Determining Optimal Locations for Free Standing Ambulance Stations to Minimize First Responder Wait Times

Final Report

Lily Lin (88911094)
Zoë Eshan (79909628)

1 Introduction

In this report we develop a model that offers a possible method for Vancouver to meet the national goal for emergency response time of eight minutes and fifty-nine seconds. According to Ambulance Paramedics of BC, this goal is only being met 30 percent of the time in the province of British Columbia. It seems that this discrepancy is due to a number of factors including a province-wide shortage of paramedics and the subsequent closure of many ambulance stations. [1]

A large factor in responding to emergency calls quickly is the time it takes for an ambulance to reach the callers location. A proposed way of alleviating the response time problem is to have so called free standing ambulance stations, which are places were a ambulance can park to be closer to possible call sites and respond more quickly. These stations can be staffed with paramedics who might not have full medical degrees but would be able to administer emergency first aid and get patients to hospitals quickly. As hospitals are already seeing a shortage of workers, a natural goal of the city would be to achieve the response time goal with as few additional staff needed using a few low staff ambulance stations.

In order to properly minimize the number of ambulance stations needed, we developed a series of models for a hospital system in Vancouver which optimize for this exact problem. We began with a basic set covering model which aimed to cover the entire city of Vancouver so that everyone would be theoretically located within an 8 minute and 59 second emergency response radius. Later we focused on P-Median models to account for average response times as well as some real world factors such as population density and senior populations.

We will explain how the set covering and p-median models work and compare and contrast our two models while assessing their impact on the real world emergency response system in Vancouver. Finally, we develop a model for adding free standing ambulance stations to Vancouver and compare the result with the "ideal" results produced by the aforementioned models.

2 Set Covering Model

The Set Covering problem is a well known discrete optimization problem where given a set of subsets of a universal set U, the objective is to choose the fewest number of subsets such that all elements of U belong to at least one subset. Set Covering models in Location Theory are adaptations of the Set Covering problem to the idea of choosing the fewest number of locations to cover a given area. The problem is modelled as a set of potential supply locations and a set of demand locations. Each potential supply location "covers" a certain subset of demand locations based on factors such as distance or travel time. These subsets are then interpreted as a Set Covering problem, essentially choosing the fewest number of supply locations to cover all demand locations.

One notable drawback of this model is that a Set Covering model does not distinguish between customers who might be covered 'better' because either they are covered by multiple service locations or are closer to a service than customers on the edges of a radius. [2]

In general, covering problems are divided into two categories based on their objectives. The first time seeks to minimize the number of stations needed to cover an entire population (set covering), and the second starts with a set number of stations and optimizes the location of those stations to maximize the amount of the population that can be covered (maximum covering). [2]

For our project we began by looking at the first category: the traditional set covering problem. We used the Torgas et al's 1971 formulation which was first developed specifically for modeling coverage for emergency service vehicles, so stretching it to fit our project wasn't too big of a leap. [2]

The model can be formulated as a integer programming problem like so:

$$\begin{aligned}
&Minimize: \quad && Z = \sum_{j \in J} x_j \\
&S.T: && \sum_{j \in N_i} x_j \geq 1, \qquad \forall i \in I \\
& && x_j \in \{0,1\}, \quad \forall j \in J
\end{aligned} \tag{1}$$

where

J = the set of eligible sites for an ambulance station (indexed by j);

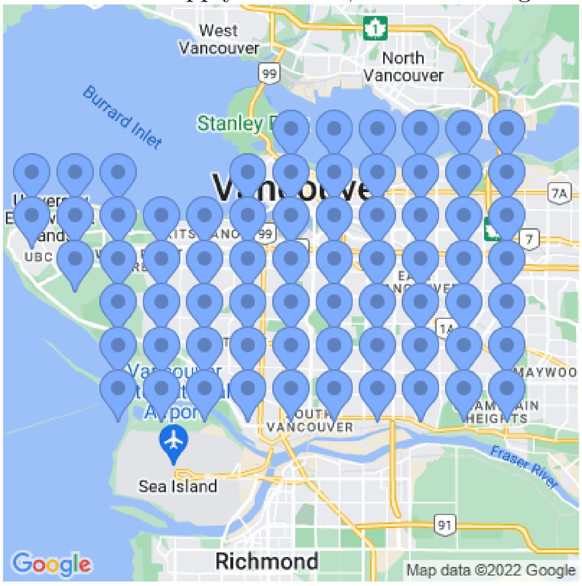
I = the set of demand nodes;

$x_j = 1$ if a station is located at node j and 0 otherwise;

$N_i = [j|d_{ji} \leq S]$ the set of sites that cover demand node i with d_{ji} = the shortest distance from a potential ambulance station located at j to demand node i, and S = radius of 8 minutes and 59 seconds.

For simplicity, the set of eligible sites and the set of demand nodes were set to be equal to each other, and the set was built by filling the area representing the city of Vancouver with equidistantly spaced points as shown in Figure 1

Figure 1: The base set of demand and supply locations, rendered using the Google Maps Static API



A customer is considered ‘covered’ if their home is within an eight minute and fifty-nine second approximate travel time from the nearest hospital or freestanding ambulance station. In our first model this corresponded to being at most 3km away.

Solving the resulting integer programming problem using scipy we get the solutions shown in Figure 2 and 3. In Figure 2 is the set of 9 locations that optimally cover the set of points and in Figure 3 a visualization of the coverage of each hospital.

