

# Homomorphisms and quotient groups

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## 1 Homomorphisms and quotient groups

### 1.1 Generators and group presentations

We can imagine the subgroup generated by  $x$  as  $x$  being thrown in a box with itself and shook around. So what if we throw in more elements to be shook together?

**Definition:** Subsets as group generators

The subgroup generated by a subset  $S$  of  $G$ ,  $\langle S \rangle$ , is the set of finite products between elements of  $S$  and their inverses.

- $S = [a, b]$ , then  $\langle S \rangle$  is stuff like  $abababa$ ,  $a^5b^3ab^2$ ,  $a^{-1}bab^{-100}$
- If  $\langle S \rangle = G$ , then  $S$  is a **set of generators** for  $G$

**Example:**  $\mathbb{Z}$

$\langle 1 \rangle = \mathbb{Z}$ , because each integer can be written as a bunch of 1's or a bunch of  $-1$ 's