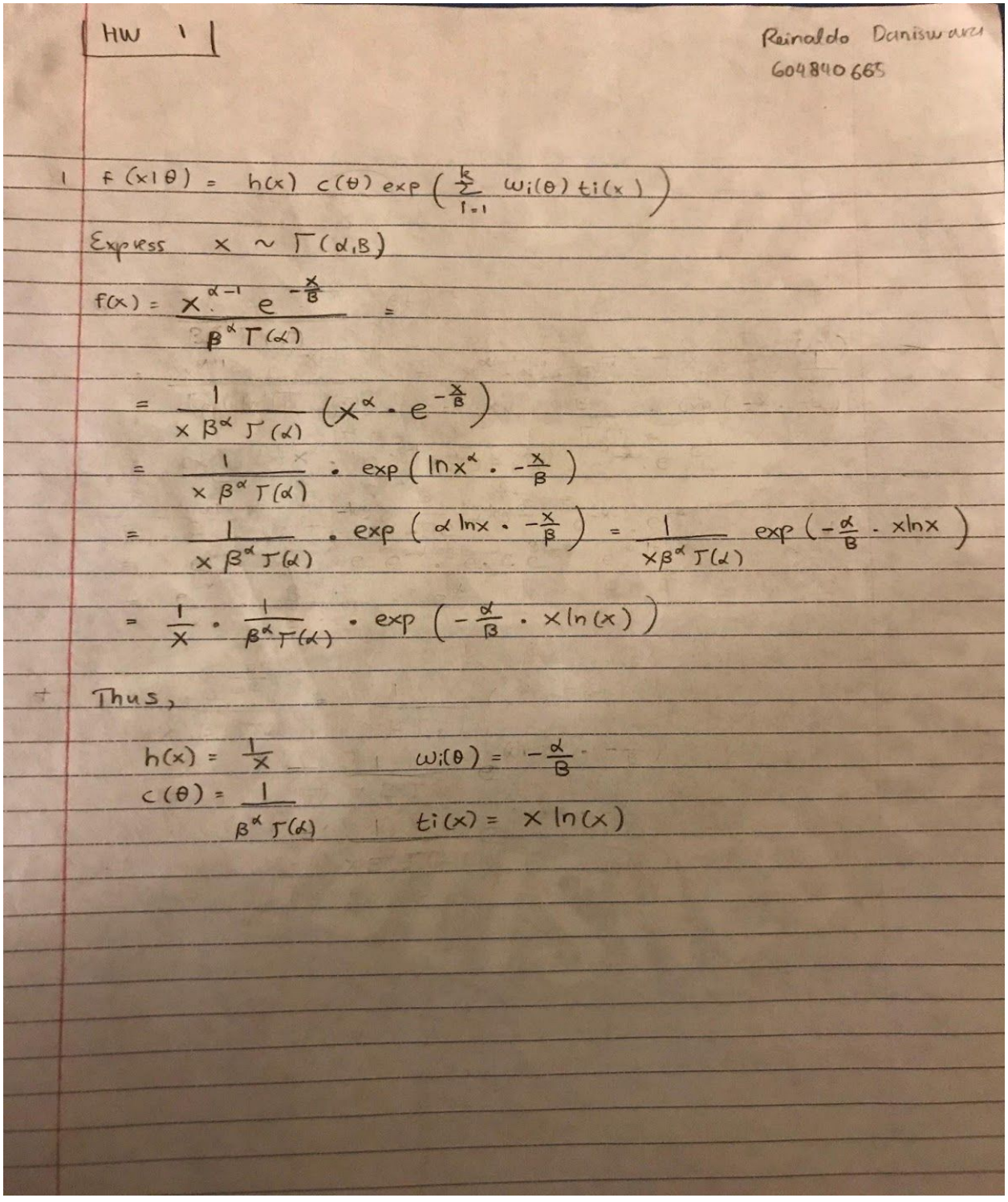
Homework 1 Stat 100B
Reinaldo Daniswara - 604840665



HW 1

Reinaldo Daniswara
604840665

1 $f(x|\theta) = h(x)\, c(\theta) \exp\left(\sum_{i=1}^{k} w_i(\theta)\, t_i(x)\right)$

Express $x \sim \Gamma(\alpha, \beta)$

$$f(x) = \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)} =$$

$$= \frac{1}{x\,\beta^{\alpha}\Gamma(\alpha)} \left(x^{\alpha} \cdot e^{-\frac{x}{\beta}}\right)$$

$$= \frac{1}{x\,\beta^{\alpha}\Gamma(\alpha)} \cdot \exp\left(\ln x^{\alpha} \cdot -\frac{x}{\beta}\right)$$

$$= \frac{1}{x\,\beta^{\alpha}\Gamma(\alpha)} \cdot \exp\left(\alpha \ln x \cdot -\frac{x}{\beta}\right) = \frac{1}{x\beta^{\alpha}\Gamma(\alpha)} \exp\left(-\frac{\alpha}{\beta} \cdot x \ln x\right)$$

$$= \frac{1}{x} \cdot \frac{1}{\beta^{\alpha}\Gamma(\alpha)} \cdot \exp\left(-\frac{\alpha}{\beta} \cdot x \ln(x)\right)$$

Thus,

$h(x) = \frac{1}{x}$ $\quad w_i(\theta) = -\frac{\alpha}{\beta}$

$c(\theta) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)}$ $\quad t_i(x) = x \ln(x)$

2 $f(x) = \frac{2}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, $0 < x < \infty$

$f(x) = \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\beta^{\alpha} \Gamma(\alpha)}$