Figure 2: Optimal solution to the set covering problem with straight line distances

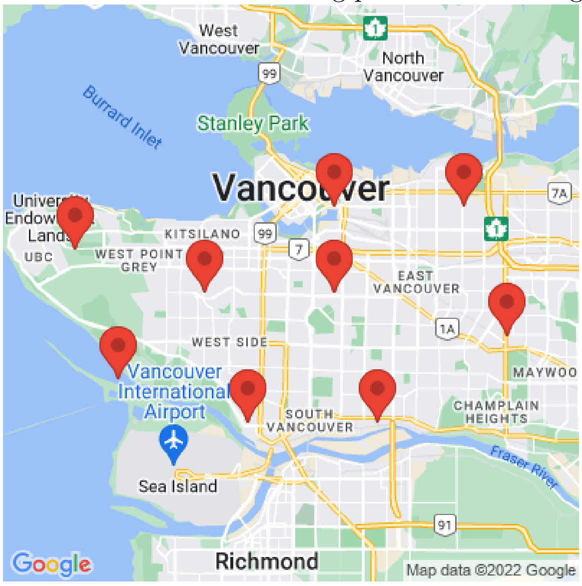
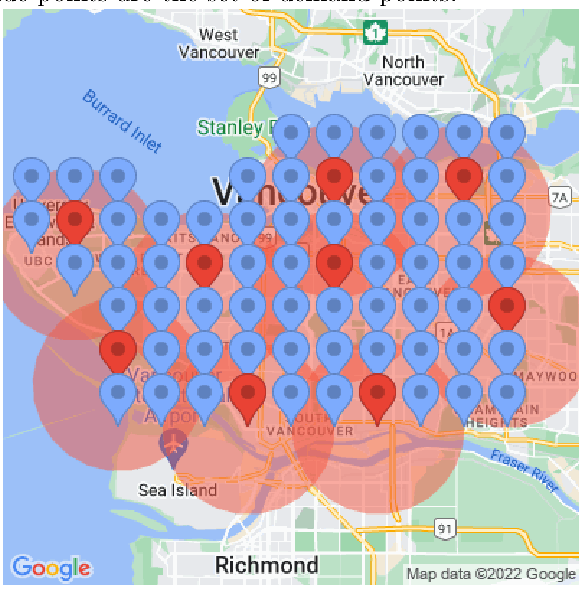


Figure 3: A visualization of the demand points and the coverage of each chosen supply location. The red points represent the chosen supply locations and the red shaded circles represent the areas covered by them. The blue points are the set of demand points.



However choosing to model travel time as proportional to the straight line distance between points is not very realistic. Vancouver roads function primarily in an irregular grid and to reach a point at a diagonal to the ambulance station would mean driving along two perpendicular roads instead of moving along the shorter but impossible straight line distance path. To accurately represent distance covered we chose to use the Google Maps Distance Matrix API, which can route a path between two points based on real roads and traffic. Using this new set of distances we recomputed our covering set N_i and produced a second set covering solution, shown in figures 4 and 5.

Figure 4: Optimal solution to the set covering problem with Google Maps distances

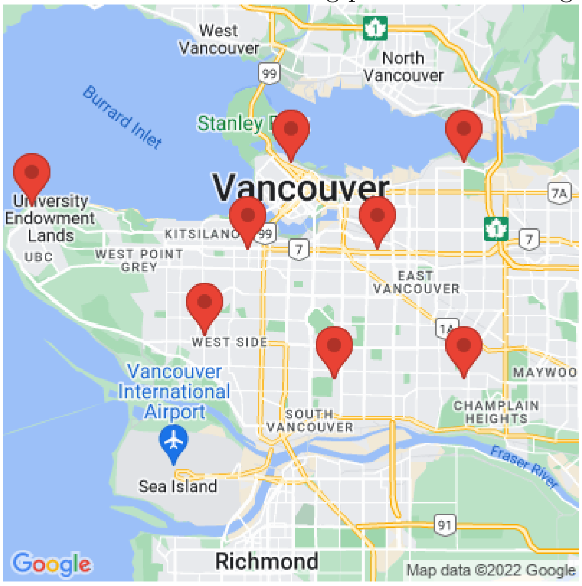
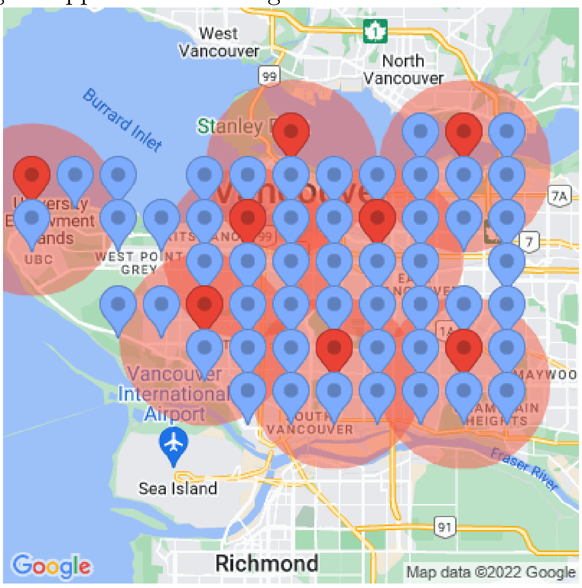


Figure 5: A visualization of the demand points and coverage of the set covering solution with Google Maps distances. Coverage is approximated using circles



We see that these two results are relatively good solutions to our problem. Both models optimize for a minimal number of locations while covering all of Vancouver, however both have a few key flaws. As mentioned earlier, in the Set Covering model the distance between a demand point and the supply point that covers it is irrelevant as long as it's within the specified bound. This does not reflect the situation we want to optimize for, which calls for minimizing this distance to respond to calls more quickly in as many cases as possible. Additionally there are many other key real world considerations that are not accounted for in this model, such as relative need by area and population density.

