CPSC 302

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1 Pre-class

1.1 Numerical algorithms and error

How to measure errors? (For v approximating u)

- Absolute
  - The **absolute error** is $|u - v|$
- Relative
  - The **relative error** is $\frac{|u-v|}{|u|}$
- Some combination

MATLAB code example

```
% some examples
u = [1;     1;     -1.5; 100;  100]
v = [0.99; 1.01; -1.2; 99.99; 99]
abs_error = abs(u-v)
relative_error = abs(u-v)./abs(u) % use ./ for vector math
```

Sources/types of errors

- In the model (how close it is to reality)
- In the input data

Approximation errors

- Discritization errors
- Convergence errors
- Roundoff errors

Example:

Given a smooth function $f(x)$. approximate the derivative at $x = x_0$

- We get a discritization error when we don't choose a small enough h for calculating the derivative, ie we discretize $\mathbb{R}$ to size h
- We can then calculate the discritization error from the Taylor series(see notebook)

Try with $f(x) = \sin(x)$ at $x_0 = 1.2$. (so we want to approximate cos)

- When h is large ($h > 10^{-8}$) we see that the error matches the discritization error
- For smaller h there is also a roundoff error
  - We see that for smaller h, around 10^{-10} the error begins to grow (not following the error pattern we expected from just discritization)
  - Why is there roundoff error? Soon tm

2 Lecture

- Q: When is absolute error more important than relative error?
- A: Like the height of a flight of stairs, you don't really care how relatively high the stairs are, just that you're not off by a certain amount
- Q: What about relative error?
- A: If you're measuring something that's 100kg, you don't necessarily care if you're off by 1g, but if you're measuring something that weighs 2g, being off by 1g matters alot.

Roundoff error

- Why does decreasing h not continue to decrease the error? Why does the error start increasing after a certain point?
- Idea: As we lower h we increase the proportion that roundoff error affects our calculations involving h, which increases the error proportionally in the result
- Q: How would you define a roundoff error?
- A: Computers don't have infinite memory, so at some point it has to round off the value, which introduces error.

Discretization error

- When you make finite approximations of continuous processes

Convergence errors

- Arise from iterative methods