

Example	A_1	A_2	A_3	Output Y
x_1	1	0	0	0
x_2	1	0	1	0
x_3	0	1	0	0
x_4	1	1	1	1
x_5	1	1	0	1

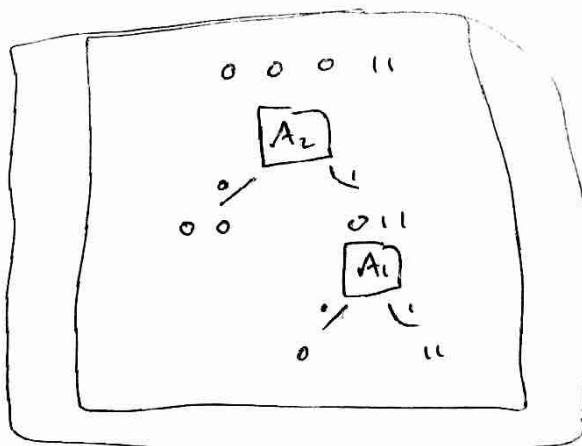
$$\begin{aligned}
 \text{gain}(A_1) &= H\left(\frac{1}{5}, \frac{2}{5}\right) - \frac{1}{5}H(0,1) + \frac{2}{5}H(0,1) \\
 &= 0.92 - 0 + 0 = 0.92
 \end{aligned}$$

$$\begin{aligned}
 \text{gain}(A_2) &= H\left(\frac{2}{5}, \frac{3}{5}\right) - \frac{4}{5}H\left(\frac{1}{2}, \frac{1}{2}\right) + \frac{1}{5}H(0,1) \\
 &= 0.97 - \frac{4}{5} = 0.17
 \end{aligned}$$

$$\begin{aligned}
 \text{gain}(A_3) &= H\left(\frac{2}{5}, \frac{3}{5}\right) - \frac{2}{5}H(0,1) + \left(\frac{3}{5}\right)H\left(\frac{1}{3}, \frac{2}{3}\right) \\
 &= 0.97 + \frac{3}{5}(0.92) = 1.522
 \end{aligned}$$

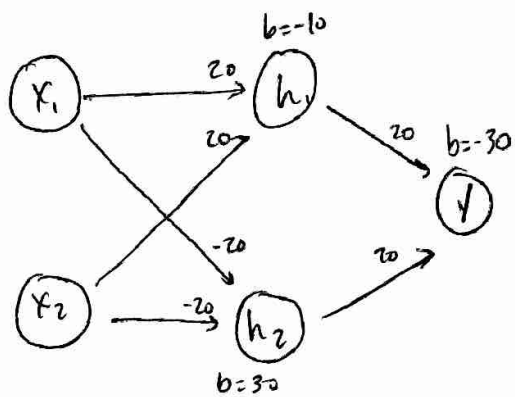
$$\begin{aligned}
 \text{gain}(A_5) &= H\left(\frac{1}{5}, \frac{2}{5}\right) - \frac{2}{5}H\left(\frac{1}{2}, \frac{1}{2}\right) + 0 \\
 &= 0.25
 \end{aligned}$$

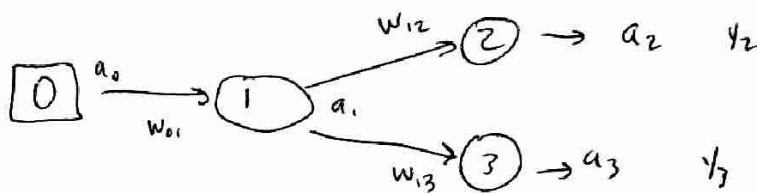
Final tree



x_0, z

x_1	x_2	z
0	0	0
0	1	1
1	0	1
1	1	0





$$w_{01} = 1 \quad w_{12} = 1$$

$$w_{13} = 2$$

$$g(x) = \frac{1}{1+e^{-x}}$$

$$\text{input} = a_0 = 1 \quad \text{Desired} = t_2 = 2, t_3 = 0$$

$$a_1 = g(w_{01} a_0) = g(1 \times 1) = \frac{1}{1+e^{-1}} = 0.73$$

$$a_2 = g(w_{12} a_1) = g(1 \times 0.73) = \frac{1}{1+e^{-0.73}} = 0.73$$

$$a_3 = g(w_{13} a_1) = g(2 \times 0.73) = \frac{1}{1+e^{-1.46}} = 0.88$$

$$\Delta_2 = (t_2 - a_2) g'(a_2)$$

$$= (2 - 0.73) g'(1.1)$$

$$= 1.27 \left(\frac{1}{1+e^{-1}} \right) \left(\frac{-e^{-1}}{1+e^{-1}} \right)$$

update

$$\alpha = 0.1$$

$$w_{12} = w_{12} + \alpha a_1 \Delta_2$$

$$= 1 + (0.1)(0.73)(-0.24)$$

$$w_{12} = 0.982$$

$$w_{13} = w_{13} + \alpha a_1 \Delta_3$$

$$= 2 + 0.1(0.73)(0.09)$$

$$w_{13} = 2.006$$

$$w_{01} = w_{01} + \alpha a_0 \Delta_1$$

$$= 1 + (0.1)(1)(0.11)$$

$$w_{01} = 1.011$$

$$\Delta_2 = -0.24$$

$$\Delta_3 = (t_3 - a_3) g'(a_3)$$

$$= (0 - 0.88) g'(2)$$

$$\Delta_3 = -0.88(-0.05) = 0.04$$

$$\Delta_1 = (w_{12} \Delta_2 + w_{13} \Delta_3) g'(a_1)$$

$$= 1(-0.24) + (2)(0.04) g'(1)$$

$$\Delta_1 = 0.11$$