Validation of High Order Theories for Sandwich Beam Bending Behavior Using Direct Image Correlation (DIC) Techniques

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Abstract

Using direct image correlation techniques, the real behavior of a sandwich beam composite was studied in comparison to the predictions made by the Classical, First Order, Elasticity, HSAPT, and EHSAPT theories. The model sandwich beam was constructed of a core of glass-embedded foam and two face sheets of woven carbon fiber. The components of the sandwich beam were bonded using an epoxy-based binder. Loading was then done on the sandwich beam using a three-point loading machine until the proportionality limit is reached. Using the loading machine control software, force and loading displacement data were collected. This data was then used to calculate normalized deflection and produce a graph comparing the normalized deflection predictions of the five theories with the normalized deflection found in the model sandwich beam. The investigation showed that the Classical Theory was inadequate in accurately predicting the bending stress response of the sandwich composite. The Elasticity, HSAPT, and EHSAPT theories all predicted much more accurately the normalized deflection of the sandwich beam than the classical theory, but their accuracy was impeded by the relatively stiff beam core used in the model sandwich beam. The low stiffness of the face sheets was advantageous to the First Order theory, and it performed the best in modeling normalized deflection of the model sandwich beam out of the five theories investigated. It is proposed that future investigations into sandwich beam theories vary the stiffness of the core and the face sheets in order to analyze the performance of the theories under a broad spectrum of conditions.

Introduction

As engineering technology advances, demand for new, innovative materials have arisen to support and realize new concepts. Many of these technologies demand material properties that no natural substance possesses. Therefore, to achieve these goals, new material breakthroughs, mainly in the form of composite materials, must be explored. One type of composite material currently being intensely studied and developed for these purposes is the sandwich panel composite.

Sandwich composites are layered structures that consist of two thin, high-strength, and stiff face sheets, typically fashioned from metal or carbon composites, attached to a thick core of light but low-strength material, usually a honeycomb structure or foam. Sandwich structures are characterized by their low density and lightweight, high stiffness and strength, and high energy-absorption capability, especially in the case of high-velocity impacts. (Yuan, *et al.*, 2015) Due to these unique properties, sandwich structures have become more widely used in aerospace, nautical, and civil applications where strength and lightweight are of great importance. The increased use of sandwich structures in industry has necessitated the development of new sandwich theories that would be able to better model the real-life performance of sandwich composites.

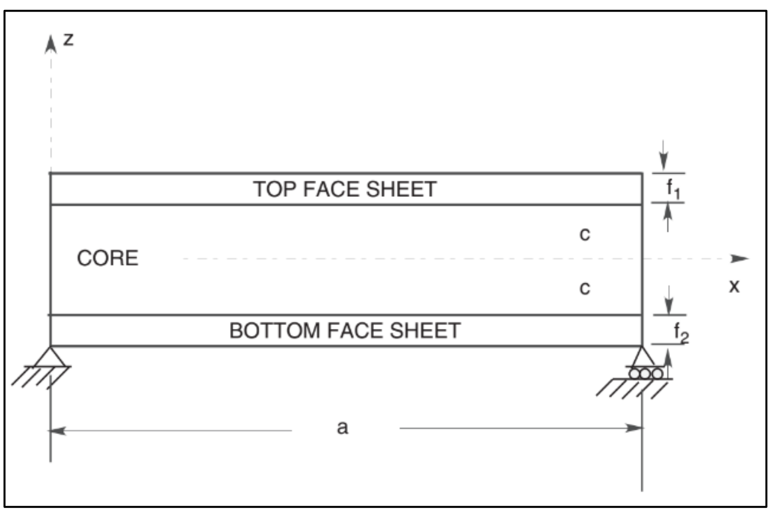

Figure 1: A diagram of a typical sandwich structure configuration. f_1 and f_2 represent the thickness of the top and bottom face sheets respectively. The height of the core is represented by 2c. (Kardomateas, *et al.*, 2011)

The conventional theory used in the analysis of sandwich composite panels is the Classical theory. This theory is based on the general Timoshenko beam theory with the Euler-Bernoulli assumptions that cross-sections of the face sheets do not deform significantly under load and that there is no rotation of the cross-section as a result of the load. This model does, however, not factor into consideration the compression of the sandwich beam core, modeling the compressible core of the sandwich structure if it were completely rigid. This model could reasonably accurately model the behavior of older metal honeycomb-cored sandwich panels, which were much less susceptible to in-plane deformation as a response to stress than newer foam and non-metallic honeycomb cores. (Forstig *et al.*, 1992) Therefore, the Classical model has serious limitations when it comes to modeling modern sandwich construction methods, as modern foam cored sandwich panels and are often flexible and compressible in all directions. (Phan *et al.*, 2012) These characteristics of the Classical theory causes it to underestimate displacement in the x-direction, as it is only capable of modeling an average linear distribution due to its limited ability to model compression in real materials. (Tessler *et al.*, 2010) Additionally, the Classical theory also overestimates the in-plane displacements of the composite face sheets under stress. (Tessler *et al.*, 2010) The tendency of the Classical theory to erroneously estimate the properties of sandwich composite materials due to its inability to properly model the effects of the non-infinite rigidity of the sandwich core indicates that new theories that properly model core displacement are needed to properly characterize sandwich-type materials.

