

# Retiring the third law of black hole thermodynamics

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joint work with Christoph Kehle (ETH Zürich)  
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# Outline

1. What is the third law?
2. Main result: the third law is false
3. Geometry and physics of third law violating spacetimes
4. Extremal black hole formation as a critical phenomenon

# Black hole thermodynamics

Commun. math. Phys. 31, 161–170 (1973)  
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## The Four Laws of Black Hole Mechanics

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Department of Physics, Yale University, New Haven, Connecticut, USA

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Fundamentally, these are statements about Einstein gravity coupled to classical matter fields (and are already interesting in vacuum!).

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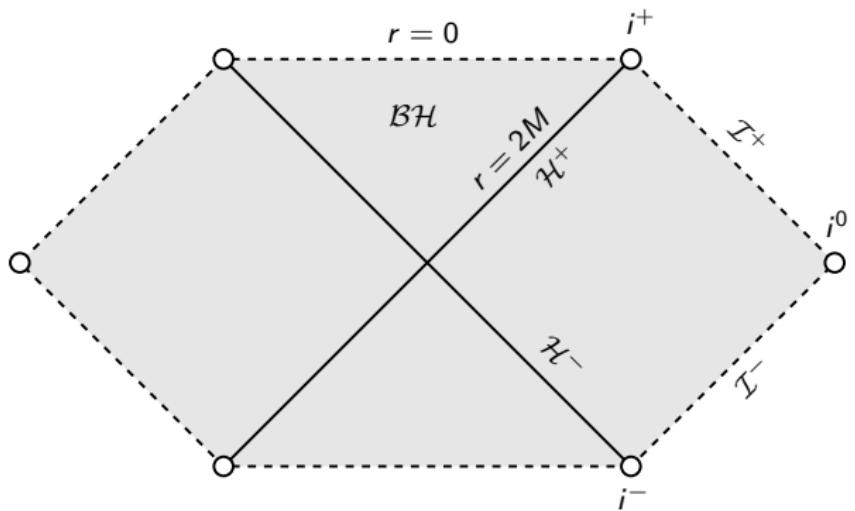
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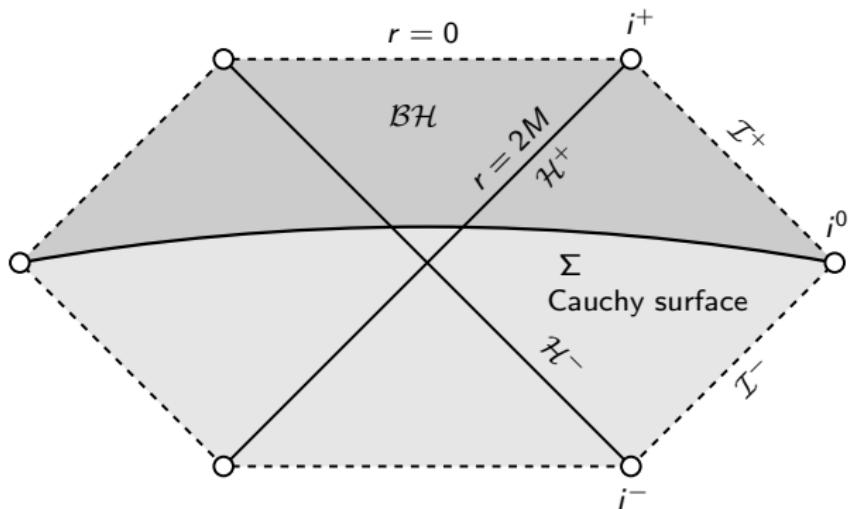
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- ▶ Laws 0, 1, and 2 proved by Hawking, Carter, Bardeen–Carter–Hawking, Wald, ...

## Refresher on Schwarzschild

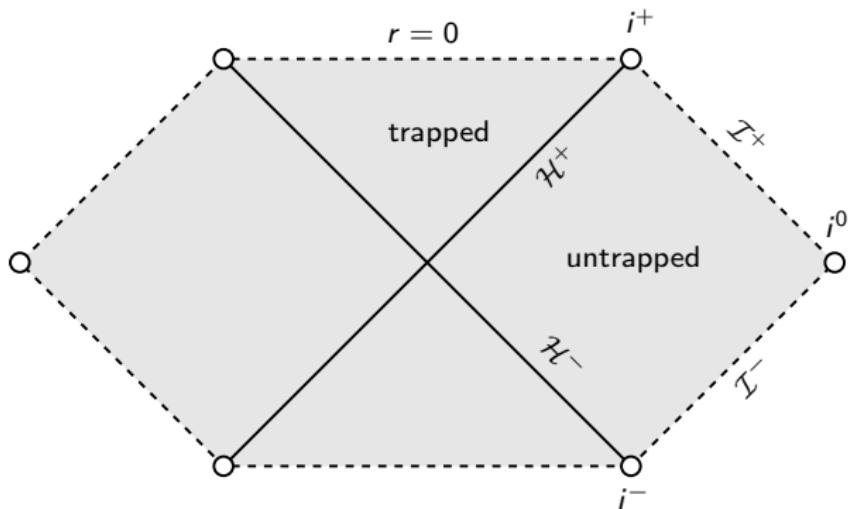


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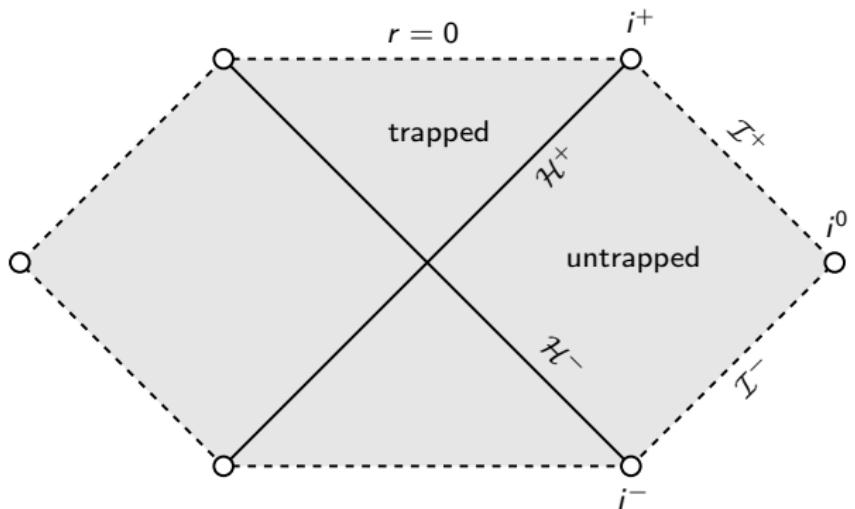
Maximally extended Schwarzschild is the unique maximal Cauchy development of the data induced on a spacelike hypersurface  $\Sigma$  as depicted here.

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The black hole interior is foliated by **trapped surfaces** (both future null expansions negative).

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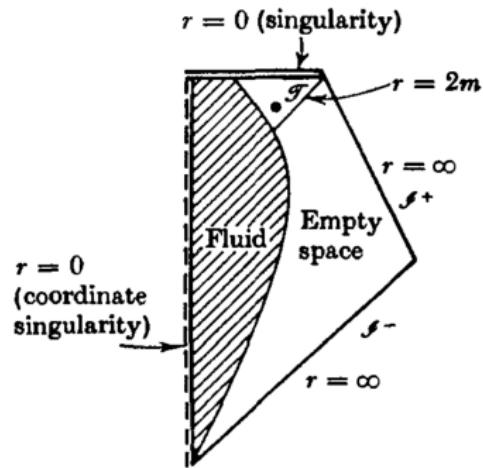


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**Theorem (Penrose's incompleteness theorem).**

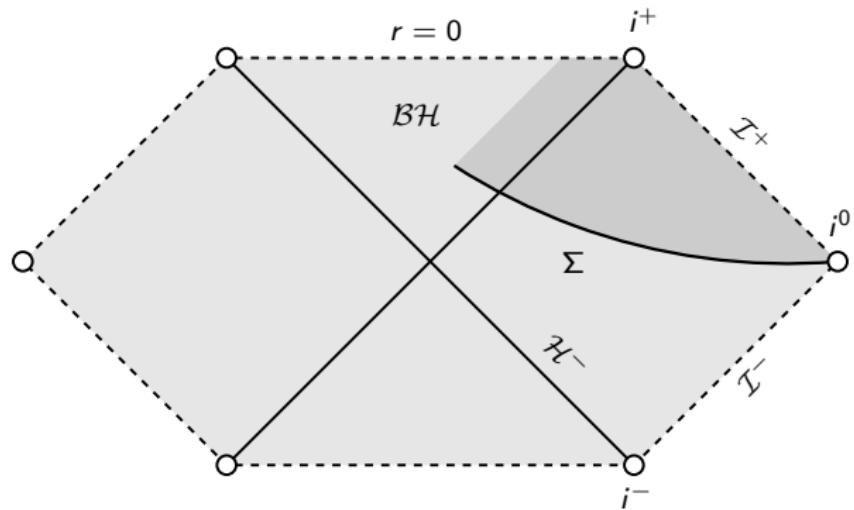
*If an asymptotically flat spacetime contains a trapped surface and satisfies the null energy condition, it is geodesically incomplete.*

## Refresher on gravitational collapse

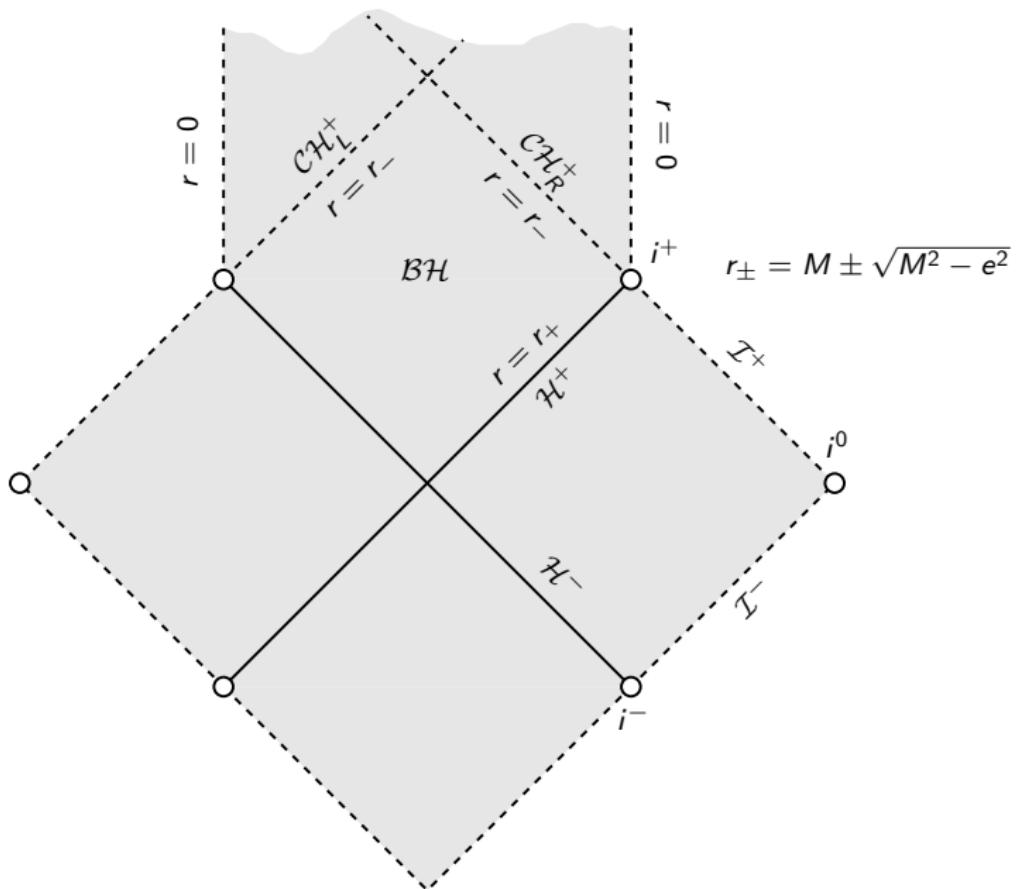


Penrose diagram of gravitational collapse. One-ended Cauchy data!

