu = E(Y) = \int_{-\infty}^{\infty} y f(y) dy = \int_{0}^{\infty} \frac{y e^{-y/2}}{2} dy$$

$$\because \int y e^{ay} dy = \frac{e^{ay}}{a^2}(ay - 1)$$

By substituting $a = -\dfrac{1}{2}$, we obtain

$$\boxed{\mu = \int_{0}^{\infty} \frac{y e^{-y/2}}{2} dy = \frac{1}{2}(4) = 2}$$

課本例 Example 5.5

To find σ^2, we will first find $E(Y^2)$ by making use of the general formula

$$\because \int y^m e^{ay} dy = \frac{y^m e^{ay}}{a} - \frac{m}{a} \int y^{m-1} e^{ay} dy$$

Then with $a = -\frac{1}{2}$ and $m = 2$, we ca write

$$E(Y^2) = \int_{-\infty}^{\infty} y^2 f(y) dy = \int_0^{\infty} \frac{y^2 e^{-y/2}}{2} dy = \frac{1}{2}(16) = 8$$

$$\boxed{\begin{aligned} \sigma^2 &= E(Y^2) - \mu^2 = 8 - 2^2 = 4 \\ \sigma &= \sqrt{\sigma^2} = \sqrt{4} = 2 \end{aligned}}$$

課本例 Example 5.6

Find $P(\mu - 2\sigma < Y < \mu + 2\sigma)$ from Example 5.5

Solution :

We showed in Example 5.5 that $\mu = 2$ and $\sigma = 2$

Therefore, μ-2σ = 2-4 = -2 and $\mu + 2\sigma = 6$, then

$$
\begin{aligned}
&P(\mu - 2\sigma < Y < \mu + 2\sigma) \\
&\qquad = \int_0^6 f(y)dy = \int_0^6 \frac{e^{-y/2}}{2} dy \\
&\qquad = -e^{-y/2}\Big]_0^6 = 1 - e^{-3} \\
&\qquad = 1 - 0.049787 = 0.950213
\end{aligned}
$$

連續型機率分佈

連續型機率分佈

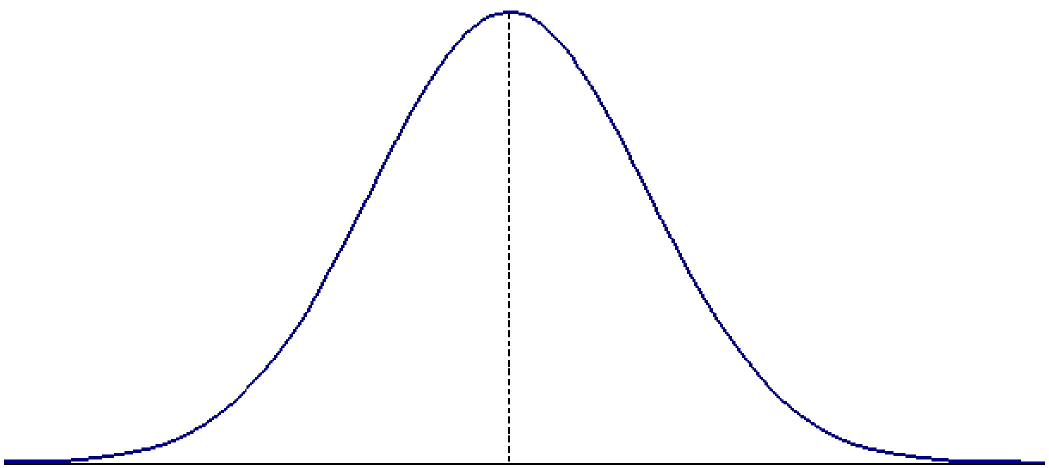
- **常用的連續型機率分佈**
  1) 常態分佈(Normal Distribution)
  2) 對數常態分佈(Lognormal Distribution)
  3) 齊一分佈(Uniform Distribution)
  4) 珈瑪分佈(Gamma Distribution)
  5) 指數分佈(Exponential Distribution)
  6) 韋伯分佈(Weibull Distribution)
  7) 貝塔分佈(Beta Distribution)

常態分佈
(Normal Distribution)

常態分佈(Normal Distribution)

- **何謂常態分佈？**

自然界所觀察到的許多連續型隨機變數常呈<u>**鐘形**</u>分佈，如下圖所示。此鐘形分佈又稱為<u>**常態**</u>分佈（或高斯分佈）。



Carl Friedrich Gauss

Carl Friedrich Gauss (1777–1855), painted by Christian Albrecht Jensen

Source : http://en.wikipedia.org/wiki/Carl_Friedrich_Gauss

常態分佈(Normal Distribution)

- **常態機率分佈**

$$f(x)=\frac{1}{\sqrt{2\pi\sigma}}e^{-(x-\mu)^2/2\sigma^2} \qquad ,-\infty<x<\infty$$

