

Homomorphisms and quotient groups

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1 Homomorphisms and quotient groups

1.1 Generators and group presentations

We can imagine the subgroup generated by x as x being thrown in a box with itself and shook around. So what if we throw in more elements to be shook together?

Definition: Subsets as group generators

The subgroup generated by a subset S of G , $\langle S \rangle$, is the set of finite products between elements of S and their inverses.

- $S = [a, b]$, then $\langle S \rangle$ is stuff like $abababa$, $a^5b^3ab^2$, $a^{-1}bab^{-100}$
- If $\langle S \rangle = G$, then S is a **set of generators** for G

Example: \mathbb{Z}

$\langle 1 \rangle = \mathbb{Z}$, because each integer can be written as a bunch of 1's or a bunch of -1 's