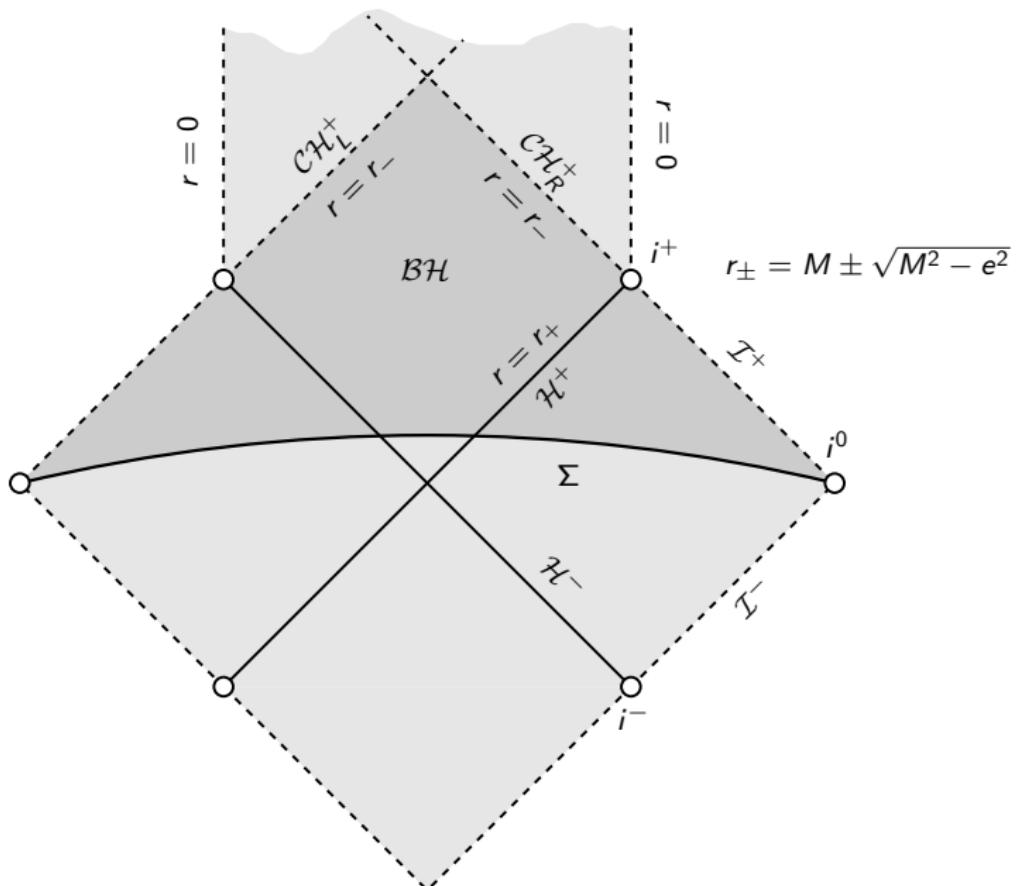
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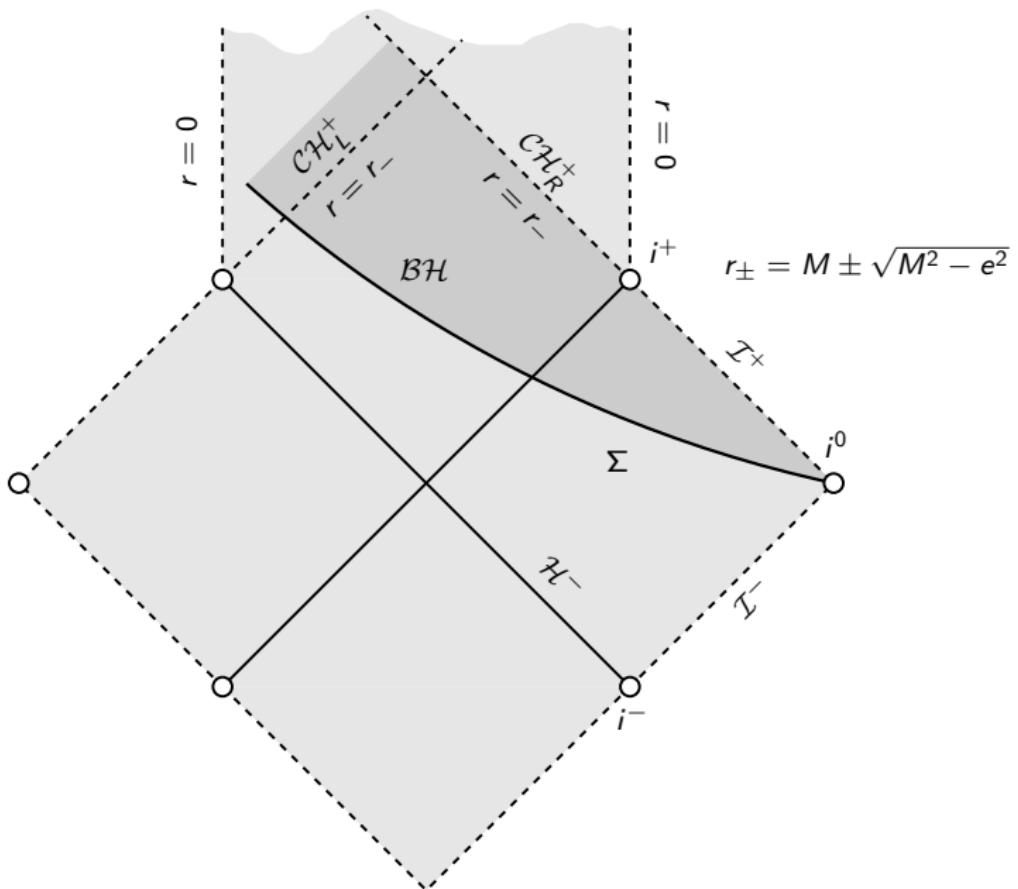
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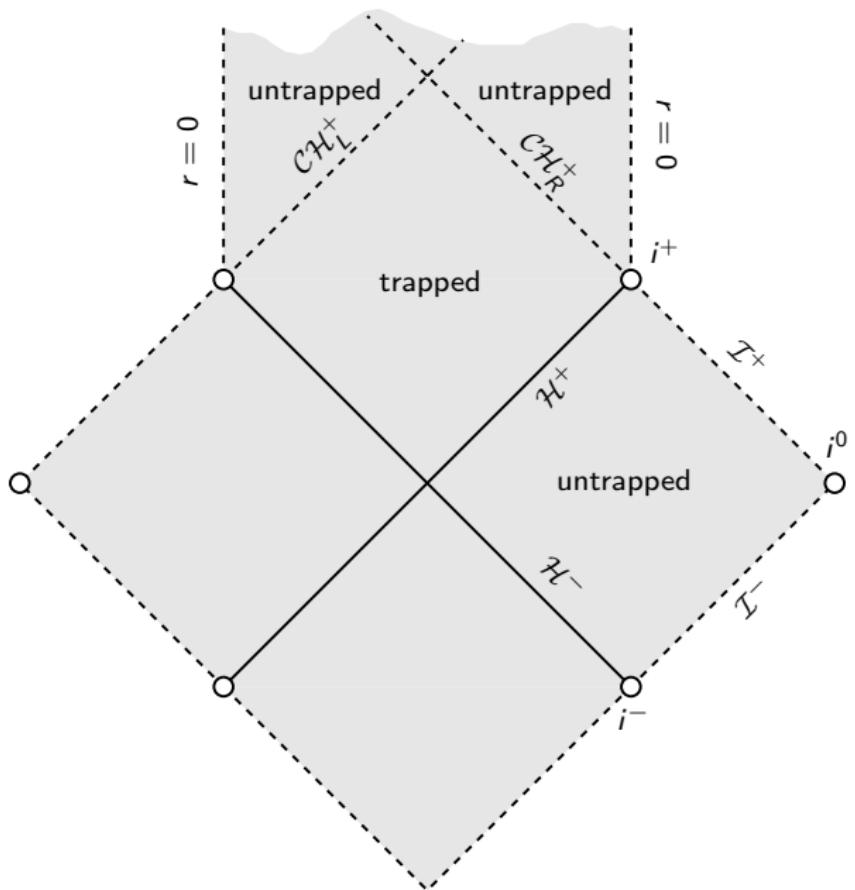
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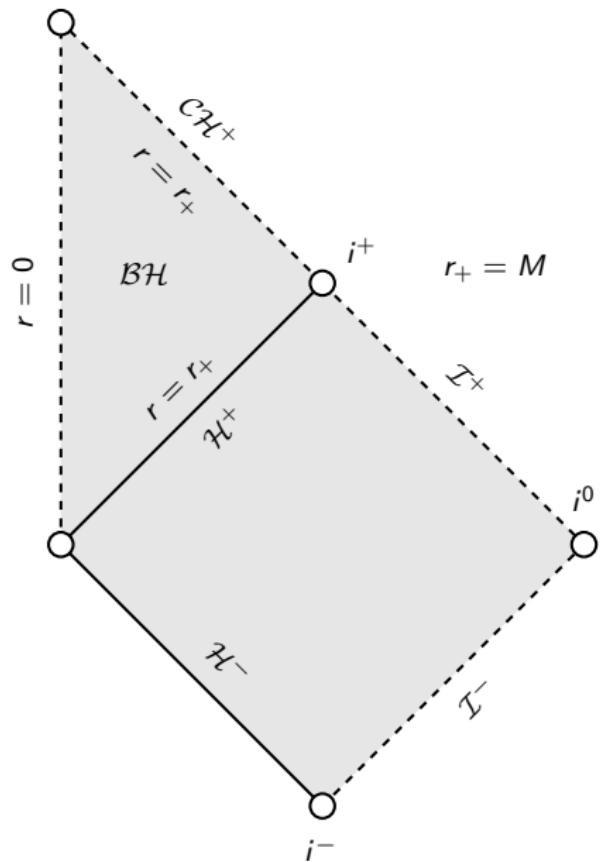
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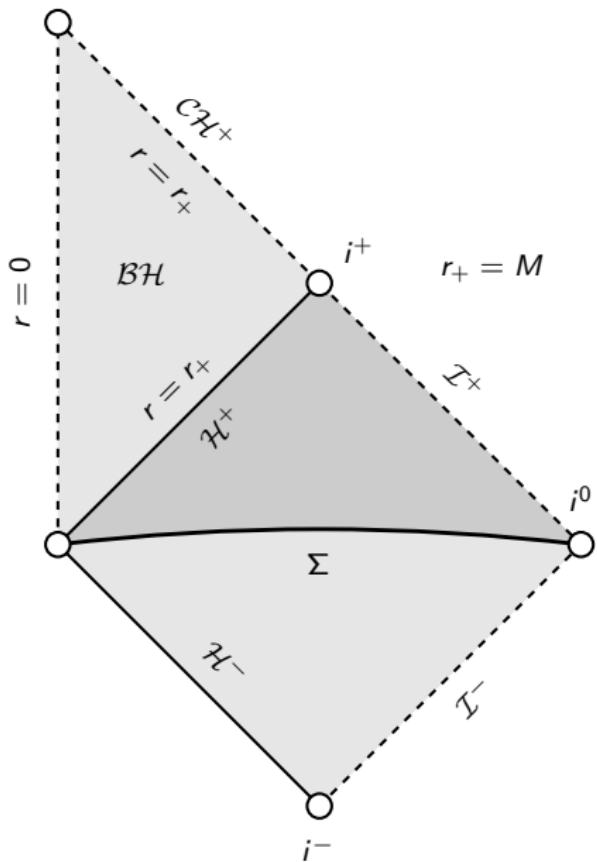
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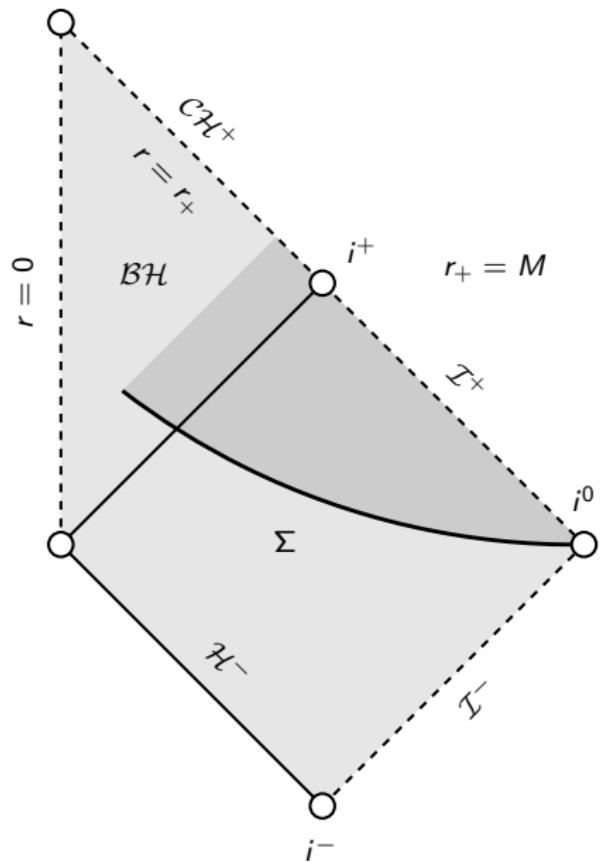
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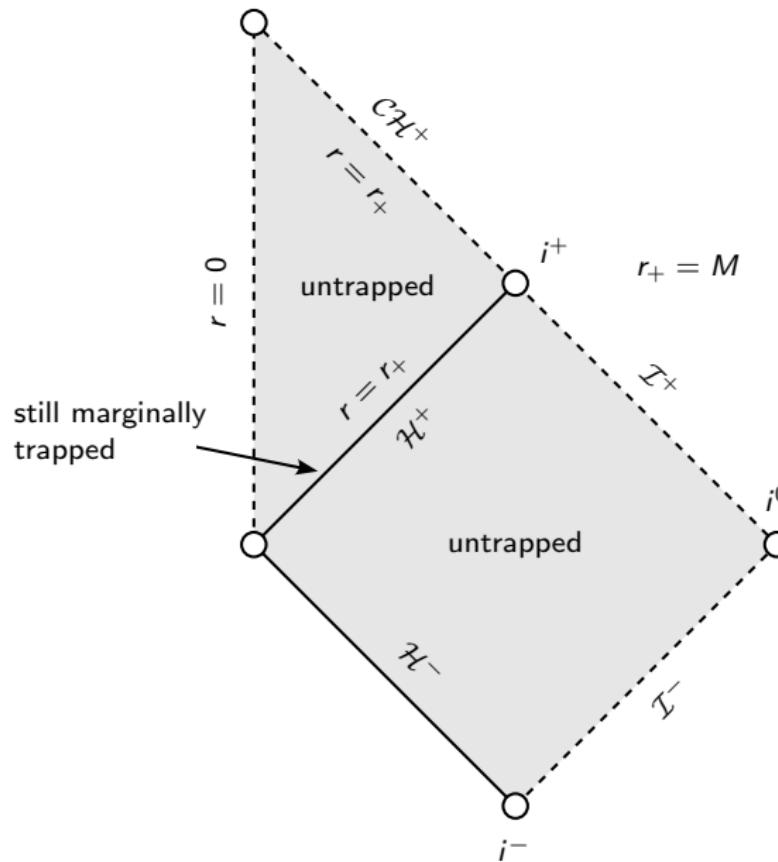
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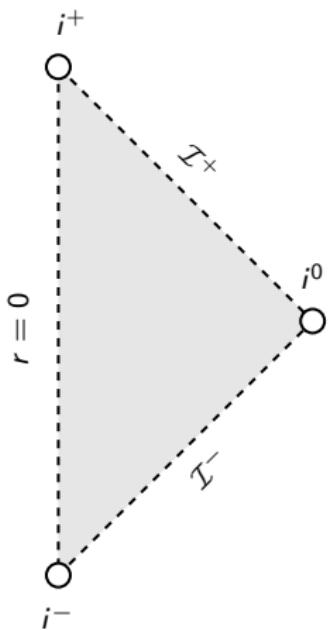
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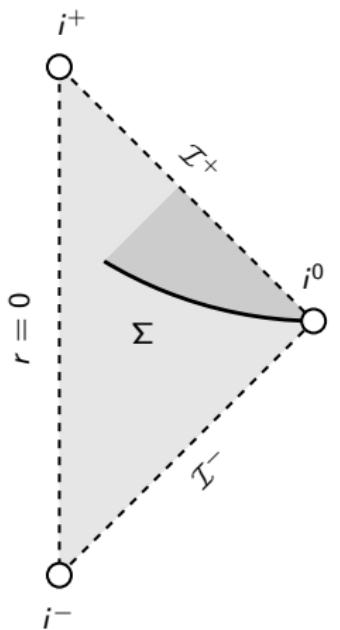
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# Refresher on superextremal Reissner–Nordström: $0 < M < |e|$



