The Behavior of Metric Dimension on Random Geometric Graphs

Evan Alba and Matthew Hernandez
Advisor: Dr. Richard C. Tillquist

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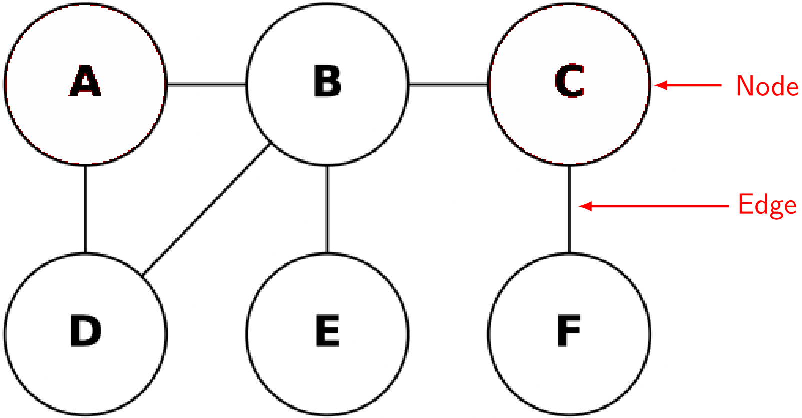
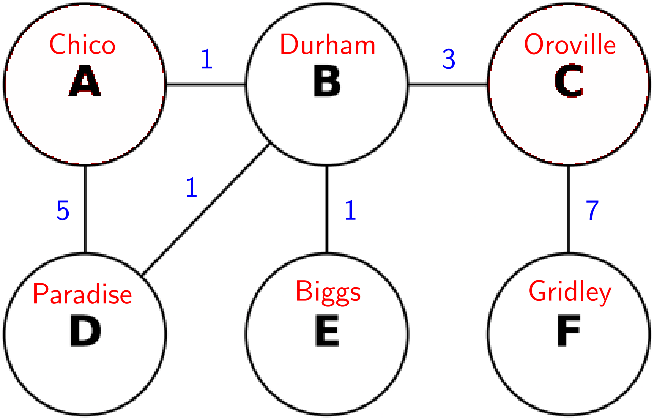
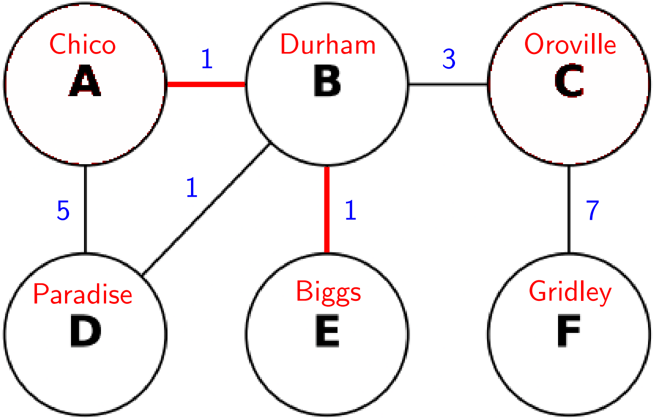
Introduction to Graph Theory



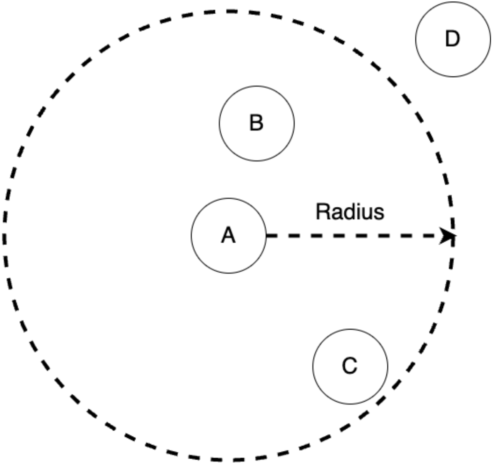


What is the shortest path from Chico to Biggs?

