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Time-resolved velocity and ion sound from simultaneous bow shock imaging and inductive probe measurements

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Abstract

- We present a technique to measure the time-resolved velocity and ion sound speed in magnetized, supersonic high-energy-density plasmas.
- Inductive probes measure the magnetic field advected by a plasma with frozen-in flux ($R_M \gg 1$). We estimate the bulk flow velocity from the transit time of a magnetized fluid parcel traced by the probes.
- The supersonic flow also generates a hydrodynamic bow shock around the probe. From the the shock Mach angle, we determine the upstream Mach number, and ion sound speed from the known upstream velocity.
- We demonstrate this diagnostic technique in a supersonic $M_A \sim 8$ super-Alfvénic $M_A \sim 2$ aluminum plasma generated during the ablation stage of an exploding wire array on the MAGPIE generator (1.4 MA, 250 ns).
- Velocity and ion sound speed measured using this technique agree well with optical Thompson scattering measurements reported in literature, as well as with 3D resistive MHD simulations in GORGON.

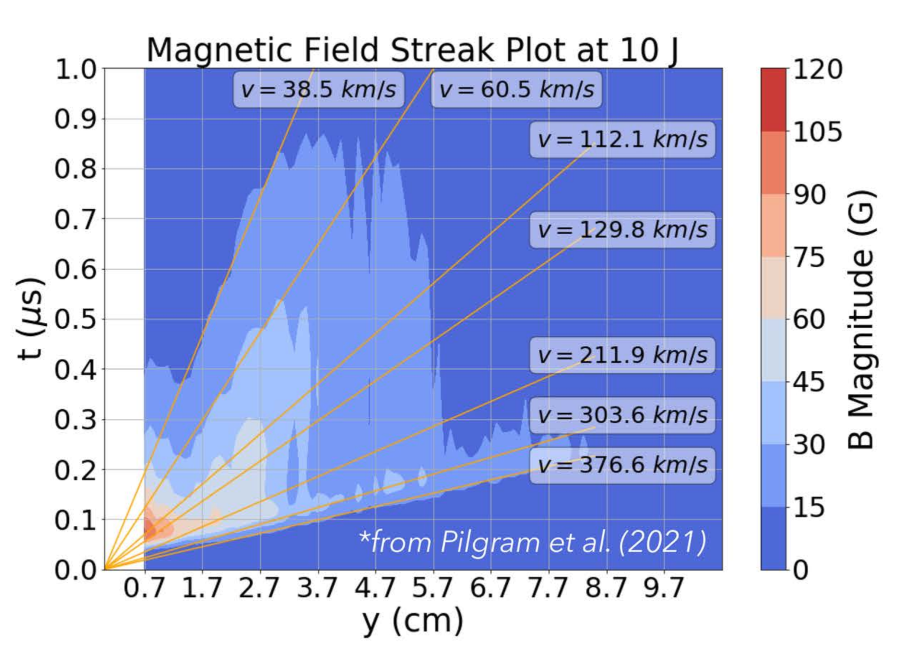
Velocity from Frozen-In Magnetic Field

Plasma flows advect magnetic fields when resistive diffusion rate is small ($R_M \gg 1$):

$$R_M = \frac{UL}{\bar{\eta}} \gg 1$$ (Magnetic Reynolds number)

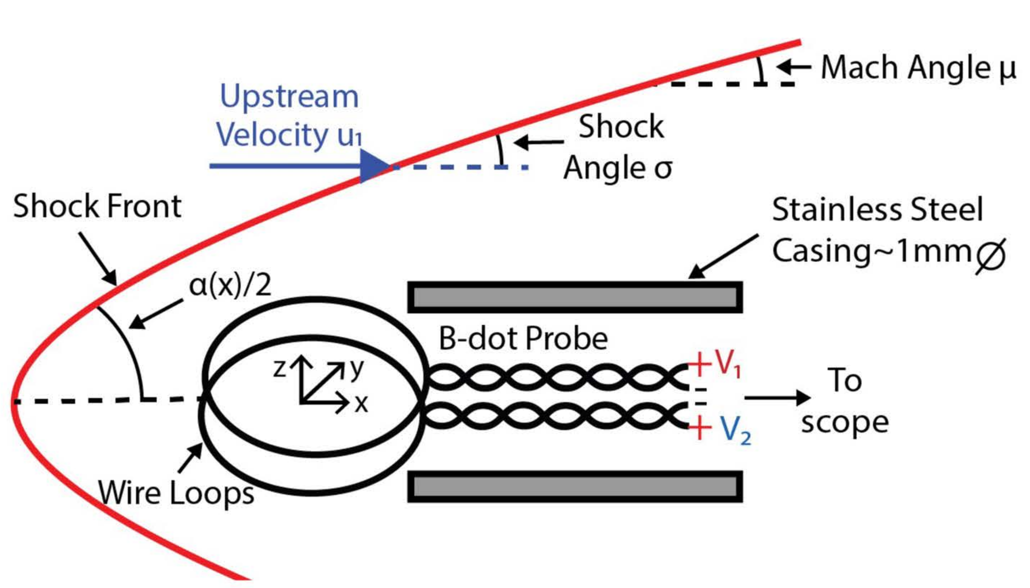
Frozen-in magnetic fields can serve as tracers of high-R_M plasma flow.

