Method of Moments

Moment

The j th moment of X is $\alpha_j = \mathbb{E}\left[X^j\right]$

With the pdf f_θ, this is function of θ:

$$\alpha_j = \int x^j f_\theta(x)dx$$

With the plug-in method, we can estimate this as:

$$\hat{\alpha_j} = \frac{1}{n}\sum_{i=1}^{n} X_i^j$$

We can therefore solve for θ by setting $\hat{alpha_j} = \alpha_j$

- If we have multiple values in θ, we will need as many different moments in our system of equations

For a normal random variable with mean μ and variance σ^2, we can compute the moments as:

$$\begin{aligned}\alpha_1 &= \mu \\ \alpha_2 &= \mu^2 + \sigma^2\end{aligned}$$

We can then solve this to get:

$$\begin{aligned}\mu &= \alpha_1 \approx \hat{\alpha_1} \\ \sigma^2 &= \alpha_2 - \alpha_1^2 \approx \hat{\alpha_2} - \hat{\alpha_1}^2\end{aligned}$$

This was solvable in closed form, but generally we might have to do other things to solve this

Asymptotic Properties of Method of Moments

The method of momenys estimator is consistent and asymptotically normal