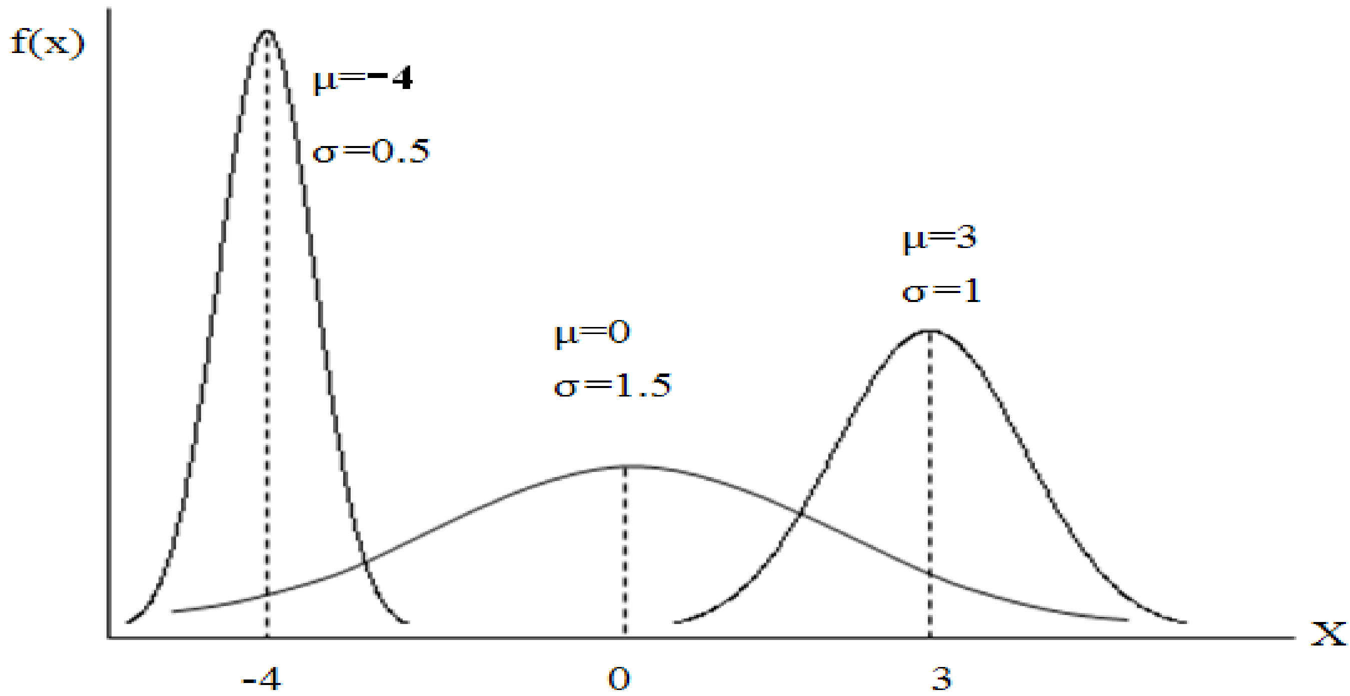
其中

- π = Mathematical constant approximated by 3.1416
- e = Mathematical constant approximated by 2.718
- μ = Population mean or the true mean
- σ^2 = Population variance

It is denoted by $N(\mu,\sigma)$

常態分佈(Normal Distribution)

- **常態曲線**
  - 原始資料之μ與σ值不同時，其常態曲線之變化亦不同。

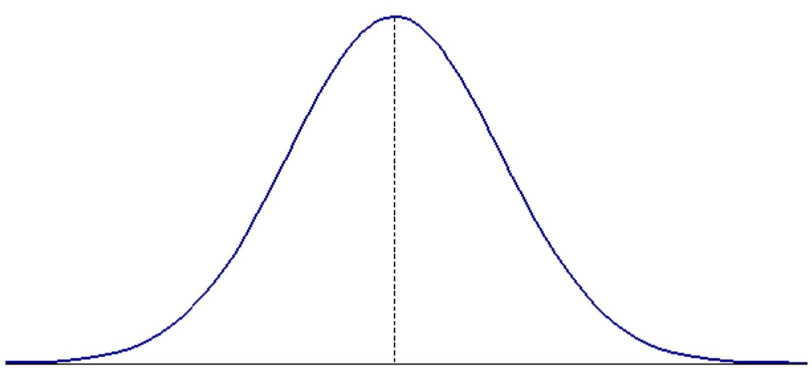


每一常態分佈，可以 $N(\mu,\sigma)$ 表之。

常態分佈(Normal Distribution)

- **$N(\mu,\sigma)$ 的特性**

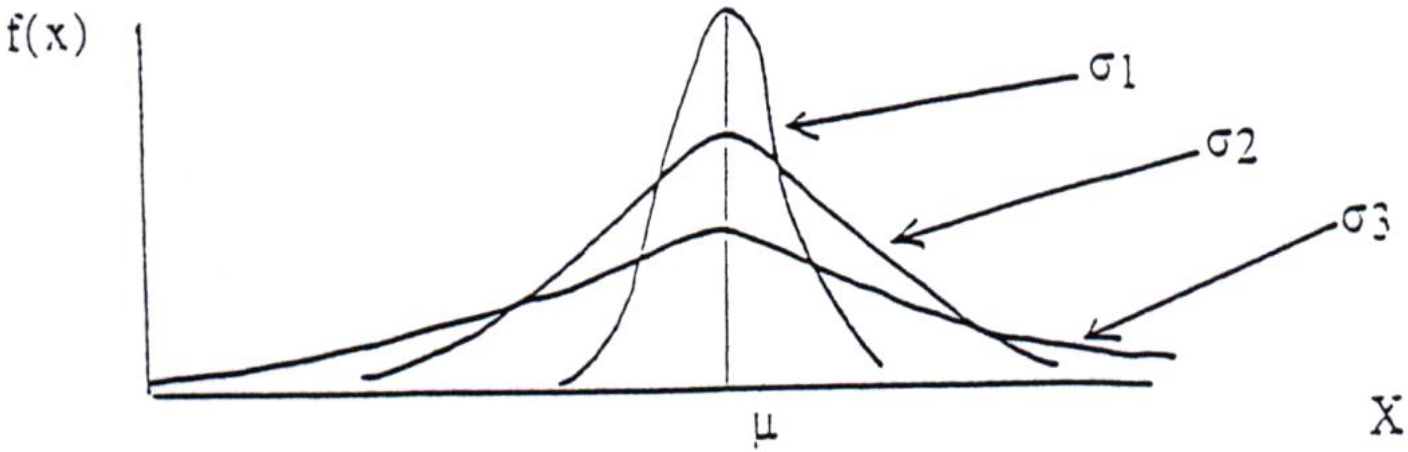
1) 對稱於 μ。
2) 隨機變數 x 之值可由 $-\infty$ 至 $+\infty$ 。
3) 鐘形分佈。
4) 曲線下之面積為 1 。
5) 集中趨勢的三個量數(平均數、中位數及眾數)是一致的。



平均數=中位數=眾數

常態分佈(Normal Distribution)

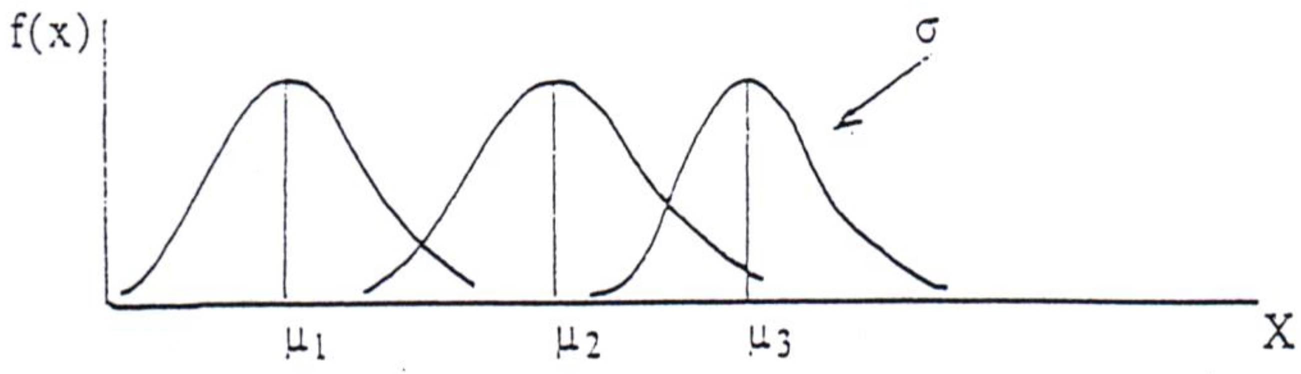
- ***μ* 與 *σ* 如何影響常態曲線**
  1) 下圖是三條有相同平均數μ但不同標準差 ($\sigma_1, \sigma_2, \sigma_3, \sigma_1<\sigma_2<\sigma_3$) 之常態曲線



