Changing Perspectives: A Simple Method for Improving Numerical Estimation and Reducing Overconfidence

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Abstract

The purpose of this study is to examine the impact of putting numbers into perspective on people's ability to estimate measurements. Past research has suggested that context surrounding numbers in news articles makes them easier to understand, recall in the future, and estimate. Through the use of *perspectives*, or additional sentences that express the value of a similarly scaled item, comprehension of large measurements or unknown quantitative units can be improved. It is hypothesized that the presence of perspectives will result in more accurate estimations and reduced overconfidence. A survey was created for three different conditions where respondents were tasked to 1) estimate raw numbers, 2) provide a confidence in which the correct estimate falls, or 3) estimate a value in terms of the perspective. These three surveys were evenly distributed to 614 participants who were asked to estimate measurements. They were then randomly assigned to one of two versions: Perspectives or Non-Perspectives. Through T-tests, the accuracy of responses in each version were compared to find that perspectives were helpful in improving the accuracy of estimations only for certain questions. Perspectives were most helpful for the largest and most uncommon measurements, while there was typically no significance of perspectives for estimations that people commonly face. Additionally, perspectives were correlated to the size and accuracy of the confidence interval and reduced overconfidence. The applications of this study range from increasing the accuracy of project planning times and retirement budgeting to improving AI in chatbots and automated journalism.

Introduction

Background

From predicting the number of minutes it will take to get to work to guessing the cost of a grocery bill, humans estimate numbers on a daily basis. However, when it comes to larger quantities and infrequently used units, it is much more difficult to estimate accurately. Large numbers, such as in the billions or trillions, are more abstract because we do not have much experience with them in personal contexts. In a study conducted by Brain Butterworth, author of The Mathematical Brain, which involved distributing a math test to assess brain damage on hospital patients, it was found that Italian patients were superior to other ethnicities in comparing and understanding big numbers because of their experience with large quantities through the use of their former currency. While an American would pay for a car in thousands of dollars, an Italian would pay millions or even billions of lire.² This study supports the idea that it is more difficult to work with larger numbers when there is a lack of familiarity. This disconnect is exemplified by former Vice President of the United States Joe Biden, who failed to grasp larger quantities during an interview. He mistakenly stated that an opponent spent over \$200 billion in political advertising. Biden meant to say \$200 million, but he did not catch his mistake, and even referred to that incorrect figure a second time during the same interview.³ While this slip up occurred on an important public stage, his confusion between million and billion is commonplace. Additionally, the general population lacks in comprehension of larger numbers when faced with unfamiliar measurements in the news. The New York Times recognized this deficiency when declaring a new policy to "put large numbers in context". 4 To revert back to the political ad example, the New York Times would argue that if Biden provided context to the large sum of money he was dropping, he would have made a more memorable and impactful argument, and would have avoided the mistake, preventing confusion and reputation damage. They might suggest he say "To put \$200 million into perspective, this is equal to the production budget for the movie Titanic". Providing a different perspective to numbers has been found to improve people's ability to understand those numbers. Previously conducted research, such as

¹ Factors in Computing Systems - CHI 16, 2016.

² Butterworth, Brian. *The Mathematical Brain*. London: Papermac, 2000.

³ Shear, Michael. "\$200 Billion in Ads, Mr. Biden? That's Real Money." The New York Times. The New York Times, October 22, 2010.

⁴ Factors in Computing Systems - CHI 16, 2016.

⁵ "Movie Budgets." The Numbers - Where Data and Movies Meet. Accessed October 2, 2019.

that of Pablo Barrio, Dan Goldstein, and Jake Hoffman on improving numerical comprehension in the news, will be explored.

What has been only lightly touched upon, however, goes beyond comprehension and into estimation. Most people estimate smaller numbers on a daily basis, but this is not the case for larger numbers. We are likely to have difficulty estimating larger measurements if we exhibit an inability to understand them. While one may have lived in the State of Vermont for their entire life, when tasked with estimating the square mileage of their home state, they'd most likely become flustered and make what feels like an embarrassingly wild guess. The estimation of larger numbers is a task that very few people feel comfortable with and can perform accurately. For example, a study about numerical estimation performed on undergraduates at The University of Oklahoma found that about 50% of the participants were unable to accurately estimate numbers on a 1,000—to-1 billion number line. The difficulty exhibited during numerical estimation has been recognized and addressed by researchers in the past.

