

# **CS 585: Homework #0**

Due on Friday, September 8, 2017

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## 1 Domain of a joint distribution

### 1.1

The joint distribution  $P(A, B)$  defines probabilities for  $4 \times 3 = 12$  possible outcomes.

### 1.2

The joint distribution  $P(A_1, A_2, \dots, A_n)$  defines probabilities for  $2^n$  possible outcomes.

## 2 Independence versus Basic Definitions

### 2.1

Which of the following statements is always true?

1.  $P(A|B) = P(B|A)$  False
2.  $P(A, B) = P(A|B)P(B)$  True
3.  $P(A, B) = P(A)P(B)$  False
4.  $P(A|B) = P(A)$  False
5.  $P(A, B, C) = P(A)P(C)$  False
6.  $P(A, B, C) = P(A)P(B)P(C)$  False
7.  $P(A, B, C) = P(A)P(B|A)P(C|A, B)$  True
8.  $P(A) = \sum_{b \in \text{domain}(B)} P(A, B = b)$  True
9.  $P(A) = \sum_{b \in \text{domain}(B)} P(A|B = b)P(B = b)$  True

Now assume that  $A$ ,  $B$ , and  $C$  are all independent of each other. Which of these statements is true?

1.  $P(A|B) = P(B|A)$  False
2.  $P(A, B) = P(A|B)P(B)$  True
3.  $P(A, B) = P(A)P(B)$  True
4.  $P(A|B) = P(A)$  True
5.  $P(A, B, C) = P(A)P(C)$  False
6.  $P(A, B, C) = P(A)P(B)P(C)$  True
7.  $P(A, B, C) = P(A)P(B|A)P(C|A, B)$  True
8.  $P(A) = \sum_{b \in \text{domain}(B)} P(A, B = b)$  True
9.  $P(A) = \sum_{b \in \text{domain}(B)} P(A|B = b)P(B = b)$  True

### 3 Logarithms

#### 3.1 Log-probs

The range of possible values for  $\log(p)$  is  $(-\infty, 0]$ .

#### 3.2 Prob ratios

The range of possible values for  $\frac{p}{q}$  is  $[0, \infty)$ .

#### 3.3 Log prob ratio

The range of possible values for  $\log(\frac{p}{q})$  is  $(-\infty, \infty)$ .

## 4 Deriving Bayes Rule

$$P(A, B) = P(A)P(A|B)$$

Definition of conditional probability

$$P(B, A) = P(B)P(B|A)$$

Definition of conditional probability

$$P(A, B) = P(B, A)$$

Commutative property

$$P(A)P(A|B) = P(B)P(B|A)$$

Substitution

$$P(B|A) = \frac{P(A)P(A|B)}{P(B)}$$

Divide by  $P(B)$