In order to fully model the real-world behavior of the compressible core, a higher-order beam theory is required. In one theory proposed by Forstig *et al.*, the shear strain in the sandwich core is deemed to be constant, with the assumption that the in-plane rigidity of the core is negligible. This model was able to more accurately model softer cores as it utilizes separate differential equations to model the top and bottom face sheets and the core. (Forstig *et al.*, 1992) However, this model is limited in that it does not take into account the in-plane rigidity and compressibility of the core, and therefore may not be able to fully model the effects of a real sandwich panel using a soft core.

Another theory proposed by Li *et al.* proposes a higher-order model that improves the modeling of the in-plane rigidity – the deformation of the sandwich panel along its length. This model has since been extended by Phan *et al.* to include cases of a negative discriminant. This is important in sandwich construction as negative discriminants frequently result from the core being more rigid in the transverse direction than in-plane. This model, known as the HSAPT (High-Order Sandwich Panel Theory), can very accurately models the behavior of very soft cores, but decreases accuracy as the stiffness of the sandwich core increases. (Phan *et al.*, 2012) This is often a problem as many sandwich panel designs that include stiffer core materials will fail to be modeled accurately by the HSAPT theory.

More recently, an extension to the HSAPT theory was proposed by Phan *et al.* This theory takes into consideration the axial, transverse, and rotational stresses at the centroid of the sandwich beam, rather than only transverse strain at the midpoint of the beam, as many other contemporary theories do. The Extended-HSAPT (EHSAPT) theory assumes that face sheets follow the Euler-Bernoulli kinematic assumptions. The EHSAPT theory also assumes that the core material has non-negligible transverse, in-plane, and shear rigidities. Finally, the theory also assumes that the face sheets and the core material are perfectly bonded.

Because of the EHSAPT theory's modeling of transverse shear in the core due to its inclusion of higher-order differential equations, as well as its inclusion of modeling of core shear stiffness and face sheet stiffness, the EHSAPT theory is projected to be more accurate in modeling the response of sandwich beams to loading. The EHSAPT theory generally predicts that sandwich beams will have greater response to loading than that predicted by the much more conservative First-Order Shear and Classical theories. (Figure 2).

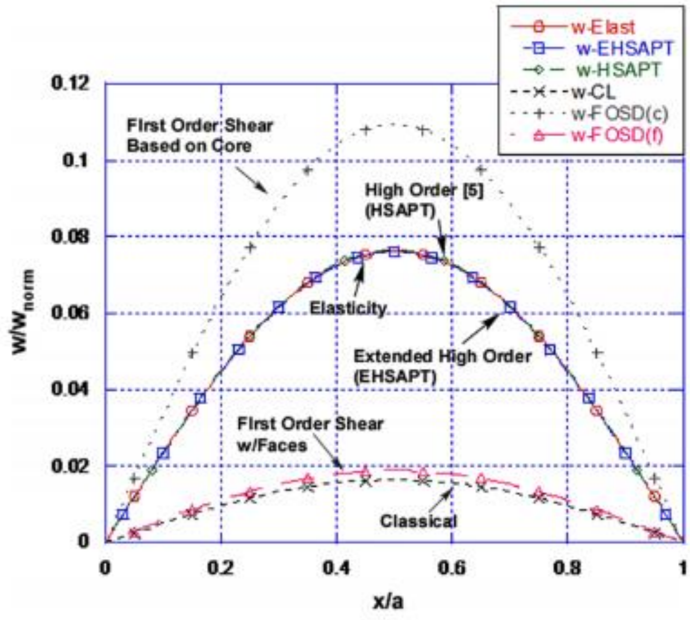

Figure 2: Predictions of normalized deflection by different theories on a theoretical sandwich panel structure. (Phan *et al.*, 2012)

Of note in Figure 2 is the difference in predictions of normalized deflection by the First Order Shear theory that is based only on the structural rigidity of the core and that which also includes prediction of the face sheets. The First Order Shear theory based on the sandwich core gives a much higher prediction of normalized deflection when compared to that which also models the stiffness of the face sheets. This indicates that the First Order Shear model may have certain deficiencies when modeling real sandwich beams that possess rigid face sheets.

