$$U(\omega) = \beta(\omega) - \omega$$