While the more sophisticated Google Maps API alleviates some of the concerns about realism, Google Maps APIs are not free and in the process of experimenting with variations on the second model we used up the free credits available to us. So due to the aforementioned fundamental flaws in the Set Covering formulation as well as the lack of a budget for this project, no further Set Covering based models were considered.

3 P-median Models

Distinct from the Set Covering problem, P-median models focus on minimizing the travel time or distance from a customer to a location where services are provided. The problem can be formulated as follows. First the model is given a set of patients and a set of possible hospital locations. The decision variables are then to decide which of the given possible hospital locations should be allocated and which hospital each patient should be assigned to. These decision variables must satisfy that exactly n hospitals are allocated and each patient is allocated to a hospital that exists. The objective function is then to minimize the sum of the distances between each patient and their assigned hospital.

The P-median model can be expressed as an integer programming problem like so:

$$\begin{aligned}
&Minimize: \quad Z = && \sum_{i=1}^{m}\sum_{j=1}^{n} Y_{ij} cost(i,j) && \\
&S.T: && \sum_{i=1}^{m} Y_{ij} \leq mX_j && j = 1,2,...,n \\
& && \sum_{j=1}^{n} Y_{ij} = 1 && i = 1,2,...,m \\
& && \sum_{j=1}^{n} X_j = p && \\
& && X_j \in (0,1) && i = 1,2,...,m \\
& && Y_{ij} \in (0,1) && i = 1,2,...,m, j = 1,2,...,n
\end{aligned} \tag{2}$$

Where:

$X_j = 1$ if there is a station assigned to location j, and 0 otherwise

$Y_{ij} = 1$ if the customer i is assigned to ambulance station j, and 0 otherwise

$m =$ the total number of patients

$n =$ the total number of potential free standing ambulance sites

$p =$ the number of hospitals to allocate

$cost(i, j) =$ the cost to travel from patient i to location j

The first condition ensures that if a patient is assigned to hospital j, then $X_j = 1$. The second ensures that patient i is assigned to exactly one hospital. Finally the third condition ensures that exactly p hospitals are allocated.

In the earlier Set Covering model the choice of demand and potential supply locations did not significantly affect the resulting model due to how the Set Covering model computes its solution. The important factors in the Set Covering model are ensuring that the chosen demand points properly fill the desired region and that the chosen supply points are dense enough to provide an accurate solution.

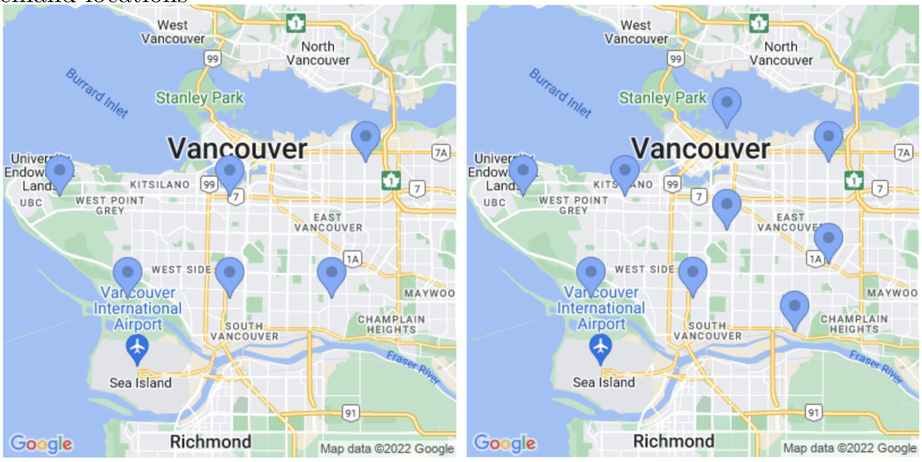
In the P-median model we see that the choice of potential supply locations still has this property, but the choice of demand locations is of much greater importance. Essentially these demand locations represent the population that the model will minimize the distance to, so we considered a number of variations on demand locations which we will outline in the following section. The demand locations considered were based on the following four variations:

1. Uniform demand
2. Population density based demand
3. Senior population based demand
4. Balanced demand

3.1 Demand location variations

In the first variation, we chose our demand locations to be the same uniform set of 69 points seen in Figure 1. This produces a set of hospitals that are uniformly spread throughout the region, as expected.

Figure 6: Optimal 6 hospital (left) and 9 hospital (right) placements for the p-median model with uniform demand locations



However this obviously produces a result unlike what we want, since we know that people in Vancouver are not uniformly distributed across the region. Notably we see that due to the uniformity of the demand locations, the model chooses to place hospitals in areas with no population at all such as in Pacific Spirit park and into the Fraser river.

To incorporate population density into the model we chose to adjust the demand locations to more accurately represent the distribution of people in Vancouver. To do this we collected census data from CensusMapper, a website for visualizing Canadian Census data, and created a mapping of census region to population within that region. Then using population as a weight we determined how many randomly chosen Vancouver residents should reside in each census region. As a simplifying assumption, all residents of a region were assumed to be located at the same latitude and longitude since census regions are relatively small. The results from this model can be found in Figure 8.