It is seen that predictions made by the EHSAPT, HSAPT, and Elasticity theories for the theoretical sandwich panel presented by Phan *et al.* generally fall upon the same regression line. This is rather unsurprising, as the three theories are closely related to one another, with the EHSAPT theory being a derivative of the HSAPT.

Of all theories applied to the theoretical sandwich beam by Phan *et al.*, the Classical theory presented the weakest results, greatly underestimating the normalized deflection of the sandwich panel. This indicates that the Classical theory is too conservative when it comes to modeling normalized deflection in sandwich panels to be accurate and useful.

Therefore, given the weakness of the Classical and First Order Shear theories, the EHSAPT, HSAPT, and Elasticity theories promise to present a much more accurate model of the response of sandwich beam structures to loading stresses, which is extremely important to incorporating sandwich panels into designs and real-world applications if the results presented are valid when compared to the response of real sandwich beams to loading stress. Therefore, this study seeks to use a model sandwich beam structure to investigate the accuracy of predictions by the EHSAPT, HSAPT, and Elasticity theories for sandwich structures.

Material Fabrication and Methodology

The sandwich sample was constructed of a core of glass-embedded foam with Young's modulus of 40MPa and two face sheets of woven carbon fiber with a thickness of 1.25 millimeters each. Glass-embedded foam had the advantage of having randomly dispersed reflective particles within its structure that would allow Direct Image Correlation (DIC) techniques to be applied in the analysis of the sandwich panel under stress without the need to apply a speckle pattern using paint, which would likely affect the mechanical properties of the sandwich panel.

To bond, the face sheets with the core, epoxy glue was applied to the bottom face sheet and then evenly spread. The foam core was then placed on top. Epoxy was then applied to the top face sheet and evenly spread. The top face sheet was then stacked on top of the core. Weights were placed evenly on top of the completed sandwich panel and then left for 24 hours in order to ensure as many air gaps were removed as possible, and that the epoxy was completely cured before experimentation. After the 24-hour setting period, the sides of the sandwich composite structure were cut using a band saw and then sanded to remove any epoxy residue that spilled over the side of the sandwich panel during the setting process. This was done in order to ensure that that spillover epoxy would not affect the mechanical properties of the sandwich panel, as well as to ensure that reflective glass particles would be exposed throughout the entire cross-section of the sandwich panel. The completed sandwich panel had dimensions of 254mm of length, 40mm of width, and 34.90mm of height.

The completed sandwich panel was then placed in a three-point loading machine. Cylindrical bottom supports of a radius of 3.175mm were placed at each end of the sandwich composite structure as shown in Figure 3.

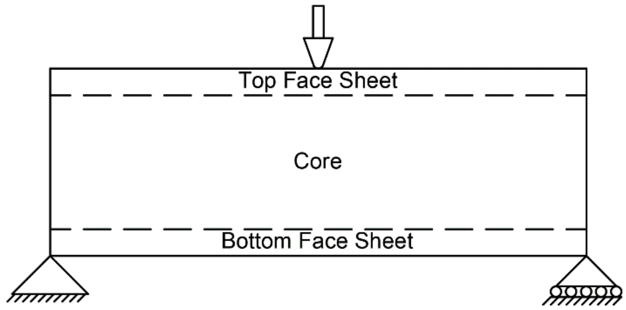

Figure 3: Schematic of the 3 point bending test applied to the composite sandwich structure.

Cylindrical supports were chosen due to the fact that a cylinder with its side tangent to the sandwich panel theoretically will contact the sandwich composite panel along one infinitesimally thin line, and therefore will minimize the influence of the bottom supports on the shear and bending response of the sandwich panel under stress.

Once placed in the loading machine, a downward force was applied by lowering a load arm upon the sandwich panel at an average speed of 0.1mm/second. A load cell with a maximum measurable load of 10kN was attached to the load arm, with force readings being fed to the control software of the loading machine. At an interval of every 100N of downward force applied, loading was paused, and an image was taken with a Cannon EOS 5DS R DSLR camera with a EF 100-400mm f 4 5-5 6L IS II USM Lens. Loading and imaging at 100N intervals were continued until the sandwich structure reached the proportionality limit, and the structure reached the plastic state, where nonreversible structural changes begin to take place. The proportionality limit was determined to have been reached by monitoring the stress-strain graph output in the loading machine control software. After the proportionality limit was reached, data collection was stopped, and the loading machine was returned to its zero position.

The loading machine control software exports a data set that relates compression force to the displacement of deflection. The data set is then correlated with the stress-strain curve created by the control software in order to find the domain of the elastic portion of the stress-strain curve. This data was then used to derive the Young's Modulus in accordance with Equation 1, where F represents the force exerted on the sandwich beam at a specific point, L_0 represents the initial length of the beam. A represents the cross-sectional area of the sandwich beam, while ΔL represents the change in length of the beam.

$$E = \frac{FL_0}{A\Delta L}$$

Equation 1: Derivation of the Young's Modulus.