由此圖中你觀察到什麼？ ______________________

常態分佈(Normal Distribution)

2) 下圖是三條有相同標準差σ但不同平均數 ($\mu_1,\mu_2,\mu_3,$ $\mu_1<\mu_2<\mu_3$) 之常態曲線



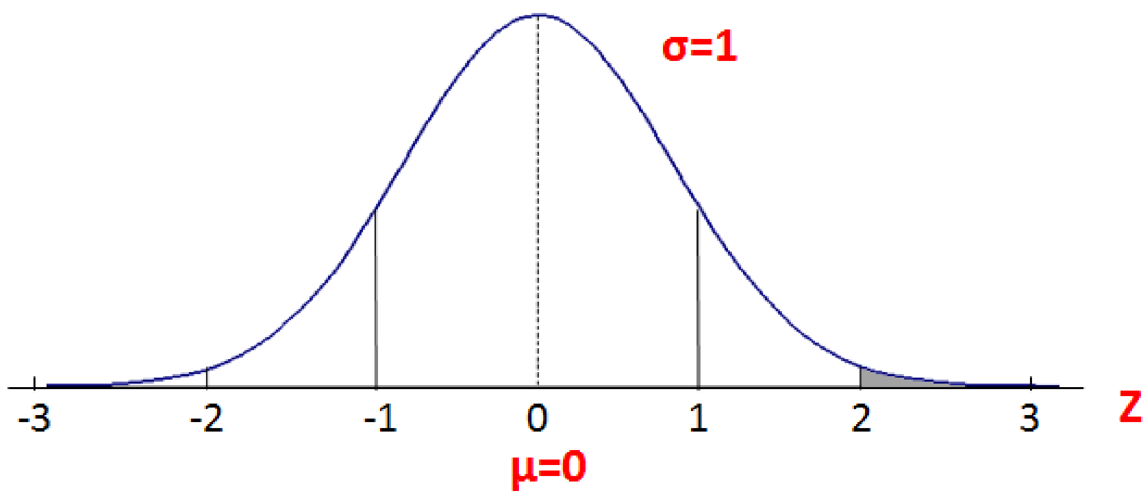
由此圖中你觀察到什麼？ ______________________

由1)與2)可知： ________ — 位置參數(Location parameter)

________ — 變異參數(Dispersion parameter)

如何使用常態機率表

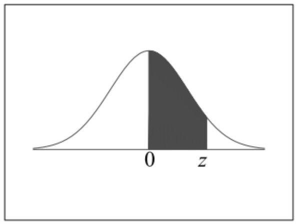
- **何謂標準常態分佈？**
  - 平均數為0、標準差為1之常態分佈稱為標準常態分佈，以 ***N(0,1)*** 表之。
  - 例：令Z為 $N(0,1)$ 之隨機變數，亦即$Z\sim N(0,1)$，其常態曲線如下圖。則，$P(2\leqq Z\leqq 3)$ = 曲線下介於_____與_____之間的面積＝陰影部份之面積



常態分佈(Normal Distribution)

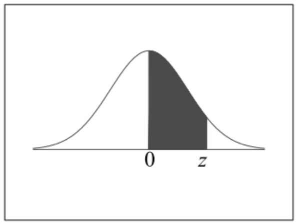
- **如何利用表查出標準常態之機率**
  - 下頁表為標準常態分佈 $N(0, 1)$ 之機率表。
  - 設 $Z \sim N(0, 1)$，請利用表找出下列之機率：
    1) $P(0 \leqq Z \leqq 1.96)=$
    2) $P(-1.81 \leqq Z \leqq 1.81)=$
    3) $P(0.53 \leqq Z \leqq 2.42)=$
    4) $P(Z \geqq -0.36)=$

Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Standard Normal Distribution Table

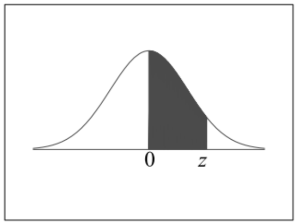


利用表求

$P(0 \leqq Z \leqq 1.96) =$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

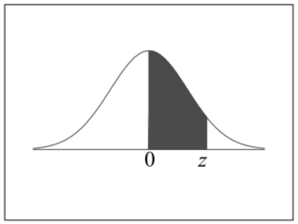
Standard Normal Distribution Table



利用表求
P(-1.81≦Z≦1.81)=

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Standard Normal Distribution Table

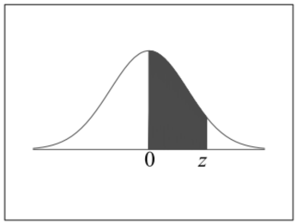


利用表求

$P(0.53 \leqq Z \leqq 2.42)=$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Standard Normal Distribution Table



利用表求

$P(Z \geqq -0.36)=$

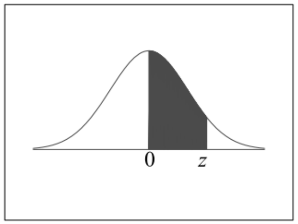
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

常態分佈(Normal Distribution)

– 設 $Z \sim N(0, 1)$，請利用表 C 值：

1) $P(Z < C) = 0.95$
2) $P(Z > C) = 0.7019$
3) $P(Z > C) = 0.1379$
4) $P(Z < C) = 0.0110$

Standard Normal Distribution Table

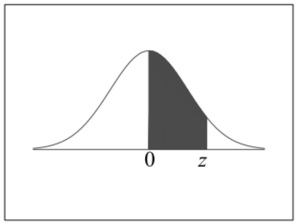


利用表求 C

P(Z < C) = 0.95

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Standard Normal Distribution Table

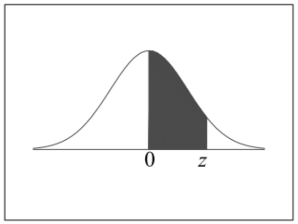


利用表求 C

P(Z > C) = 0.7019

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Standard Normal Distribution Table