## Refresher on superextremal Reissner–Nordström: $0 < M < |e|$



## Surface gravity $\kappa$ of Reissner–Nordström

- RN with mass  $M$  and charge  $e$ ,  $|e| \leq M$ , has

$$\kappa = 2\pi T = \frac{\sqrt{M^2 - e^2}}{(M + \sqrt{M^2 - e^2})^2}$$

- **Subextremal:**  $\kappa > 0$
- **Extremal:**  $\kappa = 0$

## The third law

Original formulation of Bardeen–Carter–Hawking:

Extending the analogy even further one would postulate:

### *The Third Law*

It is impossible by any procedure, no matter how idealized, to reduce  $\kappa$  to zero by a finite sequence of operations.

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  - ▶ This parenthetical remark is related to the “mistake” in Israel’s paper and we will discuss it later.

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### **Conjecture (The third law, BCH '73, Israel '86).**

*A subextremal black hole cannot become extremal in finite time by any continuous process, no matter how idealized, in which the spacetime and matter fields remain regular and obey the weak energy condition.*

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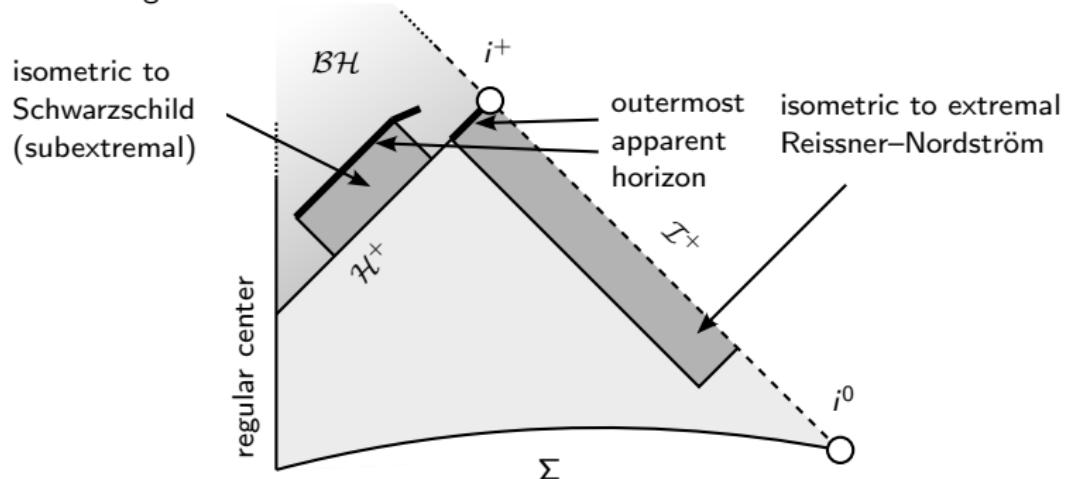
### **Theorem (Kehle–U. '22).**

*Subextremal black holes can become extremal in finite time, evolving from regular initial data. In particular, the “third law of black hole thermodynamics” is false.*

More precisely, there exist regular solutions of the Einstein–Maxwell-charged scalar field (self-gravitating charged massless scalar field) system with the following behavior:

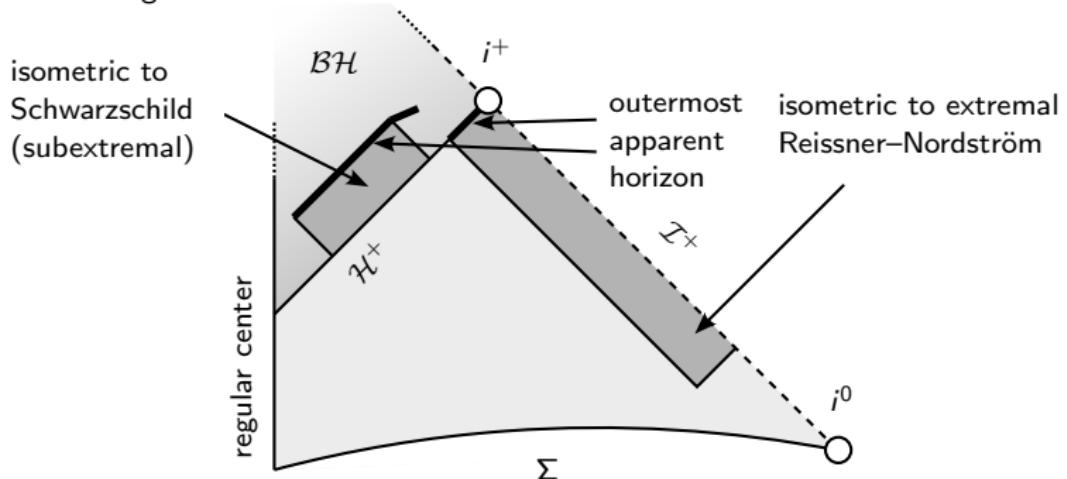
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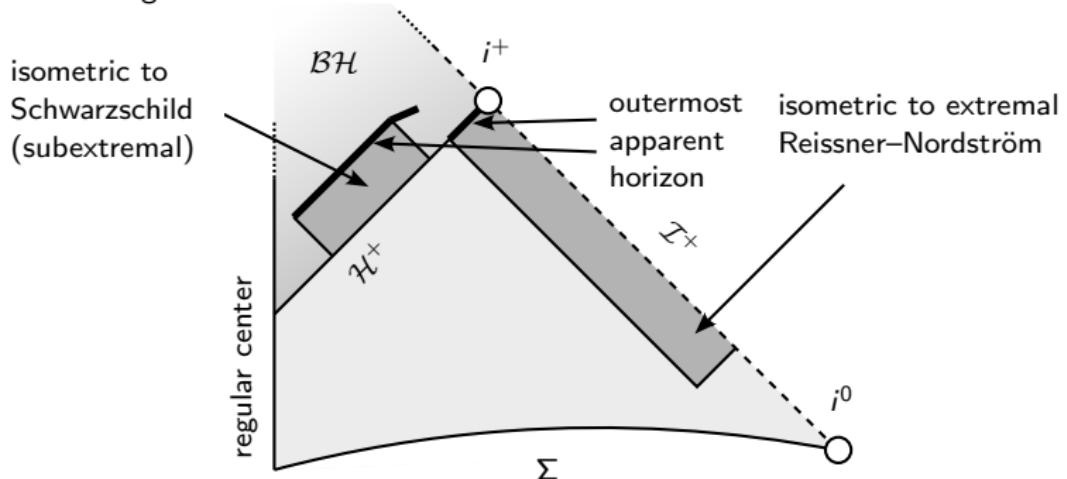
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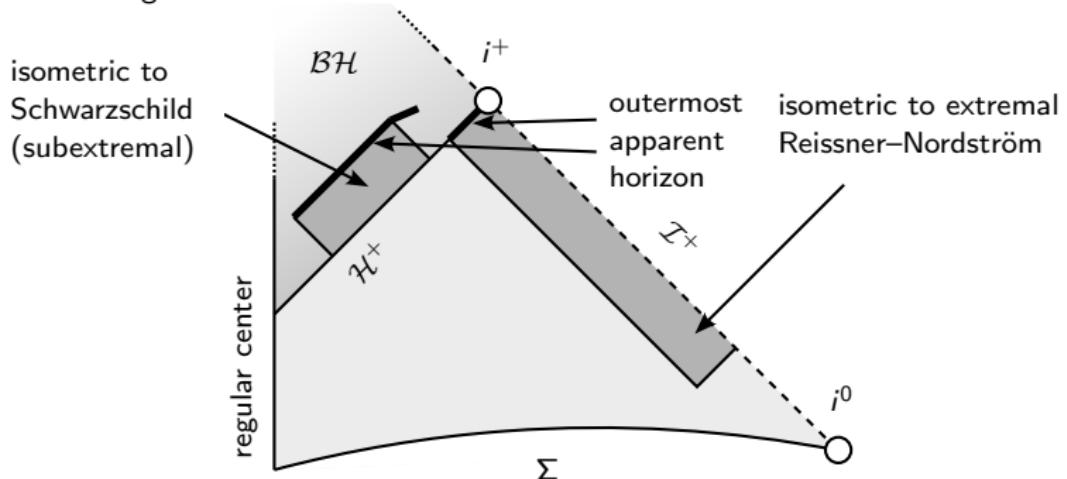
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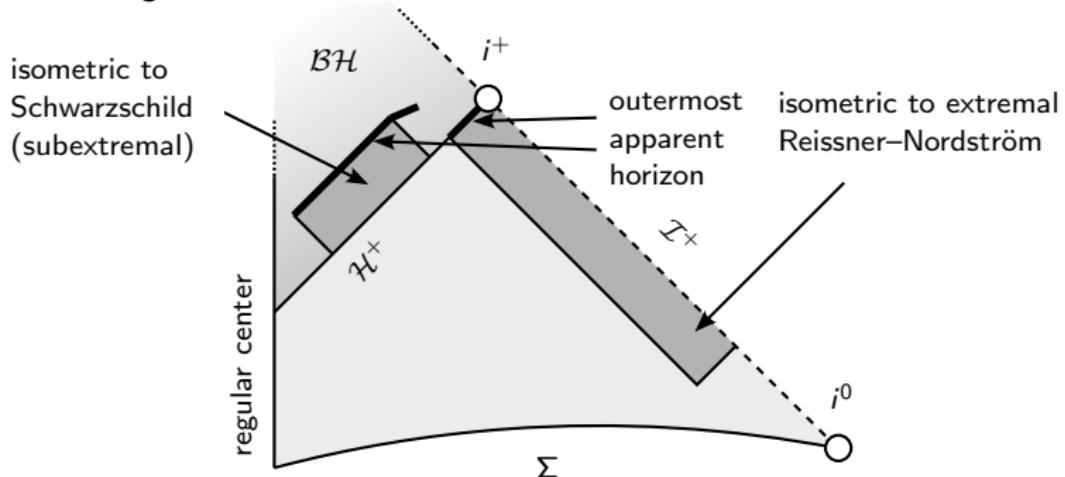
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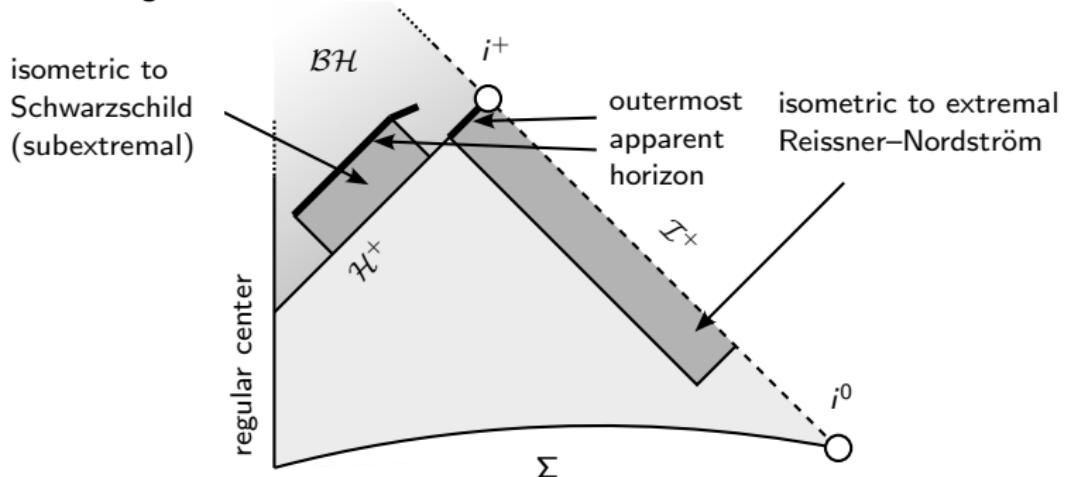
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- ▶ Model satisfies the dominant energy condition.