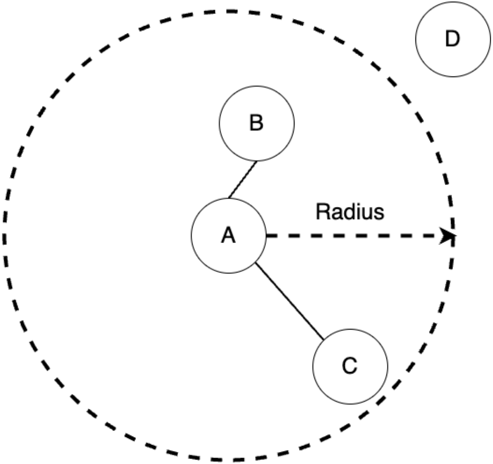


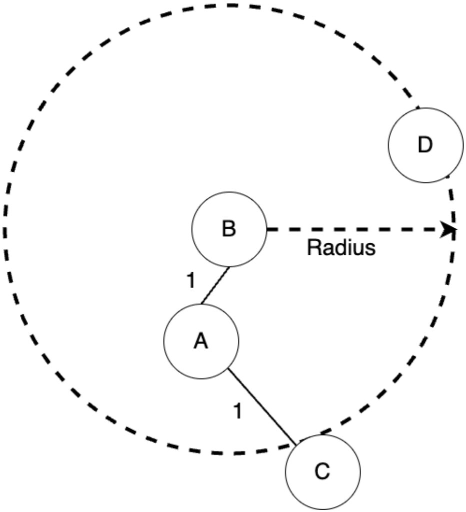
What is the shortest path from Chico to Biggs?

$1 + 1 = 2$

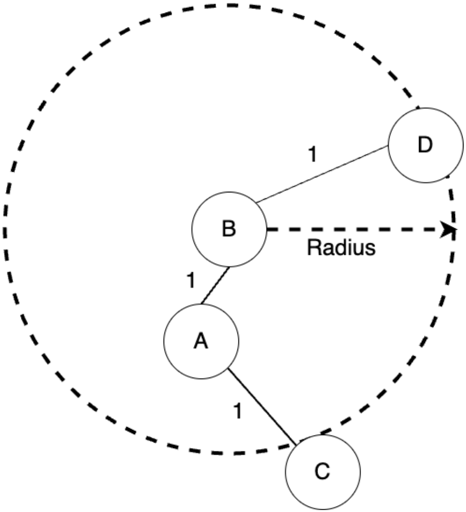


D
B
Radius
A
C

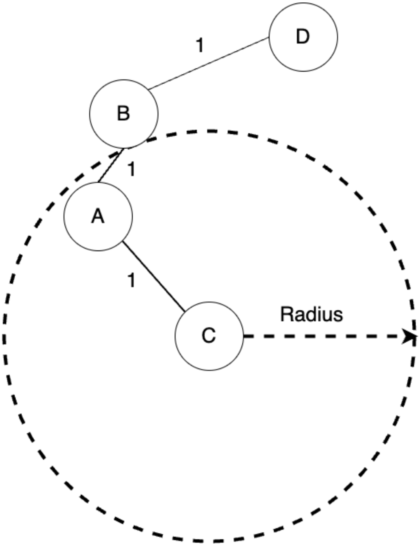

D
B
Radius
A
C



D
B
Radius
1
A
1
C



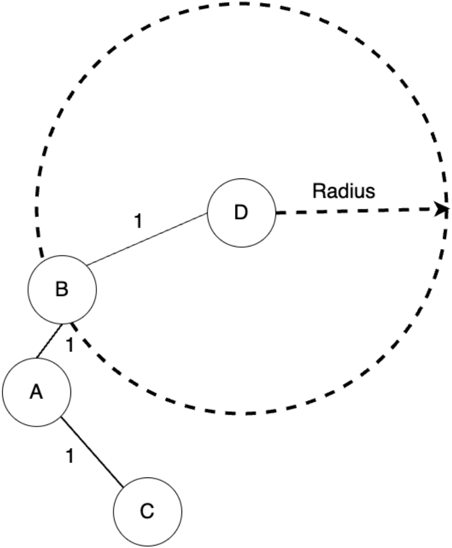
D
1
B
Radius
1
A
1
C



D
1
B
1
A
1
C
Radius

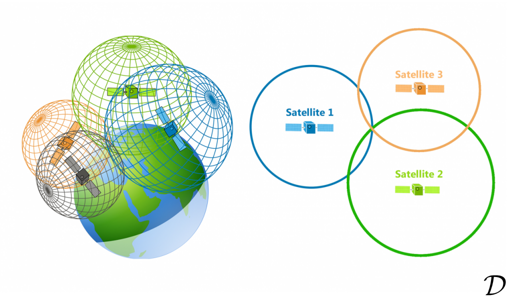
Modeling Physical Networks

Random Geometric Graphs

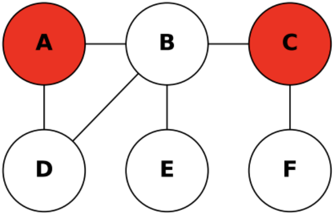


Metric Dimension

Intuition



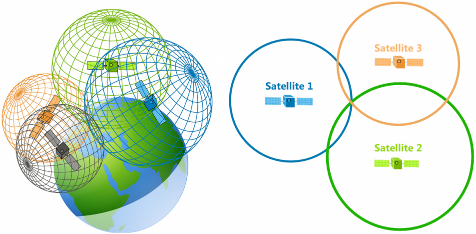
gisgeography.com/trilateration-triangulation-gps/