Perspectives

The term *perspective* is defined as an additional sentence tacked onto a measurement to give it context and make it easier to understand. A perspective can be expressed in the form of a ratio, rank, or unit change. A few examples of perspectives are: "to put the area of France into perspective, it is about the size of Texas" and "fifteen centimeters is about equal to the length of half a foot." The effect of perspectives on numerical comprehension has been studied by researchers Pablo Barrio, Daniel Goldstein, and Jake Hoffman from Columbia University and Microsoft Research. They used Amazon's Mechanical Turk to crowdsource a plethora of perspectives for numbers in quotes from news articles, and sifted out the most helpful perspectives from that large pool. They concluded that perspectives improved readers' ability to recall measurements, estimate measurements, and detect errors in given statistics from news article quotes. Additionally, researchers from Stanford University took this one step further and developed a formula to automatically generate perspectives. Their algorithm was created with

⁶ Thompson, Clarissa A., and John E. Opfer. "How 15 Hundred Is Like 15 Cherries: Effect of Progressive Alignment on Representational Changes in Numerical Cognition." Society for Research in Child Development. John Wiley & Sons, Ltd (10.1111), November 15, 2010.

⁷ Factors in Computing Systems - CHI 16, 2016.

⁸ Factors in Computing Systems - CHI 16, 2016.

the goal of helping people understand numbers. Although the research world has recognized the importance of perspectives, there is still much to be done involving their applications.

Research has begun to investigate how perspectives can be practically applied to improve everyday activities. At the University of Washington, they personalized news articles according to the readers' location. Using specifically distance and area measurements, the researchers developed an algorithm which generated spatial analogies with landmarks familiar to the respondent. Users who received the spatial analogy aid rated the news articles more helpful, suggesting that perspectives aid spatial awareness. 10 Similarly, Fanny Chevalier, Romain Vuillemot, and Guia Gali developed a framework for visually re-expressing large or unfamiliar measurements through graphic compositions. Specifically directed towards graphic designers, they suggest design strategies to improve the effectiveness of accurately portraying a complex measurement and its effect on the accuracy of their models. 11 Finally, Microsoft developed an AI tool called the Perspective Engine, which automatically contextualizes large numbers. It has been implemented in their search engine, Bing, as well as on their presentation software, PowerPoint. As a result of the Perspective Engine, both applications generate perspective sentences when a user enters or inquires for a measurement.¹² Various measures have been taken to investigate and apply perspectives, demonstrating the urgency of improving numerical comprehension and further experimentation.

Estimation and Overconfidence

Perspectives caused a significant improvement in numerical estimation. This could be extremely helpful in reducing overconfidence, which has caused major disasters in history such as the sinking of the Titanic and the nuclear accident at Chernobyl. Research conducted by Daniel Kahneman, author of *Thinking Fast and Slow*, reveals that when people express an estimation in terms of a 90% confidence interval, the right answer only falls in that interval 50%

⁹ Chaganty, Arun, and Percy Liang. "How Much Is 131 Million Dollars? Putting Numbers in Perspective with Compositional Descriptions." *Proceedings of the 54th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, 2016.

¹⁰ Kim, Yea-Seul, Jessica Hullman, and Maneesh Agrawala. "Generating Personalized Spatial Analogies for Distances and Areas." *Proceedings of the 2016 CHI Conference on Human*

¹¹ Chevalier, Fanny, Romain Vuillemot, and Guia Gali. "Using Concrete Scales: A Practical Framework for Effective Visual Depiction of Complex Measures." *IEEE Transactions on Visualization and Computer Graphics* 19, no. 12 (2013): 2426–35.

¹² "Perspectives Engine." Microsoft Research, June 19, 2019.

¹³ More, Don A. "Overconfidence." Psychology Today. Sussex Publishers, January 22, 2018.

of the time.¹⁴ The researchers Don Moore and Paul Healy found that, when faced with a difficult task, participants were overconfident in the accuracy of their performance. They also concluded that within the subcategories of overconfidence, the presence of over precision, or the excessive belief that one is correct, reduced overestimation of actual performance.¹⁵ Research on overconfidence can be applied to estimating numbers. By combining the findings on the benefits of perspectives with the issue of overconfidence, perspectives could possibly reduce both overconfidence and improve accuracy in numerical estimations.