Figure 7: CensusMapper regions and populations

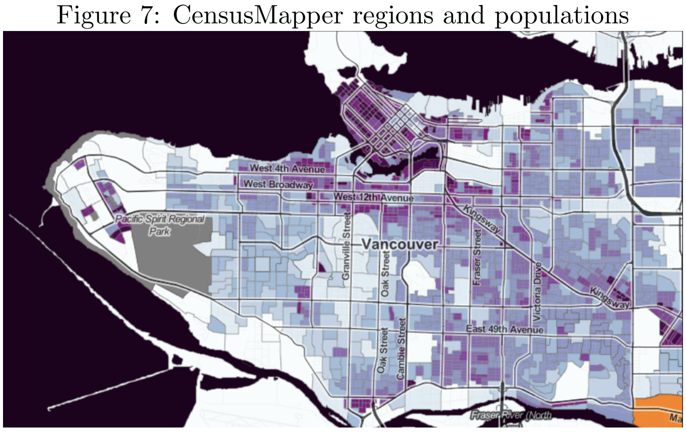
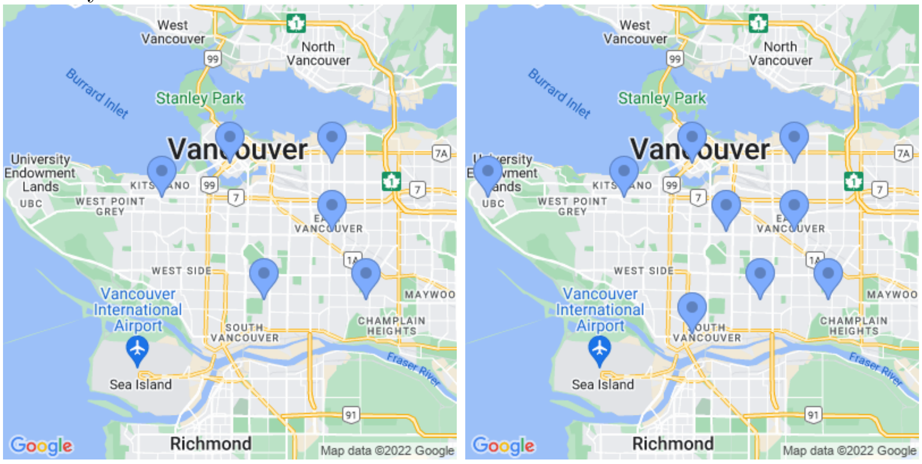
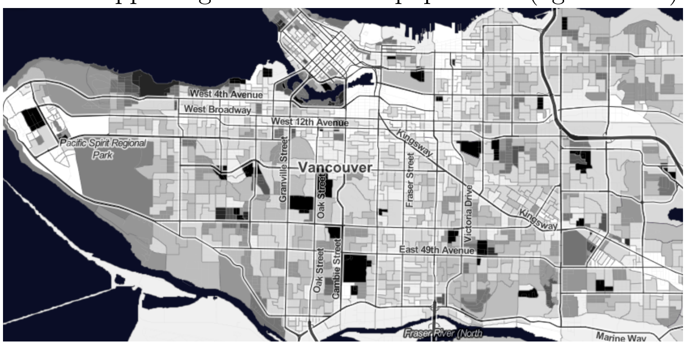


Figure 8: Optimal 6 hospital (left) and 9 hospital (right) placements for the P-median model with population density based demand locations



In the prior model the implicit assumption was made that emergency calls correlated directly with the population of a given region, however that is likely not true as shown by a case study in Germany which found that while people above the age of 75 made up only 9 percent of the population, they made 33 percent of all emergency calls in 2012 [3].

Figure 9: CensusMapper regions and senior population (age over 65) per region



Looking at a CensusMapper map of the senior population in each census region (replicated in Figure 9), we see that the distribution is quite different from the population distribution map found in Figure 7, furthering the argument for including this data in choosing the demand locations. Based on this we produced two more variations on demand locations, one entirely based on the population density of seniors in Vancouver as well as a balanced model where 30 percent of calls originate from senior populations and 70 percent from the general population.

Figure 10: Optimal 6 hospital (left) and 9 hospital (right) placements for the P-median model with senior population density based demand locations

