

Unibot Notes

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1 Equations for Precession

1.1 Precession Angular Acceleration

1.1.1 Variables

- τ - Torque about wheel base caused by point mass and link2 mass
- l_2 - Length of link2
- α - Angle of link2, w.r.t. link1 about +x axis pointing out of paper
- m - Mass of point mass
- θ - Precession angle
- ω_p - Precession angular velocity ($\dot{\theta}$)
- I - Wheel moment of inertia w.r.t. center of wheel
- ω - Wheel angular velocity w.r.t. +y axis pointing rightward
- L - Wheel angular momentum ($I\omega$)

1.1.2 Equations v1 (Hyperphysics)

With the help of <http://hyperphysics.phy-astr.gsu.edu/hbase/top.html>

$$\begin{aligned}\tau &= mgl_2 \sin \alpha = \frac{\Delta L}{\Delta t} \\ \Delta \theta &= \arctan \frac{\Delta L}{L}\end{aligned}\tag{1}$$

For small $\frac{\Delta L}{L}$, $\arctan \frac{\Delta L}{L} \approx \frac{\Delta L}{L}$

$$\Delta \theta \approx \frac{\Delta L}{L}$$

$$\begin{aligned}\omega_p &= \frac{\Delta \theta}{\Delta t} \\ &= \frac{\Delta L}{L \Delta t} \\ &= \frac{\tau}{L}\end{aligned}$$

$$\dot{\omega}_p = \frac{mgl_2}{L} \cos \alpha \dot{\alpha} - mgl_2 \sin \alpha \frac{\dot{\omega}}{I\omega^2}\tag{2}$$