Research Questions and Hypotheses

Based on the literature, this study aims to investigate how perspectives impact the accuracy of people's numerical estimations and level of overconfidence. It touches upon two main research questions: (1) Can people more accurately estimate measurements when they are provided with a perspective? (2) When expressing their answers in terms of 90% confidence intervals, are people better calibrated when perspectives are present? This study hypothesizes that: (1) perspectives will have a greater effect on estimates in the millions, billions and trillions. (2) The presence of perspectives will make the accuracy estimations fall closer to the 90% confidence interval than the 50% confidence interval.

Methodology

Survey

The survey questions were developed through the process of a pilot experiment, which acted as a smaller scale version of the study for the purpose of understanding what types of questions generate interesting results. There were 30 participants, and the data was not statistically analyzed or recorded. However, it provided a loose demonstration of the difference in accuracy based on the presence of perspectives and allowed for the weeding out or tweaking of defective questions. Data for the "real version" of the study was collected through the distribution of an original survey on Amazon's Mechanical Turk to 614 US residents over age 18. Mechanical Turk is a crowdsourcing website which can be used to distribute surveys and pay workers to complete them. In the case of this study, they were paid \$0.25 upon completion. After answering a few questions verifying their understanding that searching the internet for the

¹⁴ Kahneman, Daniel. *Thinking, Fast and Slow*. New York: Farrar, Straus and Giroux, 2015.

¹⁵ Moore, A., Don, Healy, and Paul J. "The Trouble with Overconfidence." figshare. Carnegie Mellon University, June 30, 2018.

correct answers was prohibited, the participants were randomly assigned to one of three survey conditions.

Condition A: Raw Number Estimates

40% of the participants were randomly assigned to Condition A. In this condition, they were given a survey which asked them to estimate quantities in terms of a raw number, such as the diameter of Jupiter in miles or the amount of water in the Long Island Sound in gallons. This condition contained A/B testing, meaning that within the condition, people were randomly assigned to a version of the survey which included perspectives in the questions, or a version of the survey without perspectives (*Figure 1*).

Condition B: Confidence Interval Point Estimates

40% of the participants of this study were randomly assigned to Condition B. This condition included the same set of 7 questions in Condition A, but people answered in terms of a 90% confidence interval (I am 90% confident that New York is between X and Y times the size of Vermont) instead of point estimates. It also had A/B testing; within this condition, people were randomly assigned to answer questions with a perspective present or absent (*Figure 1*).

Figure 1: Survey Questions for Conditions A & B

	Question	Perspective
1	Estimate the volume of water in gallons in the <u>Long</u> <u>Island Sound</u> .	For reference, the Chesapeake Bay contains 18 trillion gallons of water.
2	Estimate the size of New York State in square miles.	For reference, the State of Vermont is 10,000 square miles.
3	Estimate <u>Jupiter</u> 's diameter in miles.	For reference, the diameter of Earth is about 8,000 miles.
4	Estimate how many gallons of <u>paint</u> are necessary to completely cover the surface area of the Empire State building.	For reference, it takes about a dozen gallons of paint to cover the exterior of a house.
5	Estimate the maximum outside temperature reached on September 27, 2018 in <u>Paris</u> , France. Please answer in fahrenheit degrees.	For reference, the weather in New York City on that date was 72 degrees.

6	Pretend you work for a hedge fund which is considering purchasing the company <u>Snapchat</u> . Snapchat has \$1.3 billion of revenue. What should your first offer be to buy Snapchat?	For reference, Twitter is worth \$25 billion, which is 8 times its revenue.
7	Suppose you make \$65,000 a year. How much should you budget for your monthly <u>car</u> lease payment?	For reference, people often spend less than 50% of their after tax income on costs that stay the same every month, including rent (or mortgage payments), cable bill, car payments, etc.

Condition C: Confidence Interval Ratio Estimates

Condition C did not involve A/B testing, so only 20% of the participants were assigned to this Condition. The participants were instructed to answer estimation questions in terms of perspective, as well as in 90% confidence intervals. (*Figure 2*).

Figure 2: Survey Questions for Conditions C

	Question
1	I am 90% confident that the Long Island Sound is between and times the size of Chesapeake Bay
2	I am 90% confident that New York is between and times the size of Vermont.
3	I am 90% confident that Jupiter's diameter is between and times the size of Earth's diameter.
4	I am 90% confident that it would take between and times more paint to cover the Empire State Building than to cover a typical house.