Direct Image Correlation (DIC) techniques were used to compare the response of the sandwich beam between loading cases. DIC is a technique that allows a correlation between two different loading cases based on a speckle pattern that is present on a cross-section of the sandwich beam being analyzed. As the sandwich beam deforms under stress, the speckle patterns will deform with the beam. Digital photos are taken at given intervals of loading force or displacement. A software solution was then used to correlate between different images using the speckle pattern within the sandwich beam.

The distributed load q_0 can be calculated for the sandwich beam using Equation 2. F represents applied force, a represents the cross-sectional area of the sandwich beam, *b* represents the length of the base of the specimen, and *h* represents the length of the height of the specimen.

$$q_0 = \frac{3Fa}{4bh^2}$$

Equation 2: Calculation of distributed load

The calculation of Young's Modulus and distributed load are necessary in order to calculate the variable of comparison between the theories and the experimental results derived from the sandwich beam. This calculation is shown in Equation 3, where a represents the length of the sandwich beam and f represents the thickness of the face sheets. E and q_0 represent the previously derived values of Young's Modulus and distributed load respectively.

$$w_{norm} = \frac{3q_0a^4}{2\pi^4Ef^3}$$

Equation 3: Calculation of normalized deflection

The calculation of normalized deflection for the model sandwich beam allows comparison with the normalized deflection presented by the theoretical prediction of the theories under consideration. Graphs were then created presenting the relationship between normalized deflection and the distance along the beam, presented as a percentage of total beam length.

Results

Using Direct Image Correlation, comparisons were made between 0N and 100N, 100N and 200N, 0N and 500N, and 0N and 1000N cases. A comparison was also made between 0N and 7mm deflection, where the sandwich beam has already reached plastic state. The normalized deflection was then plotted against the percent length of the sandwich beam in order to compare practical results against the predictions made by the HSAPT, EHSAPT, First Order Shear (FOS), Elasticity, and Classical theories.

The DIC results between the 0N and 100N loading cases (Figure 4) indicate that the FOS model overestimates the normalized deflection of the sandwich beam throughout its entire length. Meanwhile, the Classical Theory overestimates normalized deflection from 10% to 15% length, while underestimating from 15% to 85%. The HSAPT, EHSAPT, and Elasticity theories all provide similar estimates for normalized deflection, and accurately predicts the real behavior of the sandwich beam from 40% to 55%, while overpredicting for all other ranges.

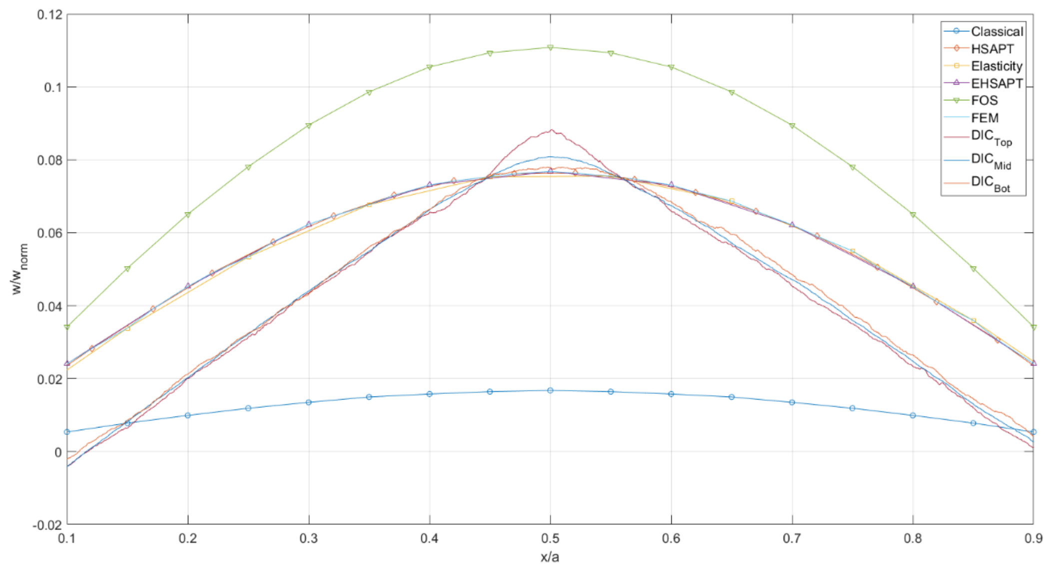

Figure 4: DIC comparison results between the unloaded beam and 100N loading force.

