HPP Lattice-Gas Cellular Automata

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1 ABSTRACT

The goal of this assignment is to program the HPP LG CA in MatLab and Anylogic. For this purpose we will first describe general information about Cellular Automata and the HPP model of Lattice-Gas Cellular Automata. After the implementation of the model in the two systems we are providing screenshots with our results and comments about the code and implementation method in MatLab and Anylogic.

2 WHAT ARE CELLULAR AUTOMATA (CA) IN GENERAL

A cellular automaton is a collection of "coloured" cells on a grid of specified shape that evolves through a number of discrete time steps according to a set of rules based on the states of neighbouring cells. The rules are then applied iteratively for as many time steps as desired. [Mathworld]
CA can have the following 5 characteristics. Not all of these must be always fulfilled.

- CA are regular arrangements of single cells of the same kind.
- Each cell holds a finite number of discrete states.
- The states are updated simultaneously (‘synchronously’) at discrete time levels.
- The update rules are deterministic and uniform in space and time.

- The rules for the evolution of a cell depend only on a local neighbourhood of cells around it. [LG CA - Introduction]

Cellular automata come in a variety of shapes and varieties. One of the most fundamental properties of a cellular automaton is the type of grid on which it is computed. The simplest such "grid" is a one-dimensional line. In two dimensions, square, triangular, and hexagonal grids may be considered. Cellular automata may also be constructed on Cartesian grids in arbitrary numbers of dimensions. As our problem needs to be handled with a two-dimensional automata, we will only describe this one here more detailed. [Mathworld]
There is much more freedom for arranging the cells and defining the neighbourhoods for the updating rules in two dimensions. There are two types of neighbourhoods in two dimensions. There is the von Neumann neighbourhood and the Moore neighbourhood.
Despite of their simple update rules cellular automata can display complex behaviour which is a prerequisite to use them as a simulation tool for physical (biological, chemical, ...) phenomena like, for example, fluid flow. CA are very easy to implement and are especially well suited for massively parallel computers because of the local character of the update rules. By construction they are unconditionally numerically stable. [LG CA - Introduction]

3 HPP LG CA

Lattice gas cellular automata are a type of cellular automaton used to simulate fluid flows.
As a cellular automaton, these models comprise a lattice, where the sites on the lattice can take a certain number of different states. In lattice gas, the various states are particles with certain velocities. Evolution of the simulation is done in discrete time steps. After each time step, the state at a given site can be determined by the state of the site itself and neighbouring sites, before the time step. The state at each site is purely boolean. At a given site, there either is or is not a particle moving in each direction.
In papers published in 1973 and 1976, Hardy, Pomeau and de Pazzis introduced the first Lattice Boltzmann model, which is called the HPP model after the authors. HPP model is a two-dimensional model of fluid particle interactions. In this model, the lattice is square, and the particles travel independently at a unit speed to the discrete time. The particles can move to any of the four sites whose cells share a common edge. Particles cannot move diagonally. If two particles collide head-on, for example a particle moving to the left meets a particle moving to the right, the outcome will be two particles leaving the site at right angles to the direction they came in. [Wikipedia - LG automata]

4 IMPLEMENTATION RULES

In this model the lattice takes the form of a two-dimensional square grid, with particles capable of moving to any of the four adjacent grid points which share a common edge, and particles cannot move diagonally. This means each grid point can only have one of sixteen possible interactions.

- Particles exist only on the grid points, never on the edges or surface of the lattice.

- Each particle has an associated direction (from one grid point to another immediately adjacent grid point).
- Each lattice grid cell can only contain a maximum of one particle for each direction, i.e, contain a total of between zero and four particles.

The following rules also govern the model:

1. A single particle moves in a fixed direction until it experiences a collision.
2. Two particles experiencing a head-on collision are deflected perpendicularly.
3. Two particles experience a collision which isn't head-on simply pass through each other and continue in the same direction.
4. Optionally, when a particles collides with the edges of a lattice it can rebound.

