Parameters of Quadratization of Scalar Polynomial ODE's: Research Plan

Foyez Alauddin

In the category of Mathematics

1 Research Question

In my research, we strive to investigate quadratization and its efficiency. The problem we look at is what is the form of a scalar polynomial ODE such that it can be quadratized using exactly 1 new variable.

2 Rationale

Quadratization is a process that allows us to use new variables to reduce a defined scalar polynomial ODE and the Lie derivatives of the new variables into a quadratic form. The reason we strive to do this is because there are already well-established techniques for solving quadratics. Solving such quadratics can aid in finding critical points for optimization, a very important tool in mathematical modeling in science and engineering. Quadratization has been recently used for model dimension reduction in [x]. The idea is that although a quadratized system is larger than the original one, there are more powerful reduction techniques available for quadratic systems.

3 Procedure

My experiment is mostly computer based. I first will test by hand whether it is possible to quadratize using some n^{th} degree scalar polynomial in x using a new variable z that I choose. I will first look to obtain a pattern. I then plan to use a programming software called SageMath to write a program that computes matrices of a matrix using Groebner Bases. What I am essentially doing here is to find linear dependence. Linear dependence is crucial in understanding quadratization as we strive to determine whether the terms present in our scalar polynomial ODE and the Lie derivative of our new variable can written as a quadratic combination of our new variable and x. The code I use is provided in my paper.

4 Risks and Safety

There are no potential risks as my experimentation relies solely on a computer and hand-written testing.

5 Data Analysis

What the Groebner Basis code will give us are conditions on the coefficients of our defined scalar polynomial ODE. We will analyze conditions to see which coefficients must tend to zero in order for the determinant of our matrices to equal zero. We will then attempt to prove a pattern, if one exists.

6 Bibliography

I primarily used the following links and abstracts to aid in my preliminary knowledge and understanding. I was formally introduced to the idea and process of quadratization by my research mentor Professor Gleb Pogudin. Here are the links that I used:

1. https://arc.aiaa.org/doi/10.2514/1.J057791

2. https://math.berkeley.edu/ bernd/what-is.pdf