Statistical Analysis

The survey data was uploaded and analyzed on the programming language R. First, it was divided into data frames pertaining to each version (perspectives or non-perspectives) of each condition (A, B, or C). For conditions B and C which required the answer in the form of a confidence interval, the midpoint for each response was found. The margin of error was calculated for each data frame using the formula Order of Magnitude Error (OME) =

[abs(log10(estimate/true)], comparing the respondent's estimate to the actual value and quantifying the accuracy of each response.

Through the *gplot* function in R, graphical analyses of the distribution of responses were performed for each condition. The data included values in the millions, billions and trillions, so the log of each datapoint was graphed instead of the raw numbers for convenience.

The first measure of statistical analysis compared the presence of perspectives to accuracy of the response. A t-test was performed to compare the accuracy, expressed as OME, of responses influenced by perspectives compared to the absence of perspectives for Conditions A and B. In addition to calculating the correlation between perspectives and accuracy for each condition as a whole, t-tests were performed for each type of question to see which types of estimations benefited the most from the aid of perspectives.

To address the research question regarding which condition generated the most accurate responses with the help of perspectives, the mean accuracy was calculated for Perspectives Condition A, Perspectives Condition B, and Condition C. However, before the means were compared, Condition C was normalized to match the other conditions. Since Survey C asked for answers in terms of the perspective, the midpoints of the intervals were transformed back to the original units to allow for comparison across conditions. For example, if a participant said they are 90% confident that New York is 2 to 6 times the size of Vermont, and Vermont is 9,623 square miles, the midpoint of their response, 4, would be multiplied by 9,623. Once Condition C shared the same units as A and B, the mean accuracies of responses across all perspective versions were compared to see which condition, A, B, or C, generated the most accurate responses with perspectives.

Additionally, it was hypothesized that participants would be truer to their confidence interval when given a perspective. Since Condition B required answers formatted as 90% confidence intervals and involved a perspectives and non-perspectives version, a proportion test was used to calculate how often the interval contained the correct answer. Thus, the use of both the t-test and proportion test on Condition B allowed us to see if people are both more accurate and more appropriately confident in their answers when thinking with perspectives than without them. Finally, the completion rates were examined to see which Condition prompted the most skipped questions. Completion rates between conditions were compared through proportion tests.

Results

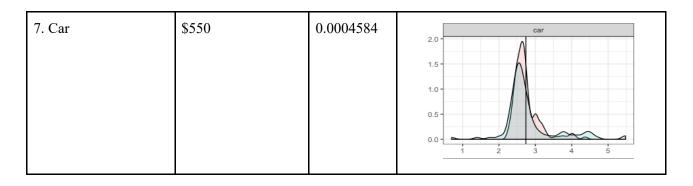
T-test

In Condition A, there was a significant correlation between the variables *perspectives* and *accuracy* (p < .001), supporting the hypothesis that perspectives have an effect on the accuracy of people's estimations. In addition to measuring the accuracy across all responses in Condition A, t-tests were performed for each survey question (*Figure 3*). Different questions were more impacted by perspectives than others. For example, question #1 which prompted the participant to estimate the volume of the Long Island Sound had a greater correlation between perspectives and accuracy (p < .001), compared to the question about estimating the diameter of Jupiter (p = .001). Some questions did not exhibit a significant correlation between perspectives and accuracy, such as the estimation of paint gallons necessary to cover the surface area of the Empire State Building (p >.05). The graphical distributions of responses in *Figure 3* demonstrates this difference, as certain questions have more overlap between the "Perspectives A" curve (blue) and the "Non-perspectives A" (red) curve.