## Einstein–Maxwell-charged scalar field system

- ▶ Lorentzian manifold  $(\mathcal{M}^{3+1}, g)$
- ▶ 2-form  $F = dA$  (electromagnetism)
- ▶ Charged (complex) massless scalar field  $\phi$

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$$R_{\mu\nu}(g) - \frac{1}{2}R(g)g_{\mu\nu} = 2 \left( T_{\mu\nu}^{\text{EM}} + T_{\mu\nu}^{\text{CSF}} \right) \quad (1)$$

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- ▶ In an appropriate gauge, this is a nonlinear hyperbolic system for  $(g, F, \phi)$ .
- ▶ Spherical symmetry,  $\phi$  degree of freedom breaks rigidity from Birkhoff.

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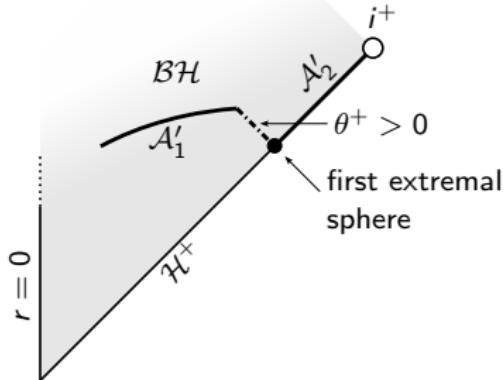
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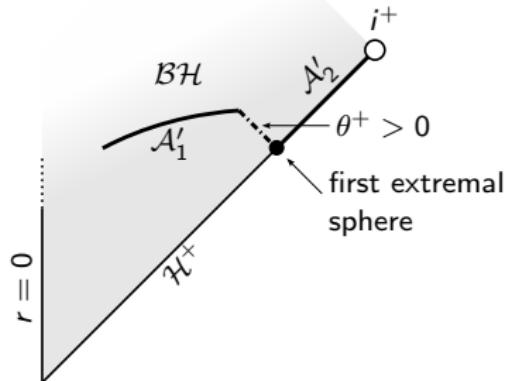
- ▶ Under this implicit assumption, Israel concluded the third law to be true on the basis of Raychaudhuri's equation.

## Israel's paper reinterpreted



Outermost apparent horizon becomes disconnected the instant the black hole becomes extremal!

## Israel's paper reinterpreted



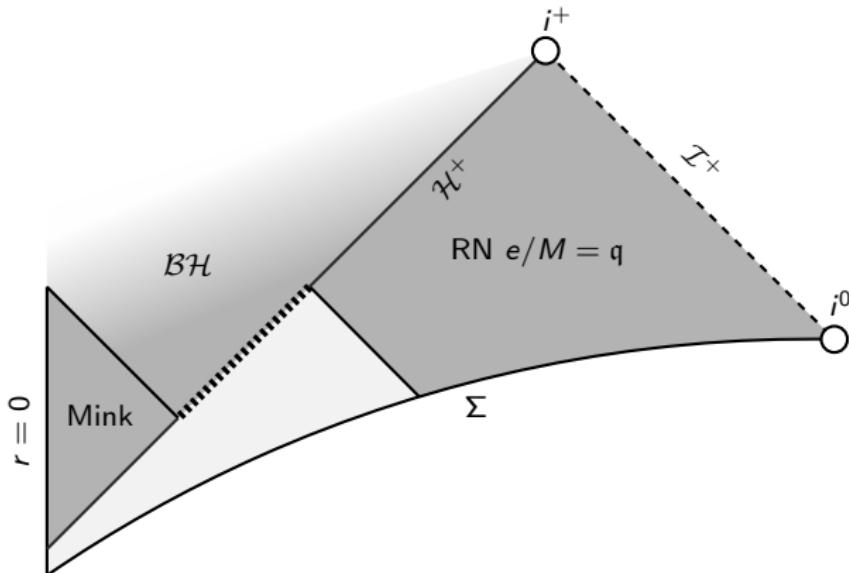
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However, this is a feature, not a glitch!

## Gravitational collapse for any charge to mass ratio

### Theorem (Kehle–U.).

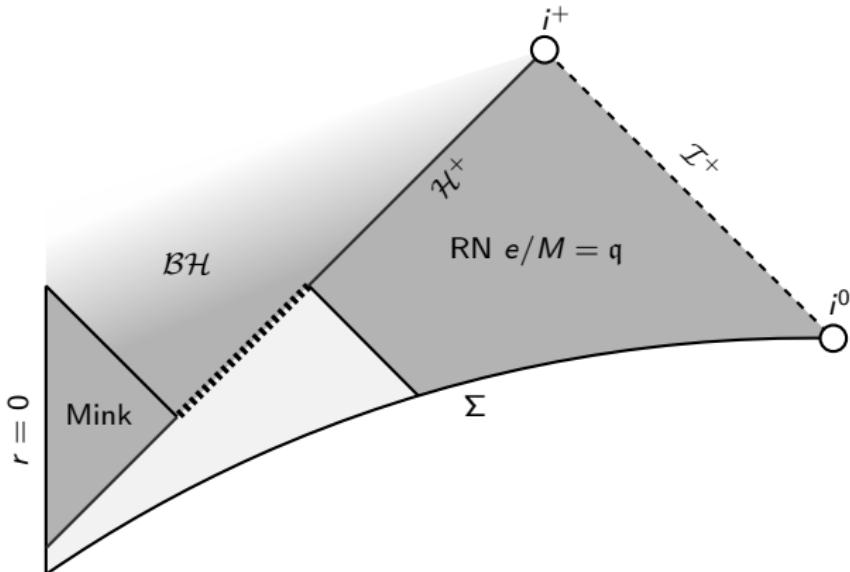
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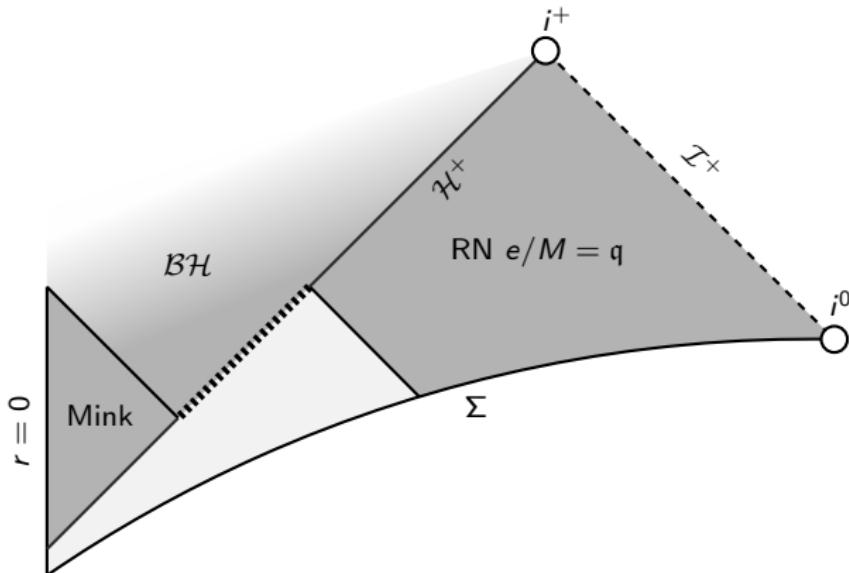


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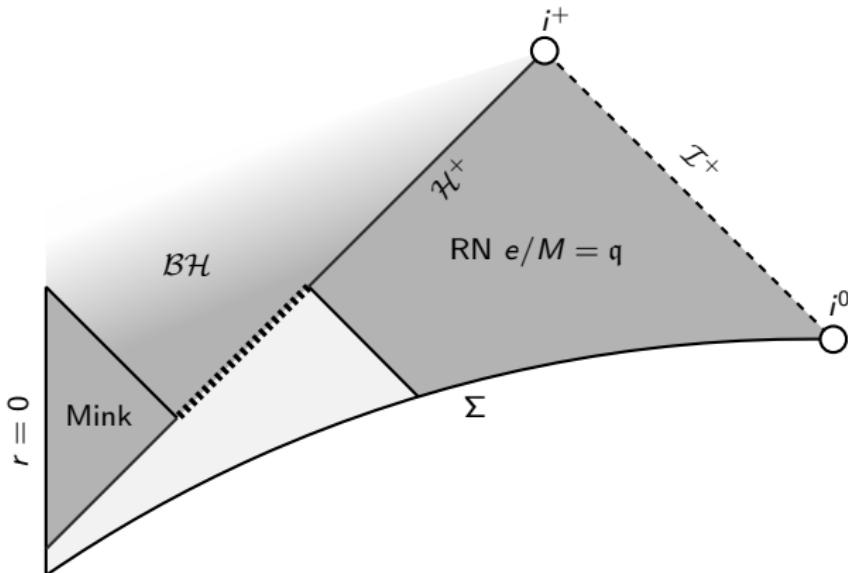