The HPP models follows a two stage update process. [Wikipedia HPP-Model]

4.1 Collision Step

In this step the above rules, 2., 3. and 4. are checked and applied if any collisions have occurred. This results in head-on collision particles changing direction, pass-through collisions continuing unchanged, or non-colliding particles simple remaining the same. [Wikipedia HPP-Model]

4.2 Transport Step

The second step consists of each particle moving one lattice step in the direction they are currently travelling, which could have been changed by the above Collision Step. [Wikipedia HPP-Model]

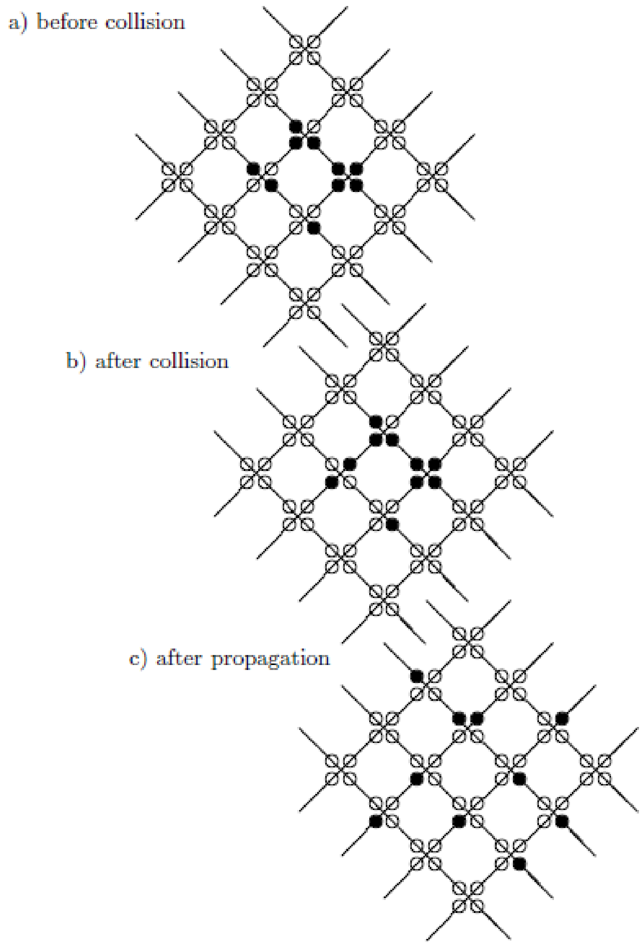

Figure 4.1: HPP: collision and propagation. Filled circles denote occupied cells and open circles empty cells. a) Part of the lattice before collision (e.g. after propagation); there is only one collision configuration (two particles in opposite cells at the same node; on the left). b) After collision (e.g. before propagation): the configuration of the cells at node on the left side has changed. c) After propagation: all particles have moved along the links to their nearest neighbours (the lattice outside the part shown was assumed to be empty, e.g. no propagation of particles from 'outside'). [LG CA - Introduction]

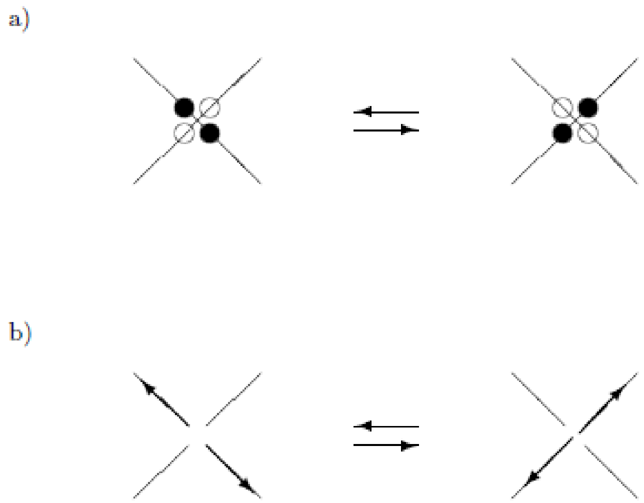

Figure 4.2: HPP: collisions. a) There is only one collision configuration (head-on collision) for HPP: two cell on opposite links are occupied and the two other cells are empty. After collision the formerly empty cells are occupied and vice versa. b)Same as a) but showing the associated momentum vectors. Both momentum vectors are rotated by 90°. Mass and momentum are conserved. [LG CA - Introduction]

5 Screenshots, Results and Comments

The simulation was created in two ways. There is one solution with Anylogic and a second with MatLab. On the following pages you can see some screenshots with comments of the simulations with these two programs.