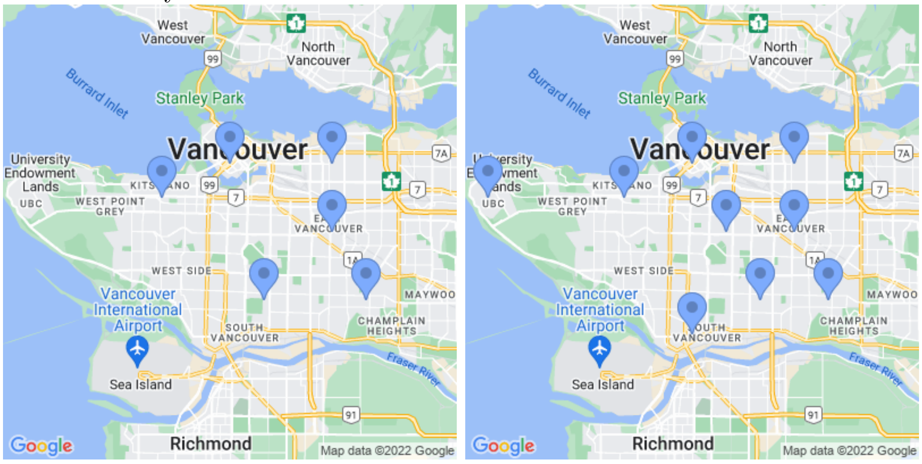
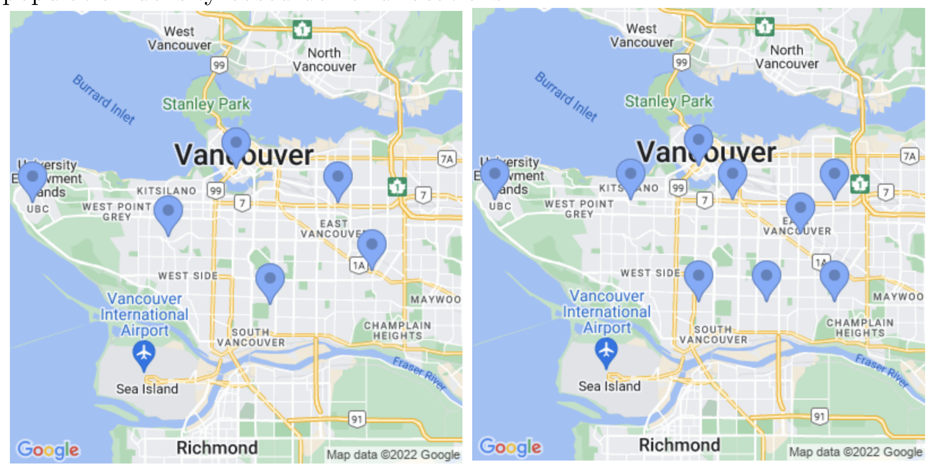


Figure 11: Optimal 6 hospital (left) and 9 hospital (right) placements for the P-median model with balanced population density based demand locations



3.2 Fixed point P-median

In the introduction we proposed the usage of free standing ambulance locations as additions to the Vancouver hospital system to lower the average response time. In order to optimize for this modified problem we first developed a modified version of the P-median model. To optimize for a system with L locations that must be allocated by adding the condition $\sum_{i=1}^{L} X_i = L$.

This ensures that the first L locations are allocated in the solution, while still allowing the decision variables to decide where each patient should be assigned, as well as where the remaining hospitals should be allocated. Using this modified problem we computed the optimal placement of free standing ambulance stations with a balanced population sample as the demand locations, the results of which can be seen in Figure 13.

Figure 12: The current Vancouver hospital network. From north to south these are Lion's Gate hospital, St. Paul's hospital, Vancouver general hospital, UBC hospital, BC women's hospital, Burnaby hospital, and Richmond hospital

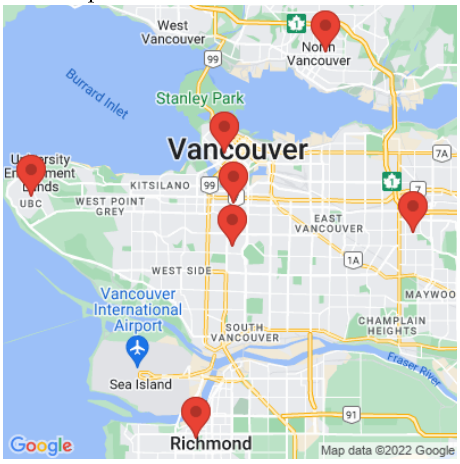
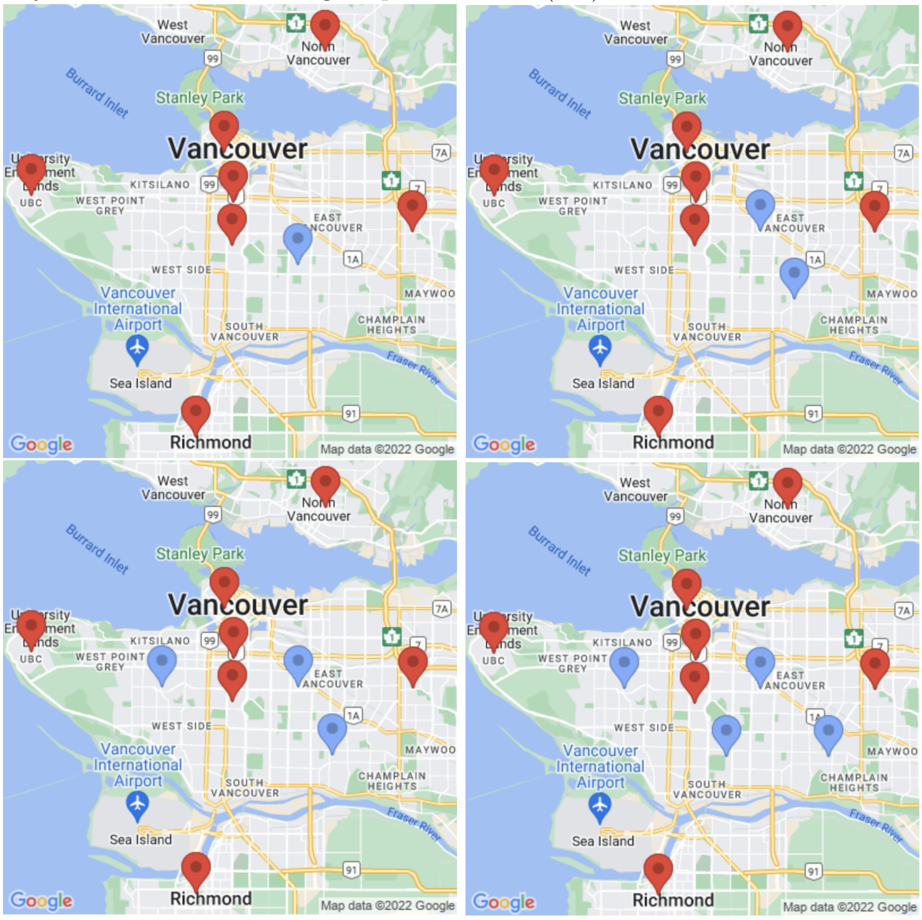


