

SEC 1-FAE1-DELA ROSA, R

Github Link: <https://github.com/rddelarosa/APM1110/blob/main/FA4/DELA%20ROSA%2C%20R-FA4.Rmd>

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5. A geospatial analysis system has four sensors supplying images. The percentage of images supplied by each sensor and the percentage of images relevant to a query are shown in the following table.

```
# Given
sensors <- data.frame(
  Sensor = 1:4,
  Percent_Supply = c(15, 20, 25, 40),
  Percent_Relevant = c(50, 60, 80, 85)
)
sensors
```

| ## | Sensor | Percent_Supply | Percent_Relevant |
|------|--------|----------------|------------------|
| ## 1 | 1 | 15 | 50 |
| ## 2 | 2 | 20 | 60 |
| ## 3 | 3 | 25 | 80 |
| ## 4 | 4 | 40 | 85 |

What is the overall percentage of relevant images?

```
# Compute
prob_relevance <- sum(sensors$Percent_Supply * sensors$Percent_Relevant) / sum(sensors$Percent_Supply)

# Output the result
cat("The overall percentage of relevant images is ", prob_relevance, "%.")
```

```
## The overall percentage of relevant images is 73.5 %.
```

6. A fair coin is tossed twice. Let E_1 be the event that both tosses have the same outcome, that is $E_1 = (HH, TT)$. Let E_2 be the event that the first toss is a head, that is, $E_2 = (HH, HT)$. Let E_3 be the event that the second toss is a head, that is, $E_3 = (TH, HH)$. Show that E_1, E_2 , and E_3 are pairwise independent but not mutually independent.

```
# Define the sample space
sample <- c("HH", "HT", "TH", "TT")

# Define events
E1 <- c("HH", "TT") # Both tosses are the same
E2 <- c("HH", "HT") # First toss is a head
E3 <- c("HH", "TH") # Second toss is a head

# Calculate probabilities
P_E1 <- length(E1) / length(sample)
P_E2 <- length(E2) / length(sample)
P_E3 <- length(E3) / length(sample)
P_E1_E2 <- length(intersect(E1, E2)) / length(sample)
P_E1_E3 <- length(intersect(E1, E3)) / length(sample)
P_E2_E3 <- length(intersect(E2, E3)) / length(sample)
P_E1_E2_E3 <- length(intersect(intersect(E1, E2), E3)) / length(sample)

# Check for pairwise independence
pairwise_independent <- (P_E1_E2 == P_E1 * P_E2) &
  (P_E1_E3 == P_E1 * P_E3) &
  (P_E2_E3 == P_E2 * P_E3)

# Check for mutual independence
mutual_independent <- (P_E1_E2_E3 == P_E1 * P_E2 * P_E3)

# Display results
cat("P(E1) =", P_E1, "\n")
```

```
## P(E1) = 0.5
```

```
cat("P(E2) =", P_E2, "\n")
```

```
## P(E2) = 0.5
```

```
cat("P(E3) =", P_E3, "\n")
```

```
## P(E3) = 0.5
```

```
cat("P(E1 and E2) =", P_E1_E2, "\n")
```

```
## P(E1 and E2) = 0.25
```

```

cat("P(E1 and E3) =", P_E1_E3, "\n")

## P(E1 and E3) = 0.25

cat("P(E2 and E3) =", P_E2_E3, "\n")

## P(E2 and E3) = 0.25

cat("P(E1 and E2 and E3) =", P_E1_E2_E3, "\n")

## P(E1 and E2 and E3) = 0.25

cat("Pairwise Independent:", pairwise_independent, "\n")

## Pairwise Independent: TRUE

cat("Mutually Independent:", mutual_independent, "\n")

## Mutually Independent: FALSE

# Conclusion
cat("\n The events E1, E2, and E3 are pairwise independent but not mutually independent.")

##
## The events E1, E2, and E3 are pairwise independent but not mutually independent.

```