Figure 3: The Significance of Perspectives in Condition A

Question	Correct answer	P-value	Graphical Distribution Perspectives vs. Non-Perspectives Non-perspectives A Perspectives A X-axis = log10(response) Y-axis = density
1. Long Island Sound	18 Trillion Gallons	< 2.2e-16	0.9 - longisland

2. New York	55,000 square miles	2.171e-15	0.75 - 0.50 - 0.25 - 0.00 - 2 4 6 8
3. Jupiter	87,000 miles	.0007833	0.6 jupiter 0.4 0.2 0.0 10 20
4. Paint	6,750 gallons	.9656	0.3 - 0.1 - 0.0 - 0.0 2.5 5.0 7.5 10.0 12.5
5. Paris	77°	0.3488	paris 0.5 1.0 1.5 2.0
6. Snapchat	\$10 billion	0.035	0.4 - 0.3 - 0.2 - 0.1 - 0.0 - 8 - 12 - 16



Condition B, which dealt with the midpoint of an interval as opposed to raw numbers in Condition A, also had an overall significant correlation between perspectives and accuracy (p < .001). In terms of t-tests for each isolated survey question, the results are very similar to those of Condition A. The specific p-values corresponding to each question are listed, alongside the graphical distribution of responses for that question (*Figure 3*).

Figure 3: The Significance of Perspectives in Condition B

Question	Correct answer	P-value	Graphical Distribution Perspectives vs. Non-Perspectives condition Non-Perspectives B Perspectives B X-axis = log10(response) Y-axis = density
1. Long Island Sound	18 Trillion Gallons	< 2.2e-16	1.5 - 1.0 - 0.5 - 0.0 - 4 8 12 16 20
2. New York	55,000 square miles	3.303e-07	0.8 - 0.6 - 0.4 - 0.2 - 0.0 - 2.5 - 5.0 - 7.5

3. Jupiter	87,000 miles	0.006011	0.4 0.3 0.2 0.1 0.0
4. Paint	6,750 gallons	0.4683	0.3 - paint_mp 0.2 - 0.1 - 0.0 - 3 - 6 - 9
5. Paris	77°	0.0004511	7.5
6. Snapchat	\$10 billion	0.9232	0.4 0.2 0.0 4 6 8 10 12 14
7. Car	\$550	0.03607	1.5 - car_mp 1.0 - 0.5 - 0.0 - 1 2 3 4 5

T-tests were not performed for Condition C because there was only one version of the survey involving answering in terms of the perspective; while Conditions A and B randomly provided the participant with a survey with perspectives either present or absent. However, the OME of the responses were still plotted to visualize how close the estimations were to the correct answer (*Figure 6*). Additionally, the median response for each question was calculated (*Figure 7*).

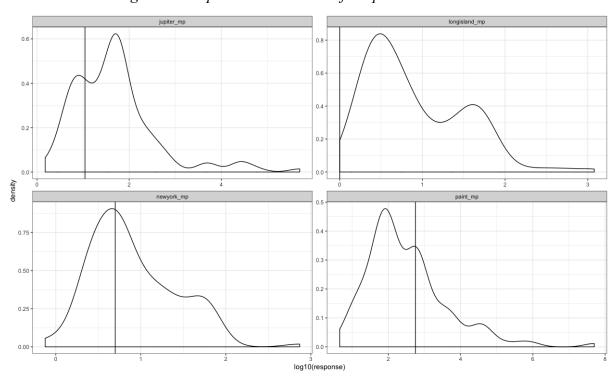


Figure 6: Graphical Distribution of Responses Condition C

Figure 7: Median Response per Question in Condition C

Question	Correct answer	Median Response
1. Long Island Sound	1	5.5
2. New York	5	6
3. Jupiter	11	45
4. Paint	563	150

Mean Accuracy

To address the research question inquiring which Condition generated the most accurate responses, the accuracies of the "Perspectives" versions of Condition A and B, and the overall accuracy of Condition C were compared. According to Order of Magnitude Error, the closer to zero the value is, the more accurate it is. Respectively, the means for each were 0.677, 0.738, and 0.749. Therefore, the Perspectives in Condition A had the most accurate responses.

Proportion Tests

In order to know which condition had the greatest completion rate, proportion tests were performed. The proportion consisted of blank answers, or "0"'s, out of the total number of responses. For Conditions A, B, and C were 0.48%, 0.98%, and 0.78% respectively. However, there was not a significant difference between the proportions (p > .05), making the difference in completion rates obsolete.

A proportion test was also used in Condition B to deal with the confidence intervals and to address the research question: "When expressing their answers 90% confidence intervals, are people better calibrated (in terms of percentage of intervals bracketing the correct answer) when perspectives are present?" For ideal calibration, 90% confidence intervals would bracket the correct answer. However, the results were much lower than 90% correct. For Non-Perspectives Condition B, the correct answer fell into the respondents' intervals 18.8% of the time. Perspectives Condition B demonstrated a 6.1% improvement, as the intervals contained the correct answer 24.9% of the time. While the respondents were far from 90% accurate in their responses, there was a significant improvement in accuracy (p <.05).