Inductive probe measurements of frozen-in fields have previously been used to measure flow velocity in laser-driven [1,2] and low-beta subsonic plasmas [3,4].



Bow Shocks in Supersonic Flows

Supersonic flows generate bow shocks around b-dot probes.



Mach Number from Shock Shape

Shocks are almost **hydrodynamic** when the probe size is smaller than or comparable to the resistive diffusion length l_η [4].

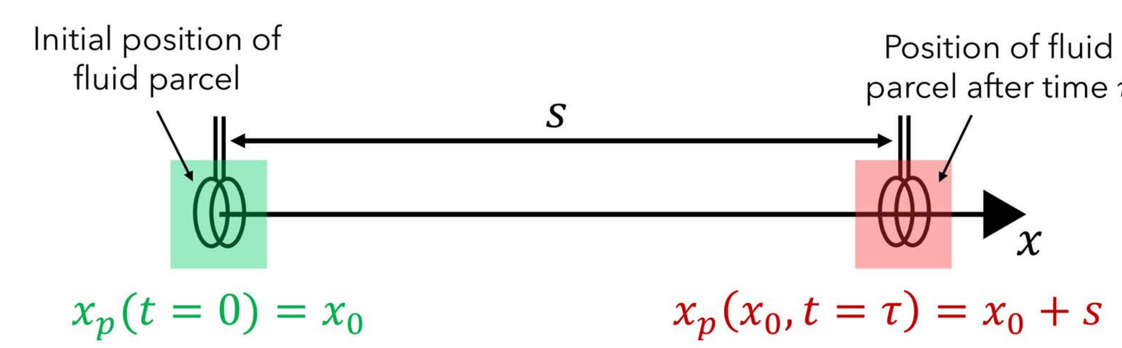
$$l_\eta = \frac{\bar{\eta}}{U}$$ (Resistive Diffusion Length)

For hydrodynamic bow shocks, upstream sonic Mach number M_S is:

$$\boxed{\sin\mu = 1/M_S}$$

Estimation of Velocity

For a magnetized fluid parcel in a 1D velocity field $u(x,t)$:



$x_p(t=0) = x_0$ $\quad$ $x_p(x_0, t=\tau) = x_0 + s$

$$\boxed{\bar{u} = \frac{s}{\tau} = \frac{1}{\tau}\int_0^\tau \dot{x}_p(x_0,t)dt}$$ (Average Lagrangian Velocity of Fluid parcel)

Where

$$\dot{x}_p(x_0,t) = u(x,t)_{x=x_p(t)}$$ (Fluid Parcel Velocity)

Assuming a simple Eulerian velocity field of the form:

$$u(x,t) = (mx + U_0)e^{\nu t}$$

Ratio of average Lagrangian velocity to Eulerian velocity at probe location $x = x_0 + s$ is:

$$\boxed{R \equiv \frac{\bar{u}}{u(x = x_0 + s, \tau)} = \frac{e^{-\nu\tau}}{m\tau}\left[1 - e^{\frac{m\tau}{\nu\tau}(1-e^{\nu\tau})}\right]}$$

The value of R is the extent to which this technique under- or over-estimates the Eulerian velocity at the probe location.

For this simple model, R depends only on dimensionless quantities $m\tau$ and $\nu\tau$

$m\tau, \nu\tau > 0 \Rightarrow R < 1$ (spatially- and temporally increasing velocity)

