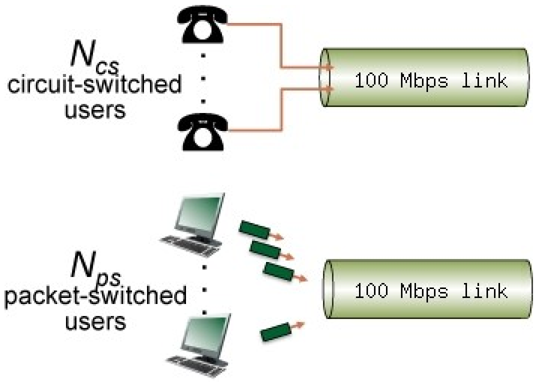
Quantitative Comparison of Packet Switching and Circuit Switching

This question requires a little bit of background in probability (but we'll try to help you though it in the solutions). Consider the two scenarios below:

- A circuit-switching scenario in which N_{cs} users, each requiring a bandwidth of 10 Mbps, must share a link of capacity 100 Mbps.
- A packet-switching scenario with N_{ps} users sharing a 100 Mbps link, where each user again requires 10 Mbps when transmitting, but only needs to transmit 10 percent of the time.



Answer the following questions:

a. When circuit switching is used, what is the maximum number of circuit-switched users that can be supported? Explain your answer.
b. For the remainder of this problem, suppose packet switching is used. Suppose there are 19 packet-switching users (i.e., N_{ps} = 19). Can this many users be supported under circuit-switching? Explain.
c. What is the probability that a given (*specific*) user is transmitting, and the remaining users are not transmitting?
d. What is the probability that one user (*any* one among the 19 users) is transmitting, and the remaining users are not transmitting? When one user is transmitting, what fraction of the link capacity will be used by this user?
e. What is the probability that any 10 users (of the total 19 users) are transmitting and the remaining users are not transmitting? (Hint: you will need to use the binomial distribution [1, 2]).
f. What is the probability that *more* than 10 users are transmitting? Comment on what this implies about the number of users supportable under circuit switching and packet switching.

Solution:

a. When circuit switching is used, at most 10 circuit-switched users that can be supported. This is because each circuit-switched user must be allocated its 10 Mbps bandwidth, and there is 100 Mbps of link capacity that can be allocated.
b. No. Under circuit switching, the 19 users would each need to be allocated 10 Mbps, for an aggregate of 190 Mbps - more than the 100 Mbps of link capacity available.

c. The probability that a given (*specific*) user is busy transmitting, which we'll denote p, is just the fraction of time it is transmitting, i.e., 0.100. The probability that one specific other user is not busy is $(1-p)$, and so the probability that *all* of the other $N_{ps}-1$ users are not transmitting is $(1-p)^{N_{ps}-1}$.Thus the probability that one specific user is transmitting and the remaining users are not transmitting is $p^1(1-p)^{N_{ps}-1}$, which has the numerical value of 0.0150094635297. This user will be transmitting at a rate of 10 Mbps over the 100 Mbps link, using a fraction 0.1000 of the link's capacity when busy.

d. The probability that exactly one (*any* one) of the N_{ps} users is busy is N_{ps} times the probability that a given specific user is transmitting and the remaining users are not transmitting (our answer to (c) above), since the one transmitting user can be any one of the N_{ps} users. The answer to (d) is thus $N_{ps}p^1(1-p)^{N_{ps}-1}$, which has the numerical value of 0.2851798070643.

e. The probability that 10 specific users of the total 19 users are transmitting and the other 9 users are idle is $p^{10}(1-p)^9$. Thus the probability that *any* 10 of the 19 users are busy is $choose(19,10)p^{10}(1-p)^9$, where $choose(19,10)$ is the $(19,10)$ coefficient of the binomial distribution [1, 2]). The numerical value of this probability is 3.5789129932842E-6.

f. The probability that more than 10 users of the total 19 users are transmitting is $\Sigma_{i=11,19} choose(19,i)p^i(1-p)^{19-i}$. The numerical value of this probability is 3.509693338438E-7. Note that 10 is the maximum number of users that can be supported using circuit switching (the answer to part (a)). With packet switching, nearly twice as many users (19) are supported with a small probability that more than 10 of these packet-switching users are busy at the same time.