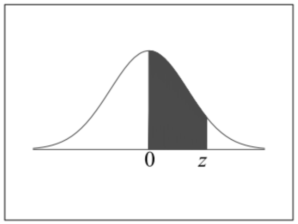


利用表求 C

P(Z > C) = 0.1379

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Standard Normal Distribution Table



利用表求 C

$P(Z < C) = 0.0110$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

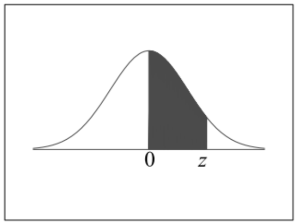
常態分佈(Normal Distribution)

- **如何求出一般常態變數之機率**
  - 作法：先將其標準化(Standardize)，轉換成標準常態變數後，再求其機率。標準化之公式如下：

$$\boxed{Z=\frac{X-\mu}{\sigma} \qquad ,\text{其中 } Z\sim N(0,1)}$$

- **例：設 $X\sim N(10,2)$ ，平均數=10，標準差=2**
  a) 請找出 X 介於 11 與 13.6 間之機率
  b) 請找出 X 大於 12 之機率

Standard Normal Distribution Table

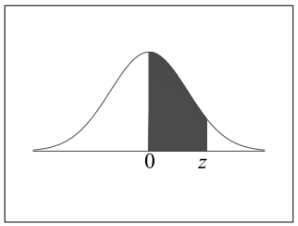


X~N(10,2)，利用右表
求 P(11≦X≦13.6) = ?

$$P(11 \le X \le 13.6)$$
$$= P(\frac{11-10}{2} \le Z \le \frac{13.6-10}{2})$$
$$= P(0.5 \le Z \le 1.8)$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Standard Normal Distribution Table



X~N(10,2)，利用右表求 P(X > 12) = ?

$$P(X > 12)$$
$$= P(Z > \frac{12-10}{2})$$
$$= P(Z > 1)$$
$$= 1 - P(Z < 1)$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
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1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
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2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

常態分佈(Normal Distribution)

- **例：**
  - 假設某產品之長度資料呈常態分佈，其平均數為38.5公分，標準差為2.5公分。若此產品之規格界限為38±2，請問此產品之不良率為何？

規格界限為38±2

長度 X~N(38.5,2.5)，
良率 P(36≦X≦40) = ?

$$P(36 \le X \le 40)$$
$$= P(\frac{36-38.5}{2.5} \le Z \le \frac{40-38.5}{2.5})$$
$$= P(-1 \le Z \le 0.6)$$
$$= 0.3413 + 0.2257$$
$$= 0.567$$

不良率= 1 — 良率
= 1-0.567
= 0.433

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

檢查數據是否呈常態分佈

檢查數據是否呈常態分佈

1) 利用直方圖

– 只要出現鐘形分佈圖形，即判定數據呈常態分佈

2) 利用常態機率圖

– 只要圖形呈直線，即判定數據呈常態分佈

3) 利用統計檢定

– 只要顯著度 p-value > 0.05，即判定數據呈常態分佈

a. 卡方適配度檢定(Chi-Square Goodness-of-fit Test)

b. K-S檢定(Kolmogorov-Smirnov test)

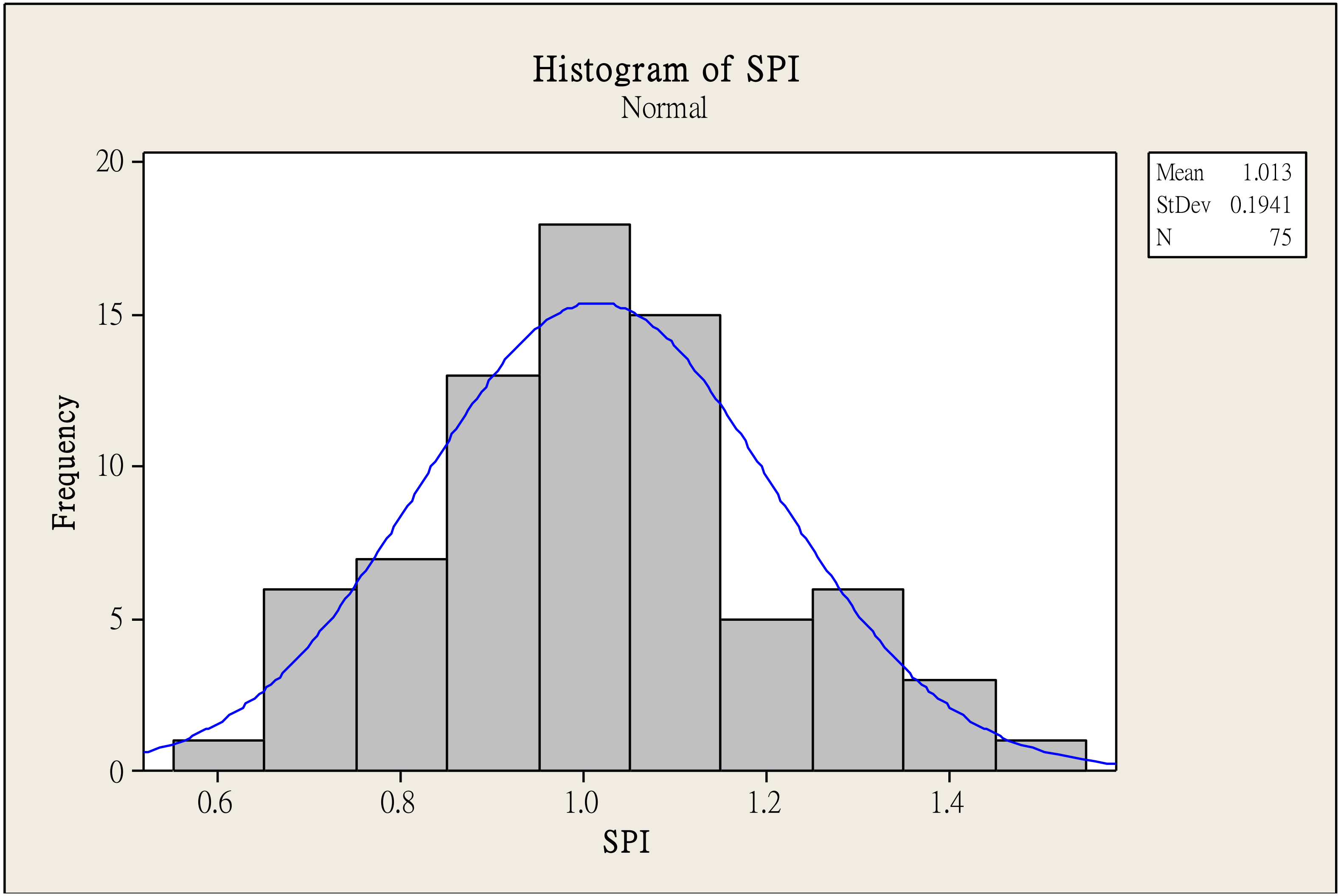
c. A-D檢定(Anderson-Darling Test)

