ECE 9305A: Introduction to Probability Theory I
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Homework Assignments

[Each homework assignment begins on a new page]

Homework 1 (2018/09/19): Simulating the frequentist definition of probability

- Generate N binary digits with $P_1 = 0.5 + 0.045 * 8 = 0.86$, $P_0 = 0.14$ for N = 10, 100 ...
- Observe the relative frequencies (F) of the two outcomes, 1 and 0 (table below).

Outcome	N = 10 Relative freq.	N = 100 Relative freq.	N = 1000 Relative freq.	N = 10000 Relative freq.	N = 100000 Relative freq.
1	0.9	0.89	0.859	0.8603	0.8599
0	0.1	0.11	0.141	0.1397	0.1401

- It was observed that as this experiment was repeated with increasing N, the two relative frequencies got closer and closer to the original probabilities $P_1 = 0.86$ and $P_0 = 0.14$.
- At around N = 100000, the changes to the relative frequencies as N was doubled (to 200000) were very small (~0.0001).
- Therefore, N = 100000 was considered as the infinity for which the following quantity approaches the limit. This quantity is the probability of the outcome according to the frequentist definition of probability.

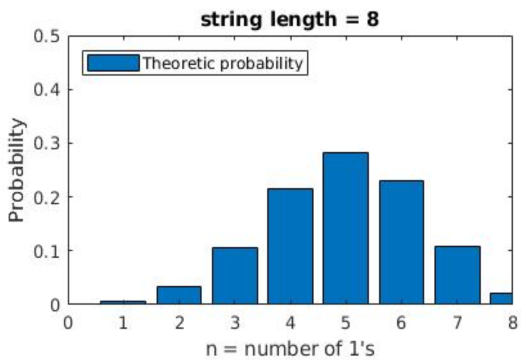
$$Probability\ of\ outcome = \lim_{N \to inf} Relative\ frequency\ of\ outcome$$

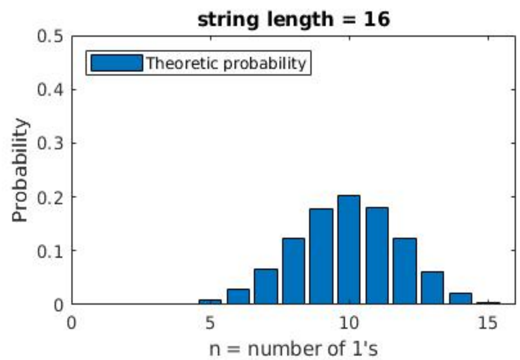
Homework 2 (2018/09/26): Probability of n number of 1's in a binary string of length N

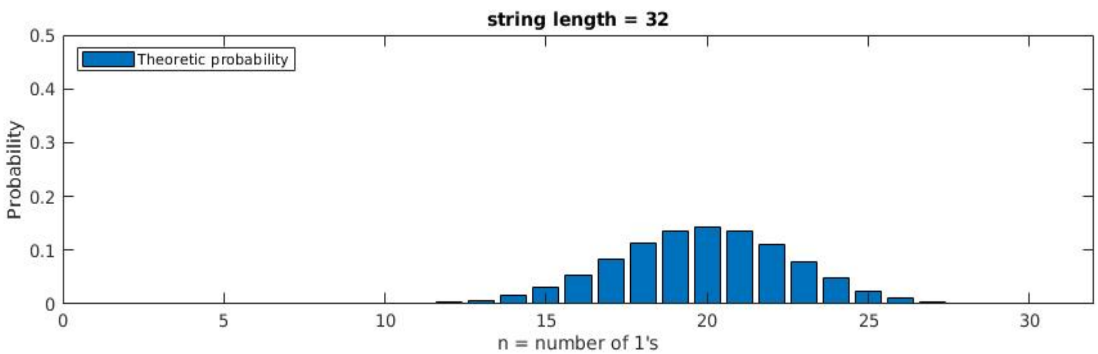
- A binary digit string of length N is generated (consider cases N = 8, 16, 32).
  $P_0 = 0.3 + 0.01 * 8 = 0.38$, $P_1 = 0.62$.
- Part 1: the theoretic probability of having exactly n number of 1's in this string is given by