Discussion

The results shed light on the types of estimations that people struggle with and demonstrate the positive effects of putting numbers into perspective. Perspectives were most helpful for the largest and most uncommon measurements, while there was typically no significance of perspectives for estimations that people commonly face. For example, estimations of the sizes New York and the Long Island Sound both benefited greatly from perspectives, while the estimation of the temperature of Paris on a given date did not benefit from perspectives in both Conditions A and B. Perspectives weren't as helpful for the paint

gallon question, most likely because coming up with the answer involved complicated mental math even with the perspective present. If too much math is involved, then the purpose of using perspectives is defeated, as mental calculations are just as intimidating as estimating unfamiliar values. Overconfidence was reduced by the use of perspectives, but the proportion of correct intervals was still far less than 90%. Finally, although Condition C was expected to generate the most accurate results, Condition A ended up having the mean accuracy closest to zero. This could be because the raw numbers have a better chance of being closer to the correct answer than the midpoint of an interval, as the participant's answer did not necessarily have to be directly in the middle of their interval.

This study could be made specific to certain environments, such as software planning, consulting, and urban planning. For a software development company, this study could be used to determine the time it takes to build a project for a customer. For the mayor of New York City, perspectives could be used when announcing the estimated completion time for construction on the subways. The estimated completion time for public works projects is often underestimated because it is calculated by adding up the time it will take to complete each step of the process.¹⁶ Instead, a perspective could be used by providing the completion time of a former project of similar scale, enabling the estimation to be based on facts rather than predictions. Additionally, one could estimate the necessary funds to allocate to their retirement by putting their spending in context with more familiar monetary values. One could estimate their necessary retirement savings by considering it to be fifteen times their average annual expenses, or four times their average salary. Perspectives could also be used in situations that are infrequently occurring and where there are emotional factors at play. People have trouble evaluating end-of-life care options, but if the numbers involved were put into perspective, this process could be less daunting. For example, if a patient has terminal pancreatic cancer, he might be presented with data that says, "chemo treatment A costs \$450,000 and has a 40% chance of extending life by an additional five months, and 60% chance of no extension." In order to make this decision more tangible, a perspective could be added: "would you trade the savings you made working for 10 years for a 40% chance of an additional five months of life?" In addition to estimations in planning and budgeting, this study could be applied to the world of Artificial Intelligence. With

¹⁶ Dubner, Stephen J., and Alvin Melathe. "Here's Why All Your Projects Are Always Late - and What to Do About It (Ep. 323)." Freakonomics. Alvin Melathe, March 7, 2018.

the verification that perspectives cause a significant improvement in estimations and overall understanding of numbers, perspectives could be applied to chatbots or automated journalism to make computers even more understandable and human-like.

Although a pilot experiment was conducted, there were still some undetected issues with the consistency and phrasing of the survey questions. One of the problems with conditions A and B was the lack of clarity in the required scale of responses. For example, if a participant decided that their estimate of the volume of the Long Island Sound was 12 trillion gallons, they might have just entered "12", while others would enter "12,000,000,000." To account for this in the statistical analysis, responses for the Long Island Sound question and the value of Snapchat question were multiplied by one trillion or one billion if they were less than 1,000. Also, despite the anonymity and question at the beginning of the survey disallowing the respondent to look up the answers on the internet, they could have still felt compelled to look up answers because of the level of difficulty of answers.

In the future, this study should be conducted in a controlled environment where internet access is not an option, such as tacked onto the end of a standardized test. Additionally, conducting this experiment on younger age groups would be helpful in understanding how experience with numbers impacts numerical estimation. It is important to examine other characteristics that contribute to one's ability to estimate large measurements, such as how often one reads the newspaper. Future research can also define "larger" and "smaller" numbers in terms of numerical comprehension by testing exactly where on the number line people start to lose touch with numbers. Specifying the range that constitutes smaller versus larger numbers is a natural next step in this field of research. Finally, while this study required estimations in terms of a 90% confidence interval, an additional study could ask the participant to provide their confidence interval and measure how they vary with and without perspectives. If further research is conducted on this topic, the perspectives technique can be more commonly accepted and used to help a larger number of people understand and solve problems.

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