5.1 Anylogic

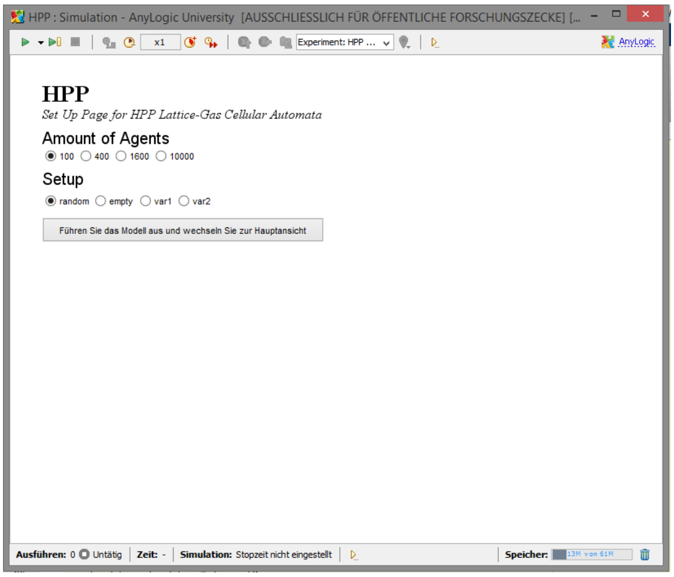

Figure 5.1: Startpage of the Anylogic example. You can choose between different amounts of agents and setups for the HPP Lattice-Gas Simulation.

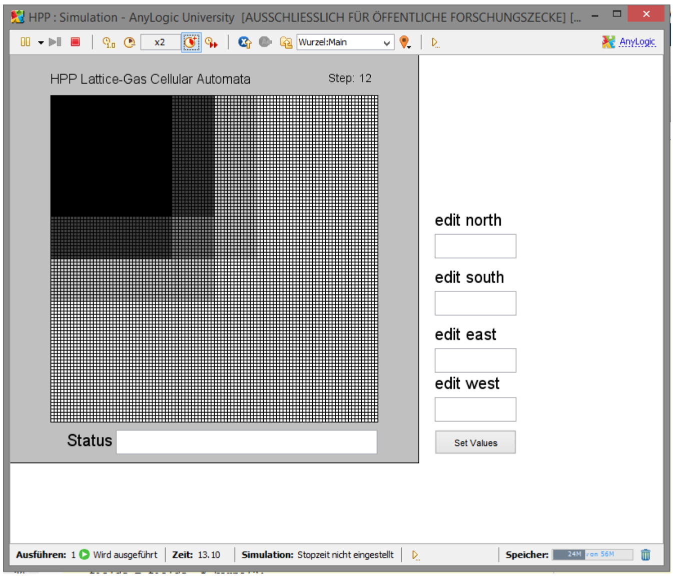

Figure 5.2: Begin of Anylogic Simulation with 10000 Agents and Setup var1. You can edit the values by entering 0 or 1 in the edit field and press set values to see the changes. The status field show if the values are added correctly.

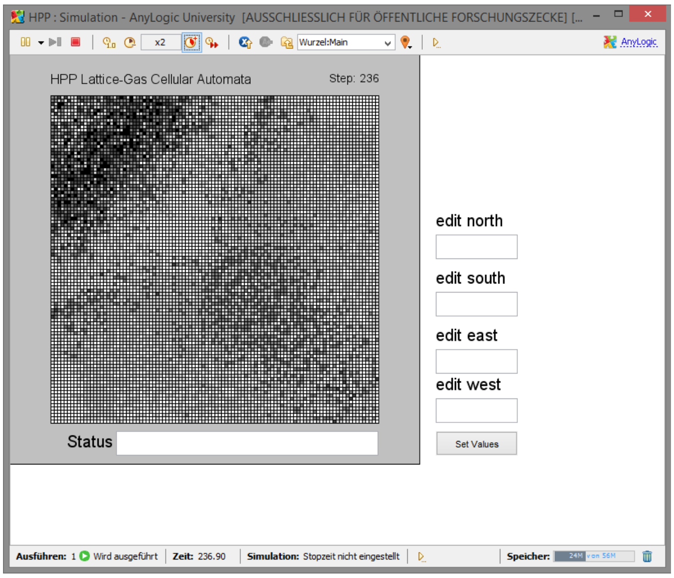

Figure 5.3: Anylogic Simulation with 10000 Agents and Setup var1 after 236 steps. Compared with the start image you can see how the gas has distributed.