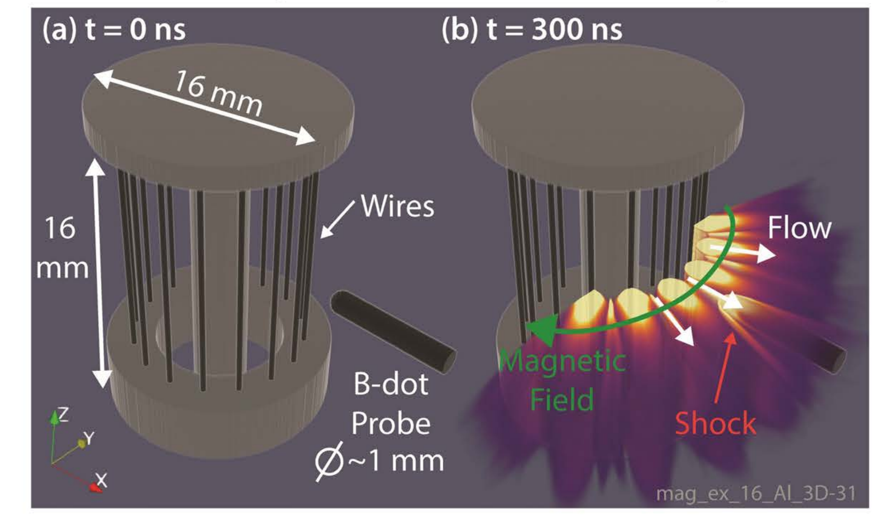
$m\tau, \nu\tau < 0 \Rightarrow R > 1$ (spatially- and temporally decreasing velocity)

$m\tau, \nu\tau \to 0 \Rightarrow R \to 1$ (spatially- and temporally constant velocity or $\tau \to 0$)

Estimating R requires some prior knowledge of the variation of the velocity field, which may be informed from simulations, previous experiments, or from analytical models.

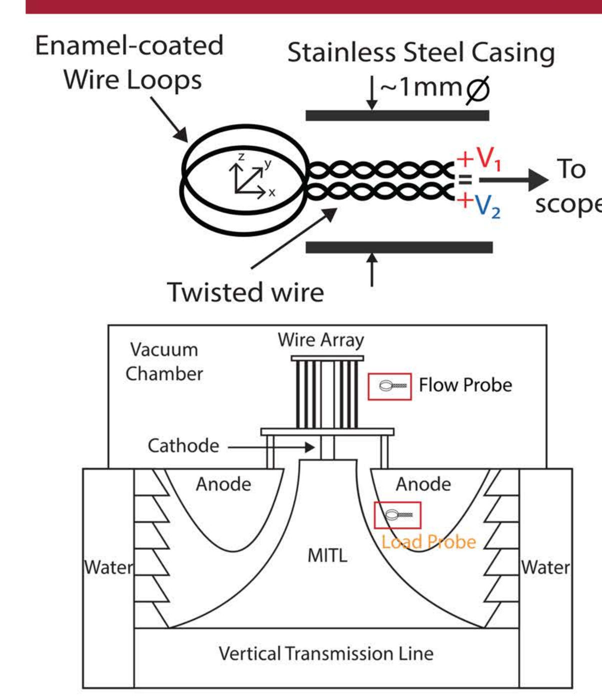
Demonstration on MAGPIE

Exploding wire array with 16, 30 µm Aluminum wires driven by 1.4 MA, 250 ns current pulse



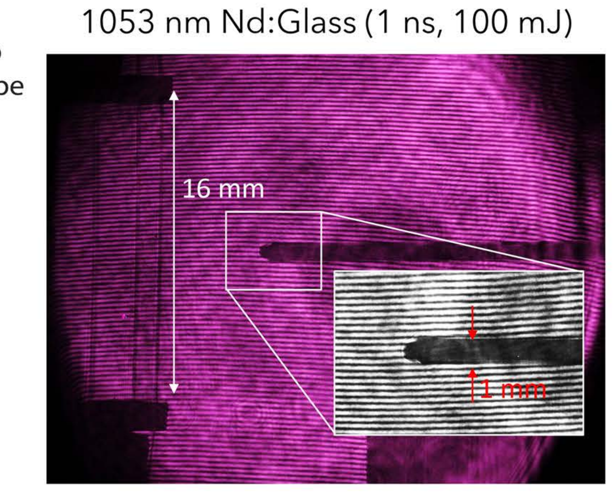
$M_S \sim 8$, $M_A \sim 2$, $R_M \sim 20$, $l_\eta \sim 0.3 - 0.6$ mm,
$U \sim 60\ km s^{-1}$, $T_e \sim T_i \sim 10 - 15\ eV$, $n_e \sim 10^{18} cm^{-3}$