In the 100N versus 200N DIC comparison (Figure 5), the Classical model again greatly underestimates normalized deflection across the entire length of the sandwich beam. The HSAPT, EHSAPT, and Elasticity models again provide similar predictions, this time slightly underestimating normalized deflection across the entire sandwich beam length. The First Order Shear model provides the most accurate prediction here, successfully predicting normalized deflection from 10% to 30%, while slightly underestimating from 30% to 90% length.

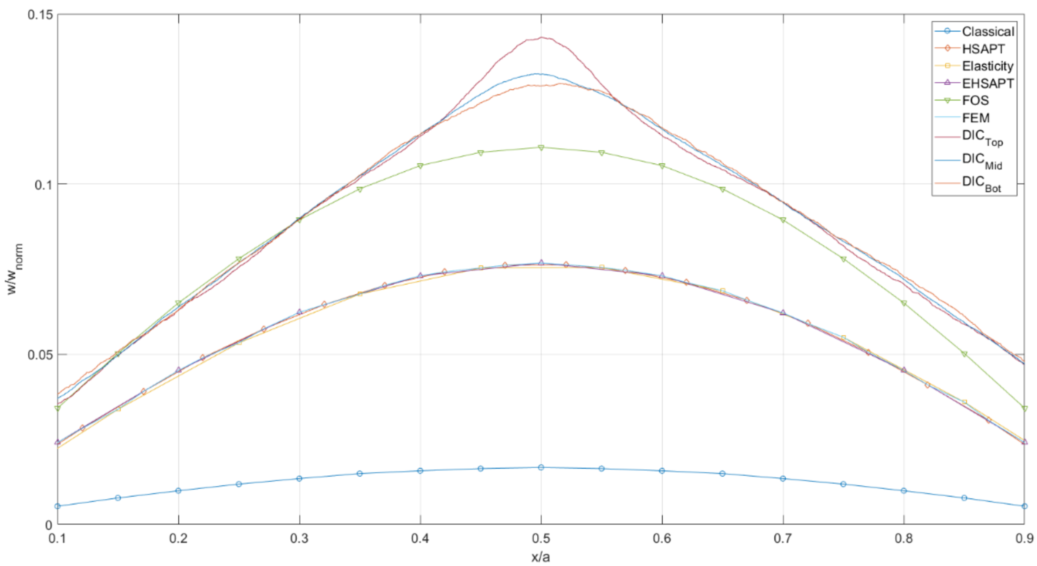

Figure 5: DIC comparison results between the beam at 100N loading force and 200N.

DIC comparison between 0N and 500N loading cases (Figure 6) yields similar results as the DIC comparison the 100N and 200N case. The Classical theory again greatly underestimates normalized deflection, while the HSAPT, EHSAPT, and Elasticity theories slightly underestimate normalized deflection across the entire length of the sandwich beam. The First Order Theory provides the best prediction of normalized deflection in this loading case, accurately predicting normalized deflection from 10% to 35%, and again from 65% to 90%, while slightly underestimating from 35% to 65% length.

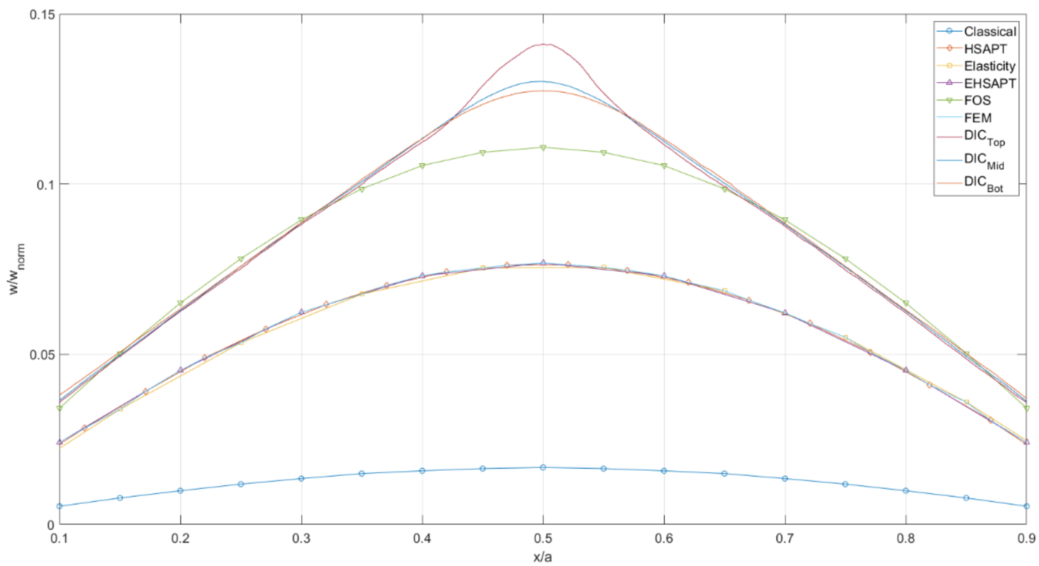

Figure 6: DIC comparison results between the unloaded beam and 500N loading force.