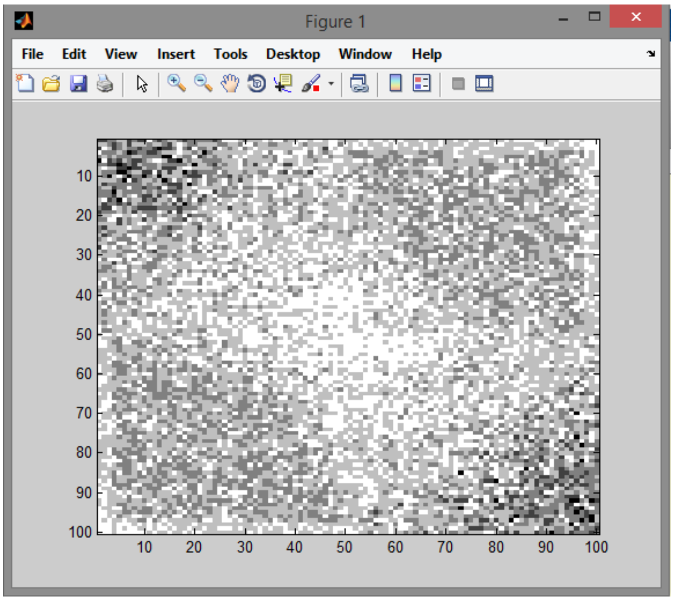

Figure 5.4: MatLab Simulation with duration of 300 height and width 100 and scenario 3. These 3 values can be changed when the MatLab Simulation is started.

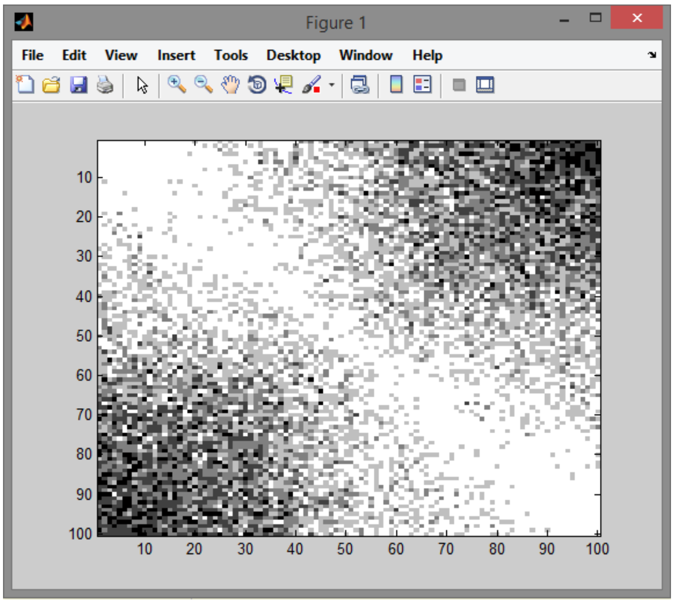

Figure 5.5: MatLab Simulation with duration of 300 height and width 100 and scenario 3. This image shows the state of the HPP LG CA after the duration of 300.

5.3 Conclusion

The HPP-Lattice Gas Cellular Automaton is an established method to learn about the flows of fluids and gases. Using different initial states one can easily study the different behaviours based on simple collision and propagation rules. The HPP simulation was quite easy to implement, but doesn't fully represent the original characteristics of such fluids or gases. For example, it lacks rotational invariance, making the model highly anisotropic. This results in strange square-shapes, which are unnatural for fluids and gases. More precise models are created using hexagonal grids and are called FHP cellular automata, but these are subject to further studies.

LIST OF FIGURES

REFERENCES

[Mathworld] "'Mathworld"', http://mathworld.wolfram.com/CellularAutomaton.html, 05.02.2014

[Wikipedia HPP-Model] "'Wikipedia"', http://en.wikipedia.org/wiki/HPP_model, 05.02.2014

[Wikipedia - LG automata] "'Wikipedia"', http://en.wikipedia.org/wiki/Lattice_gas_automaton, 05.02.2014

[LG CA - Introduction] "'LGCA"', Lattice-Gas Cellular Automata and Lattice Boltzmann Models - An Introduction, 26.06.2005