Pair_Programming_Exercises_Module 05

HD Sheets

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Pair exercises, Module 5

Drawn from Chapter 3 of Open Intro to Statistics

These problems mostly require a written discussion, although there are some calculations as well. Add calculation cells as needed, but always be sure to give me a discussion

3.2 Roulette wheel.

The game of roulette involves spinning a wheel with 38 slots: 18 red, 18 black, and 2 green. A ball is spun onto the wheel and will eventually land in a slot, where each slot has an equal chance of capturing the ball. {assume the wheel is “fair” and all slots are really equally likely}

a. You watch a roulette wheel spin 3 consecutive times and the ball lands on a red slot each time. What is the probability that the ball will land on a red slot on the next spin?

```
18/38
```

```
## [1] 0.4736842
```

About 47%. These are all independent trials, it doesn’t matter how many times you spin, the odds are the same each time.

b. You watch a roulette wheel spin 300 consecutive times and the ball lands on a red slot each time. What is the probability that the ball will land on a red slot on the next spin?

```
18/38
```

```
## [1] 0.4736842
```

This is the same as spinning 3 times and getting a red on the fourth. The probability of getting red does not change, it is independent of the spin count.

c. Are you equally confident of your answers to parts (a) and (b)? Why or why not?

Yes, both A and B represent the same underlying probability, B just shows a more extreme scenario, but the number of spins does not impact the probability of getting a red.

A long series of positive outcomes is called a “run”

##3.6 Dice rolls.

If you roll a pair of fair dice, what is the probability of getting specific values or outcomes for the total of the two dice?

Do this by enumeration, writing down the combinations that give you the desired result, and then divide by the total number of combinations.

How may possible combinations are there of two dice rolls?

1, 1 1, 2 1, 3 1, 4 1, 5 1, 6 2, 2 2, 3 2, 4 2, 5 2, 6 3, 3 3, 4 3, 5 3, 6 4, 4 4, 5 4, 6 5, 5 5, 6 6, 6

There are 21 different combinations

(a) getting a sum of 1?

This is impossible, the lowest sum is 2.

(b) getting a sum of 5?

2, 3 and 1, 4 are the only options, therefore two combinations.

(c) getting a sum of 12?

6,6 is the only possible option, therefore one combination.

3.8 Poverty and language.

The American Community Survey is an ongoing survey that provides data every year to give communities the current information they need to plan investments and services. The 2010 American Community Survey estimates that 14.6% of Americans live below the poverty line, 20.7% speak a language other than English (foreign language) at home, and 4.2% fall into both categories

(a) Are living below the poverty line and speaking a foreign language at home disjoint?

They are not disjoint because there is overlap between the number of Americans living below the poverty line and those speaking a foreign language at home.

(b) Draw a Venn diagram summarizing the variables and their associated probabilities.

Do this on paper, agree on it with your partner, but do not turn it in,

(c) What percent of Americans live below the poverty line and only speak English at home?

```
14.6-4.2
```

```
## [1] 10.4
```

10.4% of Americans live below the poverty line and speak English at home. The percentage of people in both categories can't be counted in this scenario.

(d) What percent of Americans live below the poverty line or speak a foreign language at home?

```
14.6+20.7-4.2
```

```
## [1] 31.1
```

The number of Americans that live below the poverty line OR speak a foreign language at home does not include the 4.2% in both categories.

(e) What percent of Americans live above the poverty line and only speak English at home?

What percentage of Americans live above the poverty line?

100-14.6=85.4

What percentage don't speak English at home and are above the poverty line?

20.7-4.2=16.5

```
100-14.6-(20.7-4.2)
```

```
## [1] 68.9
```

f. Is the event that someone lives below the poverty line independent of the event that the person speaks a foreign language at home?

State your evidence for this from the problem statement.

No, they are joint, therefore they are dependent on each other. If they were independent, we would be taking the product of the probabilities.

3.13 Joint and conditional probabilities.

P(A) = 0.3, P(B) = 0.7

(a) Can you compute P(A and B) if you only know P(A) and P(B)?

This one has my solution, read through it.

No, since we don't know if they are independent or not.

P(A AND B) = P(A)P(B) only when they are indepenent

b. Assuming that events A and B arise from independent random processes,

i. what is P(A and B)?
ii. what is P(A or B)?
iii. what is P(A|B)?