$$\frac{N!}{n!(N-n)!} . P_1^n . P_0^{N-n}$$

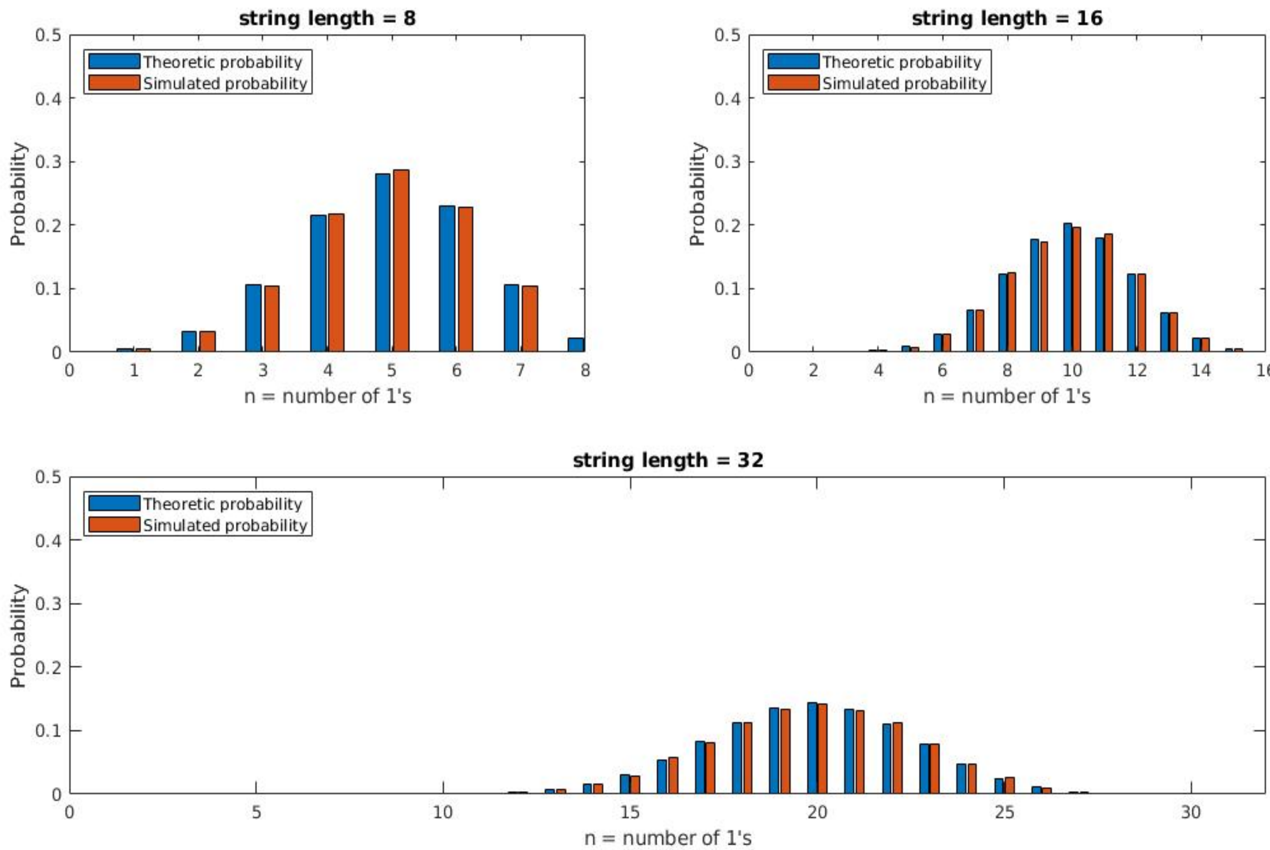
- This probability is calculated and plotted for below $n \in [0, N] \, for \, N = 8, 16, 32$.







- Part 2: the probability of having exactly n number of 1's in the string is estimated experimentally by generating a large number of strings (10000 trials in this experiment), and counting the number of 1's in the string in each trial.
- The probabilities estimated in this way are plotted against the theoretic values in the graph below (next page).
- We can clearly see that the theoretic values and the experimental values are approximately equal. It was also seen that when the number of trials was increased to 1 million, the experimental values got even closer to the theoretic values.


string length = 8
Theoretic probability
Simulated probability
Probability
0.5
0.4
0.3
0.2
0.1
0
0 1 2 3 4 5 6 7 8
n = number of 1's
string length = 16
Theoretic probability
Simulated probability
Probability
0.5
0.4
0.3
0.2
0.1
0
0 2 4 6 8 10 12 14 16
n = number of 1's
string length = 32
Theoretic probability
Simulated probability
Probability
0.5
0.4
0.3
0.2
0.1
0
0 5 10 15 20 25 30
n = number of 1's

Homework 3 (2018/10/03):