檢查數據是否呈常態分佈

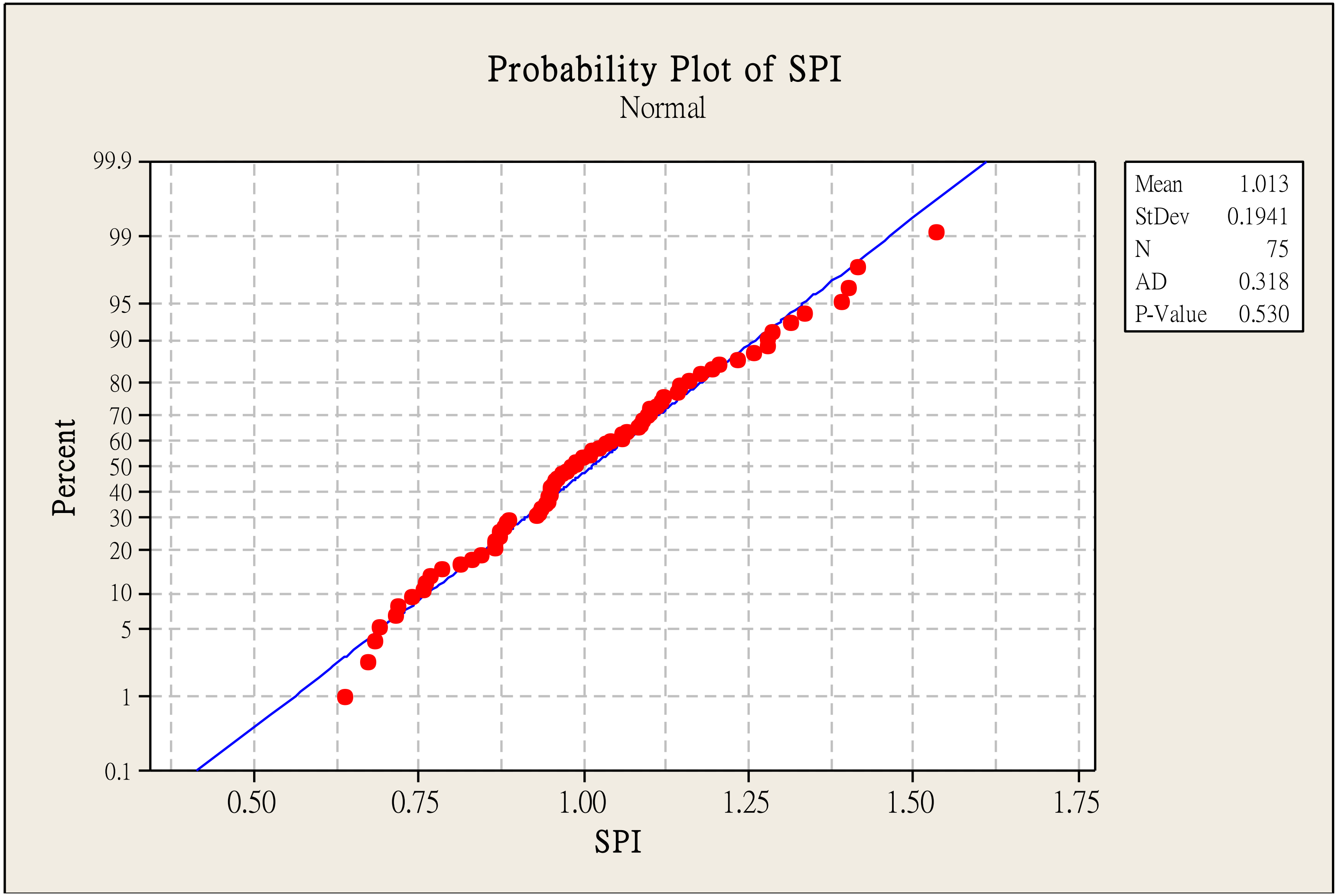
- 下列 75 筆數據為某模具上的孔徑尺寸值(mm)，請檢查數據是否呈常態分佈？

0.88	0.87	1.09	1.10	1.20
0.95	0.69	1.15	1.12	0.77
0.72	0.89	1.00	0.94	0.79
1.39	0.96	0.93	1.15	1.10
0.81	1.15	1.32	1.34	1.28
0.88	1.26	1.24	0.98	1.13
0.94	1.18	1.07	0.74	1.06
1.12	0.85	1.03	1.28	0.83
0.69	0.87	0.89	1.16	0.76
0.95	0.76	1.09	0.99	0.67
0.98	0.95	1.04	1.40	1.10
1.29	0.64	0.95	0.95	1.42
1.54	1.01	0.72	1.06	0.88
0.87	0.95	1.21	0.96	1.04
1.09	0.96	1.02	0.99	0.97

繪製直方圖


Histogram of SPI
Normal
Mean 1.013
StDev 0.1941
N 75
Frequency
20
15
10
5
0
0.6
0.8
1.0
1.2
1.4
SPI

繪製常態機率圖



Probability Plot of SPI
Normal
Percent
99.9
99
95
90
80
70
60
50
40
30
20
10
5
1
0.1
0.50
0.75
1.00
1.25
1.50
1.75
SPI
Mean 1.013
StDev 0.1941
N 75
AD 0.318
P-Value 0.530

連續型機率分佈

- **常用的連續型機率分佈**
  1) 常態分佈(Normal Distribution)
  2) 對數常態分佈(Lognormal Distribution)
  3) 齊一分佈(Uniform Distribution)
  4) 珈瑪分佈(Gamma Distribution)
  5) 指數分佈(Exponential Distribution)
  6) 韋伯分佈(Weibull Distribution)
  7) 貝塔分佈(Beta Distribution)

對數常態分佈
(Lognormal Distribution)

對數常態分佈 (Lognormal Distribution)

- **The log-normal distribution occurs when the logarithm of a random variable has a normal distribution.**
- **X is called a log-normal random variable if and only if**