Figure 13: The optimal placements of one, two, three, and four free standing ambulance stations (blue) in conjunction with the existing hospital network (red)



4 Model evaluation

To evaluate our models we computed different samples of locations and computed the average distance to the nearest hospital for those samples in the given model. The evaluation samples chosen mirror the different demand populations we considered for the p-median models, namely a uniformly spread population, a population density based sample, a senior population based sample, and a balanced sample. The method for choosing these evaluation populations was the same as the previously outlined procedure for choosing a demand population.

Since each P-median model is optimizing for its corresponding evaluation sample, we expect to see that mirrored in the results, which we do in Figure 14

Figure 14: Table of results by various evaluation populations. The best model for each evaluation method is highlighted in blue

model name	uniform sample	density sample	senior population sample	balanced sample
Uniform demand, 6 hospitals	1.993	1.986	1.967	2.021
Uniform demand, 9 hospitals	1.712	1.609	1.616	1.643
Population density demand, 6 hospitals	2.607	1.723	1.756	1.684
Population density demand, 9 hospitals	1.98	1.387	1.461	1.413
Senior population demand, 6 hospitals	2.291	1.733	1.721	1.743
Senior population demand, 9 hospitals	1.829	1.515	1.446	1.477
Balanced population demand, 6 hospitals	2.316	1.732	1.75	1.751
Balanced population demand, 9 hospitals	1.851	1.417	1.465	1.412

As expected, the population based samples all perform similarly poorly on the uniform sample based evaluation, since they all only consider locations where people actually live, rather than any point in the Vancouver region.

Notably we see in Figure 15 that there are significant diminishing returns for more than 6 hospitals, leading us to the conclusion that 6 hospitals is the most realistic option for the city to pursue. As expected, more hospitals are always better so the 9 hospital solutions all perform the best in their respective categories.

Figure 15: Table of results for the balanced P-median model on the balanced sample for 1-9 hospitals showing the diminishing returns

model name	balanced sample
balanced_sample_1	4.114
balanced_sample_2	2.871
balanced_sample_3	2.446
balanced_sample_4	2.119
balanced_sample_5	1.855
balanced_sample_6	1.751
balanced_sample_7	1.597
balanced_sample_8	1.459
balanced_sample_9	1.412

This same evaluation was performed on the result of the fixed point P-median model results and the earlier set covering models, shown in Figures 16 and 17. Only the balanced sample evaluation is shown on the table to reduce clutter.

Figure 16: Table of results by the balanced sample population for the fixed point P-median models

model name	balanced sample
Real world hospital system with 1 additional	2.123
Real world hospital system with 2 additional	1.876
Real world hospital system with 3 additional	1.711
Real world hospital system with 4 additional	1.589

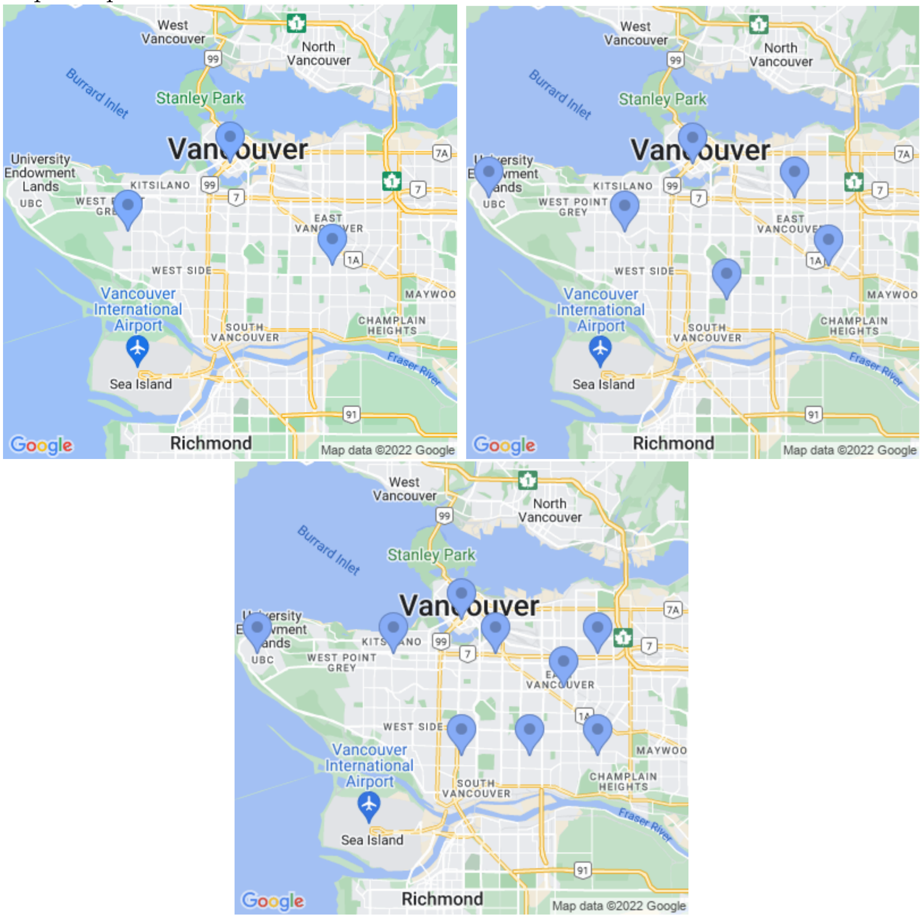
Figure 17: Table of results by the balanced sample population for the set covering models

model name	balanced sample
Google Maps set covering (8 hospitals)	1.651
Straight line set covering (9 hospitals)	1.654