A DIC comparison between the unloaded beam and the beam under 1000N of loading force (Figure 7) shows that all theories underestimate normalized deflection. The classical theory greatly underestimates normalized deflection, while the HSAPT, EHSAPT, and Elasticity theories continue to slightly underestimate normalized deflection. The First Order Shear theory performed the best in this case, being the closest prediction to the actual loading response of the sandwich beam. However, the First Order Shear theory also underestimates normalized deflection in the 0N versus 1000N loading case.

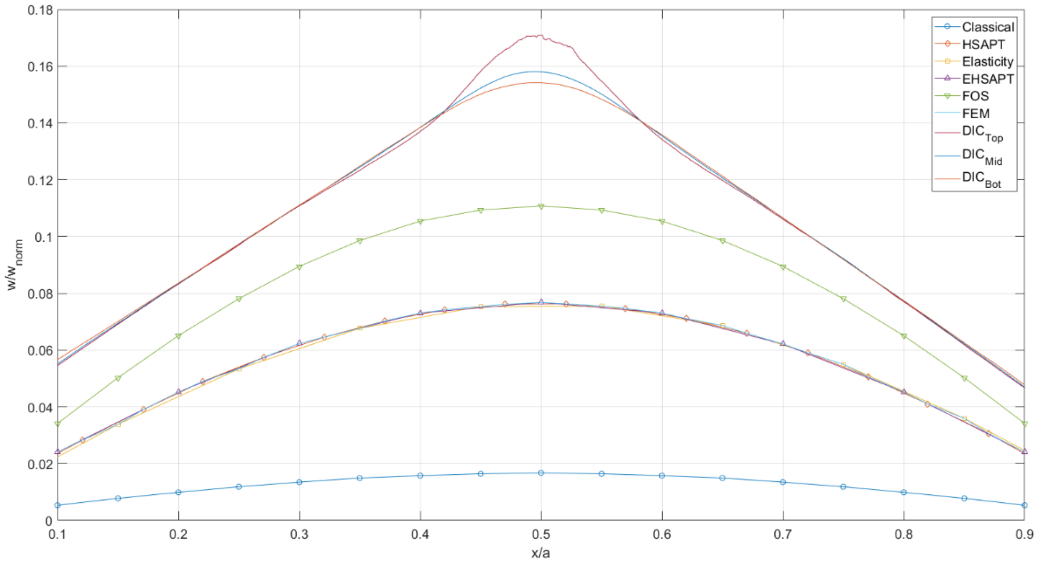

Figure 7: DIC comparison results between the unloaded beam and 1000N loading force.

An investigation was also made into the performance of the various theories in predicting normalized deflection past the proportionality limit, where the geometry and properties of the sandwich beam has been irreversibly altered. These results are shown in Figure 8. All theories fail to correctly estimate normalized deflection under plastic state. The First Order Shear theory continued to perform the best, while the Classical theory performed the worst out of the five theories investigated. However, none of the theories estimated the normalized deflection of the sandwich panel to any acceptable accuracy. As none of the five theories were formulated to model response of sandwich panels after the proportionality limit, and the breakdown of the theories under the permanent deformation is not surprising.