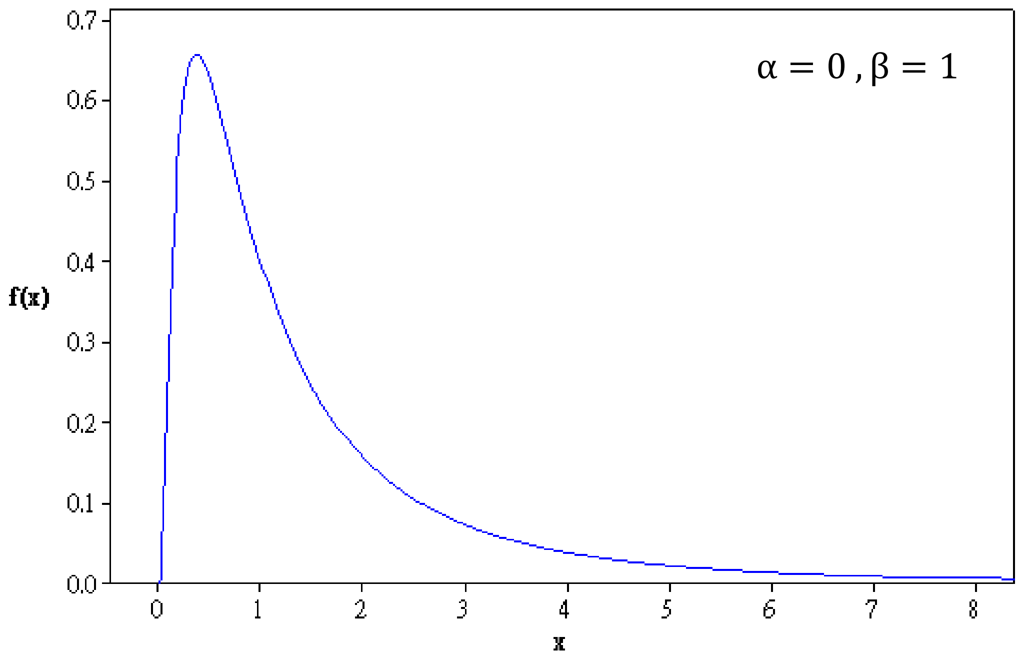
$$f(x) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\beta} x^{-1} e^{-\frac{(\ln x - \alpha)^2}{2\beta^2}} & x > 0, \beta > 0 \\ 0 & \text{otherwise} \end{cases}$$

Where $\ln x$ is the natural logarithm of X

對數常態分佈 (Lognormal Distribution)

- **The Mean and the Variance of Log-normal Distribution**

$$\mu = e^{\alpha+\frac{1}{2}\beta^2} \qquad \sigma^2 = e^{2\alpha+\beta^2}(e^{\beta^2}-1)$$



對數常態分佈 (Lognormal Distribution)

- **例 1**

  The current gain of certain transistors is measured in units which make it equal to the logarithm of $\mathbf{I_0/I_i}$, the ratio of the output to the input current. If it is normally distributed with $\mu = 2$ and $\sigma^2 = 0.01$, find

  1) The probability that $\mathbf{I_0/I_i}$ will take on a value between 6.1 and 8.2.
  2) The mean and the variance of the distribution of $\mathbf{I_0/I_i}$.

齊一分佈
(Uniform Distribution)

齊一分佈 (Uniform Distribution)

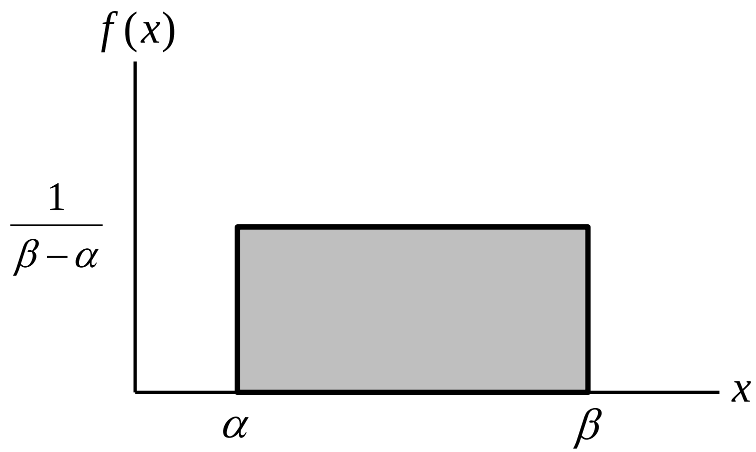
- The continuous random variable **X** is called a **uniform** random variable if and only if **X** is **uniformly** distributed over the interval (α, β), i.e., the density for X is

$$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & \alpha < x < \beta \\ 0 & \text{otherwise} \end{cases}$$

- The Mean and Variance for the Uniform R.V.

$$E(X) = \mu = \frac{\alpha + \beta}{2}$$

$$Var(X) = \sigma^2 = \frac{(\beta - \alpha)^2}{12}$$



齊一分佈 (Uniform Distribution)

- **例 2**

  If X is uniformly distributed over (0, 10), calculate the probability that

  a) $X < 3$

  b) $X > 6$

  c) $3 < X < 8$

齊一分佈 (Uniform Distribution)

- **例3**

Buses arrive at a specified stop at **15-minute intervals** starting at **7, 7:15, 7:30, 7:45**, and so on. If a passenger arrives at the stop at a time that is uniformly distributed between **7** and **7:30**, find the probability that he waits

a) **less than 5** minutes for a bus

b) **More than 10** minutes for a bus

課本例 Example 5.7

- **Suppose the research department of a steel manufacturer believes that one of the company's rolling machines is producing sheets of steel of varying thickness. The thickness Y is a uniform random variable with values between 150 and 200 mm. Any sheets less than 160 mm thick must be scrapped, since they are unacceptable to buyer.**

a) Calculate the mean and standard deviation of Y, the thickness of the sheets produced by this machine. Then graph the probability distribution, and show the mean on the horizontal axis. Also show 1 and 2 standard deviation intervals around the mean.

b) Calculate the fraction of steel sheets produced by this machine that have to be scrapped.