$$\mathcal{D} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 0 & 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 2 & 2 & 1 \\ 1 & 1 & 2 & 0 & 2 & 3 \\ 2 & 1 & 2 & 2 & 0 & 3 \\ 3 & 2 & 1 & 3 & 3 & 0 \end{bmatrix} \end{array}$$

Metric Dimension

Intuition



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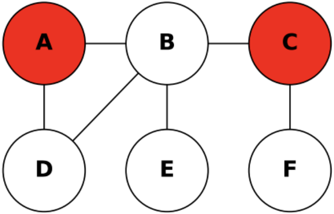


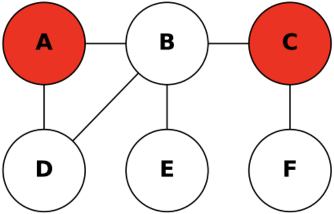
$$\mathcal{D} = \begin{array}{cc} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} & \begin{bmatrix} 0 & & 2 & & & \\ 1 & & 1 & & & \\ 2 & & 0 & & & \\ 1 & & 2 & & & \\ 2 & & 2 & & & \\ 3 & & 1 & & & \end{bmatrix} \end{array}$$

Definition (Resolving Set)

Given a graph $G = (V, E)$, a subset of nodes $R \subseteq V$ is resolving if for all distinct $u, v \in V$ there is an $r \in R$ such that $d(u, r) \neq d(v, r)$.

Definition (Metric Dimension)

The metric dimension of a graph $G = (V, E)$, denoted $\dim(G)$, is the size of smallest resolving sets of G.

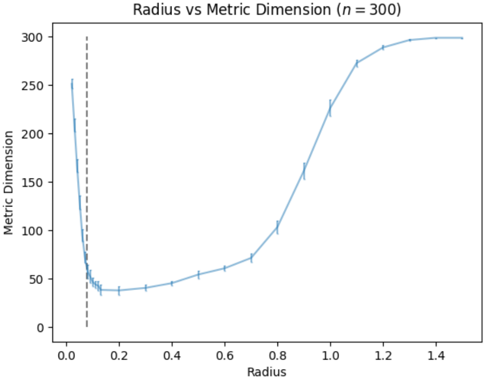


- Finding $\dim(G)$ on arbitrary graphs is an NP-hard problem
  - There are no known efficient algorithms for solving the problem
  - Exact values and bounds are known for specific families of graphs
- The Information Content Heuristic (ICH) algorithm finds small resolving sets
  - The algorithm has a $O(\log(n))$ approximation ratio
  - The runtime is $O(n^3)$

Motivating Questions

- **Question #1:** How accurately can we predict $\dim(G)$ when G is a RGG?
- **Question #2:** Can we find small resolving sets on RGGs more efficiently than the ICH algorithm?

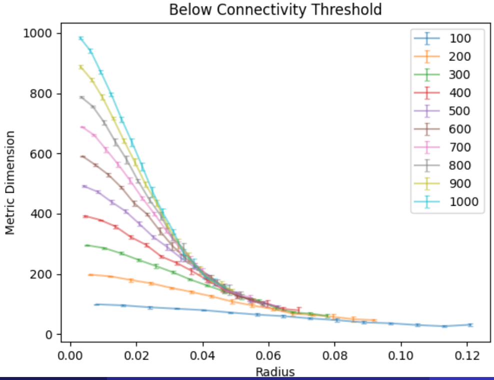
- Explore RGGs and search for patterns that we might be able to take advantage of



- There is a sharp connectivity threshold in RGGs at

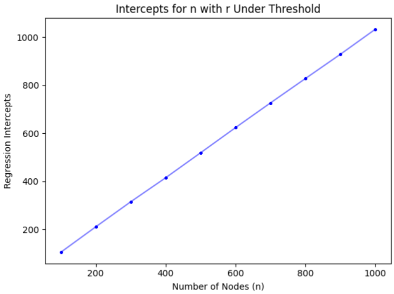
$$r = \sqrt{\frac{\log(n)}{\pi n}}$$

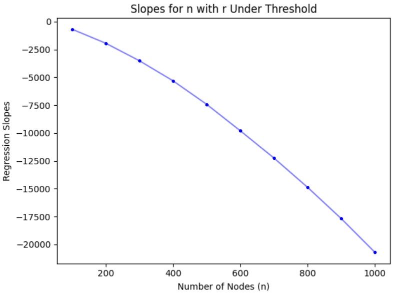
- What happens below this threshold?