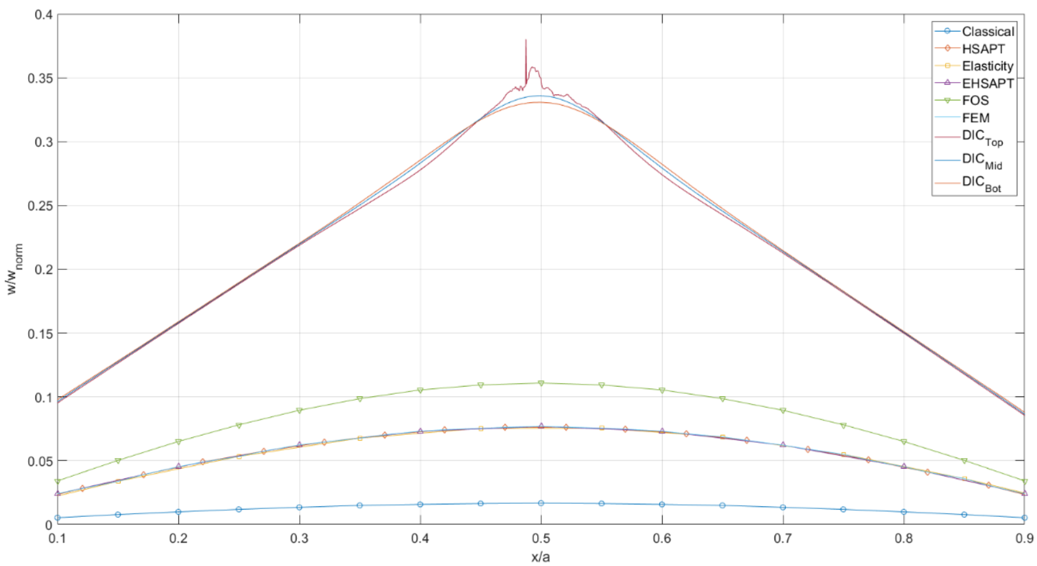

Figure 8: DIC results between the unloaded beam and the beam at 7mm of deflection.

Discussion and Future Studies

The HSAPT, EHSAPT, and Elasticity theories were able to deliver much-improved accuracy when modeling the normalized deflection of a sandwich beam than the Classical theory. This is due to these three theories' ability to model the in-plane bending response of the experimental sandwich beam's flexible foam core, which the Classical theory had assumed to be incompressible and failed to model. In the 0N and 100N loading case, the HSAPT, EHSAPT, and Elasticity theories provided the best estimation for the normalized deflection for the real sandwich beam out of all the five theories considered. However, other loading cases show that the HSAPT, EHSAPT, and Elasticity theories consistently underestimate the normalized deflection in the sandwich core. This may be due to the relatively rigid foam material used for the core of the sandwich beam. The glass-imbedded foam that was chosen due to its reflective properties also had a high Young's Modulus, and therefore higher structural rigidity, than a regular foam. As the HSAPT theory, and by extension its derivative EHSAPT theory, is most accurate for modeling softer sandwich structure cores, with accuracy decreasing as core stiffness increases (Li *et al.*, 2008), the stiffer sandwich core may have contributed to the loss of accuracy of the theories in modeling the model sandwich beam.

The First Order Shear (FOS) model consistently provided the most accurate results for normalized deflection. In all cases except for the 0N versus 100N case and the 7mm case, the FOS model provided the closest estimation out of all the five theories considered, with the FOS theory modeling nearly perfectly the normalized deflection in the left and right ends of the sandwich beam in the 100N versus 200N and 0N versus 500N cases. In the 0N versus 100N case, the FOS theory provided an overestimation of normalized deflection in the core, but this overestimation is not so great as to jeopardize the accuracy of the FOS model. All five theories broke down past the proportionality limit. However, this is hardly surprising, given that all the theories being considered were derived for use after the proportionality limit has been reached, and the complete loss of accuracy after that point should not be considered a fault of any of the theories considered in this study.

Important to note for the results presented by the FOS model is that this study utilized face sheets with relatively low Young’s moduli when compared to the core. As shown in Figure 2, the FOS model breaks down greatly in accuracy when it attempts to predict the response to loading stress of sandwich beams with stiff face sheets. Therefore, it is possible that the choice of face sheets in the creation of this model sandwich beam contributed to the more accurate prediction of the FOS model in this case. To investigate this possibility, it is proposed that future investigations into sandwich beam theories utilize model sandwich beams with stiffer face sheets, in order to investigate the real-world response of the FOS model when used to predict stress response of sandwich beams that utilize different types of face sheet materials with varying stiffness.

Out of all theories, the Classical theory was consistently the poorest performing beam theory when compared to the real-world properties of the sandwich beam. Due to its many limitations, most glaring of which its inability to properly model the compressible sandwich beam core, it consistently failed to provide an accurate model of the sandwich panel, with its predictions for normalized deflection always being a gross underestimate of the real performance of the sandwich structure. Due to its inability to accurately model the rigidity of the sandwich core, the Classical theory sees a very shallow rise in predicted normalized deflection at the center of the sandwich beam, while other theories are able to much more accurately model the increase in the model sandwich beam’s normalized deflection. Therefore, the Classical theory is wholly unsuitable for providing an accurate prediction of the properties of sandwich beams with nonrigid cores.

In all loading cases, the five theories all failed to accurately model the normalized deflection in the center of the beam, between approximately 40%-60% beam length. Additionally, the DIC results for the top face sheet of the sandwich beam show a sharp increase in normalizes deflection at the center of the beam, with a peak occurring at approximately 50% beam length. DIC analysis also showed a slight increase in normalized deflection at the center of the sandwich beam for the sandwich core and bottom face sheet. None of the theories were able to successfully predict this sharp increase in normalized deflection. However, this is not hypothesized to be a fault of any of the theories being considered. All theories work under the assumption that the load to the sandwich beam is concentrated at a single, infinitely thin axis of loading at the center of the sandwich beam. However, this is incredibly difficult to achieve in real life. This study was conducted using a top-loading point with a diameter of 3.175mm, which may still have been too wide to allow the theories to provide an accurate model of the sandwich beam's response to load at the center of the beam. This also explains the dramatic rise in normalized deflection of the sandwich beam's top face sheet near the loading point, as the loading provided by the loading point used in the study caused disproportionate deflection in the top face sheet at the center of the beam as compared the other sections of the sandwich beam at other x positions.