課本例 Example 5.7

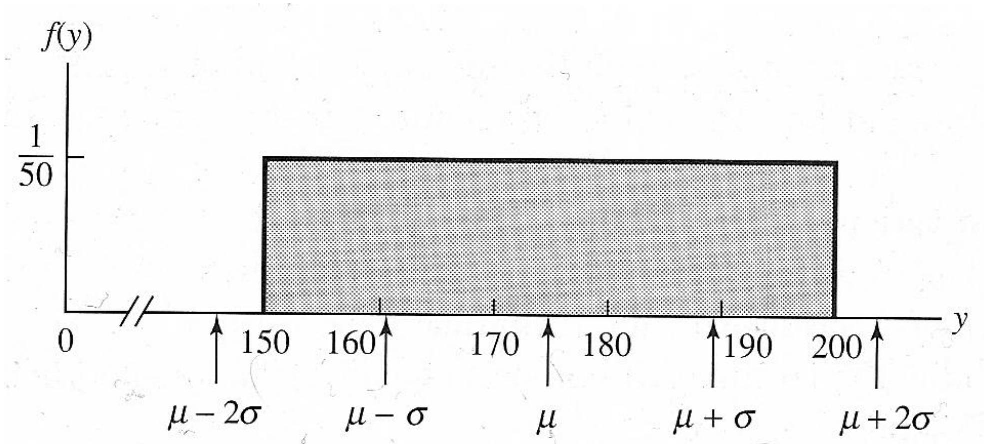
Solution :

a) $$\mu = \frac{\alpha+\beta}{2} = \frac{150+200}{2} = 175\,(\text{mm})$$

$$\sigma = \sqrt{\frac{(\beta-\alpha)^2}{12}} = \frac{\beta-\alpha}{\sqrt{12}} = \frac{200-150}{\sqrt{12}} = 14.43\,(\text{mm})$$

The uniform probability distribution is

$$f(y) = \frac{1}{\beta-\alpha} = \frac{1}{200-150} = \frac{1}{50}$$

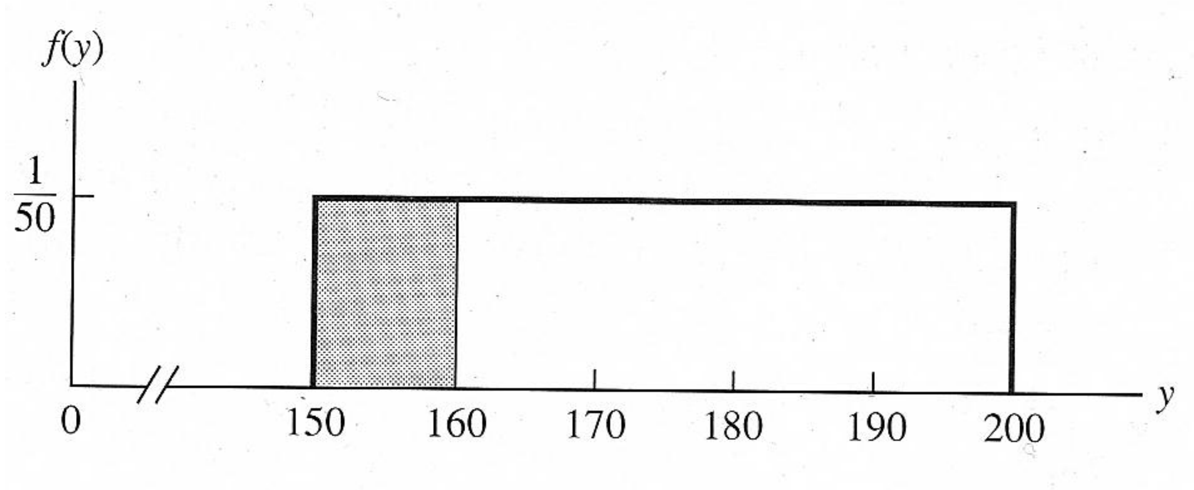


課本例 Example 5.7

Solution :

b) $$P(Y < 160) = (160 - 150) \times (\frac{1}{50}) = \frac{1}{5}$$

That is, 20% of all the sheets made by this machine must be scrapped.



珈瑪分佈
(Gamma Distribution)

珈瑪分佈 (Gamma Distribution)

- **Several important probability densities (such as Exponential, Weibull) are special cases of the gamma distribution.**
- **Def: X is called a Gamma random variable if and only if**

$$f(x) = \begin{cases} \dfrac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}} & x > 0, \alpha > 0, \beta > 0 \\ 0 & \text{otherwise} \end{cases}$$

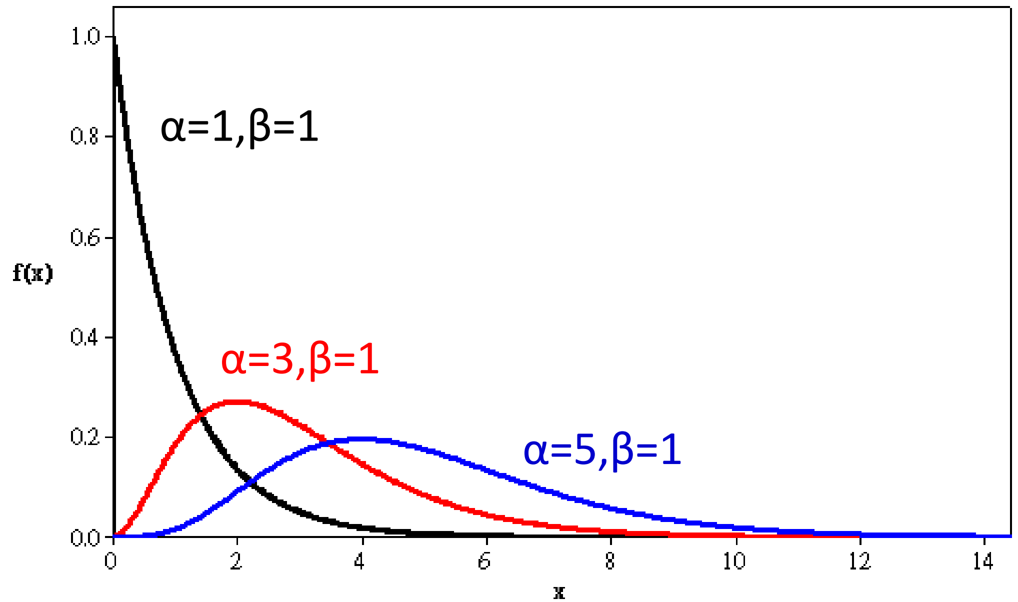
Where $\Gamma(\alpha)$ is the value of the gamma function

- **Gamma function**

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx \quad where \;\; \alpha > 0$$

珈瑪分佈 (Gamma Distribution)

- **Properties of the Gamma function**
  1) $\Gamma(\alpha) < \infty, if\ \alpha > 0$
  2) $\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1), \text{if } \alpha > 1$
  3) $\Gamma(\alpha) = (\alpha - 1)!, \text{if } \alpha \text{ is a positive integer}$
- **The Mean and Variance for the Gamma R.V**
  - $E(X) = \alpha\beta$
  - $Var(X) = \alpha\beta^2$



珈瑪分佈 (Gamma Distribution)

- **The Chi-Square Probability Distribution**
  - A Chi-square random variable is a gamma-type random variable X with $\alpha = \nu / 2$ and $\beta = 2$

$$f(x) = c(x)^{(\nu/2)-1} e^{-x/2} \qquad (0 \le x^2 \le \infty)$$

$$\text{where} \quad c = \frac{1}{2^{\nu/2}\Gamma(\nu/2)}$$

The parameter ν is called the number of degrees of freedom.