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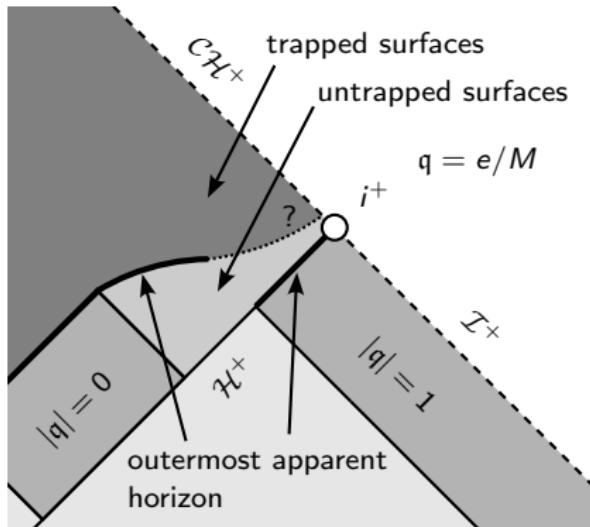
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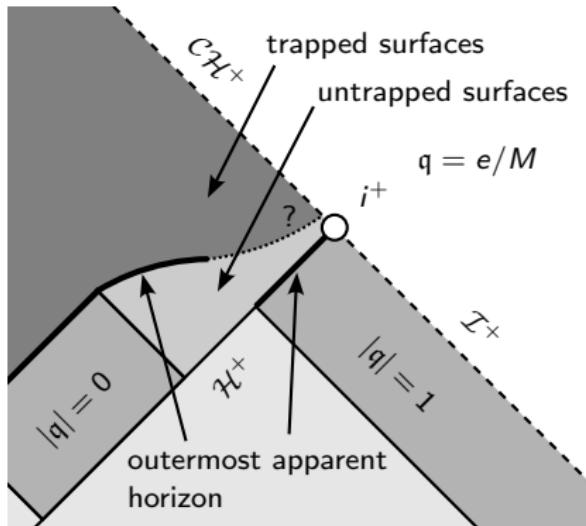


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## Interior structure of third law violating solutions

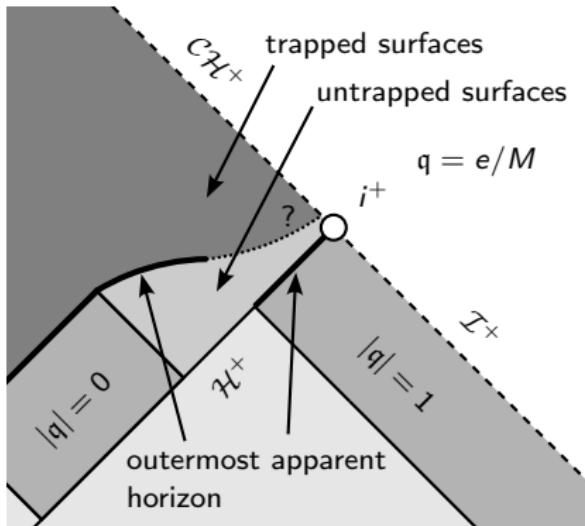


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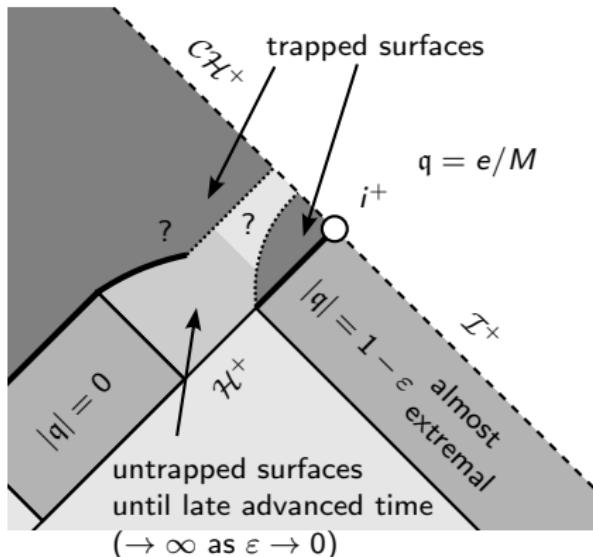
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- ▶ The outermost apparent horizon becomes disconnected, yet the spacetime is highly regular.
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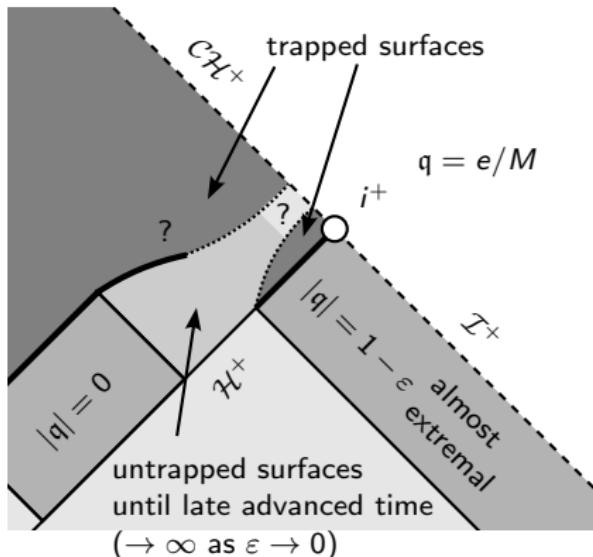
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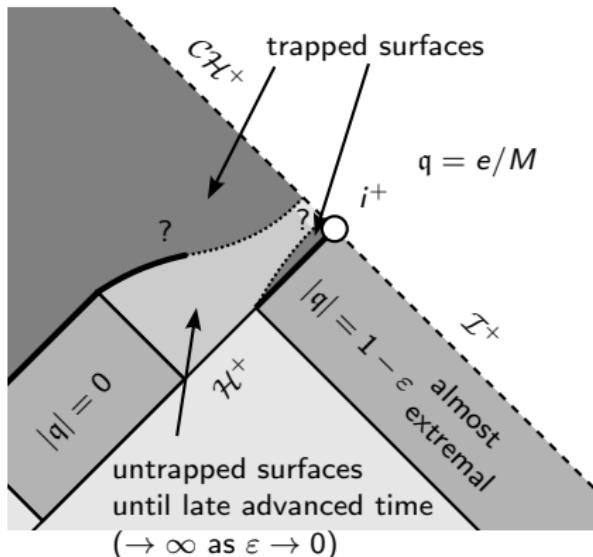
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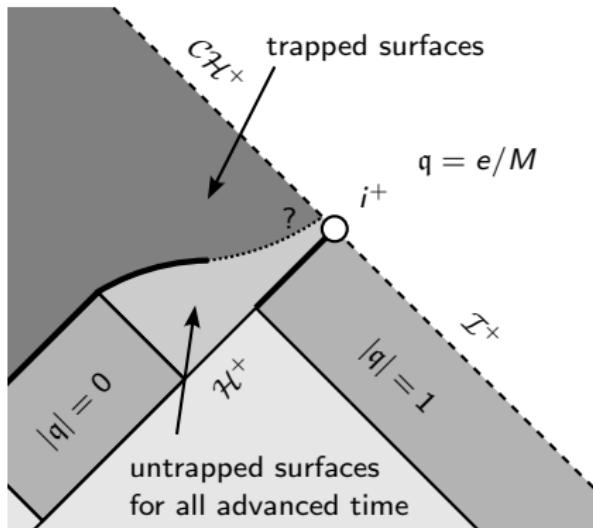
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## On the (non)genericity of our examples

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It is impossible by any procedure, no matter how idealized, to reduce  $\kappa$  to zero by a finite sequence of operations.

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- ▶ However, such a reformulation would be completely trivial (and has nothing to do with  $T = 0$ ).

## The third law and supercharging

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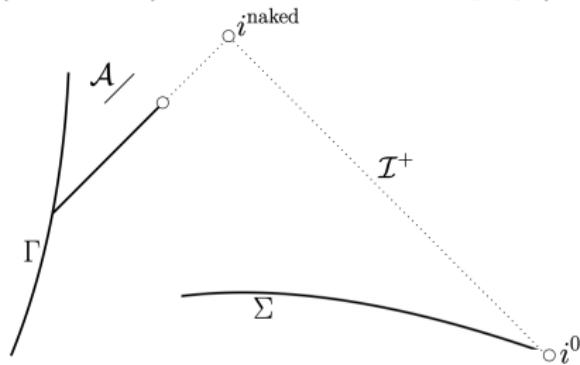
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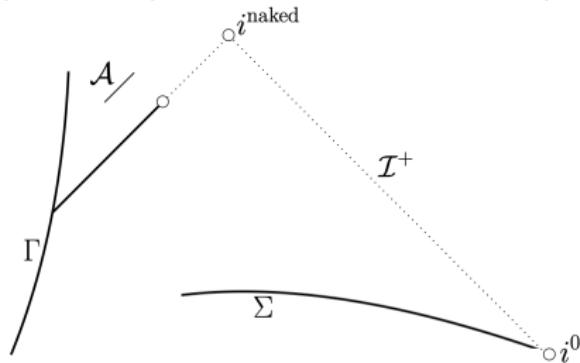


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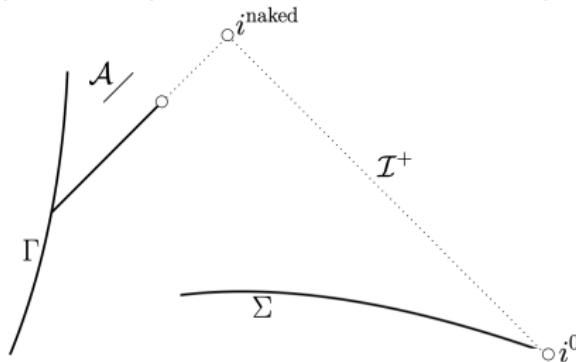
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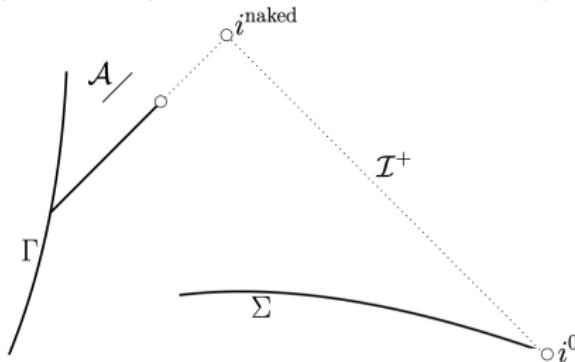
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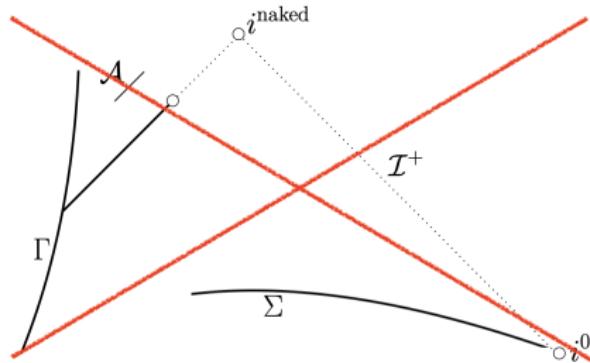
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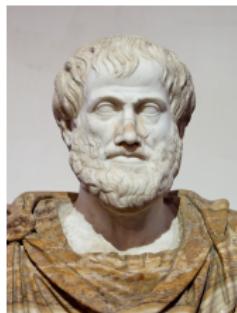
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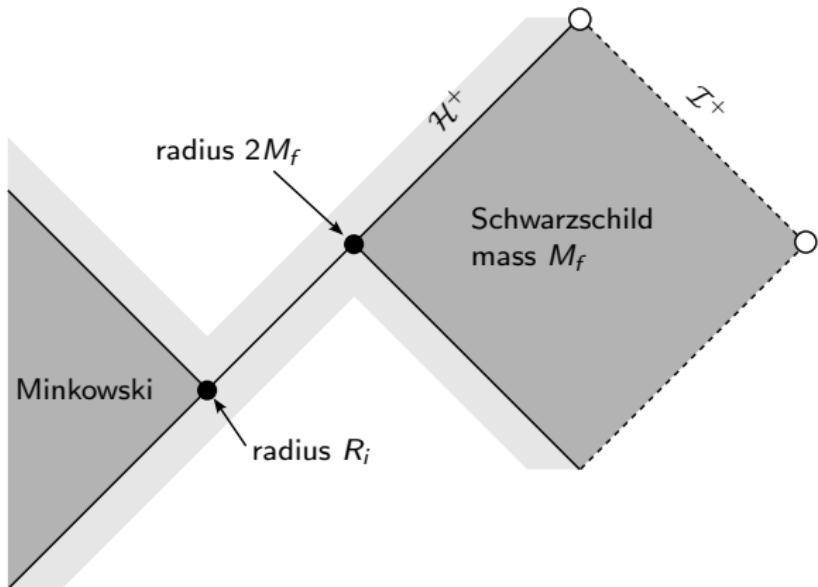
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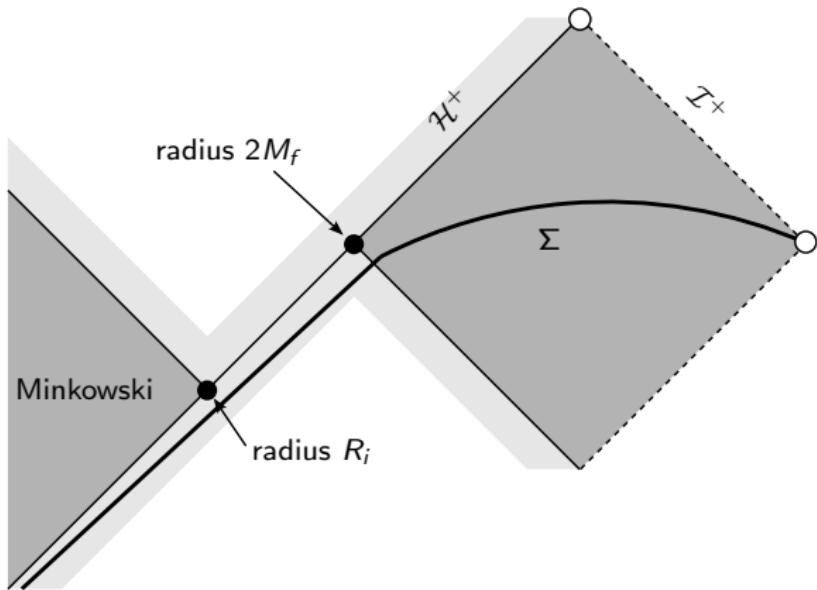


The black hole region  $\mathcal{BH}$  and event horizon  $\mathcal{H}^+$  of a spacetime are **teleological** notions—they can't be located *without knowing the entire future of the spacetime*.