5 Discussion

When looking at the population based P-median models with 6 or more hospitals in Figure 18 we see that they generally follow a similar shape, and that they all find that a hospital in Downtown and a hospital at UBC are optimal. Both of these placements make sense. UBC is a remote location with a high population that is separated from the rest of Vancouver by a large unpopulated region in Pacific Spirit Park, therefore it makes sense that the optimal location for the hospital is centered in UBC. Similarly, downtown Vancovuer has such a dense population that a centrally located hospital in downtown is almost always part of the optimal solution, even in low hospital models.

Figure 18: Optimal placements for 3, 6, 9 hospitals using the balanced population demand locations. Note the hospitals placed in UBC and downtown



Another potential explanation for this pattern is in the geometry of Vancouver itself. We see that the Set Covering models both placed hospitals in UBC and Downtown Vancouver, suggesting that the protruding shape of those two regions is what leads to models optimizing for hospital locations there.

We had initially expected the different demand locations to produce significantly different results, especially in the resulting evaluations, however we found that the population density based models (general, senior, balanced) all performed similarly at both 6 and 9 hospitals as seen in Figure 19. This could be due to a number of factors. First, it's possible that senior populations are distributed similarly to the general population, thus making all three models essentially the same. Alternatively it's possible that with that many hospitals in the system, the differences are no longer evident, making them uninteresting factors to consider.

Figure 19: Evaluation results for the six and nine hospital P-median models.

model name	balanced sample
Population density demand, 6 hospitals	1.684
Senior population demand, 6 hospitals	1.743
Balanced population demand, 6 hospitals	1.751
Population density demand, 9 hospitals	1.413
Senior population demand, 9 hospitals	1.477
Balanced population demand, 9 hospitals	1.412

The initial set covering model concluded that the minimal number of hospitals required to fully cover the city would be 8 hospitals, however our results with the P-median model show that as few as 6 hospitals can reduce the average distance to the nearest hospital for a person living in Vancouver to under 1.5km. This demonstrates the pros and cons of Set Covering as a modelling for emergency service locations, namely that perfectly covering a region will likely require many more locations than would be necessary to reduce the average distance below the same threshold. However those hospitals will be able to guarantee a minimum distance, rather than just lowering the average.

We also see in Figure 20 that the set covering model performs worse than the P-median models with the same number of hospitals due to the points mentioned earlier.

Figure 20: Comparison of the evaluation results of set covering and balanced sample P-median

model name	balanced sample
Balanced population demand, 6 hospitals	1.751
Balanced population demand, 9 hospitals	1.412
Google Maps set covering (8 hospitals)	1.651
Straight line set covering (9 hospitals)	1.654

Finally, we see that the free standing ambulance station model performs relatively well in comparison to the "ideal" locations generated by the P-median models, indicating that this could be a viable way to reduce ambulance wait times in Vancouver

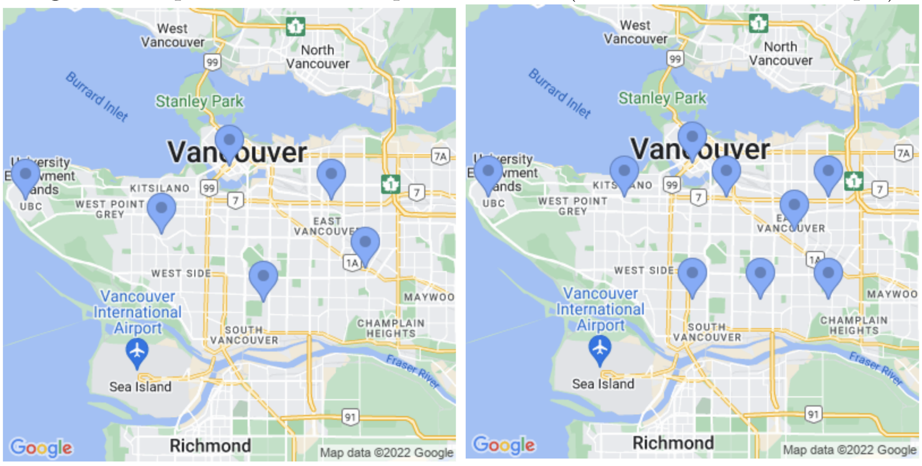
Figure 21: Comparison of the evaluation results of fixed point P-median models and balanced sample P-median models

model name	balanced sample
Balanced population demand, 6 hospitals	1.751
Balanced population demand, 9 hospitals	1.412
Real world hospital system with 1 additional	2.123
Real world hospital system with 2 additional	1.876
Real world hospital system with 3 additional	1.711
Real world hospital system with 4 additional	1.589