Diagnostics



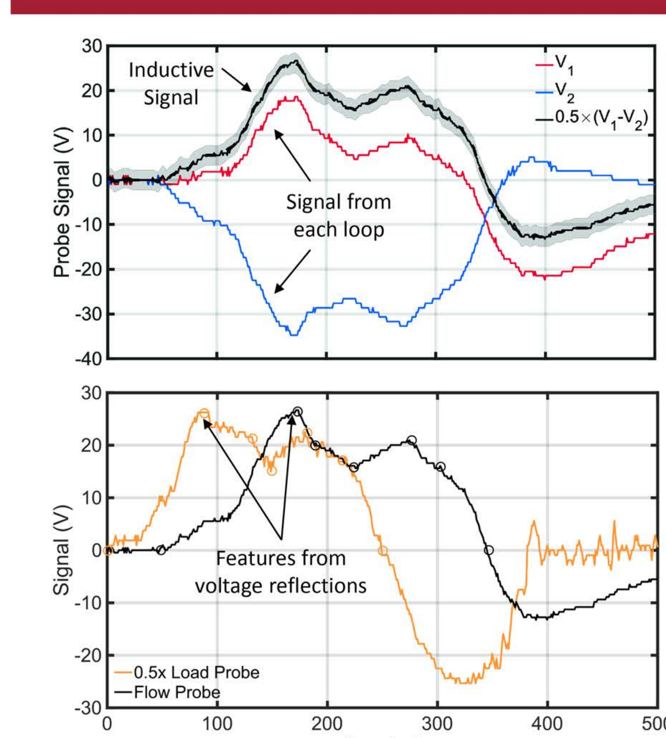
Dual-loop b-dot probe used in experiment (top). Placement of flow and load probes (bottom)

Side-on Imaging (x-z plane):
Mach-Zehnder Interferometer
1053 nm Nd:Glass (1 ns, 100 mJ)



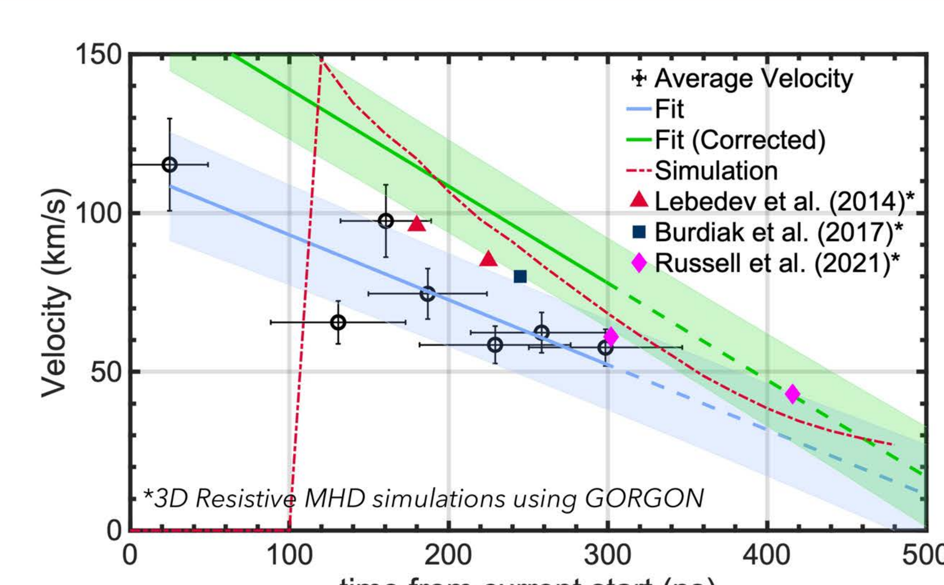
Background interferogram taken before current start showing flow probe

Inductive Probe Measurements



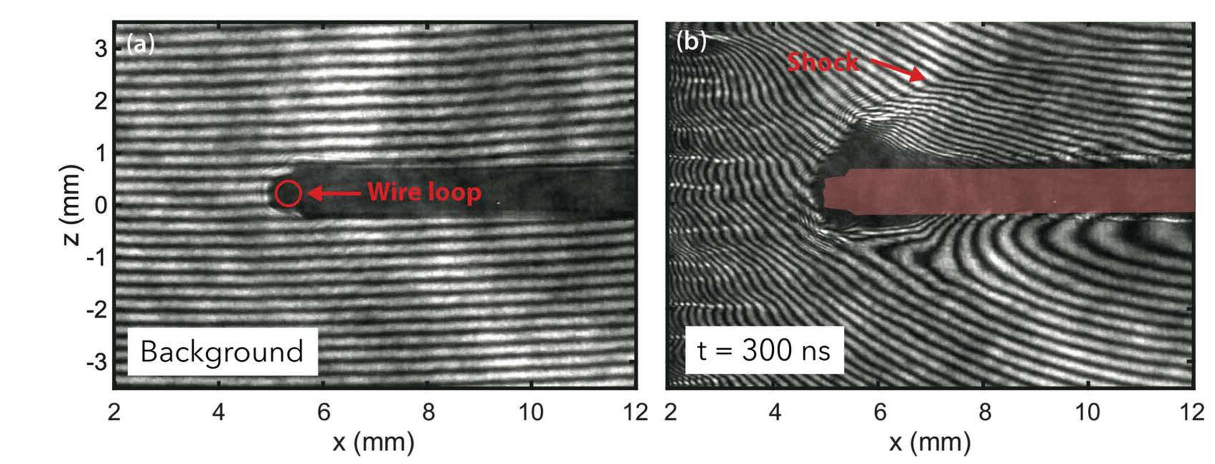
- Single-loop B-dot probe in feed gap monitors current delivered to load
- B-dot probe with two oppositely-wound loops measure magnetic field in the plasma flow (at 5.55 ± 0.25 mm from wires)
- Signals from load and flow probes have similar shape, but are displaced in time, confirming frozen-in flux