Approach of predictMD

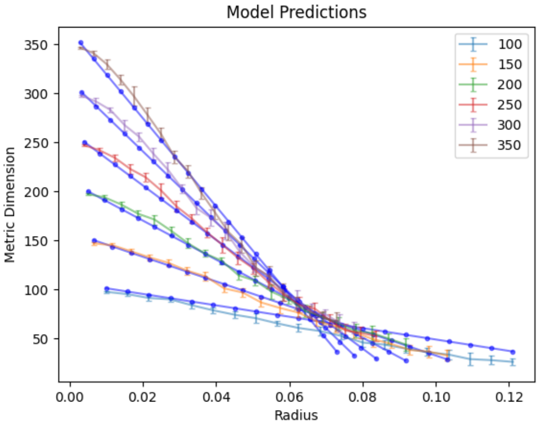
- Fit a linear regression model at every n value based on radius
- Used a linear fit on the intercepts
- Used a quadratic fit on the slopes



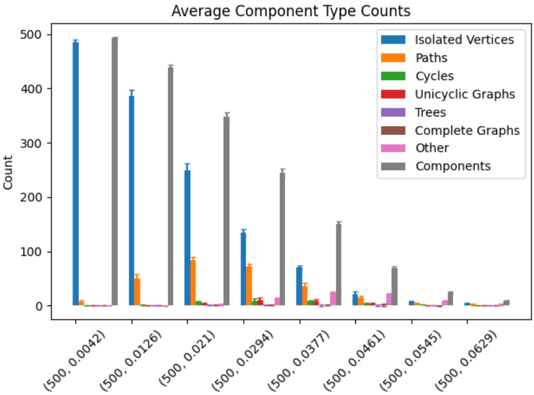


Testing predictMD

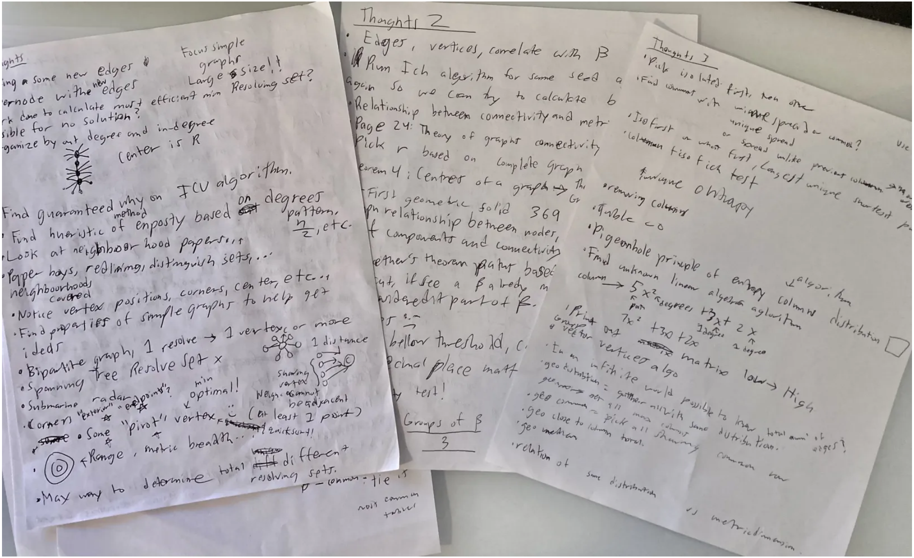
- Run the predictMD algorithm for n and r
- Run the ICH algorithm for the same n and r