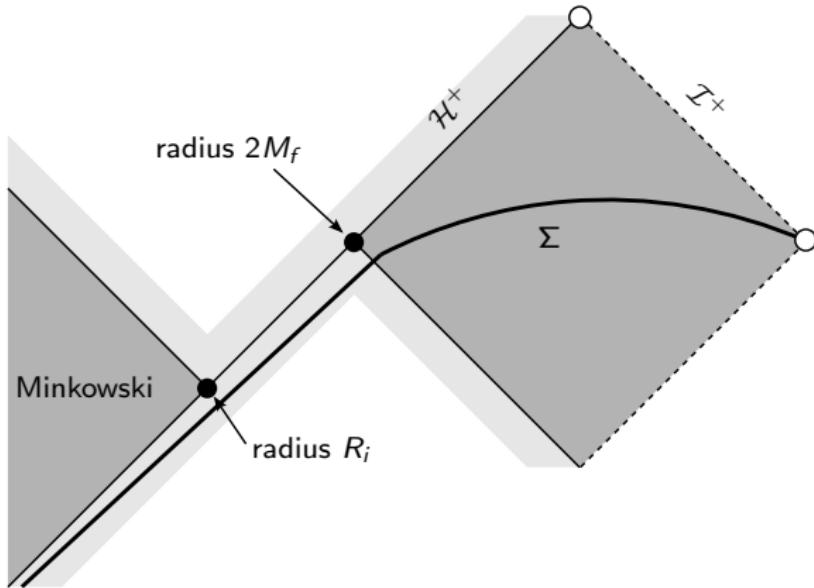
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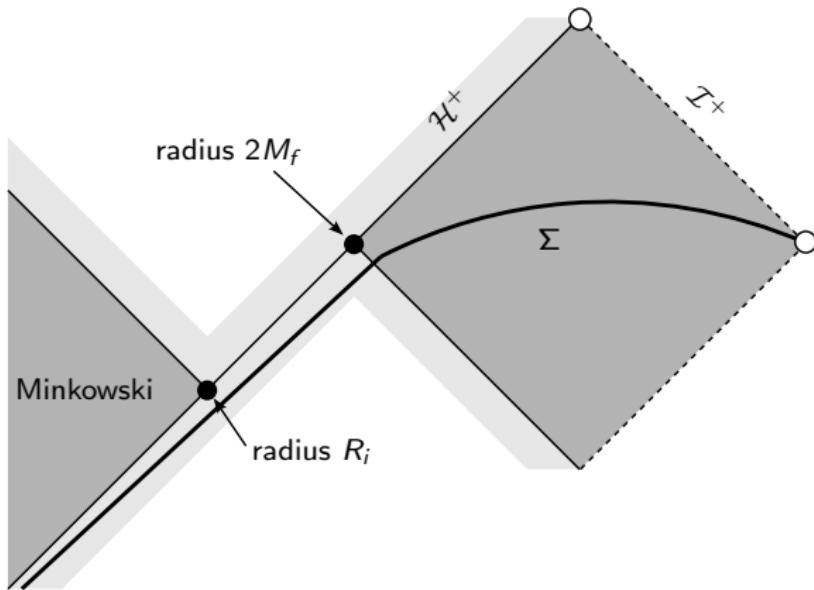


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**Characteristic gluing** for the vacuum Einstein equations recently initiated in fundamental works of Aretakis–Czimek–Rodnianski and refined by Czimek–Rodnianski, but in a **perturbative** regime around Minkowski space which is inapplicable here.

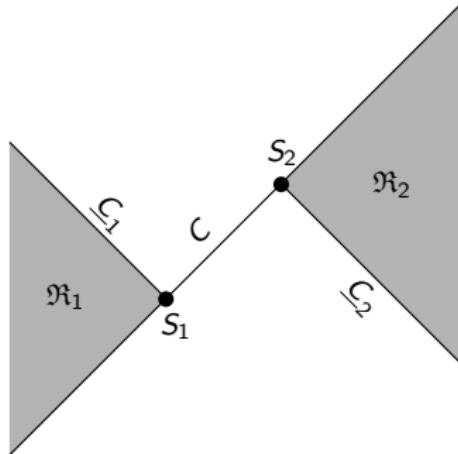
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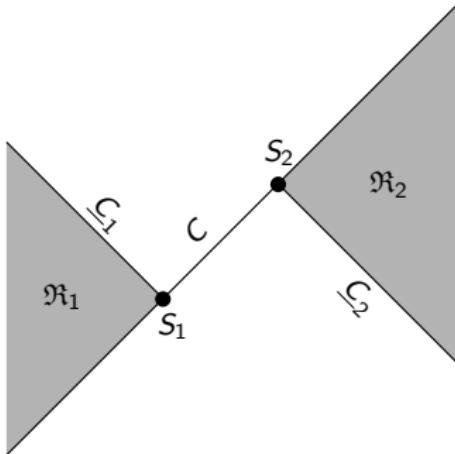
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The present problem is fundamentally **nonlinear** and **nonperturbative**, but is tractable because of the symmetry assumption.

## Characteristic/null gluing for the linear wave equation



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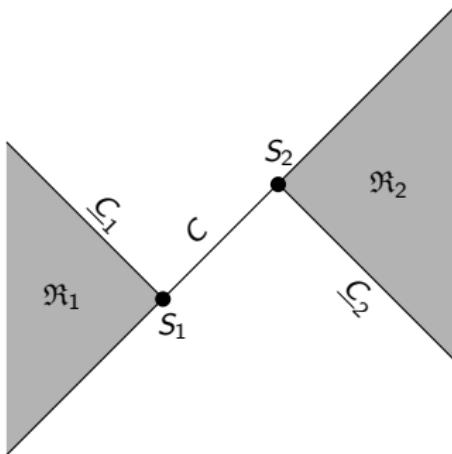


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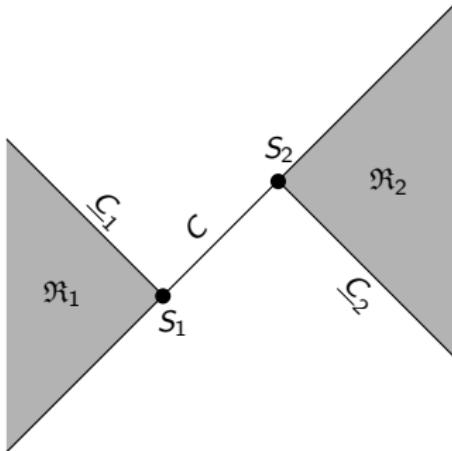
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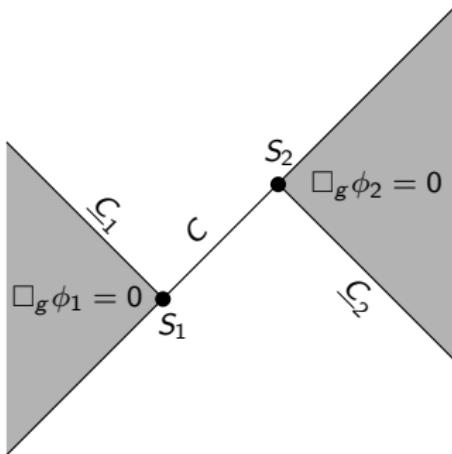
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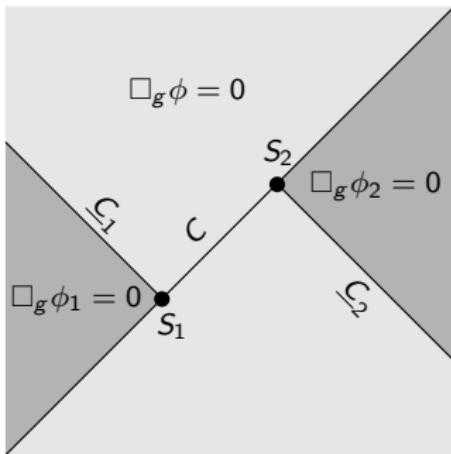
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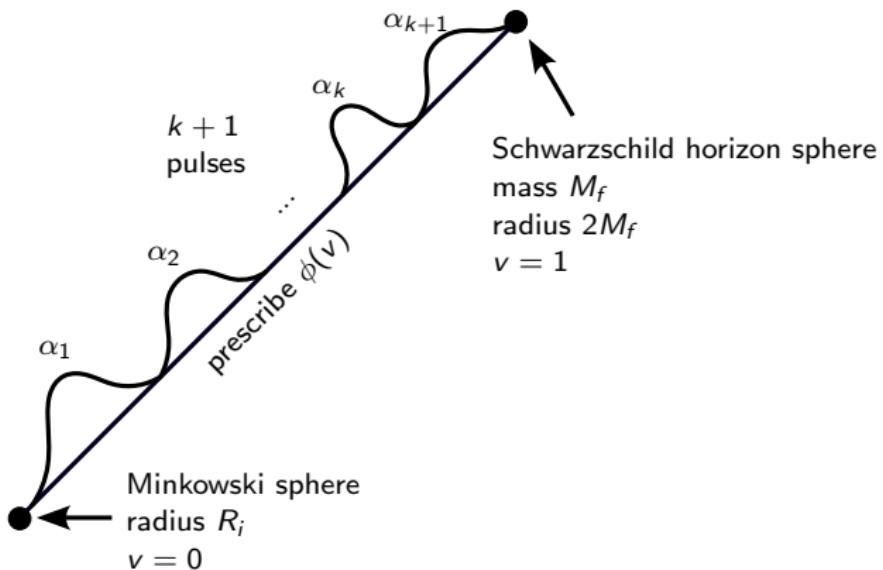
### Theorem (Kehle–U. '22).

