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HW 3

1 Let $x \sim N(\mu, \sigma)$

a. Show $ax+b \sim N(a\mu + b, a\sigma)$ $\quad M_x(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

$$M_{ax+b}(t) = M_{ax}(t) \cdot M_b(t)$$
$$= \exp(ta\mu + \tfrac{1}{2}t^2 a^2\sigma^2) \cdot \exp(bt)$$
$$= \exp(ta\mu + \tfrac{1}{2}t^2 a^2 \sigma^2 + bt)$$
$$= \exp(t\underbrace{(a\mu + b)}_{\text{constant}} + \tfrac{1}{2}t^2 a^2 \sigma^2)$$

$ax+b \sim N(a\mu + b, a\sigma)$

b PDF Normal $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

let $ax+b = y$

$x = \frac{y-b}{a}$ $\quad F_y(g) = F_x\left(\frac{y-b}{a}\right) \rightarrow f(g) = \frac{1}{a} f_x\left(\frac{y-b}{a}\right)$

plug to $f(x)$

$$f(x) = \frac{1}{a\sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\left(\frac{y-b-a\mu}{\sigma}\right)\right)^2$$
$$= \frac{1}{a\sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\left(\frac{y-(a\mu+b)}{\sigma}\right)\right)^2$$

Thus, $ax+b \sim N(a\mu + b, a\sigma)$

2 Let $\ln(x) \sim N(\mu, \sigma)$. Find Ex and $var(x)$

$M_{\ln(x)}(t) = E e^{\ln(x)t} = E(x^t)$

Since $\ln(x) \sim N(\mu, \sigma)$ have same properties with $x \sim N(\mu, \sigma)$ then,

$M_x = e^{\mu t + \frac{t^2\sigma^2}{2}} = E(x^t)$

Thus, $E(x^t) = e^{\mu t + \frac{t^2 \sigma^2}{2}}$

$Var(\ln(x)) = E(x^2) - [E(x)]^2$

$E(x) = e^{\mu + \frac{\sigma^2}{2}} \rightarrow [E(x)]^2 = e^{2\mu + \sigma^2}$

$E(x^2) = e^{2\mu + 2\sigma^2}$

$Var(x) = e^{2\mu + 2\sigma^2} - e^{2\mu + \sigma^2}$

3 $M_{x_1+x_2+\dots+x_n} = [M_{x_i}(t)]^n$

Since moment-generating function gamma $(1-\beta t)^{-\alpha}$.

then it becomes $T=(1-\beta t)^{-\alpha n}$

$$\bar{x} = \frac{(1-\beta t)^{-\alpha n}}{n}.$$