  - The mean and variance of a chi-square random variable are:

$$\mu = \nu \quad and \quad \sigma^2 = 2\nu$$

課本例 Example 5.12

- From past experience, a manufacturer knows that the relative frequency distribution of the length of time Y (in months) between major customer product complaints can be modeled by a gamma density function with $\alpha=2$ and $\beta=4$. Fifteen months after the manufacturer tightened its quality control requirements, the first complaint arrived. Does this suggest that the mean time between major customer complaints may have increased ?

課本例 Example 5.12

- **Solution**

$$\mu = \alpha\beta = (2)(4) = 8$$

$$\sigma^2 = \alpha\beta^2 = (2)(4)^2 = 32$$

$$\sigma = 5.7$$

Since Y=15 month lies barely more than 1 standard deviation beyond the mean ($\mu+\sigma = 8 + 5.7 = 13.7$ months), we would not regard 15 months as an unusually large vale of Y. Consequently, we would conclude that there is insufficient evidence to indicate that the company's new quality control program has been effective in increasing the mean time between complaints.

指數分佈
(Exponential Distribution)

指數分佈 (Exponential Distribution)

- **X is called an <u>exponential</u> random variable if and only if**

$$f(x) = \begin{cases} \dfrac{1}{\beta} e^{\frac{-x}{\beta}} & x > 0 \text{ and } \beta > 0 \\ 0 & \text{otherwise} \end{cases}$$

Exponential distribution is a special case of Gamma distribution with α = 1

- **The Mean and Variance for the Uniform R.V.**
  - $E(X) = \beta$
  - $Var(X) = \beta^2$

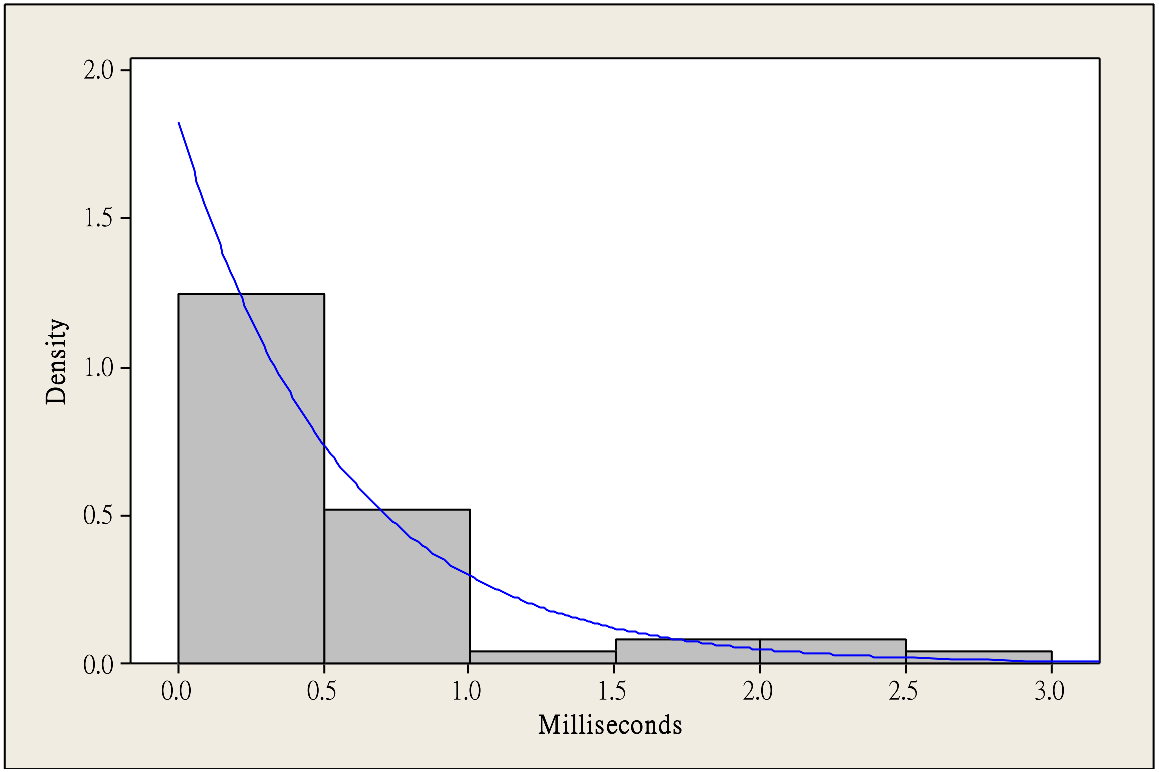
指數分佈 (Exponential Distribution)

- **例5**
  - A nuclear engineer observing a reaction measures the time intervals between the emissions of beta particles.

0.894	0.991	0.061	0.186	0.311	0.817	2.267	0.091	0.139	0.083
0.235	0.424	0.216	0.579	0.429	0.612	0.143	0.055	0.752	0.188
0.071	0.159	0.082	1.653	2.010	0.158	0.527	1.033	2.863	0.365
0.459	0.431	0.092	0.830	1.718	0.099	0.162	0.076	0.107	0.278
0.100	0.919	0.900	0.093	0.041	0.712	0.994	0.149	0.866	0.054

指數分佈 (Exponential Distribution)

- These decay times (in milliseconds) are presented as a histogram in the following figure



指數分佈 (Exponential Distribution)

- **Remark:**
  - It can be shown that in connection with Poisson processes the **waiting time** between successive arrivals has an exponential distribution.
  - More specifically, it can be shown that if in a **Poisson** process the **mean arrival rate (average number of arrivals per unit time)** is $\boldsymbol{\lambda = 1/\beta}$, the **time until the first arrival**, or the **waiting time** between successive arrivals, has an **exponential** distribution with $\boldsymbol{1/\beta}$.

指數分佈 (Exponential Distribution)

- **例6**
  - If on the average three trucks arrived per hour to be unloaded at a warehouse, what are the probabilities that the time between the arrival of successive trucks will be

  a) Less than 5 minutes

  b) At least 45 minutes

  c) What is the expected waiting time between successive arrivals?

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