Estimating Parameters

Definitions

Parameters of Interest and Nuisance Parameters

Given a statistical model with multiple parameters, there may be some parameters / function of parameters we are interested in estimating. Parameters we care about are **parameters of interest** while those we don't care about are **nuisance parameters**.

Identifiable

A parameter θ is identifiable from the statistical model P_θ if:

$$P_\theta = P_{\theta'} \Rightarrow \theta = \theta'$$

- For a normal distribution, σ^2 is identifiable, but σ is not because it could be positive or negative

Methods to Estimate Parameters

Plug-In Method

If a parameter can be represented as an expectation or sum of expectations, then just replace the expectation with the sample average:

For normally distributed X_i:
- $\mu = \mathbb{E}\left[X_1\right] \rightsquigarrow \frac{1}{n}\sum_{i=1}^n X_i$
- $\sigma^2 = \mathbb{E}\left[X_1^2\right] - \mathbb{E}\left[X_1\right]^2 \rightsquigarrow \frac{1}{n}\sum_{i=1}^n X_i^2 - \left(\frac{1}{n}\sum_{i=1}^n X_i\right)^2$

Maximum Likelihood

Likelihood

Let $f_\theta(x)$ be the PDF corresponding to P_θ. That is, it is the likelihood of observing a single sample x given that the parameters are x.

Then, the **likelihood** function is:

$$L_n(\theta) = \prod_{i=1}^n f_\theta(X_i)$$

This is the probability of observing these n samples given that we have a specific θ.

The **log-likelihood** function is:

$$\ell_n(\theta) = \log L_n(\theta) = \sum_{i=1}^n \log f_\theta(X_i)$$

Maximum Likelihood Estimator

The **maximum likelihood estimator** is the point which maximizes the function $\ell_n(\theta)$, i.e.:

$$\hat{\theta} = \arg\max \ell_n(\theta)$$