*For any  $k \in \mathbb{N}$  and  $0 < R_i < 2M_f$ , the Minkowski sphere of radius  $R_i$  can be characteristically glued to the Schwarzschild event horizon sphere with mass  $M_f$  to order  $C^k$  within the Einstein-scalar field model in spherical symmetry.*

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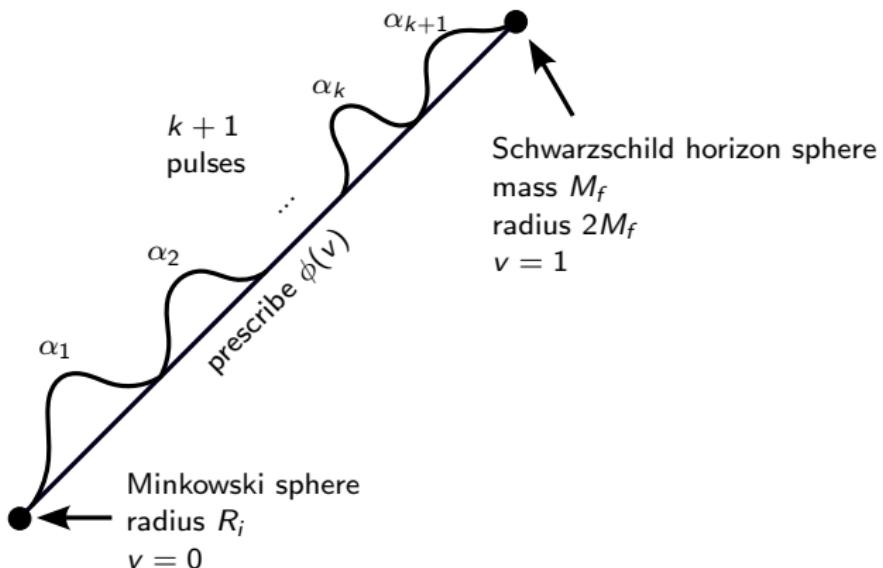
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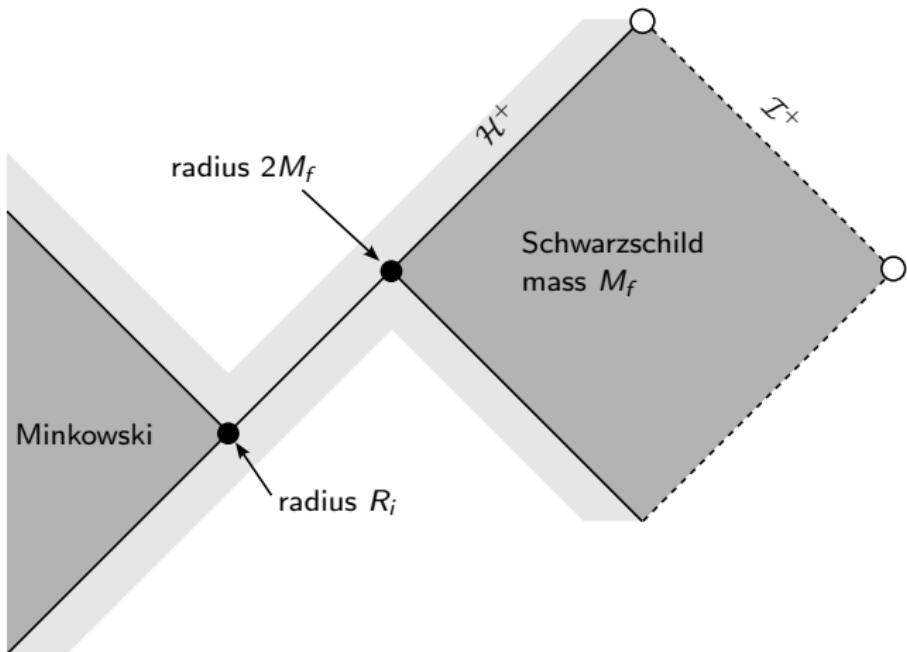
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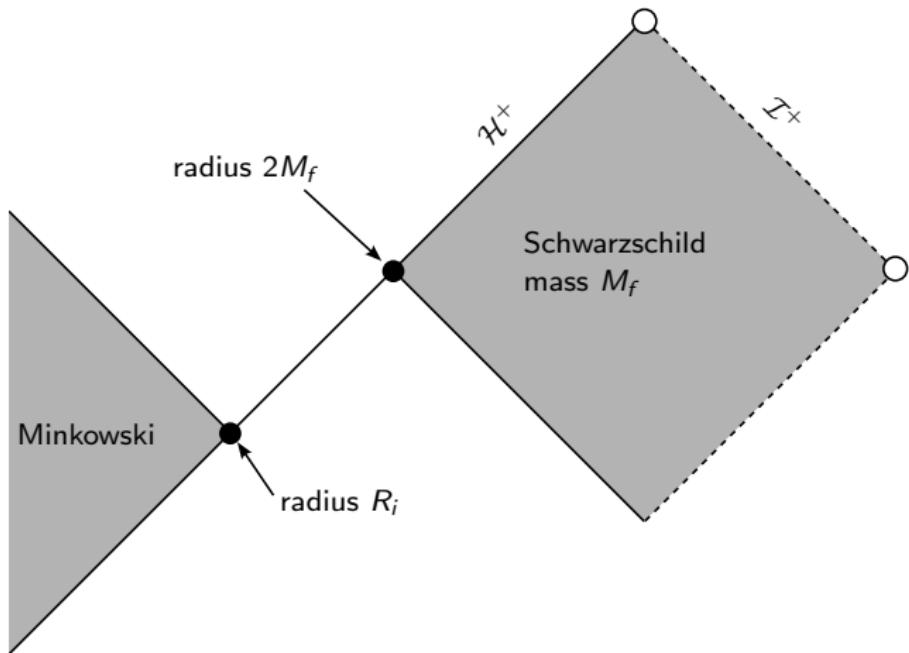


Pulse amplitudes  $\alpha_j$  are chosen using the Borsuk–Ulam theorem, making use of the symmetry  $\phi \mapsto -\phi$  of the equations.

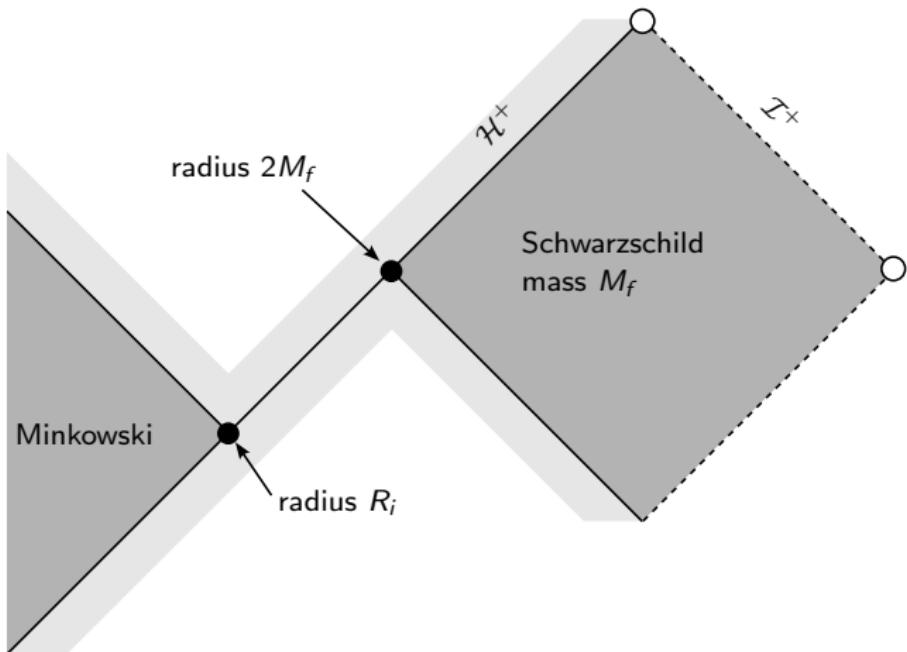
## Minkowski to Schwarzschild gluing



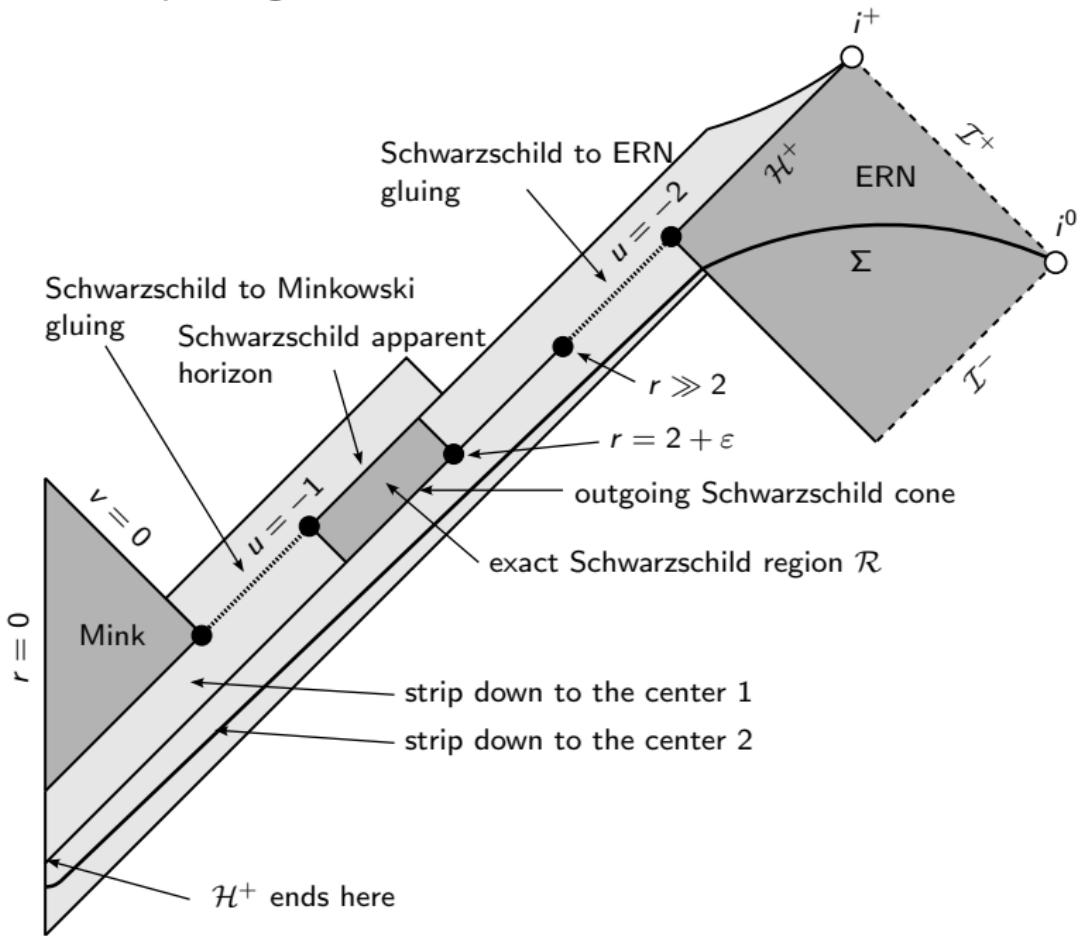
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## Blueprint to disproving the third law



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This would follow from characteristic gluing, in vacuum, of a Minkowski sphere to a Kerr exterior sphere.

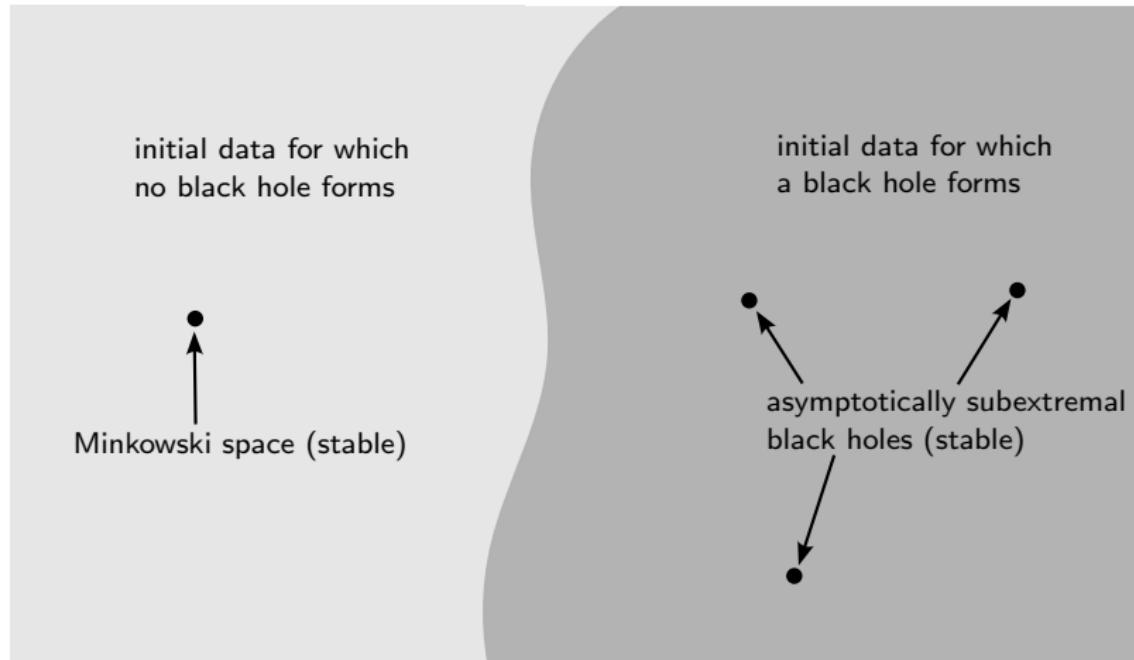
## Outlook: the “moduli space” of gravitational (non-)collapse



