T Test

- Previously in the Wald test, we assumed that n was large so sample means would approach the true mean with the CLT
- However, for small n this doesn't work
- When we know the data is normally distributed, we can instead use the t distribution with $n-1$ degrees of freedom

Student's t-distribution

This distribution with η degrees of freedom is given with pdf:

$$f(x) = \frac{\Gamma(\frac{\eta+1}{2})}{\sqrt{\eta\pi}\Gamma(\frac{\eta}{2})}\left(1+\frac{x^2}{\eta}\right)^{-\frac{\eta+1}{2}}$$

- $\sqrt{n}\cdot\frac{\overline{X}_n-\mu}{\sigma}$ is exactly distributed according to t_{n-1}
- This requires that we assume normality, but this can be checked with the KL test first