$\Gamma(\frac{1}{2}) = \sqrt{\pi}$

let $Y = x^2$

$F_Y(g) = P(Y \le g) = P(x^2 \le g) = P(x \le \sqrt{g})$

So, $F_Y(g) = F_X(\sqrt{g})$

$F_Y(g) = F_X(\sqrt{g}) \rightarrow f(g) = \frac{1}{2} g^{-1/2} f_X(\sqrt{g})$

$\rightarrow f(g) = \frac{1}{2\sqrt{g}} \cdot \frac{2}{\sqrt{2\pi}} e^{-\frac{g}{2}}$

$= g^{-1/2} \cdot \frac{e^{-\frac{g}{2}}}{\sqrt{2} \cdot \sqrt{\pi}} = \frac{g^{-1/2} e^{-\frac{g}{2}}}{2^{1/2} \cdot \Gamma(\frac{1}{2})}$

So, $\alpha = \frac{1}{2}$ $\beta = 2$

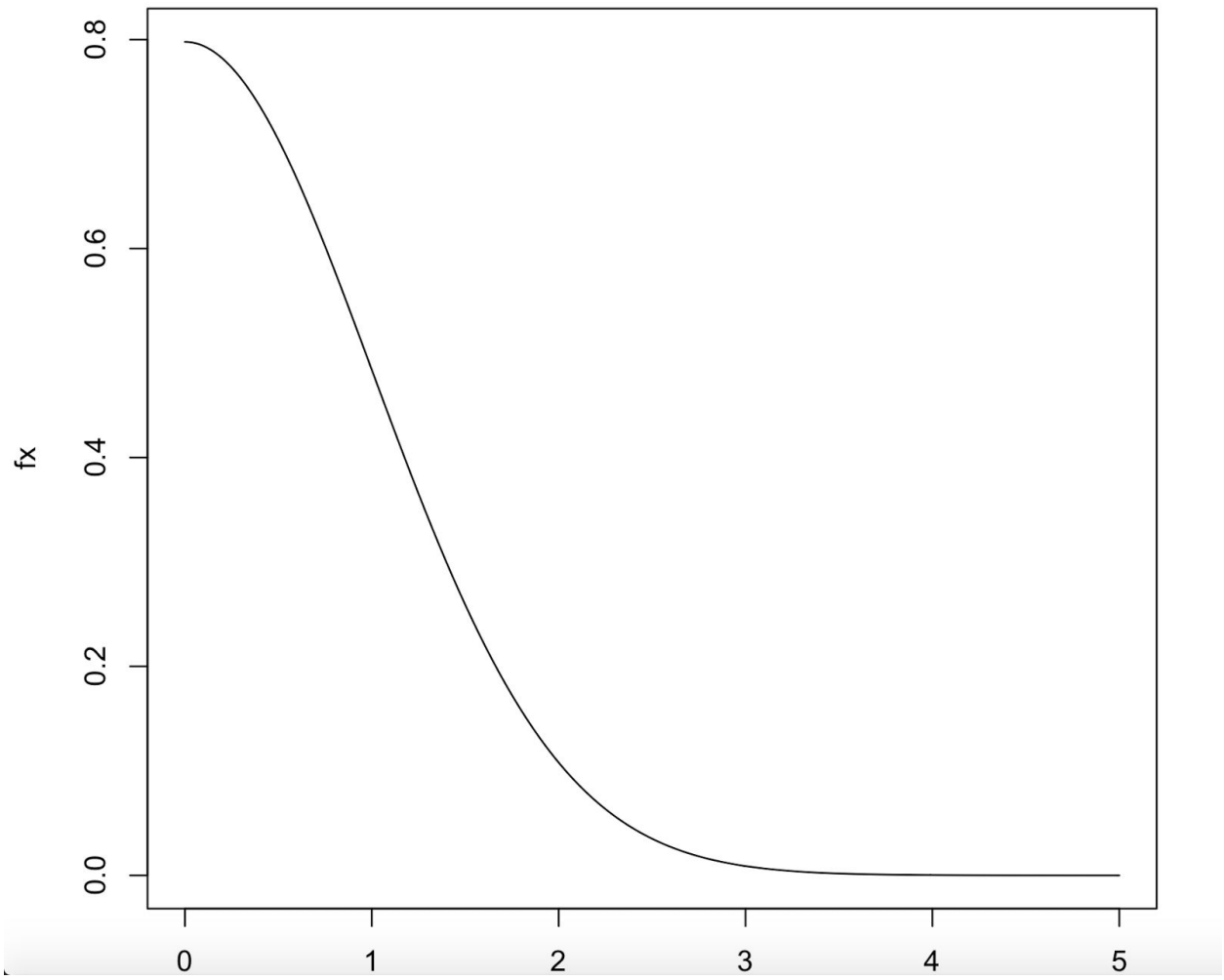
By the formula, Mean $= \alpha\beta = \frac{1}{2} \cdot 2 = 1$

Variance $= \alpha\beta^2 = \frac{1}{2} \cdot 4 = 2$

$E(x) = E(\sqrt{y}) = \frac{\Gamma(1/2 + 1/2)\, \beta^{1/2}}{\Gamma(1/2)} = \frac{\Gamma(1)\sqrt{2}}{\sqrt{\pi}} = \sqrt{\frac{2}{\pi}}$

$Var(x) = E(x^2) - [E(x)]^2 \rightsquigarrow E(x^2) = E(Y) = \alpha\beta = 1$

$= 1 - \left(\sqrt{\frac{2}{\pi}}\right)^2 = 1 - \frac{2}{\pi} = \frac{\pi - 2}{\pi}$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