Time-Resolved Velocity

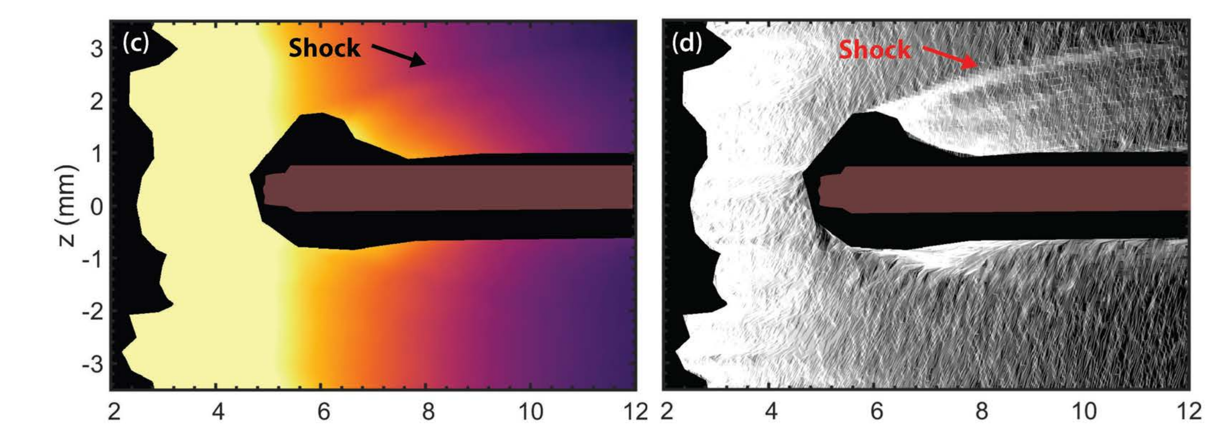


From simulation, $\nu \sim -4e6\ s^{-1}$, $m \sim 1.2e7 s^{-1}$ and $\tau \sim 78$ ns $\longrightarrow$ $R \sim 0.67$ (Consistent with simulation and OTS measurements)

Shock Imaging and Mach Number



(a) Side-on background interferogram (b) Side-on raw interferogram at 300 ns after current start. The red shaded region represents the silhouette of the probe from the background interferogram.



(c) Side-on line-integrated electron density map. (d) Enhanced image of the shock generated by taking the logarithm of the gradient of the line-integrated electron density.

From shock geometry:

Mach Angle $\mu = 7° \pm 0.5°$
Upstream Mach Number $M_S = 8.2 \pm 0.6$

Summary & Comparison

	Experiment	Experiment (corrected)	Simulation	Optical Thompson Scattering (Literature)**
Bulk velocity at probe location at 300 ns (km/s)	52 ± 14	78 ± 14*	68	61 ± 1
Mach Angle (degrees)	$7° \pm 0.5°$	$7° \pm 0.5°$	7°	n/a
Upstream Sonic Mach Number	8.2 ± 0.6	8.2 ± 0.6	$7 - 11$	n/a
Ion Sound speed (km/s)	6 ± 2	9.5 ± 2	$6 - 10$	n/a
ZT_e (eV)	10 ± 6	22 ± 9	$9 - 25$	$28 - 40$

*Experimental velocity corrected using R = 0.67
** OTS measurements from pulsed-power-driven Aluminum plasma taken at ~5mm from wires[4,5,6]

Conclusions and Future Work

- Time-resolved velocity and ion sound speed estimates from simultaneous bow shock imaging and voltage measurement of inductive probes agree well will simulation and OTS.
- We plan to utilize this technique for the MARZ experimental campaign [7] on the ~30 MA Z machine (Sandia National Labs), using an array of inductive probes at different radial locations.

References & Acknowledgements

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