Hypothesis Testing

- Answers binary questions such as "is a drug better than placebo"

Test and Rejection Region

A **test** is a function Ψ from the data to $\{0, 1\}$. The **rejection region** of a test is $R = \{\text{datasets for which } \Psi = 1\}$

- The rejection region fully characterizes Ψ

Test Statistic

Function that summarizes data and is sufficient to compute a test Ψ

Hypothesis Test

A **hypothesis test** takes the form
$H_0 : \theta \in \Theta_0$ vs $H_1 : \theta \in \Theta_1$
where Θ_0 and Θ_1 are two disjoint subsets of Θ such that their union is Θ

- We typically characterize H_0 as the null hypothesis, or the status quo
- H_1 is a revelation or something that you prove

Error Types

Type I Error

A Type I error is when H_0 is true, but the test concludes H_1 is true. This is considered more serious in things like a criminal trial.

Type II Error

A Type II error is when H_1 is true, but the test concludes H_0 is true.

The probability of a test committing an error depends on the true value of the parameter

Size and Level

The **size** of a test Ψ is:

$$\text{size}(\Psi) = \max_{\theta \in \Theta_0} P_\theta(\Psi = 1)$$

- I.e. it is the maximum possible probability of a type I error

A test is said to have **level** α if $\text{size}(\Psi) \leq \alpha$

- The maximum type I error probability is always achieved for θ on the boundary between Θ_0 and Θ_1

Power

The **power** function is defined as:

$$\text{B}(\theta) = P_\theta(\Psi = 1)$$

For a perfect test, the power function B is a step function that is zero when $\theta \in \Theta_0$ and one when $\theta \in \Theta_1$