If we know they are independent

i.) P(A and B) = P(A)P(B)= 0.3*0.7 = 0.21

ii. P(A or B) = P(A)P(not B) + P(A)P(B)+ P(not A)P(B)= 1-P(not A)P(not B)

p(not A)= 1-0.3= 0.7, p(not B) = 1-0.7= 0.3, p(not A) p(not B) = 0.21 also

c. If we are given that P(A and B) = 0.1, are the random variables giving rise to events A and B indepen- dent?

In general

P(A and B) = P(A given B)P(B)

0.1 = P(A given B) 0.7

0.1/0.7 = 0.143 = P(A given B) = P(A|B)

P(A|B) is not equal to P(A) and the events are not independent

(d) If we are given that P(A and B) = 0.1, what is P(A|B)?

See above, P(A|B)= 0.143

3.14 PB & J.

Suppose 80% of people like peanut butter, 89% like jelly, and 78% like both. Given that a randomly sampled person likes peanut butter, what's the probability that he also likes jelly? ————————————-

This is a joint and conditional probability problem, use the ideas from 3.13 above to solve it.

We are interested in P(A|B) or P(Jelly|Peanut Butter)

P(Jelly and Peanut Butter) = P(Jelly given Peanut Butter)P(Peanut Butter)

0.78/.80

```
0.78/0.80
```

```
## [1] 0.975
```

There is a 97.5% probability that he also likes Jelly

3.15 Global warming.

A Pew Research poll asked 1,306 Americans “From what you’ve read and heard, is there solid evidence that the average temperature on earth has been getting warmer over the past few decades, or not?”. The table below shows the distribution of responses by party and ideology, where the counts have been replaced with relative frequencies.40

Response Earth is Not Don’t Know warming warming Refuse Total Conservative Republican 0.11 0.20 0.02 0.33 Party and Mod/Lib Republican 0.06 0.06 0.01 0.13 Ideology Mod/Cons Democrat 0.25 0.07 0.02 0.34 Liberal Democrat 0.18 0.01 0.01 0.20 Total 0.60 0.34 0.06 1.00

ANSWER THESE QUESTIONS BASED ON THE DATA ABOVE ONLY, NOT PERSONAL PRIOR BELIEF- ie using purely frequentist statistical reasoning

(a) Are believing that the earth is warming and being a liberal Democrat mutually exclusive?

It appears that these are not mutually exclusive, as the liberal democrat response is statistically about 1/3 of the time.

b. What is the probability that a randomly chosen respondent believes the earth is warming or is a liberal Democrat? ——————————————

P(A) = .6 P(B) = .2 P(A or B) = 1 - P(not A)P(not B)

```
1 - (1-.6)*(1-0.2)
```

```
## [1] 0.68
```

c. What is the probability that a randomly chosen respondent believes the earth is warming given that he is a liberal Democrat? ——————————————————-

P(Warming and Liberal Democrat) = P(Warming given Liberal Democrat)P(Liberal Democrat) 0.18 = P(Warming given Liberal Democrat)(0.2) 0.18/0.2 = P(Warming given Liberal Democrat)

```
0.18/0.2
```

```
## [1] 0.9
```

d. What is the probability that a randomly chosen respondent believes the earth is warming given that he is a conservative Republican? ————————————

P(Warming and Conservative Republican) = P(Warming given Conservative Republican)P(Conservative Republican) 0.11 = P(Warming given Conservative Republican)(0.33) 0.11/0.33 = P(Warming given Conservative Republican)

```
0.11/0.33
```

```
## [1] 0.3333333
```

e. Does it appear that whether or not a respondent believes the earth is warming is independent of their party and ideology? Explain your reasoning. ————————————————-

No, 90% of liberal democrats believe that the earth is warming, whereas only 33% of conservative republicans believe the earth is warming.

f. What is the probability that a randomly chosen respondent is a moderate/liberal Republican given that he does not believe that the earth is warming? ————————————————

P(Not Warming and Mod/Liberal Republican) = P(Not Warming given Mod/Liberal Republican)P(Mod/Liberal Republican) 0.06 = P(Not Warming given Mod/Liberal Republican)(0.13) 0.06/0.13 = P(Not Warming given Mod/Liberal Republican)

```
0.06/0.13
```

```
## [1] 0.4615385
```