To provide a result that is more representative of the predictions of the theories, two options for continued study are proposed. Firstly, testing with different sandwich beam cores with a lower Young's Modulus and therefore lower structural rigidity may be considered. This study used a relatively rigid sandwich core with a Young's Modulus of 40MPa, and this may have impacted the results that were obtained and their conformity with the beam theories. Since the HSAPT and EHSAPT theories are proposed to be most accurate in modeling sandwich beams with softer cores, testing with a sandwich structure that possesses a softer core may be better able to show the accuracy or inaccuracy of the HSAPT and EHSAPT theories. Another possible avenue for future investigation is using progressively smaller loading points that may be able to better simulate the assumptions of infinitely thin loading points that were given in the proposition of the theories. This could also mitigate or eliminate the sharp rise in normalized deflection that is seen in the center of the top face sheet, as there would likely be a far lower impact of the loading point on the surrounding sandwich beam.

Conclusion

The real behavior of a sandwich beam composite under bending stress was studied using direct image correlation techniques. The response of the sandwich beam composite to bending stress was compared to predictions made by the Classical, First Order, Elasticity, HSAPT (High-Order Sandwich Panel Theory), and EHSAPT (Extended-HSAPT) theories. The data revealed that the Classical Theory was flawed and inadequate for the for accurately predicting the response of the sandwich composite to loading stress. The relatively stiff core of the model sandwich beam impeded the accuracy of the Elasticity, HSAPT, and EHSAPT theories, which give more accurate predictions to lower-stiffness cores. However, they still provided far more accurate predictions of the bending-stress response than the Classical Theory. The low stiffness of the face sheets was advantageous to the First Order Theory in this investigation, and it was able to perform the best in modeling the normalized deflection of the model sandwich beam out of the five theories investigated.

References

Amir Fathi, Jan-Hendrik Keller, Volker Altstaedt, (2015). “Full-field shear analyses of sandwich core materials using Digital Image Correlation (DIC)”, Composites Part B: Engineering, Volume 70, Pages 156-166, ISSN 1359-8368, https://doi.org/10.1016/j.compositesb.2014.10.045.

Auricchio, F., and Sacco, E. (2003). "Refined First-Order Shear Deformation Theory Models for Composite Laminates ." ASME. J. Appl. Mech. May 2003; 70(3): 381–390. https://doi.org/10.1115/1.1572901

Frostig, Y., Baruch, M.,Vilnay, O., and Sheinman, I., (1992). “High-Order Theory for Sandwich-Beam Behavior With Transversely Flexible Core,” J. Eng. Mech., 118(5), pp. 1026–1043.

Gdoutos, E.E., Daniel, I.M., Wang, KA. (2001). “Nonlinear behavior of composite sandwich beams in three-point bending” Experimental Mechanics, 41: 182. https://doi.org/10.1007/BF02323195

Kardomateas, G. A., & Phan, C. N. (2011). “Three-dimensional elasticity solution for sandwich beams/wide plates with orthotropic phases: The negative discriminant case.” Journal of Sandwich Structures & Materials, 13(6), 641–661. https://doi.org/10.1177/1099636211419127

Li, R., and Kardomateas, G. A. (2008). “A Nonlinear High Order Core Theory for Sandwich Plates With Orthotropic Phases,” AIAA J., 46(11), pp. 2926–2934.

Phan, C. N., Frostig, Y., and Kardomateas, G. A. (2012). "Analysis of Sandwich Beams With a Compliant Core and With In-Plane Rigidity—Extended High-Order Sandwich Panel Theory Versus Elasticity." ASME. J. Appl. Mech. July 2012; 79(4): 041001. https://doi.org/10.1115/1.4005550

Tessler, A. & Di Sciuva, Marco & Gherlone, Marco. (2010). “A consistent refinement of first-order shear deformation theory for laminated composite and sandwich plates using improved zigzag kinematics.” Journal of Mechanics of Materials and Structures. 5. 341-367. https://doi.org/10.2140/jomms.2010.5.341

Yuan, Z., Kardomateas, G.A. and Frostig, Y. (2015). "Finite Element Formulation based on the Extended High Order Sandwich Panel Theory", AIAA Journal, Vol. 53, no. 10, pp. 3006-3015, https://doi.org/10.2514/1.J053736