6 Conclusion

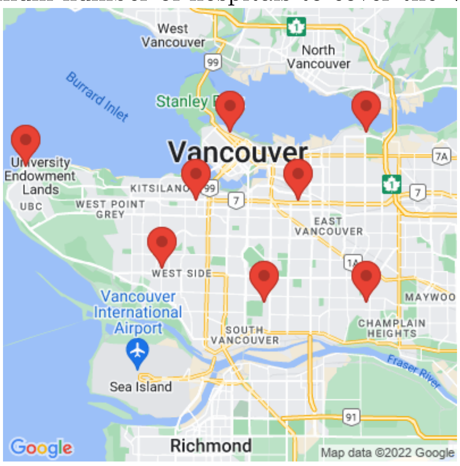
Summarizing our results, using P-median models we computed a number of optimal solutions to the problem of hospital placement in Vancouver shown in Figure 22.

Figure 22: Optimal 6 and 9 hospital locations (P-median balanced sample)



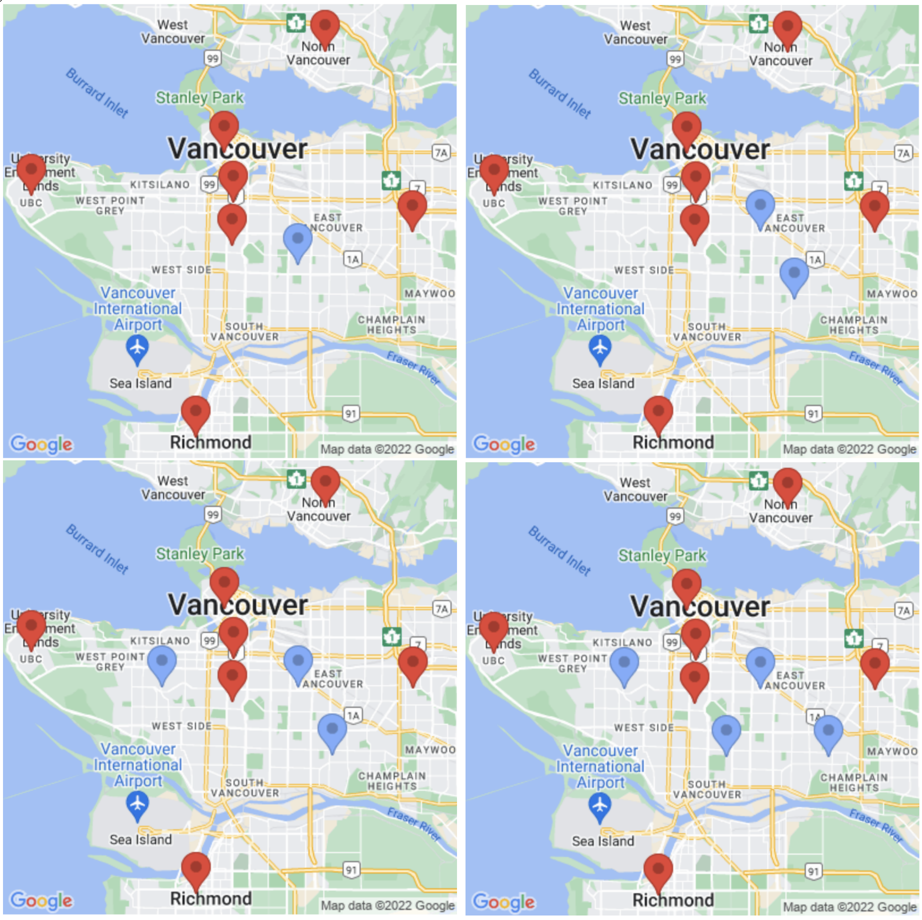
In addition we computed the minimum number of hospitals required to fully cover the Vancouver region within the prescribed wait time using a Set Covering model. The locations of these hospitals are shown in Figure 23

Figure 23: Minimum number of hospitals to cover the Vancouver region



Finally, we considered how to optimally improve Vancouver's existing hospital network, leading to the improved hospital networks shown in Figure 24

Figure 24: Optimal locations for 1,2,3,4 free standing ambulance stations added to Vancouver's hospital system



6.1 Further work

With the P-median models, the model supports weighting each demand location in the objective function, therefore possible additions to the model could be weighting each census region by additional factors such as fentanyl overdose rates.

For the Set Covering model, the placement of the demand and supply locations is known to affect the resulting solution in subtle ways. Complicated algorithms exist for filling a region with as few demand and supply locations as possible while still fully representing the geometric shape, and those algorithms are what see use in real world Set Covering models. For this project we decided to not investigate how different location constructions would affect the resulting set covering solution, however it would likely be able to produce a smaller or more realistic set cover and therefore be an interesting point of comparison.

Finally, there were a number of simplifying assumptions made in the P-median model that could be investigated further. Firstly was the grouping of the entire population of a census region to a single point, this led to highly dense locations such as downtown having a very high number of sample points all at the same location. While a population density map with smaller regions would alleviate this problem, there isn't a good way around it. One possible solution could be to use an alternative measure for estimating population density using smartphone location information. This has the added benefit of considering where people are in general rather than just where they live, which is another factor that the census-based population sampling fails to consider. Secondly, like the Set Covering problem, the set of potential hospital locations can affect the solutions. It's possible that the poor choice of potential hospital locations, which was the equidistant set of 70 points, was a significant factor in the resulting solutions. Developing a more systematic way of placing these potential locations would likely improve the resulting solutions.

References

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