Discussion 1: 11/09/2020

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August 2020

1 Discussion stuffs

Misc:

- The idea of LUB only makes sense if we assume that there already is a lower bound
- The idea of upper and lower bounds come naturally from having an ordering
- Usually there are many many upper/lower bounds (GLB and LUB are interesting)
  - For example that set A, which had an upper bound but no LUB (which motivates us extending it into the reals, which would give us a LUB $\sqrt{2}$)
- Note that the suprenum/infenum is not necessarily in/not in the set itself, though if it isn't we have this intuition that it's "really close" to the set itself
- All bounded sets of integers have a LUB and GLB, but we can make bounded sets of rationals that have neither

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Q: For that set $A = \left\{p \in \mathbb{Q} : p^2 < 2\right\}$, we proved the statement that for each element of p there is a $q \in A : p < q$. How did we choose that $q = \frac{2p+2}{p+2}$?

A: We want to choose a q that is definitely larger than p, but also it's square is less than 2, so we can do that by drawing some lines (see notebook)

- How do we know that the intersection q will be rational? We choose our horizontal line to be $y = 2$, so the intersection should be rational (?) (alternatively, just do the arithmetic and it turns out that it's rational)

Q: Why do we care so much that the rationals don't have a solution for $x^2 = 2$ (the irrationals), but not that the reals don't have a solution for $x^2 = -1$ (the complex)?

A: It's a hole vs just a missing section. The rationals pose a problem because the rationals can get arbitrary close to but not reach $\sqrt{2}$, while the reals can't get anywhere near i, so the fact that you can't doesn't really matter

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Q: How do we define order for the rationals when we define the rationals as a pair (a, b)?

A: We normally define $\frac{m}{n} < \frac{a}{b}$ if $mb < na$. Then you need to check symmetry, transitivity, and that it holds for the equivalence class of those fractions

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Q: Can we possibly order $\mathbb{R}^2$?

A: Transitivity fails unfortunately, so no

Q: Are all subsets $E \subset S$ bounded above?

A: Well of course not, $E = [-, \infty)$ is a subset of $\mathbb{R}$ and is not bounded above

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Q: What is the infenum (greatest lower bound) of $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$

A: 0. To prove it you need to check

- It's a lower bound (obvious)
- Prove that any value larger than 0 is not a lower bound
  - Pick $x > 0$
  - Fit a $\frac{1}{n_0}$ between 0 and x (should be straightforward, maybe draw a graph to help)