- Isolated vertices play an important role below the threshold



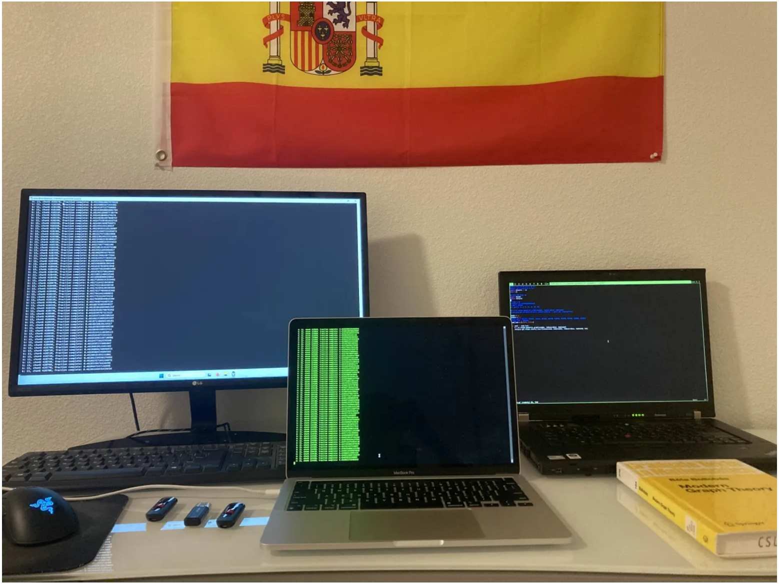
Evan Alba


Focus simple graphs
Large size!!
Resolving set?
Center is R
Find guaranteed why on ICU algorithm.
Find heuristic of enposty based on degrees pattern, n/2, etc.
Look at neighbor hood papers, ...
Paper bays, redlining, distinguish sets, ...
neighbourhoods covered
Notice vertex positions, corners, center, etc..
Find properties of simple graphs to help get ideas
Bipartite graph, 1 resolve → 1 vertex, or more
1 distance
Spanning tree Resolve set x
Submarine radar
optimal!
Corners
Some "pivot" vertex
(at least 1 point)
Range, metric breadth...
different resolving sets.
Max way to determine total
Thoughts 2
Edges, vertices, correlate with β
Run Ich algorithm for same seed again so we can try to calculate b
Relationship between connectivity and metr
Page 24: Theory of graphs connectivity
Pick r based on complete graph
Theorem 4: Centres of a graph
First geometric solid 369
relationship between nodes, components and connectivity
bellow threshold,
place matt
test!
Groups of β
3
Thoughts 3
Pick isolated first, then one
Iso first
Fick test
Pigeonhole principle of entropy column distribution
Find unknown linear algebra algorithm
$5x^2$ degrees $+3x+2x$
$7x^2+3x+2x$
Print out vertices algo matrix low → High
In an infinite world possible to have total amount of edges?
relation of same distribution
vs metric dimension

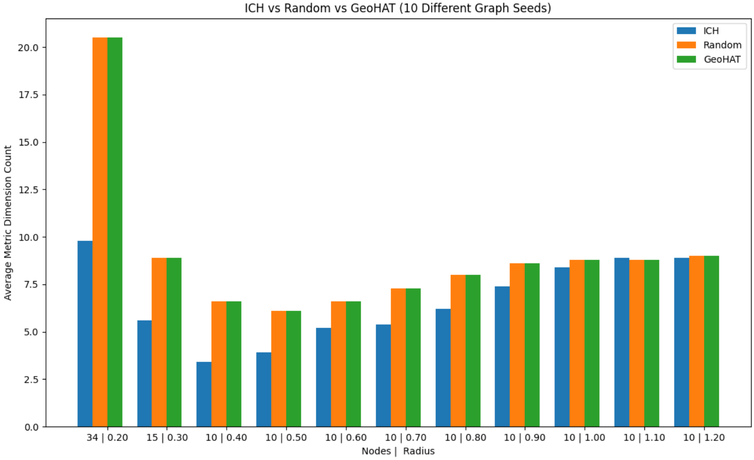
GeoHAT (Geo (Geometric) HAT (Hernandez, Alba, Tillquist))

- 1. Test for a column with all unique rows so we can find a easily a size 1 metric dimension.
- 2. Use close to unique rows columns and use if possible isolated vertices.
- 3. Use the matrix to pick columns based on the least common elements gathered in the whole matrix.

Tests



ICH vs Random vs GeoHAT



Future Work

- Find an analytic solution for metric dimension below the connectivity threshold
- Develop algorithms for predicting metric dimension above the connectivity threshold
- Explore alternative techniques for finding small resolving sets both below and above the connectivity threshold

Acknowledgements

- MESA Engineering Program
- Chico STEM Connections Collaborative

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Questions?