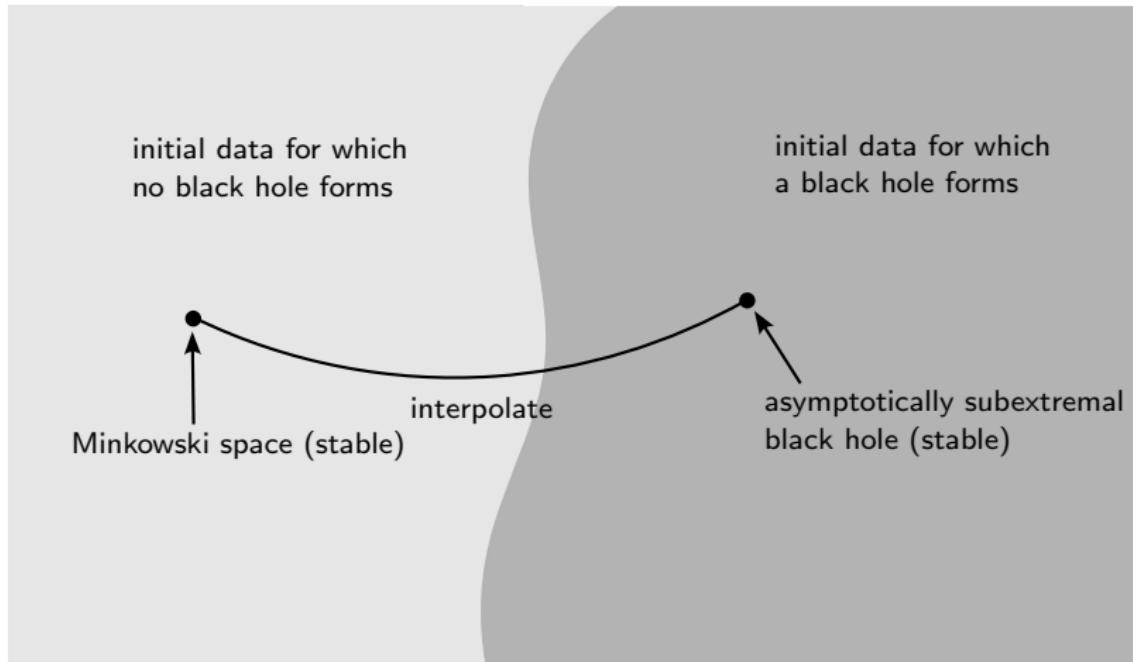
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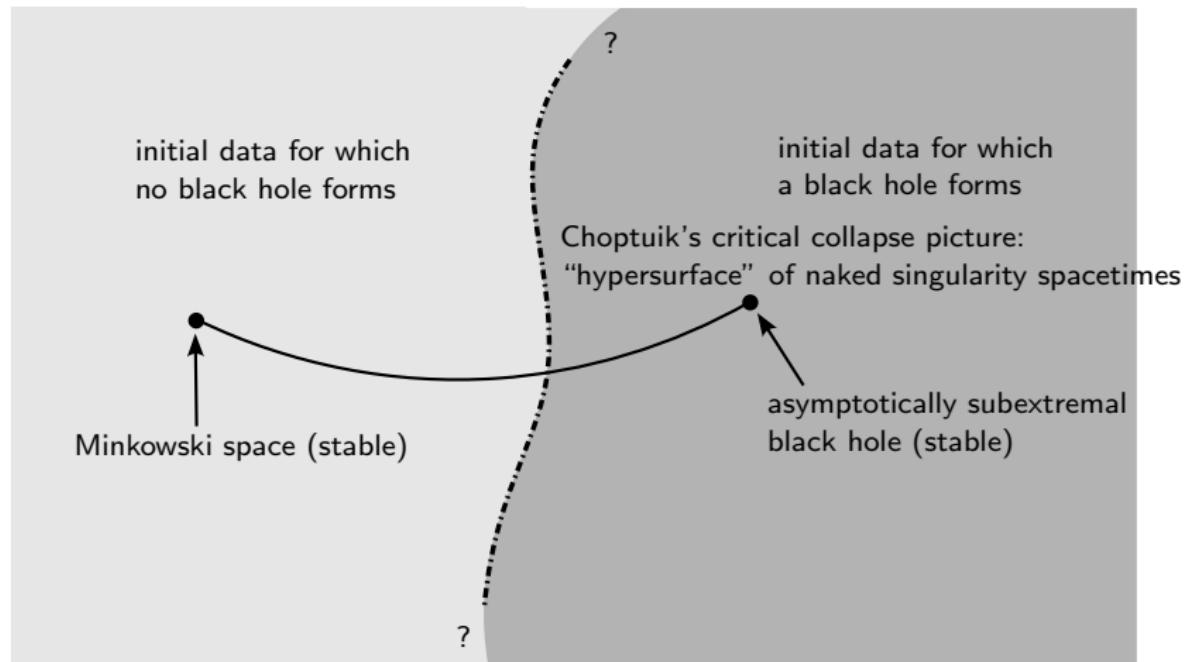
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Critical behavior: naked singularities expected when a “low energy” black hole forms.

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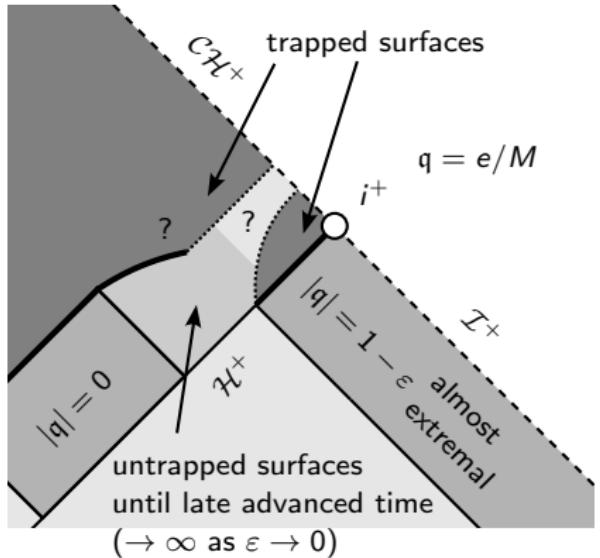
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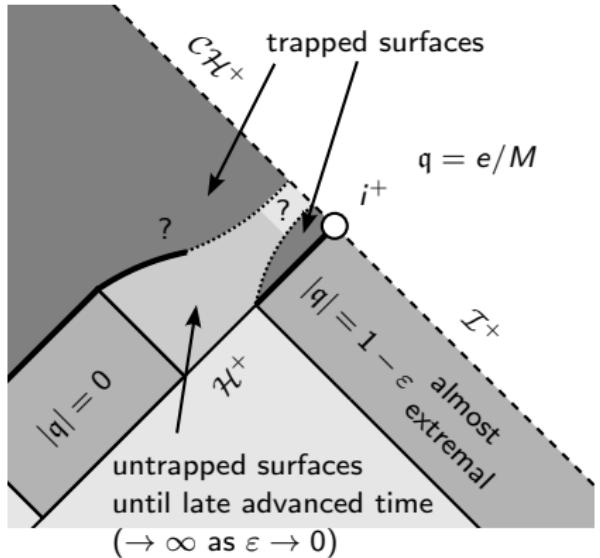
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Critical behavior: the very black-holeness of certain spacetimes is unstable.

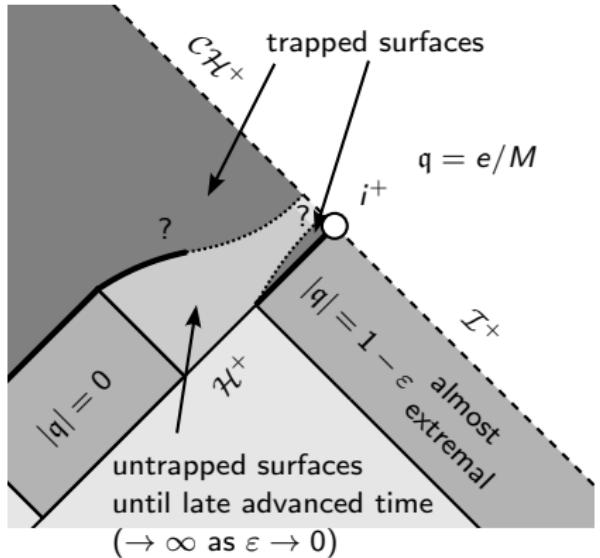
## Event horizon jumping



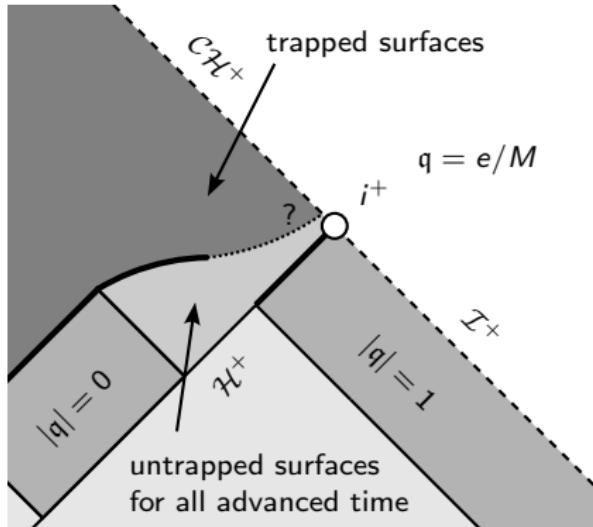
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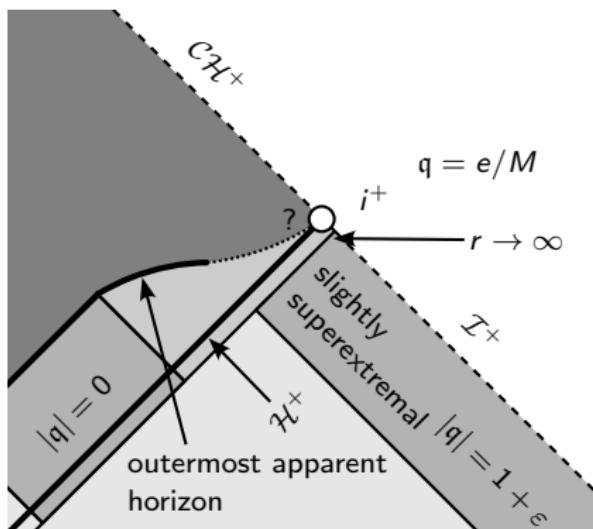
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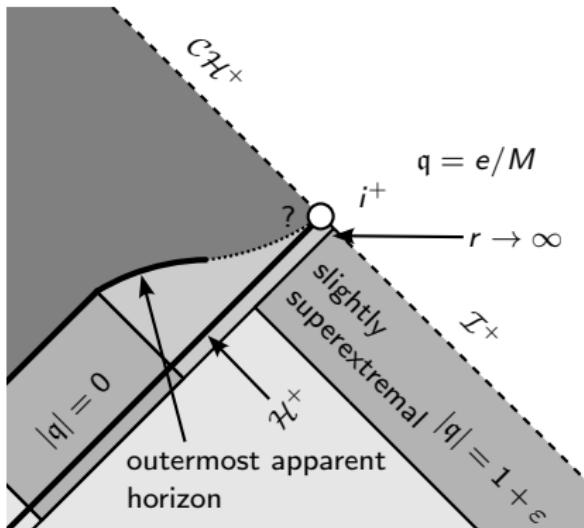
## Event horizon jumping



So what does happen if you “go past extremality”?



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Critical behavior: We expect that there are smooth 1-parameter families of solutions passing through extremality where the event horizon “jumps” as soon as the exterior becomes superextremal. There is no naked singularity.

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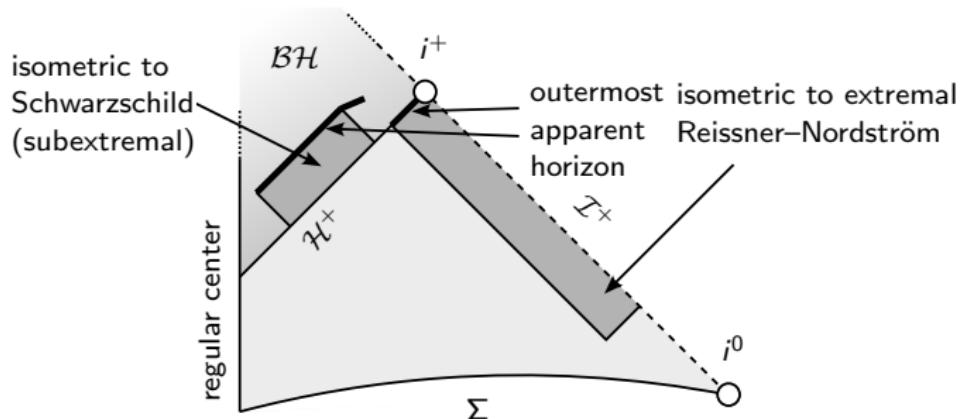
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Thank you!