Column Space

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The **column space** of a $m \times n$ matrix A is the space $C(A)$ of all vectors b such that there is some x such that $Ax = b$.

Column space is the span of the columns of A

- To express the column space succintly, we want to find the basis of the column space

Finding Basis

A set of vectors is linearly independent if there is no set of coefficients such that the linear combination of the vectors is 0

- Therefore, the null space of the matrix with these vectors is just $\{0\}$
- If we bring the matrix to RREF, then the pivot columns are linearly independent
  - The free columns are just linear combinations of the pivot columns, so these pivot columns still clearly span the same vector space as the RREF matrix
- If the pivot columns were $r_{i_1}, r_{i_2}, \ldots$, we claim that $v_{i_1}, v_{i_2}, \ldots$ in the original matrix are also linearly independent
  - Due to fact that going to RREF is always an invertible matrix operation

Guide for Finding Basis

- Create matrix with vectors forming the columns
- Bring matrix to RREF
- Find indices of columns that form pivots in RREF
- Those columns in the original matrix form a linearly independent basis